# Highlights on the mechanism of confinement from lattice simulations (in memory of Misha Polikarpov)

Vitaly Bornyakov

IHEP, Protvino and ITEP, Moscow and FEFU, Vladivostok

Quark Confinement and the Hadron Spectrum XI
St. Petersburg 12.09.14

**28.12.1952 – 18.07.2013**

It is easy to understand why the talk on the lattice studies of confinement mechanism was chosen to be devoted to Misha's memory.

Misha indeed invested a lot of time and efforts into the studies related to confinement. Out of his 250 papers more than 100 papers are on this or closely related subjects.

His contribution to this field was highly appreciated by the colleagues. More than 2000 citations.

In this talk I will briefly review lattice studies of the confinement mechanism giving special attention to the contribution of Misha to this field

Two approaches to confinement mechanism will be covered

- Dual superconductor scenario, proposed by t'Hooft and Mandelstam
- Center vortex picture, by t'Hooft; Cornwall; G. Mack; Nielsen and Olesen; Ambjorn and Olesen

Topics omitted:
Kugo-Ojima criterium
Coulomb confinement
Stochastic confinement

For comprehensive review of the confinement studies:
'An Introduction to the Confinement Problem' by Jeff Greensite, Lect. Notes Phys. 821 (2011)

First reading on the Dual superconductor scenario:
M.N. Chernodub, **M.I. Polikarpov**, 'Confinement, duality, and nonperturbative aspects of QCD', 387-414, Plenum Press 1998, hep-th/9710205

## Outline of the talk

- I. Dual superconductor scenario of confinement
  - Dirac monopole in continuum and on the lattice
  - Monopoles in nonabelian gauge theory ( t'Hooft-Polyakov monopole)
  - Abelian projection, gauge fixing
  - Why Maximally Abelian gauge ?
  - Percolation and condensation, IR density scaling
  - Abelian dominance and monopole dominance
  - Flux tube and DGL parameters
- II. Center vortex scenario of confinement
  - Density scaling
  - Center dominance
- III. Misha's heritage

Computer simulations of the nonabelian gauge theories in lattice regularization is one of the most powerful nonperturbative methods which does not use uncontrolled approximations

It allows to obtain numerically precise results for many hadronic observables

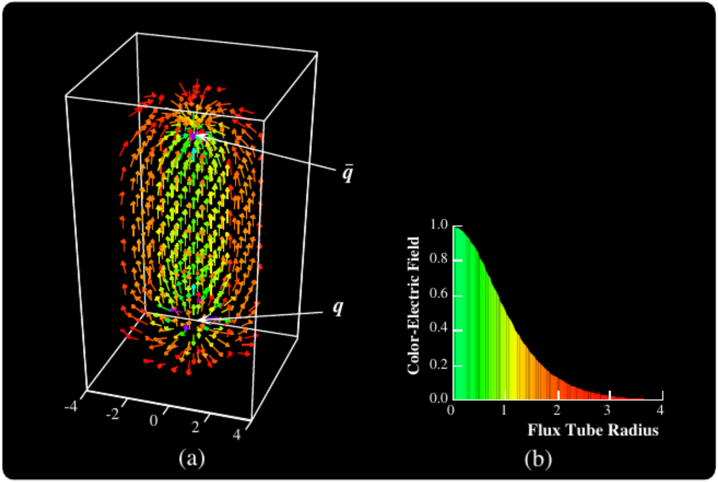
Apart from this the numerical sumulations are aimed at getting information which can be helpful for understanding the nature of the nonperturbative phenomena like confinement and chiral symmetry breaking

Dual superconductor scenario - one of the most popular ideas about nature of confinement
t' Hooft '75, Mandelstam '76

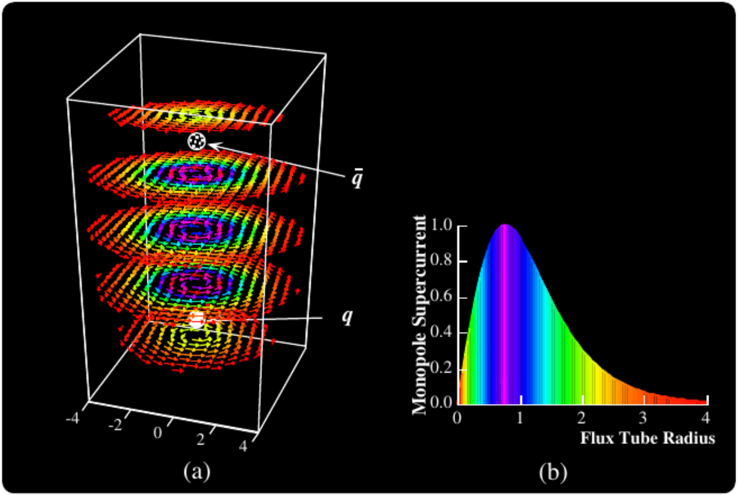
A dual superconductor is a superconductor in which the roles of the electric and magnetic fields are exchanged.

Formation of the Abrikosov-Nilsen-Olesen string in a usual superconductor due to condensation of electric charges is dual to formation of the flux tube in QCD due to condensation of color-magnetic monopoles

Superconductor is described by Landau - Ginzburg model ( Abelian Higgs model )
Dual superconductor - by dual Landau - Ginzburg model ( dual Abelian Higgs model )
It is yet unsolved task to rigorously prove that infrared QCD is dual to Abelian Higgs model




The profile of the color-electric field. Koma, 2001




The profile of the monopole currents. Koma, 2001

Lattice simulations demonstrated that

- in the confinement phase color-magnetic monopoles are condensed
- monopoles are not condensed in the deconfinement phase and the temperature of their condensation transition coincides with confinement-deconfinement phase transition temperature
- Abelian and monopole dominance for the string tension and other IR relevant quantities
- monopoles are interrelated with instantons/calorons/dyons

At present, there is no analytical proof of the existence of the condensate of abelian magnetic monopoles in gluodynamics and in chromodynamics.

However, in all theories allowing for an analytical proof of confinement, the latter is due to the condensation of monopoles.

These analytical examples are: compact electrodynamics (Polyakov ’75) , the 3D Georgi–Glashow model ( Polyakov ’77), and super-symmetric Yang–Mills theory ( Seiberg and Witten ’94).

$$\vec{A}(\vec{x}) = \frac{g_m}{4\pi}\frac{sin\theta}{r(1+cos\theta)}\vec{e}_\phi\,, \quad \vec{e}_\phi = (-sin\phi, cos\phi, 0),$$

$$\vec{H}(\vec{x}) = \vec{\partial}\times\vec{A}(\vec{x}) + \vec{H}_{st}(\vec{x}) = \frac{g_m}{4\pi r^2}\frac{\vec{r}}{r}\,,$$

$$\vec{H}_{st}(\vec{x}) = g_m\vec{e}_z\int_{-\infty}^{0} dz'\delta\left(\vec{x}-\vec{R}(z')\right)\,, \qquad \vec{R}(z') = \{0,0,z'\}\,.$$

In general case

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + F_{\mu\nu,st}(x)$$

## Lattice definitions

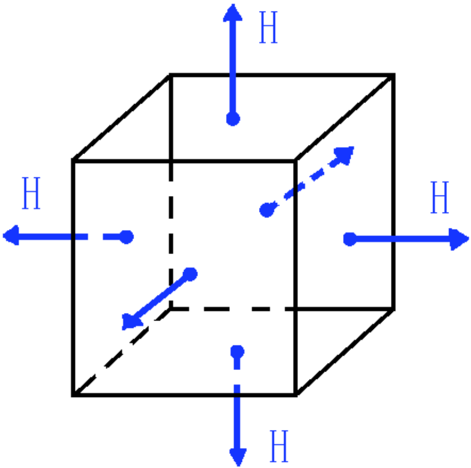
$$U_\mu(s) = exp(i\theta_\mu(s))\,, \quad \theta_\mu(s) \in [-\pi, \pi)$$

$$\theta_{\mu\nu}(s) = \partial_\mu \theta_\nu(s) - \partial_\nu \theta_\mu(s)$$

$$\overline{\theta}_{\mu\nu}(s) = \theta_{\mu\nu}(s) + 2\pi m_{\mu\nu}(s)\,,$$

$$-\pi \leq \overline{\theta}_{\mu\nu}(s) < \pi\,, \quad m_{\mu\nu} = 0, \pm 1, \pm 2$$

$\overline{\theta}_{\mu\nu}(s)$ is gauge invariant electromagnetic flux
$m_{\mu\nu}(s)$ counts number of Dirac strings through plaquette $(s, \mu\nu)$
The restriction of $\overline{\theta}_{\mu\nu}(s)$ to the interval $[-\pi, \pi)$ is natural since the Abelian action for the compact fields $\theta_l$ is a periodic function of $\overline{\theta}_{\mu\nu}(s)$ .


H
H
H
H

definition of 4D magnetic currents, DeGrand and Toussaint, 1980

$$k_\mu(s^*) = \frac{1}{4\pi}\varepsilon_{\mu\nu\rho\sigma}\partial_\nu\overline{\theta}_{\rho\sigma}(s) = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\partial_\nu m_{\rho\sigma}(s)$$

$s^*$ - site on the dual lattice
sites of the dual lattice are defined by the shift $\rho = \{1/2, 1/2, 1/2, 1/2\}$

conservation law:

$$\sum_\mu \partial_\mu k_\mu(s^*) = 0\,.$$

magnetic currents form closed loops, loops are combined into clusters.

$$\mathcal{L} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2}(D_\mu\phi)^a(D_\mu\phi)^a + \frac{\lambda}{4}(\phi^a\phi^a - \mu^2)^2$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + e\epsilon^{abc}A^b_\mu A^c_\nu$$

$\phi^a(x)$ - adjoint scalar field

topological origin:
non trivial homotopy $\pi_2$: a non trivial mapping of the sphere $S^2$ at spatial infinity onto $SU(2)/U(1)$

Global $SU(2)$ is broken down to $U(1)$ which direction is determined by scalar field direction at infinity.
$U(1)$ gauge invariant Abelian $F_{\mu\nu}$ can be defined

$$F_{\mu\nu} = \hat{\phi}^a F^a_{\mu\nu} - \frac{1}{e}\epsilon^{abc}\hat{\phi}^a D_\mu \hat{\phi}^b D_\nu \hat{\phi}^c$$

Magnetic field

$$eH_i = \frac{1}{r^2}\hat{x}_i + O(e^{-m_W r})$$

Then magnetic charge

$$g_m = \frac{4\pi}{e}$$

In the unitary gauge $\phi^1 = \phi^2 = 0$

$$e\vec{A^3}(\vec{x}) = -\frac{sin\theta}{r(1+cos\theta)}\vec{e}_\phi$$

i.e. form of Dirac monopole with charge $g_m = \frac{4\pi}{e}$

Without scalar field solution also exist.
$A_4^a(x)$ plays role of scalar field
In the unitary gauge

$$A_4^1 = A_4^2 = 0$$

$$g\vec{A}^3(\vec{x}) = -\frac{sin\theta}{r(1+cos\theta)}\vec{e}_\phi$$

Note that in this gauge it also satisfies Maximally Abelian gauge (MAG) condition:

$$\left(\partial_\mu \delta_{kl} + \epsilon_{k3l} A_\mu^3(x)\right) A_\mu^l(x) = 0\,, \quad k = 1, 2$$

t'Hooft's idea: Partial gauge fixing

$$X(x) \to X'(x) = g(x)X(x)g^{\dagger}(x)\,, \quad X(x) = X_a(x)T_a$$

gauge fixing condition: $g(x) : X'(x)$ is diagonal
Gauge freedom is fixed up to $U(1)^{N_c-1}$ which is maximal Abelian subgroup or Cartan subgroup.
Gauge field has Abelian components $a^i_\mu(x) \equiv (A_\mu(x))_{ii}$

$$a^i_\mu(x) \to a^i_\mu(x) + \frac{1}{g}\partial_\mu \alpha_i$$

and off-diagonal components
$c^{ij}_\mu(x) \equiv (A_\mu(x))_{ij}\,, i \neq j$

$$c^{ij}_\mu(x) \to e^{i\left(\alpha_i(x)-\alpha_j(x)\right)} c^{ij}_\mu(x)$$

There is a singularity at locations where two or more eigenvalues are equal.
In the vicinity of such singularity gauge field has a form of the t'Hooft - Polyakov monopole, i.e. it has a magnetic charge.

$$A^3_{sing} T_3 = -\frac{1}{g} \vec{e}_\phi \frac{1 + cos\theta}{r sin\theta} T_3$$

$$g_m = -\frac{4\pi}{g} T_3$$

Examples of $X(x)$: $F_{12}(x); L(x)$

Thus QCD becomes equivalent to theory with color magnetic monopoles, 'photons', and charged matter fields: off-diagonal gluons and quarks.

Very successful application of the MAG to define monopoles on a lattice
MA gauge condition for $SU(3)$ :

$$\sum_{c\neq 3,8}\left(\partial_\mu\delta_{ac}+\sum_{b=3,8} f_{abc}A_\mu^b(x)\right)A_\mu^c(x)=0\,,\quad a\neq 3,8$$

solutions: extremums (over $g$) of the functional $F_{\mathrm{MAG}}[A^g]$

$$F_{\mathrm{MAG}}[A]=\frac{1}{V}\int d^4x\sum_{\mu,a\neq 3,8}[A_\mu^a(x)]^2$$

Abelian projection:
$A_\mu^a(x)T^a \rightarrow A_\mu^3(x)T^3+A_\mu^8(x)T^8$
only in observable! (coming back to this soon)

Bonati, D'Elia and Di Giacomo, 2010

It was argued that MAG is a proper Abelian gauge to find gauge invariant monopoles since monopoles can be identified in this gauge by the Abelian flux, but this is not possible in other Abelian gauges

In other words, the efficiency of the method to detect monopoles (DeGrand-Toussaint) depends on the choice of the gauge.

It was demonstrated for a class of gauges which interpolate between the Maximal Abelian gauge and the Landau gauge, how monopoles gradually escape detection.

On the lattice MAG was formulated by Kronfeld, Laursen, Schierholz, Wiese, 1989

gauge fixing functional:

$$F_{\mathrm{MAG}}[U] = \frac{1}{V} \sum_{x,\mu} \left(|U_\mu(x)^{11}|^2 + |U_\mu(x)^{22}|^2 + |U_\mu(x)^{33}|^2\right)$$

Abelian projection:
$U_\mu(x) \rightarrow u_\mu(x) \in U(1)^2$

## Abelian dominance hypothesis

Ezawa,Iwazaki ’82

Physical observables, related to the infrared properties of the theory, can be computed with the help of the Abelian variables i.e.

$$< \mathcal{O} >= \frac{1}{\mathcal{Z}} \int e^{-S} \mathcal{O}(U_\mu) \mathcal{D} U_\mu$$

and

$$< \mathcal{O} >^{Ab}= \frac{1}{\mathcal{Z}} \int e^{-S} \mathcal{O}(u_\mu) \mathcal{D} U_\mu$$

give approximately equal values of the infrared physical quantities.

Example: $O = W(r,t)$ ; static potential is derived from the Wilson loop: $V(r) = \alpha/r + \sigma r$.
Abelian projection gives very good approximation for $\sigma$ but not for $\alpha$
Suzuki and Yotsuyanagi, 1990

## Nonperturbative gauge fixing

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int DU \, e^{-S(U)} \mathcal{O}(U^{g^*})$$

$U^{g^*}$ solves $f(U) = 0$.

$$Z = \int DA \, e^{-S(A)} \, I^{-1}(A) \int Dg \, e^{-\lambda F(A^g)}$$

$$I(A) = \int Dg \, e^{-\lambda F(A^g)}$$

$\lambda$ - gauge parameter

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int DA \, e^{-S(A)} \, I^{-1}(A) \int Dg \, e^{-\lambda F(A^g)} \mathcal{O}(A^g)$$

$\lambda \to \infty$

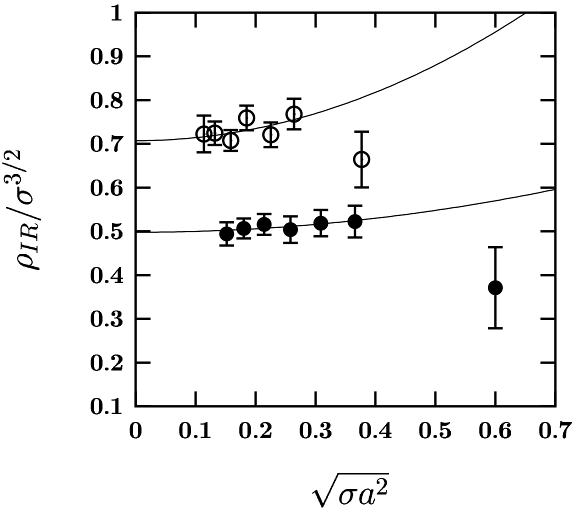
$$\int Dg \to \sum_{\textit{global minima}} F(A^g)$$

Trajectories of the Abelian monopoles form three different types of clusters:

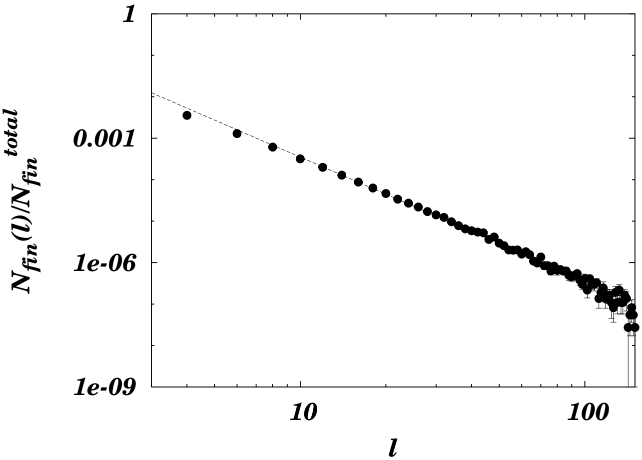
- Large cluster (one per configuration percolating cluster, of infinite size on the infinite lattice )
magnetic currents from this cluster are called IR monopoles
VB, Mitryushkin and Mueller-Preussker, 1992; Hart and Teper, 1996

- Finite size clusters with distribution of length $N(L) = C/L^3$
Hart and Teper, 1996
Both observations are in accordance with percolation theory, $1/L^3$ dependence was also derived within the polymer approach to the field theory for free or Coulomb-like interacting scalar particles
Chernodub and Zakharov, 2003

- Small clusters with length $L = O(a)$. These are UV monopoles



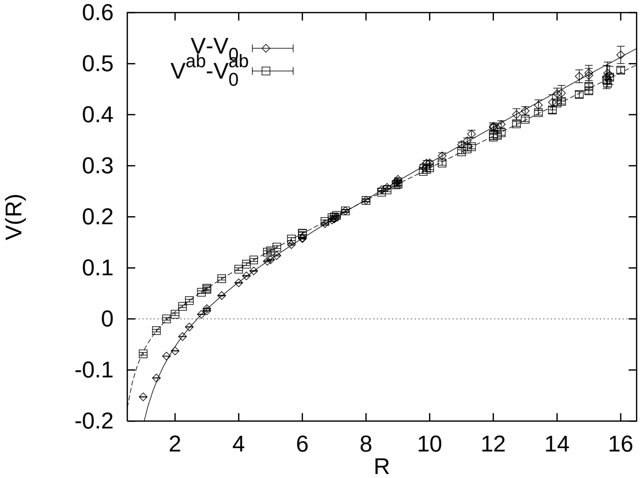

IR monopole densities obtained with TI (full symbols) and Wilson actions (open symbols). VB, Ilgenfritz, Müller-Preussker, 2005




The length distribution of finite clusters. $\beta = 2.4$, lattice $32^4$ .

VB, Boyko, **Polikarpov**, Zakharov, '03

# Abelian and monopole dominance in lattice gauge theory




Abelian and nonabelian static potentials.
Bali, VB, Mueller-Preussker, Schilling, 1996

Monopole component of the Abelian gauge field

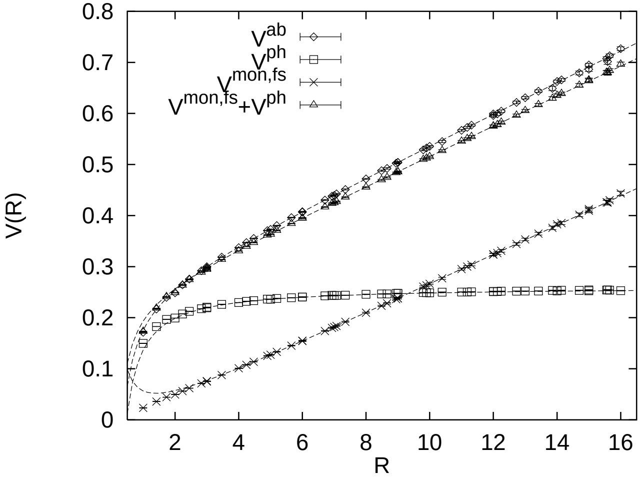
$$A_\mu^{mon}(x) = 2\pi \sum_{y,\nu} D(x-y)\partial_\nu m_{\mu\nu}(x)$$

$$A_\mu^{phot}(x) = A_\mu(x) - A_\mu^{mon}(x)$$

$$u_\mu^{mon}(x) = exp(iA_\mu^{mon}(x))$$

$$u_\mu^{ph}(x) = exp(iA_\mu^{ph}(x))$$

$$U_\mu^{mod}(x) = U_\mu(x) u_\mu^{mon,\dagger}(x)$$




Abelian static potential in comparison with 'monopole' and 'photon' static potentials

## Results in $SU(2)$:

$$\sigma^{ab}/\sigma = 0.92(4)$$

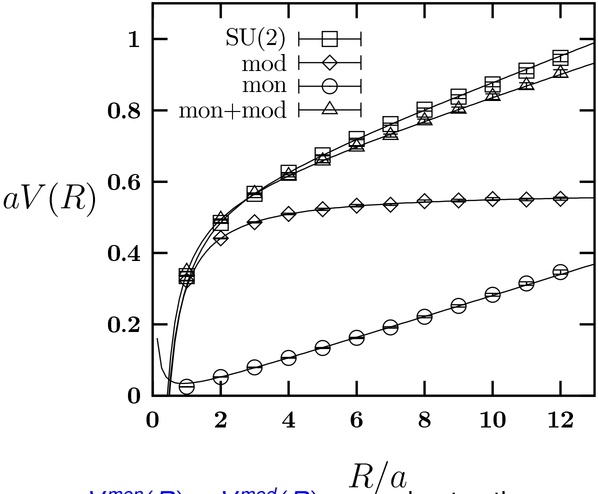
$$\sigma^{mon}/\sigma^{ab} = 0.95(2)$$

$$\sigma^{ab,2}/\sigma^{ab} = 2.23(5)$$ ( not far from 8/3 )

$\sigma^{ab}/\sigma$ was computed in the limit of infinite cutoff

$\sigma^{ab}/\sigma$ was computed for improved lattice action and universality of the Abelian dominance was demonstrated

VB, Ilgenfritz, Mueller-Preussker, 2005




the sum $V^{mon}(R) + V^{mod}(R)$ approximates the nonabelian static potential with 5% or higher precision at all distances. $\beta = 2.5, \ 24^4$ lattice

VB, **Polikarpov**, Schierholz, Suzuki, and Syritsyn, 2005

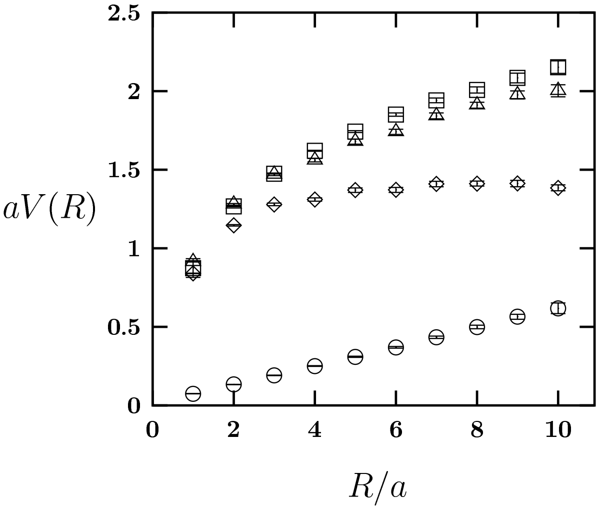
Similar result was presented for SU(3) in the talk by H. Suganuma

Known problem:
in the adjoint representation

$$W(C)_{adj} \to 1 + W_{abel,2} + W_{abel,-2} = 1 + 2cos(\phi(C))$$

The abelian projected string tension $\sigma_{abel,adj} = 0$.
This is correct result, since asymptotic string tension $\sigma_{adj} = 0$
But this does not agree with the Casimir scaling at intermediate distances

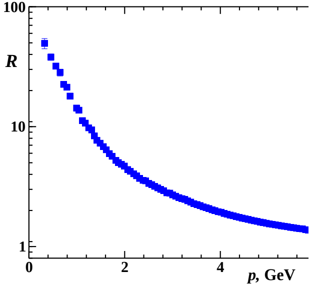
Two conclusions:
off-diagonal gluons become relevant
abelian projection procedure should be modified




$V^{mon,2}(R)$, $V^{mod,adj}(R)$ and their sum in comparison with $V^{adj}(R)$
VB, **Polikarpov**, Schierholz, Suzukid, and Syritsyn, 2005

Dominance of the diagonal gluon propagator in IR had been found
Amemiya and Suganuma, 1999 (computation in the coordinate space)
VB, Chernodub, Gubarev, Morozov and **Polikarpov**, 2003 (in momentum space)

$$R = \frac{D_{diag}(p_{min})}{D_{offdiag}(p_{min})} = 50(5), \quad p_{min} = 325 \text{ MeV}$$

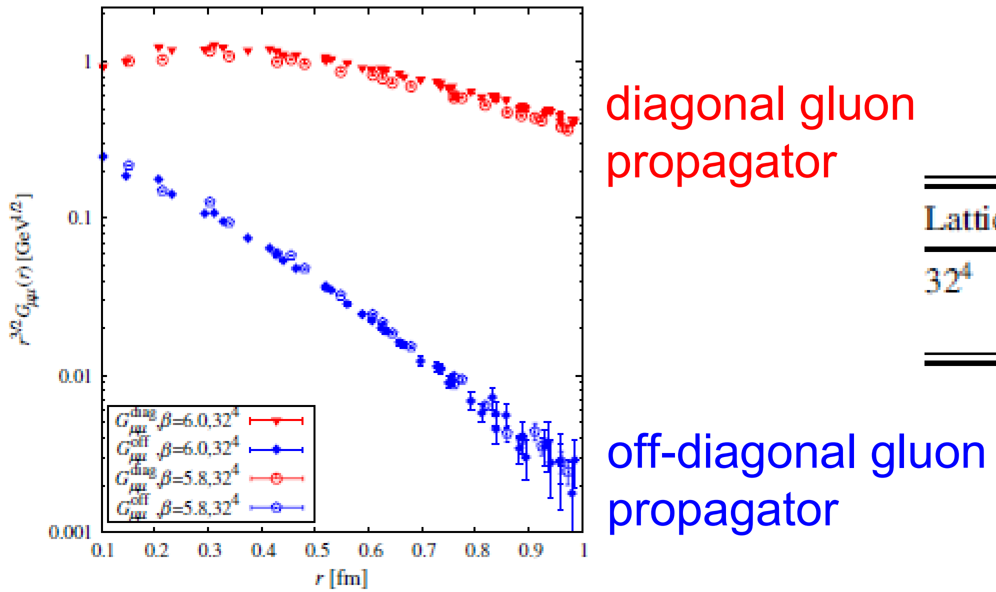



Ratio of diagonal to offdiagonal transverse propagators

# Gluon Propagators in Maximally Abelian Gauge in SU(3) Lattice QCD

S. Gongyo, T. Iritani and H. Suganuma

Abstract: We investigate gluon propagators in the maximally Abelian (MA) gauge with $U(1)^2$ Landau gauge fixing in quenched QCD, and find a *large off-diagonal gluon mass* of about 1GeV, which is responsible for *infrared Abelian dominance*.




The diagonal and off-diagonal gluon propagators (log plot of $r^{3/2}G(r)$) in SU(3) lattice QCD in the MA gauge with the $U(1)^2$ Landau gauge fixing.

| Lattice size | $\beta$ | $a$[fm] | $M_{off}$[GeV] | $M_{diag}$[GeV] |
|---|---|---|---|---|
| $32^4$ | 5.8 | 0.152 | 1.1 | 0.3 |
| | 6.0 | 0.104 | 1.1 | 0.3 |

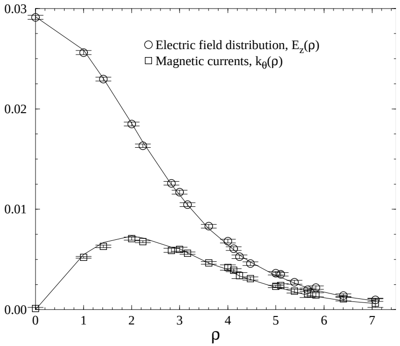
The lattice QCD result of effective masses of off-diagonal and diagonal gluons in the MA gauge. The off-diagonal gluon has a large mass of about 1GeV.

Ref. S. Gongyo, T. Iritani and H.Suganuma, PRD86 (2012) 094018, "Off-diagonal Gluon Mass Generation and Infrared Abelian Dominance in MA Gauge in SU(3) Lattice QCD", PRD87 (2013) 074506, "Gluon Propagators in MA Gauge in SU(3) Lattice QCD".

Properties of superconductors are often described in terms of a penetration depth $\lambda$ and a correlation length $\xi$ , which are equal to the inverse vector and Higgs masses.
They were computed on the lattice from the Abelian flux tube properties.
V. Singh, D. A. Browne, R. W. Haymaker, 1993
C. Schlichter, G. S. Bali, K. Schilling, 1998
F. V. Gubarev, M. Ilgenfritz, **M. I. Polikarpov**, T. Suzuki, 1999

The classical equations of motion for the Abelian Higgs model were numerically solved

$$\mathcal{L}_{AHM} = \frac{1}{4g^2} F_{\mu\nu}^2(B) + \frac{1}{2}|(\partial_\mu - iB_\mu)\Phi|^2 + \lambda(|\Phi|^2 - \eta^2)^2 \ ,$$




The lattice data for distribution of the electric flux and magnetic currents were nicely fitted by the classical equations of motion of the dual Abelian Higgs model. It was found that the mass of the vector boson is equal to the mass of the monopole (Higgs particle) within numerical errors. The effective dual Abelian Higgs Model appears to lie on the border between type-I and type-II superconductivity. The classical string tension (energy per unit length of the Abrikosov vortex) is 94% of the full non-Abelian string tension.

Abelian and monopole dominance in lattice SU(3) gluodynamics and lattice QCD

lattice formulation for $SU(3)$: Brandstaeter, Wiese and Schierholz, 1991

Most precise results on Abelian dominance in SU(3) case:
DIK (DESY-ITEP-Kanazawa) collaboration
VB, Ichie, Koma, Mori, Nakamura, Pleiter, **Polikarpov**, Schierholz, Streuer, Stueben and Suzuki, 2001

Most careful study of the DSS in $SU(3)$ gluodynamics
First study in lattice QCD ($N_f = 2$)
First study of the hadron string internal structure in Abelian projection

Results:

- The IR monopole density increases by more than a factor of two from SU(3) to QCD
- The dual Ampere law in both full QCD and in the pure SU(3) gauge theory is qualitatively satisfied
- Width of the monopole component of the flux tube does not depend on its length

| $m_\pi/m_\rho$ | $\sigma_{\rm ab}/\sigma$ | $\sigma_{\rm mon}/\sigma_{\rm ab}$ | $\delta$, fm | $\lambda$, fm |
|---|---|---|---|---|
| 0.60(1) | 0.89(4) | 0.80(4) | 0.29(1) | 0.15 |
| - | 0.83(3) | 0.84(3) | 0.29(1) | 0.17 |

- The width of the abelian flux tube $\delta = 0.29(1)$ fm in both cases; the penetration length $\lambda = 0.15(1)$ fm in full QCD and $\lambda = 0.17(1)$ fm in the quenched theory

This results in a dual photon mass of 1.3(1) GeV and 1.2(1) GeV, respectively.

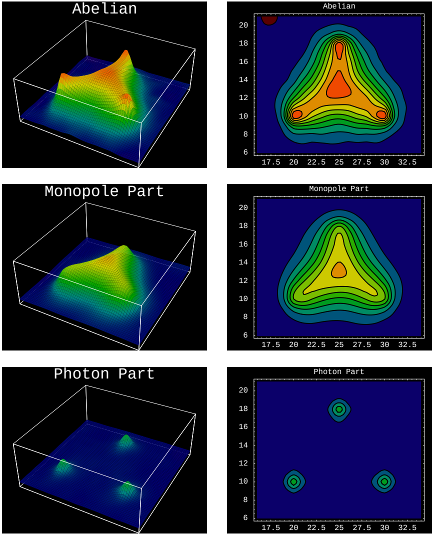
New results on Abelian dominance in SU(3) gluodynamics at this conference:
Talk by H. Suganuma
Parameters of simulations: $a = 0.057$ fm, $L = 1.8$ fm  close to continuum limit
Perfect Abelian dominance is claimed:

$$\sigma_{Ab} = \sigma$$

within error bars

I'm afraid systematic errors due to Gribov copies were not taken into account

# One more result of DIK collaboration:




Abelian action density in three-quark system (static baryon) in lattice QCD

Similarly to the dual superconductor scenario the vortex model of confinement is theoretically appealing and was confirmed by many numerical results, both in lattice Yang-Mills theory and within a corresponding infrared effective model.

The center vortex scenario of confinement is rather simple: center vortices carrying magnetic flux are condensed and the area law for Wilson loops is obtained from random fluctuations in the number of vortices topologically linked to the loop.

The vortex mechanism is a simple way to explain confinement, and is well motivated by the local, gauge-invariant order parameters for confinement ('t Hooft loop, Polyakov loop, vortex free energy).

The idea to study this mechanism via center projection after fixing center gauge was introduced in 1997 by Del Debbio, Faber, Greensite, Olejnik
This group made most work on the development of the approach

The mapping of the gauge-fixed $SU(N)$ lattice configuration to a $Z_N$ lattice configuration $U_{x,mu} \to Z_{x\mu}$ after center gauge fixing is called center projection.

Plaquettes with $Z(p) \neq 1$ on the projected lattice, are called P-plaquettes, they identify the position of thin center vortices known as P-vortices.

Direct maximal center gauge in $SU(2)$ lattice gauge theory is defined by the maximization of the following functional:

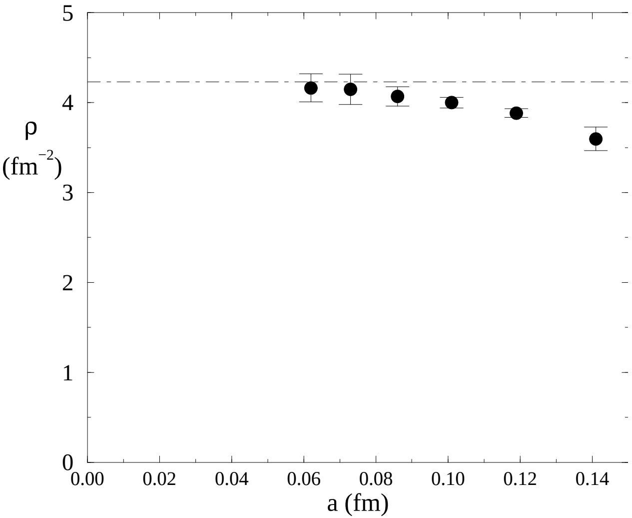
$$F(U) = \frac{1}{4V} \sum_{n,\mu} \left( \frac{1}{2} \mathrm{Tr} U_{n,\mu} \right)^2 = \frac{1}{4V} \sum_{n,\mu} \frac{1}{4} \left( \mathrm{Tr}_{adj} U_{n,\mu} + 1 \right)$$

The claim is that this procedure locates thick center vortices on the unprojected lattice; P-vortices lie somewhere in the middle of the thick vortices of the original lattice configuration.

The problem with the Casimir scaling is also present but has been solved in the model of thick center vortices.

The numerical results confirming the center vortex mechanism of confinement:

1. Scaling of density
demonstrated by Del Debbio, Faber, Giedt, Greensite, Olejnik , 1998




Density of P-vortices in SU(2) gluodynamics.
Gubarev, Kovalenko, **Polikarpov**, Syritsyn and Zakharov 2003

2. Do P-vortices really locate thick vortices on the original lattice?
From the fact that $W_n/W_0 = (-1)^n$, one concludes that P-vortices are correlated with the sign of the Wilson loop, in just the way expected if these P-vortices are correlated with the location of thick center vortices

3. Vortex Removal: after removing P-vortices from the lattice configuration one finds that confinement is lost.

de Forcrand and D'Elia, 1999

4. Center Dominance: string tension obtained from projected Wilson loops is close to nonabelian string tension

Del Debbio, Faber, Giedt, Greensite, Olejnik , 1998

5. Lattice simulations indicate that vortices are responsible for topological charge and cSB as well and thus unify all nonperturbative phenomena in a common framework

Faber, talk at Lattice 2013 and at this conference

4 January 2001

PHYSICS LETTERS B

Physics Letters B 497 (2001) 151–158

www.elsevier.nl/locate/npe

# P-vortices and drama of Gribov copies

V.G. Bornyakov [a,b], D.A. Komarov [b], M.I. Polikarpov [b]

[a] *Institute for High Energy Physics, Protvino 142284, Russia*
[b] *Institute of Theoretical and Experimental Physics, B. Cheremushkinskaya 25, Moscow, 117259, Russia*

Received 26 October 2000; accepted 16 November 2000
Editor: P.V. Landshoff

**Abstract**

We present results of the careful study of the Gribov copies problem in $SU(2)$ lattice gauge theory for the direct maximal center projection widely used in confinement studies. Applying simulated annealing algorithm we demonstrate that this problem is more severe than it was thought before. The projected (gauge noninvariant) string tension is not in the agreement with the physical string tension. We do not find any indications that P-vortices reproduce the full $SU(2)$ string tension neither in the infinite volume limit nor in the continuum limit. © 2001 Published by Elsevier Science B.V.

## 1. Introduction

The idea that the center vortices are the objects responsible for confinement in the nonabelian gauge theories is rather old [1,2]. Recently it has been argued that the center projection might provide a powerful tool to investigate this idea [3]. It is suggested that projection dependent P-vortices defined on the lattice plaquettes are able to locate thick gauge invariant center vortices and thus provide the essential evidence for the center vortex picture of confinement. So far 3 different center gauges have been used in practical computations: the indirect maximal center (IMC) gauge [3], the direct maximal center (DMC) gauge [4] and the Laplacian center gauge [5] (see also [6] for a new proposal).

IMC and DMC gauges suffer from the Gribov copies problem [7] and this technical problem gave rise to several claims and counterclaims. At present we have a drama in five acts:

(i) Initially there was the claim [3,4] that the projected string tension (the string tension which is due to P-vortices) reproduces the full $SU(2)$ string tension. Thus P-vortices are responsible for the confinement of color in gluodynamics.

(ii) The problem of Gribov copies has been raised in Ref. [8], where it has been demonstrated that there are gauge copies which produce P-vortices evidently with no center vortex finding ability since projected Wilson loops have no area law. At the same time these gauge copies correspond to higher maxima of the gauge fixing functional, than those used in [4].

(iii) In our previous publications [9,10] we confirmed the existence of two classes of gauge copies: with nonzero and zero projected string tension $\sigma_{Z(2)}$. We resolved the problem raised in [8] since we found the copies with the highest maxima of the gauge fixing functional which correspond to nonzero $\sigma_{Z(2)}$. At the same time the value of $\sigma_{Z(2)}$ was essentially lower than that obtained in [4] and in is disagreement with the physical string tension $\sigma_{SU(2)}$.

(iv) In Refs. [11,12] it has been argued that the disagreement between $\sigma_{Z(2)}$ and $\sigma_{SU(2)}$ is due to strong finite volume effects and it disappears on large enough lattices. In Refs. [9–12] the usual relaxation

0370-2693/01/$ – see front matter 
PII: S0370-2693(00)01309-5

IMC and DMC gauges suffer from the Gribov copies problem and this technical problem gave rise to several claims and counterclaims. At present we have a drama in five acts:

*i)* Initially there was the claim [3,4] that the projected string tension (the string tension which is due to P-vortices) reproduces the full $SU(2)$ string tension. Thus P-vortices are responsible for the confinement of color in gluodynamics.

*ii)* The problem of Gribov copies has been raised in Ref[8], where it has been demonstrated that there are gauge copies which produce P-vortices evidently with no center vortex finding ability since projected Wilson loops have no area law. At the same time these gauge copies correspond to higher maxima of the gauge fixing functional, than those used in [3,4] *iii)* In our previous publications [9,10] we confirmed the existence of two classes of gauge copies: with nonzero and zero projected string tension $\sigma_{Z(2)}$. We resolved the problem raised in [8] since we found the copies with the highest maxima of the gauge fixing functional which correspond to nonzero $\sigma_{Z(2)}$. At the same time the value of $\sigma_{Z(2)}$ was essentially lower than that obtained in [3,4] and was in disagreement with the physical string tension $\sigma_{SU(2)}$.

*iv)* In Refs.[11,12] it has been argued that the disagreement between $\sigma_{Z(2)}$ and $\sigma_{SU(2)}$ is due to strong finite volume effects and it disappears on large enough lattices. In Refs.[9-12] the usual relaxation plus overrelaxation (RO) algorithm was used to fix the DMC gauge.

*v)* Below we present results, obtained with the help of more powerful gauge fixing algorithm, simulated annealing (SA), and show that the problem of low value of $\sigma_{Z(2)}$ persists even on large lattices with physical extension up to $3fm$.

Our conclusion was :
The projected string tension $\sigma_{Z2}$ does not reproduce the full string tension, $\sigma_{SU2}$ . The obtained value of the ratio $\sigma_{Z2}/\sigma_{SU2} = 0.66(2)$ is rather low.

Two competing mechanisms of confinement received substantial support from the lattice results.
They have their own drawbacks and advantages

In my opinion center vortex mechanism looks simpler while dual superconductor one is more elegant
But they are interrelated: magnetic currents live on the surfaces formed by P-vortices

There are various attempts based on (or at least influenced by) lattice numerical results to develop effective infrared Yang-Mills theory.

Last years brought progress in the dual superconductor scenario development:

reasonable arguments in favor of the gauge invariance of monopoles found in MAG by Di Giacomo et al

development of the approach based on Cho-Niemi-Faddeev decomposition which is manifestly gauge invariant, by K.-I. Kondo, A. Shibata, S. Kato (sorry I was unable to cover this)

Computation of the Abelian propagators in SU(3) gluodynamics by H. Suganuma

Bose-Einstien condensation of thermal monopoles was discovered by D'Alessandro, D'Elia and Shuryak

# Misha's heritage

Misha was really a wonderful and attractive person with a gentle mind. His works on topological properties of non-perturbative QCD are world famous and I am very much grateful to him for having able to share a small part of them as an ITEP-Kanazawa collaboration. In addition to his own scientific works, it is really great that Misha has developed a big nice active lattice group in Russia, raising many brilliant young scientists who are now world-wildly working very actively.

**Suzuki Tsuneo**

For us, his students, PhD students and PostDocs, Misha Polikarpov was like a scientific father. And much more than just that, his care about us was very personal and very kind. He has deeply influenced many of us. And it is very hard to believe that we will not see Misha anymore ... at least in this world.

**Maxim Chernodub**

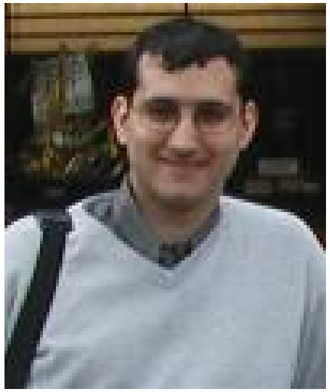
Some years ago people were hotly debating the nature of confining configurations in QCD. Misha Polikarpov and I were often on opposite sides of that discussion. You can learn a lot from debating with a worthy and honorable opponent, and that's the kind of opponent that Misha was. He wasn't interested in scoring points in some scientific contest; his interest was in getting to the bottom of things.

Later on Misha and I became collaborators on a number of scientific projects, we also worked together to organize the confinement sessions at the Confinement meetings, and I had a chance to observe Misha's role as a mentor to the younger scientists at ITEP. It was a privilege to work with him.

**Jeff Greensite**

Mikhail

Zubkov



Emil
Akhmedov

Chernodub
Maxim

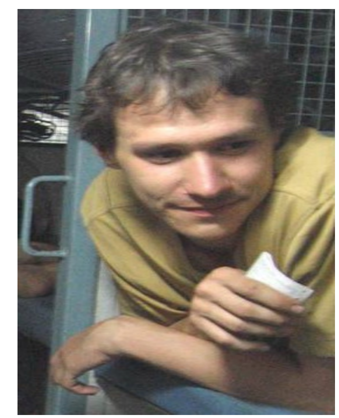
Fedor
Gubarev

Sergey
Morozov

Sergey
Syritsyn

Vladimir
Belavin

Pavel
Buividovich



Pavel Boyko

# ITEP Lattice Group: Introduction

The first ITEP publications devoted to lattice gauge theory were "Phase Transition Over Gauge Group Center And Quark Confinement In QCD" by S.B. Khokhlachev, Yu.M. Makeenko [1] and "Phase Diagram Of Mixed Lattice Gauge Theory From Viewpoint Of Large N" by Yu.M. Makeenko, M.I. Polikarpov [2]. The first numerical study of lattice theory in USSR was performed in 1982 in the "Monte Carlo Study Of Plaquette Spectral Density", Yu.M. Makeenko, M.I. Polikarpov and A.I. Veselov [3]. After several theoretical and numerical publications the ITEP lattice group was formed, the official status had been obtained by this group in 2001 when the ITEP lattice laboratory was formed (laboratory number 191).

Now the ITEP Lattice Group joins twenty researchers, the regular seminars take place in auditorium 418-419 in ITEP building 180. The prime topics of our investigations include (but not limited to):

- models of color confinement
- dynamics of topological objects in 3D and 4D systems
- nonperturbative aspects of Yang-Mills theories and, in particular, of QCD
- simulation of the QCD vacuum at zero and finite temperature with dynamical quarks
- lattice approaches to the standard model and quantum gravity.

### Contacts

Conact lab 191 secretary Olga Larina.

Secretary is located in the office 306 building 180 of ITEP (also see how to get).

Phone: +7-(499)-129-95-73, fax: +7-(499)-127-08-32.

### To call on:

в четверг 20 марта в 14:00 в корп 180 ИТЭФ состоится семинар: Е. В. Лущевская
"Решеточные вычисления свойств X(3872) (обзор литературы)"

### To thumb through:

- P. V. Buividovich, M. N. Chernodub, D. E. Kharzeev, T. Kalaydzhyan, E. V. Luschevskaya, M. I. Polikarpov, Magnetic-Field-Induced insulator-conductor transition in SU(2) quenched lattice gauge theory [arXiv:1003.2180]
- P. V. Buividovich, Y. M. Makeenko, Path integrals over reparametrizations: random walks vs. Levy flights, [arXiv:0911.1083]
- P. V. Buividovich, M. N. Chernodub, E. V. Luschevskaya, M. I. Polikarpov, Numerical evidence of chiral magnetic effect in lattice gauge theory, [arXiv:0907.0494]
- And other recent ITEP-LAT

# People with ITEP Lattice Group

| | position | office | work phone | e-mail |
|---|---|---|---|---|
| Emil Akhmedov | senior researcher at ITEP lab 192 | 409 building 138 | | akhmedov@itep.ru |
| Vladimir Belavin | senior researcher at ITEP lab 191 | 409 building 180 | | belavin@itep.ru |
| Vitaliy Bornyakov | senior researcher at IHEP and ITEP lab 191 | | | bornvit@sirius.ihep.su |
| Victor Braguta | Researcher at IHEP (Protvino) and ITEP (Moscow) at | | | braguta@mail.ru |
| Pavel Buividovich | researcher at ITEP, Moscow and JINR, Dubna | 405, building 180 | (+7) 499 1299414 | buividovich@itep.ru |
| Maxim Chernodub | senior researcher at ITEP lab 191 | 410 building 180 | +7-499-129-94-64 | maxim.chernodub@itep.ru |
| Fedor Gubarev | senior researcher at ITEP lab 191 | 410 building 180 | +7-(499)-129-94-64 | gubarev@itep.ru |
| Andrey Kotov | Student at MIPT and ITEP | 405, building 180 | | kotov.andrey.yu@gmail.com |
| Olga Larina | secretary at ITEP lab 191 | 306 building 180 | +7-(499)-129-95-73 | olarina@itep.ru |
| Elena Luschevskaya | Researcher at ITEP lab 190 | 424 building 180 | +7-(499)-129-94-14 | luschevskaya@itep.ru |
| Yurij Makeenko | senior researcher at ITEP lab 160 | 326 building 180 | | makeenko@itep.ru |
| Boris Martemyanov | senior researcher at ITEP lab 180 | building 1 (theoretical) | | martemyanov@itep.ru |
| Valentin Mitrjushkin | senior researcher at JINR and ITEP lab 191 | | | vmitr@thsun1.jinr.ru |
| Oleg Pavlovsky | Researcher at MSU and ITEP | 424, building 180 | | ovp@goa.bog.msu.ru |
| Mikhail Polikarpov | leading researcher at ITEP lab 191 | 306 building 180 | +7-(499)-129-95-73 | polykarp@itep.ru |
| Mikhail Prokudin | Researcher at ITEP | | | Mikhail.Prokudin@cern.ch |
| Roman Rogalev | senior researcher at IHEP | | | rogalyov@th1.ihep.su |
| Vladimir Shevchenko | senior researcher at ITEP lab 190 | | | shevchen@itep.ru |
| Sergey Syritsyn | PHD student at ITEP and MIT | | | syritsyn@itep.ru |
| Maxim Ulybyshev | Researcher at MSU and ITEP | 424, building 180 | | ulybyshev@goa.bog.msu.ru |
| Valentin Zakharov | senior researcher at ITEP lab 191 | 409 building 180 | | xxz@mppmu.mpg.de |
| Mikhail Zubkov | senior researcher at ITEP lab 191 | 420 building 180 | +7-(499)-129-94-93 | zubkov@itep.ru |

# ITEP Lattice preprints in 2014 (see also 2013 , 2012 , 2011 , 2010 , 2009 , 2008 , 2007 , 2006 , 2005 , 2004 , 2003 , 2002 , 2001 )

| # | Authors | Title | E-print | Journal reference |
|---|---|---|---|---|
| 2014-14 | T.G. Mertens, H. Verschelde, V.I. Zakharov | On the Relevance of the Thermal Scalar | ArXiv:1408.7012 | |
| 2014-13 | T.G. Mertens, H. Verschelde, V.I. Zakharov | Near-Hagedorn Thermodynamics and Random Walks - Extensions and Examples | ArXiv:1408.6999 | |
| 2014-12 | A. Avdoshkin, V.P. Kirilin, A.V. Sadofyev, V.I. Zakharov | On consistency of hydrodynamic approximation for chiral media | ArXiv:1402.3587 | |
| 2014-11 | T. G. Mertens, H. Verschelde, V. I. Zakharov | The thermal scalar and random walks in AdS3 and BTZ | ArXiv:1402.2808 | JHEP 1406 (2014) 156 |
| 2014-10 | M.Zubkov | Schwinger-Dyson equation and NJL approximation in massive gauge theory with fermions | ArXiv:1409.1321 | |
| 2014-09 | G.E. Volovik, M.A. Zubkov | Emergent Weyl fermions and the origin of i in quantum mechanics | | JETP Lett. 99 (2014) 481-486 |
| 2014-08 | M.A. Zubkov | Modified model of top quark condensation | ArXiv:1405.4067 | |
| 2014-07 | G.E. Volovik, M.A. Zubkov | Mirror as polaron with internal degrees of freedom | ArXiv:1404.5405 | |
| 2014-06 | G.E. Volovik, M.A. Zubkov | Emergent Weyl fermions and the origin of i in quantum mechanics | | Zh.Eksp.Teor.Fiz. 99 (2014) 552-557 |
| 2014-05 | G.E. Volovik, M.A. Zubkov | Emergent Weyl spinors in multi-fermion systems | ArXiv:1402.5700 | Nucl.Phys. B881 (2014) 514-538 |
| 2014-04 | M.A. Zubkov | Strong dynamics behind the formation of the 125 GeV Higgs boson | ArXiv:arXiv:1401.3311 | Phys.Rev. D89 (2014) 075012 |
| 2014-03 | E.V. Luschevskaya, O.V. Larina | Neutral Ï meson in a strong magnetic field in the SU(2) lattice gauge theory | | JETP Lett. 98 (2014) 652-655 |
| 2014-02 | E.V. Luschevskaya, A.A. Golubev | Current progress in laser cooling of antihydrogen | ArXiv:1406.7521 | |
| 2014-01 | V. Braguta, M.N. Chernodub, V.A. Goy, K. Landsteiner, | Temperature dependence of the axial magnetic effect in two-color quenched QCD | ArXiv:1401.8095 | Phys.Rev. D89 (2014) 074510 |

Sergey (21) is a master student at MIPT and a member of LHCb collaboration; Andrey (19) is a master student at the department of Computational Mathematics and Cybernetics of the Lomonosov Moscow State University