EBERHARD KARLS UNIVERSITÄT TÜBINGEN

MATHEMATISCH-NATURWISSENSCHAFTLICHE FAKULTÄT

Canonical Dyson–Schwinger Equations of QCD in Coulomb gauge

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Quark Confinement & Hadron Spectrum XI

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Outline

Motivation

Yang–Mills sector

Results hitherto achieved in Coulomb gauge

- IR behaviour of propagators
- linearly rising potential between static charges
- Polyakov loop potential and deconfinement phase transition (see talk by H. Reinhardt)

Fermion sector

Inclusion of quarks should explain

- constituent quark mass
- chiral condensate
- QCD phase diagram

Hamilton operator of QCD in Coulomb gauge

Steps

- start from canonically quantized theory in temporal gauge $A_0 = 0$
- physical states satisfy Gauss's law
- eliminate longitudinal degrees of freedom

$$H = \frac{1}{2}\int\left[-\mathcal{J}_A^{-1}\,\frac{\delta}{\delta A}\,\mathcal{J}_A\,\frac{\delta}{\delta A} + B^2\right] + \int \psi^\dagger(-\mathrm{i}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta m)\psi \\ - g\int \psi^\dagger\boldsymbol{\alpha}\cdot\mathbf{A}\,\psi + \frac{g^2}{2}\int \mathcal{J}_A^{-1}\,\rho\,\mathcal{J}_A\,F_A\,\rho$$

- B is the non-abelian magnetic field
- $\rho^a = \psi^\dagger t^a\psi - \mathrm{i} f^{abc}A^b\frac{\delta}{\delta A^c}$ is the colour charge density
- $F_A = (-\partial\cdot D)^{-1}(-\partial^2)(-\partial\cdot D)^{-1}$ is the Coulomb kernel

Static Green's functions

V.e.v. of an operator

$$\langle K \rangle = \int \mathcal{D}A\, \mathcal{J}_A\, \mathcal{D}\xi^\dagger\, \mathcal{D}\xi\, \Psi^*[A,\xi,\xi^\dagger]\, K\, \Psi[A,\xi,\xi^\dagger]$$

- $\mathcal{J}_A = \mathrm{Det}(-\partial \cdot D)$ (with $D = \partial + A$) is the Faddeev–Popov determinant of Coulomb gauge
- integration over transverse field configurations
- ξ and $\xi^\dagger$ are Grassmann fields
- Ψ is the vacuum wave functional

The expectation values of products of fields

$$\langle AA \rangle, \quad \langle \xi\xi^\dagger \rangle, \quad \langle \xi\xi^\dagger A \rangle, \; \ldots$$

are the static (equal-time) Green functions.

Vacuum wave functional

Formal equivalence to Lagrangian approach

Writing the vacuum wave functional as

$$|\Psi[A,\xi,\xi^\dagger]|^2 =: \exp\Big\{-S[A,\xi,\xi^\dagger]\Big\}$$

we have an Euclidean QFT defined by an “action” $S[A,\xi,\xi^\dagger]$.

Expansion of the vacuum wave functional

$$S[A,\xi,\xi^\dagger] = \frac{1}{2}\,\gamma_2 A^2 + \frac{1}{3!}\gamma_3 A^3 + \frac{1}{4!}\gamma_4 A^4 + \xi^\dagger(\bar\gamma + \bar\Gamma_0 A)\xi + \ldots$$

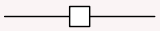
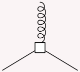
Kernels of the vacuum wave functional

$$S[A,\xi,\xi^\dagger] = \frac{1}{2}\,\gamma_2 A^2 + \frac{1}{3!}\gamma_3 A^3 + \frac{1}{4!}\gamma_4 A^4 + \xi^\dagger(\bar{\gamma} + \bar{\Gamma}_0 A)\xi + \dots$$

"Bare" vertices?

The coefficients in the vacuum wave functional play the role of the bare vertices, but

- are non-local functions
- have a non-trivial expansion in powers of the coupling
- will be represented diagrammatically by small empty boxes

$\gamma_2 =$  , $\bar{\gamma} =$  , $\bar{\Gamma}_0 =$  , $\dots$

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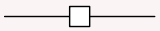
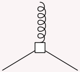
$\gamma_2 =$ , $\bar{\gamma} =$ , $\bar{\Gamma}_0 =$ , $\ldots$

- are not known... $\Rightarrow$ **variational kernels**

Canonical Dyson–Schwinger equations

Gluon and quark CDSEs are derived from the identity

$$0 = \int \mathcal{D}A\, \mathcal{D}\xi^\dagger\, \mathcal{D}\xi\, \frac{\delta}{\delta\phi} \Big\{ \mathcal{J}_A\, \mathrm{e}^{-S[A,\xi,\xi^\dagger]}\, K[A,\xi,\xi^\dagger] \Big\}$$

where $\phi \in \{A, \xi, \xi^\dagger\}$.

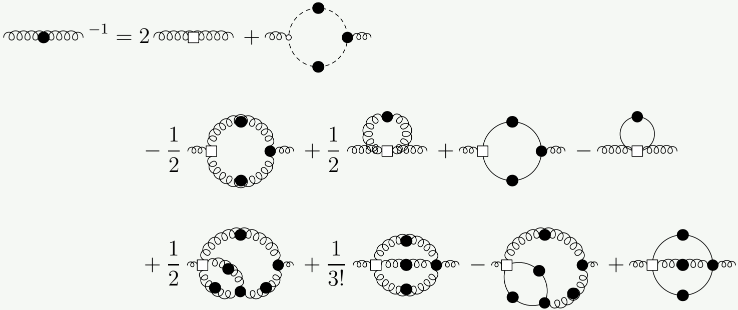
Ghost CDSEs follow from the operator identity

$$G_A = G_0 - G_0 \widetilde{\Gamma}_0 A G_A$$

where $G_A^{-1} = -\partial \cdot D$, and $\widetilde{\Gamma}_0$ is the bare ghost-gluon vertex.

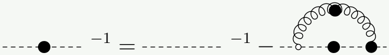
Propagator DSEs

Gluon propagator $\langle AA \rangle \equiv 1/2\Omega(\mathbf{p})$

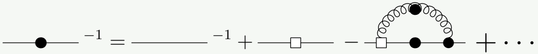


Propagator DSEs

Ghost propagator $\langle(-\partial \cdot D)^{-1}\rangle$



Quark propagator $\frac{1}{2}\langle[\psi, \psi^\dagger]\rangle$



Not quite equations of motion, rather relations between the Green functions and the so far undetermined variational kernels.

Propagator DSEs

Ghost propagator $\langle(-\partial\cdot D)^{-1}\rangle$

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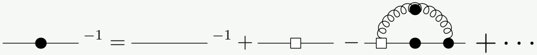

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1. Motivation
2. Canonical Dyson–Schwinger Equations
   - Hamiltonian approach to QCD
   - Vacuum wave functional and CDSEs
3. Variational solution of the Yang–Mills Schrödinger equation
   - Results for the Yang–Mill sector
   - Fermion sector
4. Conclusions

Variational method

Truncation scheme: two loops in the energy

- evaluate the energy in the state defined by the chosen Ansatz
- use the CDSEs to express the energy density as a function of the variational kernels
- minimize the energy by taking functional derivatives w.r.t. the variational kernels

$$\Downarrow$$

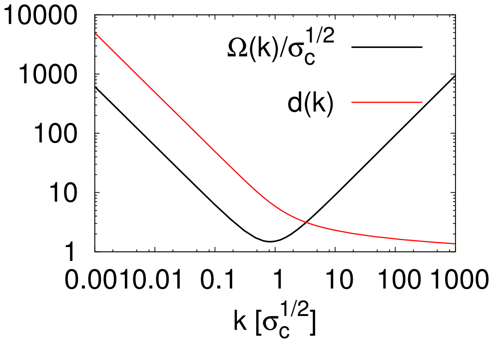
This gives a set of gap equations, which can be combined with the CDSEs.

Yang–Mills sector

Ghost and gluon propagators

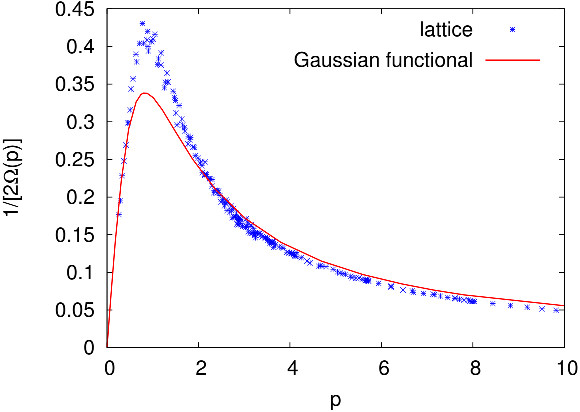
$$\langle AA \rangle = \frac{1}{2\Omega(k)}$$

$$\langle G_A \rangle = \frac{d(k)}{k^2}$$



Yang–Mills sector

Gluon propagator with non-Gaussian functional



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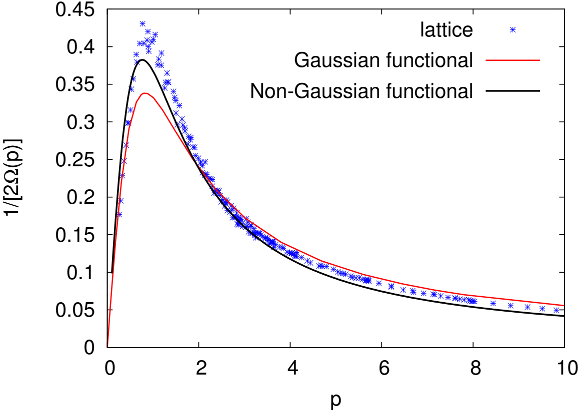
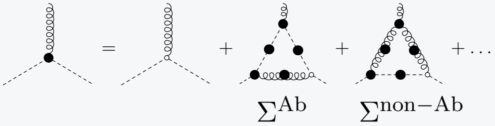
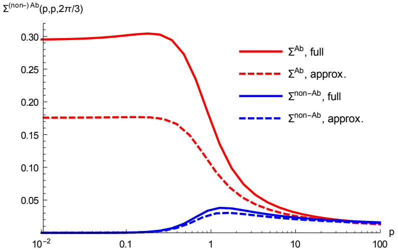


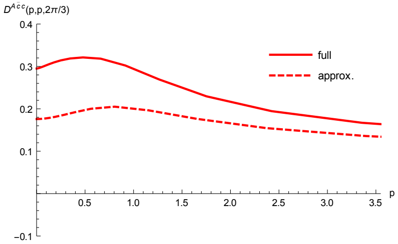
Yang–Mills sector

Ghost-gluon vertex

Truncated CDSE





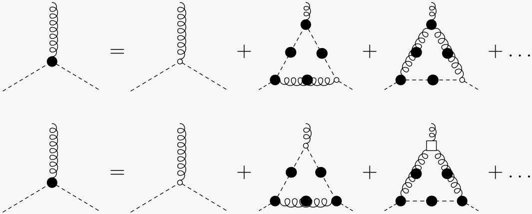


Pictures by M. Huber

Yang–Mills sector

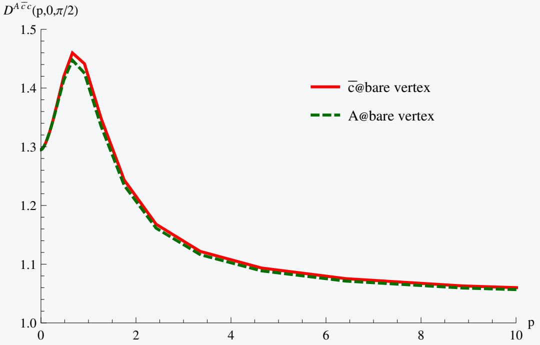
Ghost-gluon vertex

Two different CDSEs



Yang–Mills sector

Ghost-gluon vertex

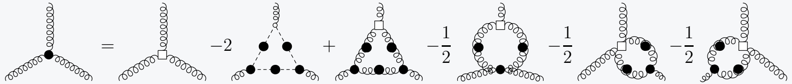


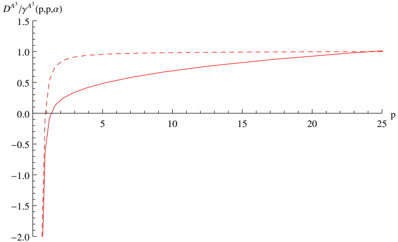
Picture by M. Huber

Yang–Mills sector

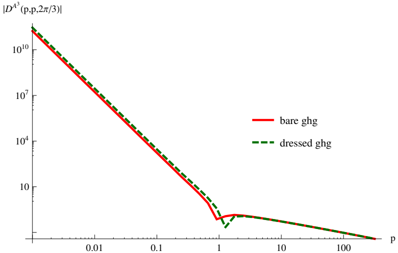
The three-gluon vertex

(Truncated) Three-gluon vertex CDSE





Dashed line: Ghost triangle only
Full line: Full CDSE



Full red line: Bare ghost-gluon vertex
Dashed green line: Full ghost-gluon vertex
Pictures by M. Huber

Fermion sector

The quark-gluon kernel

Ansatz for the quark-gluon kernel

In the exponent of the wave functional

$$S[A,\xi,\xi^\dagger] = \tfrac{1}{2}\,\gamma_2 A^2 + \tfrac{1}{3!}\gamma_3 A^3 + \tfrac{1}{4!}\gamma_4 A^4 + \xi^\dagger(\bar{\gamma} + \bar{\Gamma}_0 A)\xi$$

take the most simple Dirac and colour structure

$$\bar{\Gamma}_0 \sim \alpha_i\, t^a\, v(\mathbf{p},\mathbf{q})$$

with v being a scalar variational kernel.

Fermion sector

The quark-gluon kernel

Explicit form of the quark-gluon kernel

Variation of the energy density fixes the quark-gluon vector kernel to

$$v(\mathbf{p},\mathbf{q}) = -\frac{1 + b(\mathbf{p})\,b(\mathbf{q})}{\Omega(\mathbf{p}+\mathbf{q}) + \mathbf{p}^2/\mathcal{E}_{\mathbf{p}} + \mathbf{q}^2/\mathcal{E}_{\mathbf{q}}}$$

with

$$b(\mathbf{p}) = \frac{\mathcal{E}(\mathbf{p}) - |\mathbf{p}|}{M(\mathbf{p})}, \qquad \mathcal{E}(\mathbf{p}) = \sqrt{\mathbf{p}^2 + M^2(\mathbf{p})}.$$

Compare to the leading-order perturbative form

$$v_0(\mathbf{p},\mathbf{q}) = -\frac{1}{|\mathbf{p}+\mathbf{q}| + |\mathbf{p}| + |\mathbf{q}|}.$$

Fermion sector

Modifications to the gluon gap equation

$$\Omega^2(\mathbf{p}) = \mathbf{p}^2 + \text{Yang–Mills loop terms} \\ - 2 \int \frac{d^3q}{(2\pi)^3} \frac{X(\mathbf{p},\mathbf{q})\, v^2(\mathbf{p},\mathbf{q})}{[1+b^2(\mathbf{q})]\,[1+b^2(\mathbf{p}+\mathbf{q})]} \left[\Omega(\mathbf{p}) + \frac{\mathbf{q}^2}{\mathcal{E}(\mathbf{p})}\right]$$

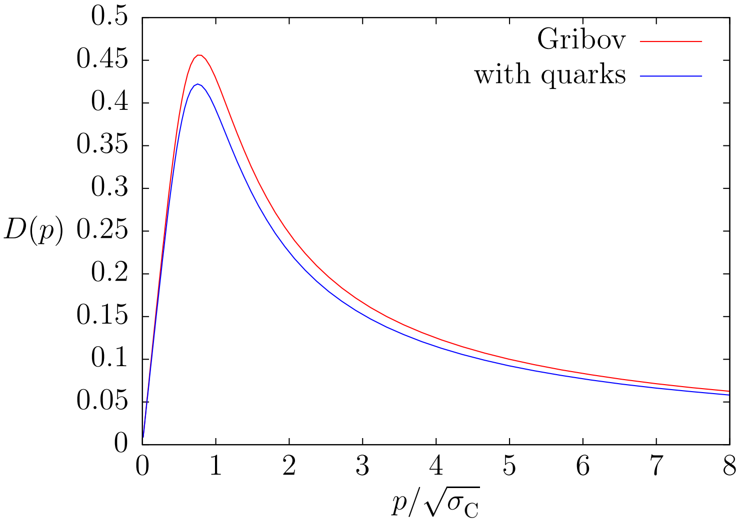
Yang–Mills terms

- ghost loop
- gluon loop
- Coulomb interaction of gluon charges

$X(\mathbf{p},\mathbf{q})$ is a scalar factor arising from the Dirac trace.

Fermion sector

Gluon propagator with quark loop



Fermion sector

Quark gap equation

$$M(\mathbf{p}) = \frac{g^2 C_F}{2} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{F(\mathbf{p}-\mathbf{q})}{\mathcal{E}(\mathbf{q})} \left[M(\mathbf{q}) - \frac{\mathbf{p}\cdot\mathbf{q}}{\mathbf{p}^2} M(\mathbf{p}) \right] + \frac{g^2 C_F}{2} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{X(\mathbf{p},\mathbf{q}) v^2(\mathbf{p},\mathbf{q})}{\Omega(\mathbf{p}+\mathbf{q})\mathcal{E}(\mathbf{q})} \mathcal{I}[M,\mathcal{E},\Omega]$$

$$g^2 F(r) = \frac{\alpha}{r} + \sigma_{\mathrm{C}}\, r$$

Still unsatisfactory results: only little enhancement (a few %) in comparison to pure Adler–Davis! ($M \simeq 120$ MeV, chiral condensate $\sim (-170\,\mathrm{MeV})^3$)

Fermion sector

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Conclusions

Summary

- standard DSE techniques can be used to treat arbitrary wave functionals
- new results for Yang–Mills vertex functions
- coupled quark-gluon system in Hamiltonian approach investigated
- renormalization still an open issue

Outlook

- include all perturbatively relevant terms in the ansatz
- possibly a dressed quark-gluon vertex