Zero modes of Overlap fermions, instantons and monopoles (II)

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Motivation

- **Our goal is to show relations between Chiral symmetry, instantons and monopoles.**
- A number of studies show, for example, by Instanton liquid model (E. V. Shuryak), the relation between Chiral symmetry breaking and instantons. Moreover, there are studies showing the relation between the instantons and monopoles.
- However, it has been difficult to directly show **the relations** by simulations, because, for example, the Chiral symmetry of the Wilson fermions is already broken in Chiral limit by discretization.

Introductions

How to show the relations?

1. Overlap Fermions

We generate quenched configurations for Wilson gauge action, and construct Overlap operator.

2. Additional monopoles

We'd like to show the quantitative relation between the number of instantons and monopoles. **Therefore, we directly add monopoles and anti-monopoles with charges to the configurations** by the way of the University of Pisa group (A. Di Giacomo, et al. Phys. Rev. D 56 (1997) 6816, Phys. Rev. D 61 (2000), 034503, C. Bonati, et al. Phys. Rev. D 85 (2012) 065001).

3. Measuring the additional monopoles

How to confirm whether we successfully add the monopoles or not?

We use techniques for measuring the monopoles (DIK collaboration, Phys. Rev. D 70 (2004) 074511, A. Bode, et al., hep-lat9312006). **Now we confirm that we are successfully adding monopoles to the configurations.**

4. Zero modes, instantons and monopoles

We'd like to find the relation among the zero modes, instantons and monopoles using the Overlap fermions as a powerful tool. We add the one monopole and one anti-monopole with several charges by the monopole creation operator, and we count the number of zero modes of the configurations. **We find that the number of zero modes increases by the number of instantons and the monopole charges.**

Our final goal: We will show the relation between the monopoles, instantons and Chiral symmetry.

Overlap operator

Overlap operator is defined by N. Neuberger (Phys. Lett. B427 (1998) 353) as follows.

$$D = \frac{1}{Ra}\left[1 + \frac{A}{\sqrt{A^\dagger A}}\right], \quad A = -M_0 + aD_W$$

A condition to a doubler of Overlap fermions: $0 < M_0 < 2$. D_W is the mass less Wilson fermion operator (r = 1).

How to compute Overlap operator in the simulations?

$$D(0) = \frac{\rho}{a}\left[1 + \frac{D_W}{\sqrt{D_W^\dagger D_W}}\right], \quad D_W = M + \frac{\rho}{a}, \ (\rho = 1.4)$$

M is Wilson's hopping term.
H_W is Hermitian Wilson Dirac operator.

$$\frac{D_W}{\sqrt{D_W^\dagger D_W}} = sgn(D_W) \equiv \gamma_5 sgn(H_W) \quad H_W = \gamma_5 D_W$$

For this function, we use Chebyshev polynomials approximation. We solve eigenvalue problems using ARPACK.

Simulation details

- O(200) ~ O(800) configurations for the Wilson gauge action are generated
- Constructing the Overlap Dirac operator from gauge links of the configurations
- Resolving eigenvalue problems using ARPACK subroutines
- Saving O(80) pairs of eigenvalues and eigenmodes, and analyzing the pair of modes

These numerical techniques have been already introduced. For example, **L. Giusti, et al. Com. Phys. Comm. 153 (2003) 31**, and the Doctoral thesis by V. Weinberg, etc.

Observable

The number of **zero modes**

n_+ : The number of zero modes has + chirality.

n_- : The number of zero modes has - chirality.

The number of **instantons**

n_+ : The number of instantons has + charge.

n_- : The number of instantons has - charge.

Topological charge: $Q = n_+ - n_-$

Topological susceptibility: $\chi/r_0^4 \equiv \frac{\langle Q^2\rangle r_0^4}{V}$

Simulation parameters

β	a/r_0	V	V/r_0^4	N_{conf}
5.789	0.279	12^4	126	200
5.812	0.266	10^4	50.0	844
		14^4	192	249
		16^4	327	274
5.846	0.248	12^4	78.9	200
		16^4	250	335
5.864	0.240	14^4	126	338
5.904	0.222	12^4	50.0	835
		16^4	158	320
5.926	0.213	14^4	78.9	277
5.989	0.190	14^4	50.0	862
6.000	0.186	12^4	25.0	402
		14^4	46.3	584
		$12^3 \times 24$	50.0	785
		16^4	78.9	380
		18^4	126	241
6.068	0.166	16^4	50.0	886

We use an analytic function from S. Necco, at al. Nucl. Phys. B622 (2002) 328 and compute the lattice spacing in all of our simulations.

Total 17 parameters.
Three different Physical volumes to check the continuum limit.

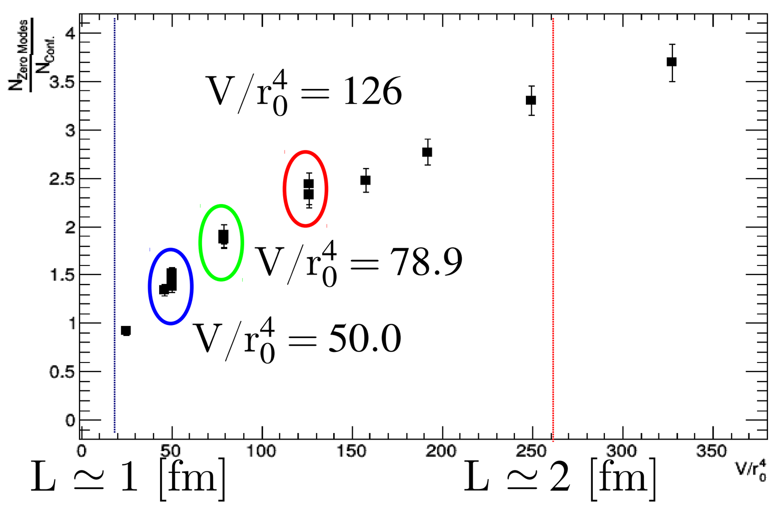
$V/r_0^4 = 50.0$

$V/r_0^4 = 78.9$

$V/r_0^4 = 126$

The number of zero modes and topological susceptibility

Number of zero modes



To get the topological susceptibility in continuum limit, we fix one physical volume changing the lattice spacing, and extrapolate to the continuum limit.

Our Result: $(\chi = 1.86(6) \times 10^2\ [MeV])^4$

L. Del Debbio, et al., PRL 94, 032003 (2005):

$$(\chi = 1.91(5) \times 10^2\ [MeV])^4$$

G. Veneziano, Nucl. Phys. B159, 213 (1979), and **E. Witten,** Nucl. Phys. B156, 269 (1979):

$$\frac{F_\pi}{6}(m_\eta^2 + m_{\eta'}^2 - 2m_K^2)|_{exp} \simeq (1.80 \times 10^2\ [MeV])^4$$

However, we never observed n_+ and n_- in the same configurations simultaneously.

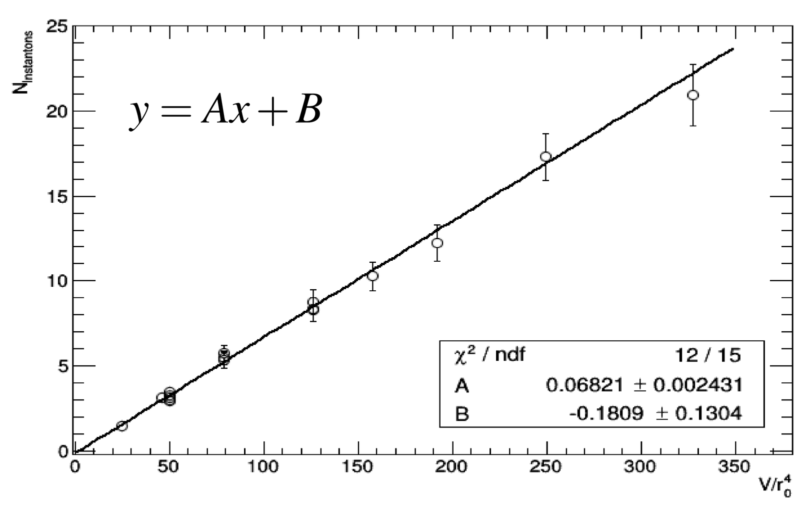
What are our zero modes?

We suppose that we observe **"net"** number of zero modes. Thus, we observe "Topological charge Q" as the zero modes 0, or $N_\pm$.

The instanton density

From analytic calculations by **A. Di Giacomo, the number of instantons is** $N_I = \langle Q^2\rangle$. Once we know this relation, we can calculate the instanton density from $\langle Q^2\rangle$.

We fit a liner function to $N_I = \langle Q^2\rangle$, and evaluate the instanton density.



Our result:

$y = 2\rho_i r_0^4 * V/r_0^4 + B$

$A = 2\rho_i r_0^4,\ A = 6.8(2) \times 10^{-2},\ B \simeq 0$

The instnaton density is

$\rho_i = 8.3(3) \times 10^{-4}\ [\text{GeV}^4],\ (r_0 = 0.5\ [\text{fm}])$.

Instanton liquid by E. V. SHURYAK

Nucl. Phys. B203 (1982) 93-115

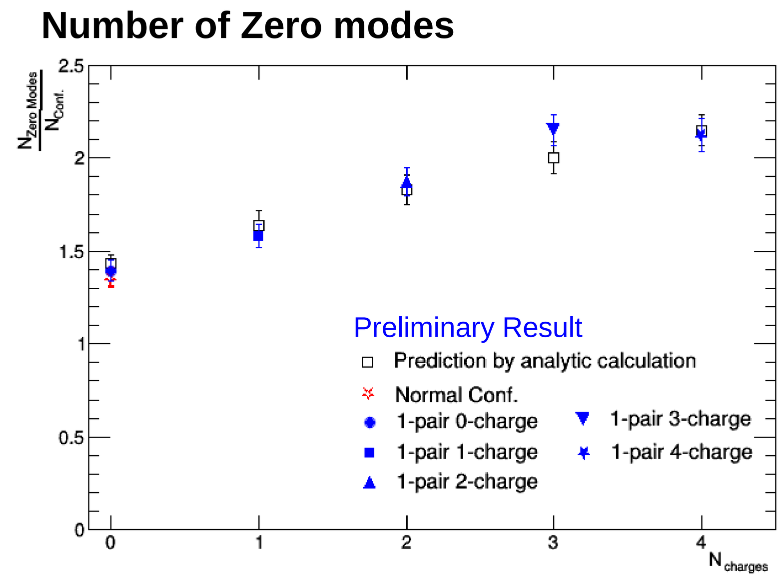
$n_c = 8 \times 10^{-4}\ [\text{GeV}^4]$

The study of monopoles by Overlap fermions

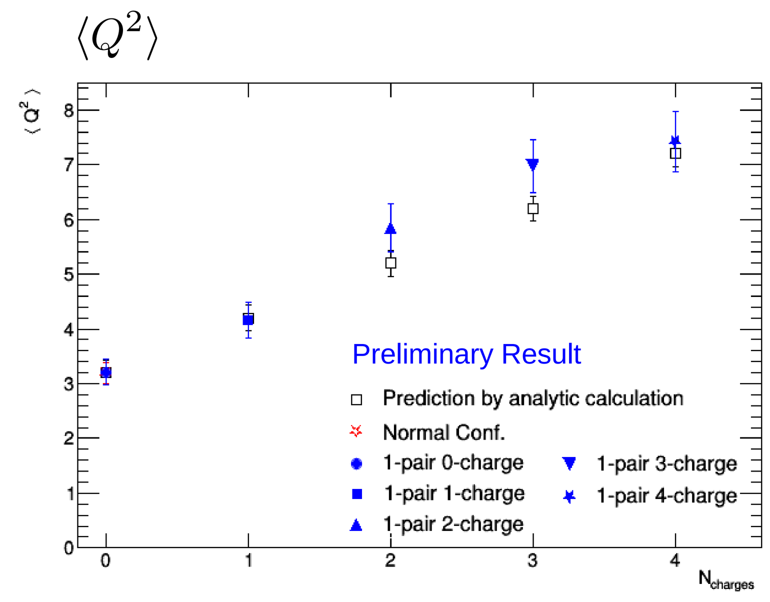
We add one monopole with + charges and one anti-monopole with – charges, and change monopoles charges from 0 to 4. First, we measure the length of the monopole loops. We confirm that **the monopole creation operator makes the long monopole loops.**

Second, we count the number of zero modes and compute $\langle Q^2\rangle$. Moreover, by analytic computations, we predict the number of zero modes and $\langle Q^2\rangle$ when we add monopoles with the several monopole charges.

Number of Zero modes



$\langle Q^2\rangle$



The numbers of zero modes (instantons) are increased by the monopoles and monopole charges.

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