Heavy Quarkonium suppression in a fireball

Miguel A. Escobedo

Physik-Department T30f. Technische Universität München

11th of September, 2014

Work done in collaboration with N. Brambilla, J. Soto and A. Vairo

Outline

Introduction

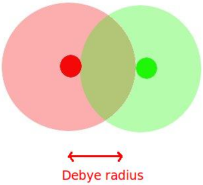
The original idea of Matsui and Satz (1986)

- Quarkonia is quite stable in the vacuum.
- Phenomena of colour screening, quantities measurable in Lattice QCD at finite temperature (static) support this. For example Polyakov loop.
- Dissociation of heavy quarkonium in heavy-ion collisions due to colour screening signals the creation of a quark-gluon plasma.

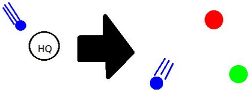
Colour screening

$$V(r) = -\alpha_s \frac{e^{-m_D r}}{r}$$

At finite temperature



Another mechanism, the decay width



- This effect makes the peak in the spectral function broader. It can arrive to a point where it is so broad that it does not make sense to speak of a bound state anymore.

Laine et al. perturbative potential (2007)

$$V(r) = -\alpha_s C_F \left[m_D + \frac{e^{-m_D r}}{r} \right] - i\alpha_s T C_F \phi(m_D r)$$

with

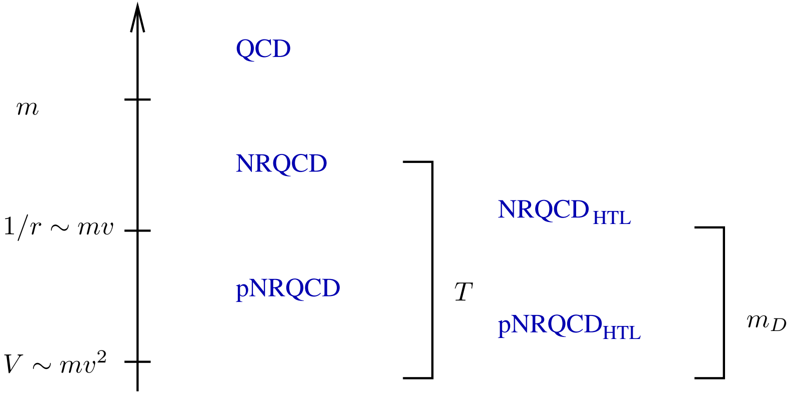
$$\phi(x) = 2\int_0^\infty \frac{dzz}{(z^2+1)^2}\left(1 - \frac{\sin(zx)}{zx}\right)$$

- This potential was obtained through the Wilson loop in Minkownski space at finite temperature.
- It has an imaginary part that has to be related with a decay width.

Question

- We talked in a generic way of a potential.
- It is historically assumed that HQ in a medium follows a Schrödinger eq. How do we show this from first principles? What is the potential that one has to put in?
- Use modern EFT techniques that have been succesful in $T = 0$ computations.

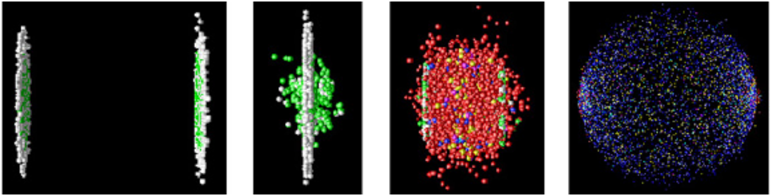
Effective field theories



(Brambilla, Ghiglieri, Petreczky And Vairo, M. A. E and Soto)

Out of equilibrium

We have a explosion



Assumption 1:

- Medium homogeneous in space and isotropic with an effective temperature T_{eff}.

What is measured?

- HQ is detected by its decay into leptons.
- Electromagnetic process, much slower than the physics that happens inside the fireball.

Assumption 2:

- HQ is measured throught the decay into leptons that happens after freeze-out.

What is measured?

$$\int d^4x\, d^4y e^{i(k_1+k_2)(x_1-x_2)} Tr(\rho J^\mu(x) J_\mu(y))$$

(in thermal eq. McLerran and Toimela (1985))
where

- J_μ is the electromagnetic current and we focuss on the component given by HQs.
- k_1 and k_2 is the momentum of the out-going leptons.
- $x_0,\ y_0 \gg t_{FO}$, where t_{FO} is the time in which we arrive to freeze-out.

Assumption 2:(in a more precise way)
In computing the lepton emission rate the integration from $x_0 = 0$ to $x_o = t_{FO}$ is negligible. The same happens for y_0.

What is measured?

$$\int d^4x\, d^4y e^{i(k_1+k_2)(x_1-x_2)} Tr(\rho J^{\mu}(x)J_{\mu}(y))$$

where

- J_{μ} is the electromagnetic current and we focuss on the component given by HQs.
- k_1 and k_2 is the momentum of the out-going leptons.
- $x_0,\ y_0 \gg t_{FO}$, where t_{FO} is the time in which we arrive to freeze-out.

Setting of the problem
Compute $Tr(\rho J^{\mu}(t_{FO}, \mathbf{x})J_{\mu}(t_{FO}, \mathbf{y}))$ assuming that we know $Tr(\rho J^{\mu}(t_0, \mathbf{x})J_{\mu}(t_0, \mathbf{y}))$ at some previous time. Assume that after t_{FO} evolution is like in the vacuum.

The case $\frac{1}{r} \gg T_{eff}$

Electromagnetic current in NRQCD

Because $M \gg \frac{1}{r} \gg T_{eff}$ we can start with the NRQCD Lagrangian at $T = 0$.

We also need to know what is J_μ in NRQCD.

$$J^0(x) = Q(\psi^\dagger(x)\psi(x) - \chi^\dagger(x)\chi(x))$$

The Fourier transform of this part does not contribute to lepton emission. Emission of soft photons.

Electromagnetic current in NRQCD

Because $M \gg \frac{1}{r} \gg T_{eff}$ we can start with the NRQCD Lagrangian at $T = 0$.

We also need to know what is J_μ in NRQCD.

$$J^i(x) = Q(e^{i2m_Q t}\psi^\dagger(x)\sigma^i\chi(x) - e^{-i2m_Q t}\chi^\dagger(x)\sigma^i\psi(x))$$

Color singlet with a spin $S = 1$.

$$Tr(\rho\psi^\dagger(x)\sigma^i\chi(x)\chi^\dagger(y)\sigma^i\psi(y))$$

Electromagnetic current in pNRQCD

Because $\frac{1}{r} \gg T_{eff}$ we can start with pNRQCD Lagrangian at $T = 0$.
Now we need the electromagnetic current in pNRQCD

$$\chi_i^\dagger(x)\sigma^i\psi_j(x) \to S_{ij}(x)$$

i and j are spinorial index that we normally do not write. Isotropy assumes that all directions of polarization are equally probable.

$$Tr(\rho S^\dagger(x)S(y))$$

- S has to be understood as the projection to the spin 1 state.
- It is the number of particles operator, which makes sense.

The Lagrangian of pNRQCD, transformation

$$\mathcal{L}_{pNRQCD} = \int d^3\mathbf{r}\, Tr\left[S^\dagger\left(i\partial_0 - h_s\right)S\right.$$
$$\left.+O^\dagger\left(iD_0 - h_o\right)O\right] + V_A(r)\,Tr(O^\dagger\mathbf{r}g\mathbf{E}S + S^\dagger\mathbf{r}g\mathbf{E}O)$$
$$+\frac{V_B(r)}{2}Tr(O^\dagger\mathbf{r}g\mathbf{E}O + O^\dagger O\mathbf{r}g\mathbf{E}) + \mathcal{L}_g + \mathcal{L}_q$$

We can consider a transformation

$$O(t) \to \Omega(t)O(t)\Omega^\dagger(t)$$
$$E^i(t) \to \Omega(t)E^i\Omega^\dagger(t)$$

such that $i\partial_t\Omega(t) = gA_0(t)\Omega(t)$.

The Lagrangian of pNRQCD, transformation

Ω can be a Wilson line connecting some time t' with t.

$$\mathcal{L}_{pNRQCD} = \int d^3\mathbf{r}\, Tr\left[S^\dagger\left(i\partial_0 - h_s\right)S\right.$$
$$\left.+O^\dagger\left(i\partial_0 - h_o\right)O\right] + V_A(r)\, Tr(O^\dagger\mathbf{r}g\mathbf{E}S + S^\dagger\mathbf{r}g\mathbf{E}O)$$
$$+\frac{V_B(r)}{2}Tr(O^\dagger\mathbf{r}g\mathbf{E}O + O^\dagger O\mathbf{r}g\mathbf{E}) + \mathcal{L}_g + \mathcal{L}_q$$

Equivalent to doing the computation in the temporal gauge.

- Advantage: It simplifies the current computation a lot.
- Disadvange: It sweeps the difficulties into the determination of the initial conditions.

Evolution of the number of singlets

$$f_s(x,y) = Tr(\rho S^\dagger(x) S(y))$$

We can use perturbation theory but expanding in r instead of α_s. In the interaction picture

$$i\partial_t S = [S, H_0]$$
$$i\partial_t \rho = [H_I, \rho]$$

Assumption 3:
We assume that HQ is comoving with the medium and that the center of mass momentum is not changed.

Evolution of the number of singlets

$$\partial_t f_S = -i(H_{eff} f_s - f_s H^{\dagger}_{eff}) + \mathcal{F}(f_o)$$

- $H_{eff} = h_s + \Sigma$ where Σ corresponds with the self-energy that can be obtained in pNRQCD by computing the time-ordered correlator.
- $\mathcal{F}(f_o)$ is a new term that takes into account the process $O \to g + S$. In ensures that the total number of heavy quarks is conserved.
- $\mathcal{F}(f_o)$ is a complicated function of $Tr(\rho O^{\dagger} O)$ and $\langle E^i E^j \rangle$. The information about the medium enters only in the chromoelectric field correlator. It will not be a Markovian process for any correlator.

Evolution of the number of singlets

Define

$$H = \frac{H_{eff} + H_{eff}^\dagger}{2}$$

$$\Gamma = i(H_{eff} - H_{eff}^\dagger)$$

$$\partial_t f_s = -i[H, f_s] - \frac{1}{2}\{\Gamma, f_s\} + \mathcal{F}(f_o)$$

- Screening.
- Decay.
- Creation.

Evolution of the number of singlets

Define

$$H = \frac{H_{eff} + H_{eff}^\dagger}{2}$$

$$\Gamma = i(H_{eff} - H_{eff}^\dagger)$$

$$\partial_t f_s = -i[H, f_s] - \frac{1}{2}\{\Gamma, f_s\} + \mathcal{F}(f_o)$$

- Screening.
- Decay.
- Creation.

Evolution of the number of singlets

Define

$$H = \frac{H_{eff} + H_{eff}^{\dagger}}{2}$$

$$\Gamma = i(H_{eff} - H_{eff}^{\dagger})$$

$$\partial_t f_s = -i[H, f_s] - \frac{1}{2}\{\Gamma, f_s\} + \mathcal{F}(f_o)$$

- Screening.
- Decay.
- Creation.

Evolution of the octet

Very similar reasoning.

$$f_o^{ab}(x,y) = Tr(\rho O^{\dagger,a}(x) O^b(y))$$

$$\partial_t f_o = -i[H_o, f_o] - \frac{1}{2}\{\Gamma, f_o\} + \mathcal{F}_1(f_s) + \mathcal{F}_2(f_o)$$

Remark:
We have this simple form because $\frac{1}{f_o}\partial_t f_o \ll E$ and we have this result because of the field redefinition we made.

Conservation of number of heavy quarks

$$\partial_t Tr(f_s) + \partial_t Tr(f_o) = 0$$

Ensured by

- Because we did not consider contact interactions in NRQCD that represent anhilation.
- Optical theorem that relates decay width and cross-section.

Application

Lindblad equation

$$\partial_t \rho = -i[H,\rho] + \sum_k \left(C_k \rho C_k^\dagger - \frac{1}{2}\{C_k^\dagger C_k, \rho\}\right)$$

- Used in open quantum systems and quantum optics. Numerical libraries available to solve it (we used qutip (Johansson, Nation and Nari (2012)).
- Introduced in the world of quarkonium by Akamatsu (2014).
- There is no prescription to find C_k or to tell how many are there.

Further simplification

$$f_s(t, \mathbf{r_x}, \mathbf{r_y})$$

Huge matrix. We can simplify it by making an expansion in spherical armonics and cutting at some point.

- If the initial condition is diagonal in the spherical harmonics space it will remain always so.
- Including up to p-wave gives a good result if you are interested in s-wave. It does not change the results a lot to include also the d-wave.
- Similar arguments apply to f_o in color space. $f_o^{ab} \propto \delta^{ab}$.

Initial conditions

Simple assumption:

- HQ is created at $t = 0$ with a the same probability as in pp collisions.
- Power counting of NRQCD tell us that at LO it will be a Dirac delta function.
- Perturbative processes creating a singlet are $\alpha_s(M_Q)$ suppressed with respect to octets.

We use

$$f_s = \alpha_s(M_Q)\delta^3(\mathbf{r_x} - \mathbf{r_y})$$
$$f_o = \frac{\delta^{ab}}{8}\delta^3(\mathbf{r_x} - \mathbf{r_y})$$

After we normalize. Results are more sensitive to the shape in r than to the fraction $\frac{f_s}{f_o}$.

Approximation to R_{AA}

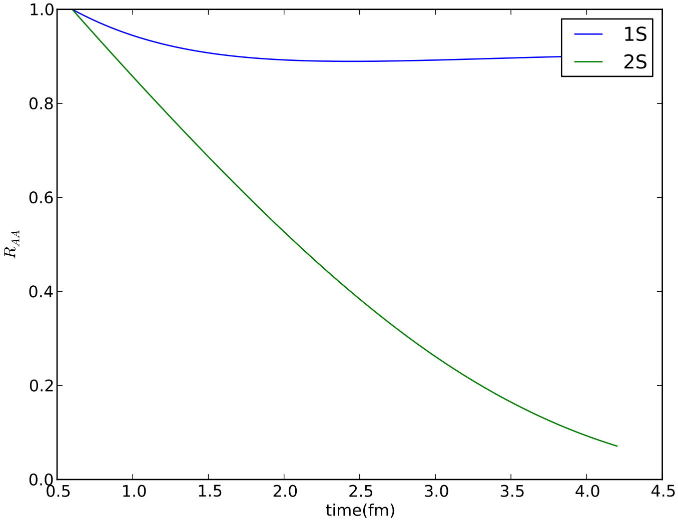
- We assume that the initia probability (up to a factor) is equal in pp and in AA.
- Both evolve in the vacuum up to $t = 0.6fm$. At that moment HQ starts to feel a thermal medium with $T = 475MeV$ that follows Bjorken evolution.
- The ratio of $\langle 1S|f_s|1S\rangle$ in AA and pp so computed will be our approximation to R_{AA}.
- Qualitative idea. We neglect CNM effects, energy loss, asymmetry, viscosity, very naive initial condition...

Toy model for $\frac{1}{r} \gg T_{eff} \gg m_D \gg E$

Nice because at LO $C^i = Ar^i$ where A is some constant.
But if you use the perturbative coupling α_s at LHC temperatures you get negative decay widths.
Toy model:do a more or less educated guess about the value of A.

Toy model for $\frac{1}{r} \gg T_{eff} \gg m_D \gg E$

Considering $C^i = 0$. Only screening.



Toy model for $\frac{1}{r} \gg T_{eff} \gg m_D \gg E$

Singlet to octet and viceversa.

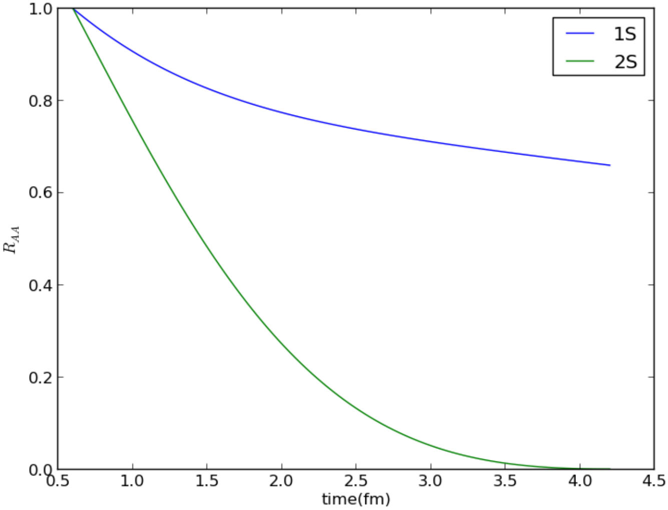
$$C^i = 10\sqrt{\frac{2C_F\alpha_s}{3}}m_D\log(2)$$

Octet to octet.

$$C^i = 10\sqrt{\frac{(N_c^2-4)\alpha_s}{6N_c}}m_D\log(2)$$

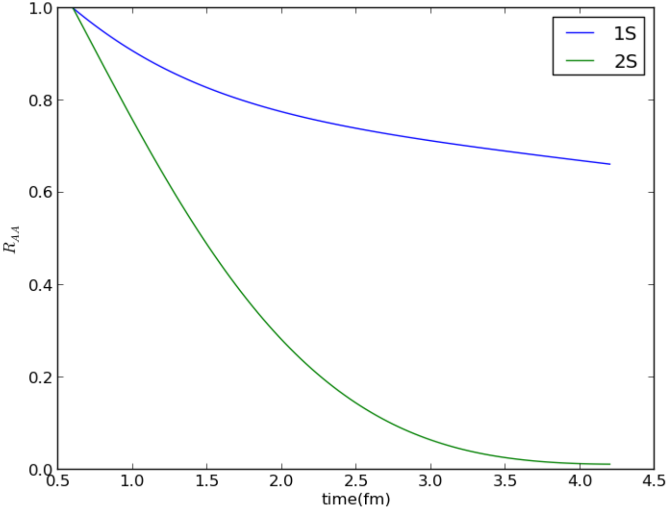
Toy model for $\frac{1}{r} \gg T_{eff} \gg m_D \gg E$

Only S and P-wave



Toy model for $\frac{1}{r} \gg T_{eff} \gg m_D \gg E$

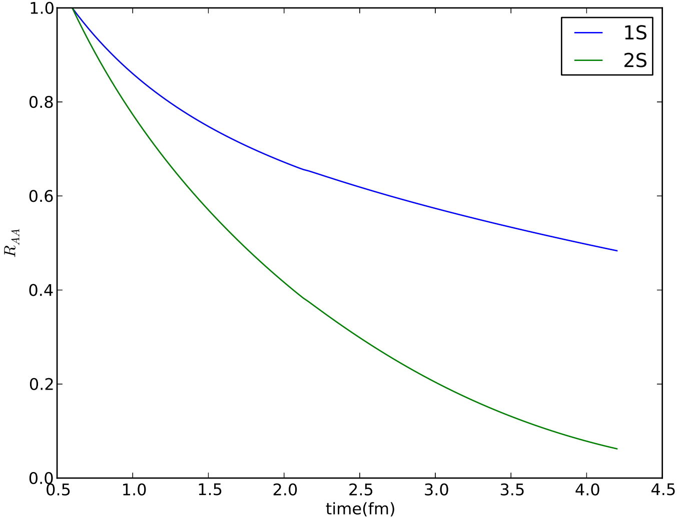
Also D-wave



The case $\frac{1}{r} \gg T_{eff} \gg E \gg m_D$

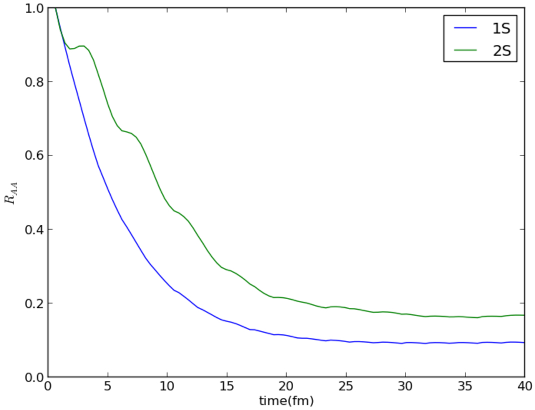
- In this case we do not encounter negative decay width with realistic couplings.
- We computed in JHEP1009(2010)038 (Brambilla, M.A.E, Ghiglieri, Soto and Vairo) the corrections to the singlet binding energy and decay width. Now we also need the octet ones. Straightforward computation.
- In is not trivial to write our equations in Lindblad form with a reasonable number of collapse operators. We need to assume quasistatic limit ($\frac{1}{T}\frac{dT}{d\tau} \ll E$)..

The case $\frac{1}{r} \gg T_{eff} \gg E \gg m_D$



Approach to equilibrium

Same as before but with a fixed temperature (the one found at 1.2*fm* with our parameters).



Conclusions

Conclusions

- R_{AA} is related with the number of singlets operator in pNRQCD. In the $\frac{1}{r} \gg T$ case a simple set of equations can be found without assuming weak coupling. All information encoded in an Hermitian effective Hamiltonian and in the correlator of chromoelectric fields.
- In some cases these equations can be written in a Lindblad form, as previously seen by Akamatsu in the high temperature regime.
- The octet thermal modifications are very important. We need to include more realistic initial conditions.
- We show results for different temperature regimes.