Few-body Physics from Lattice QCD

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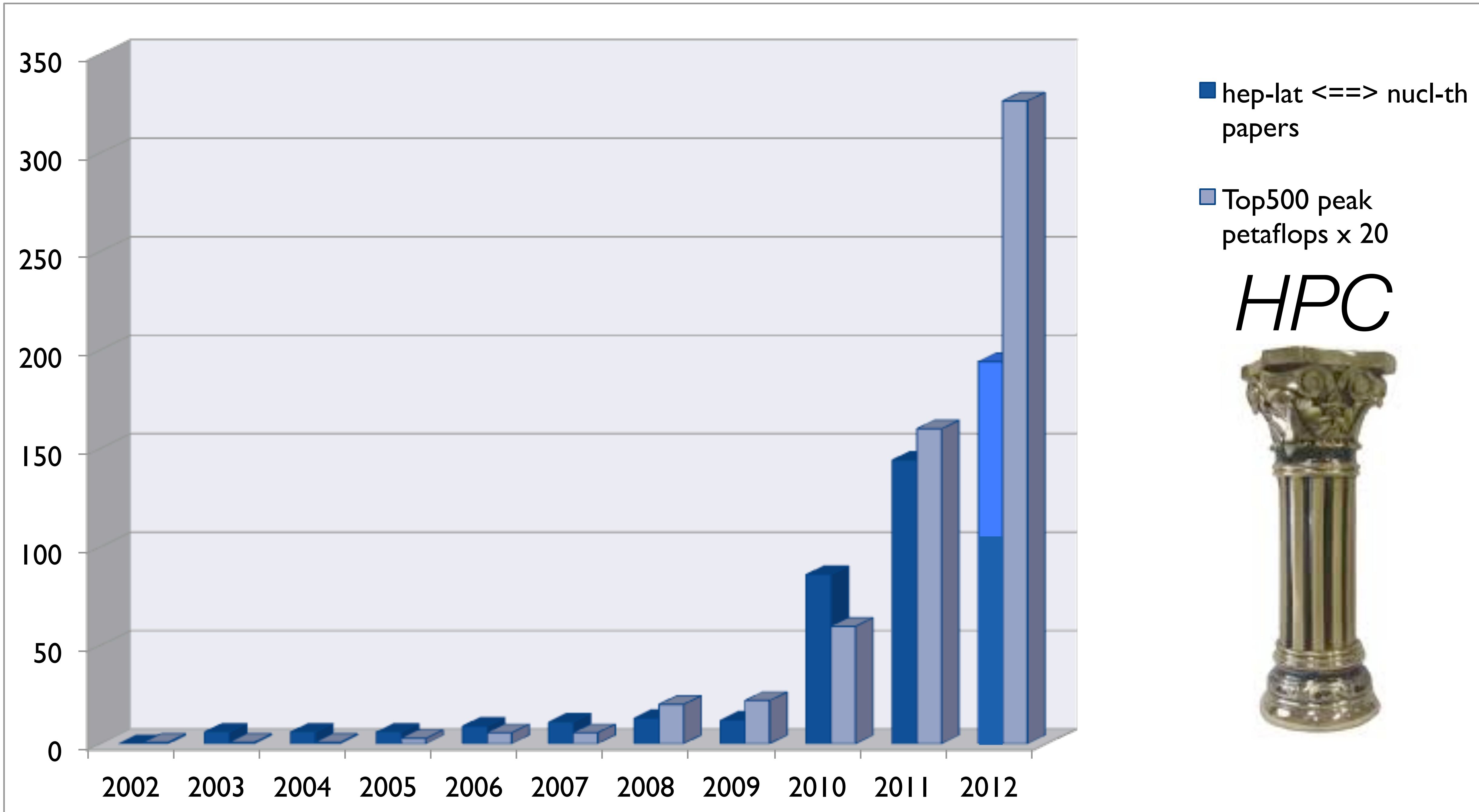
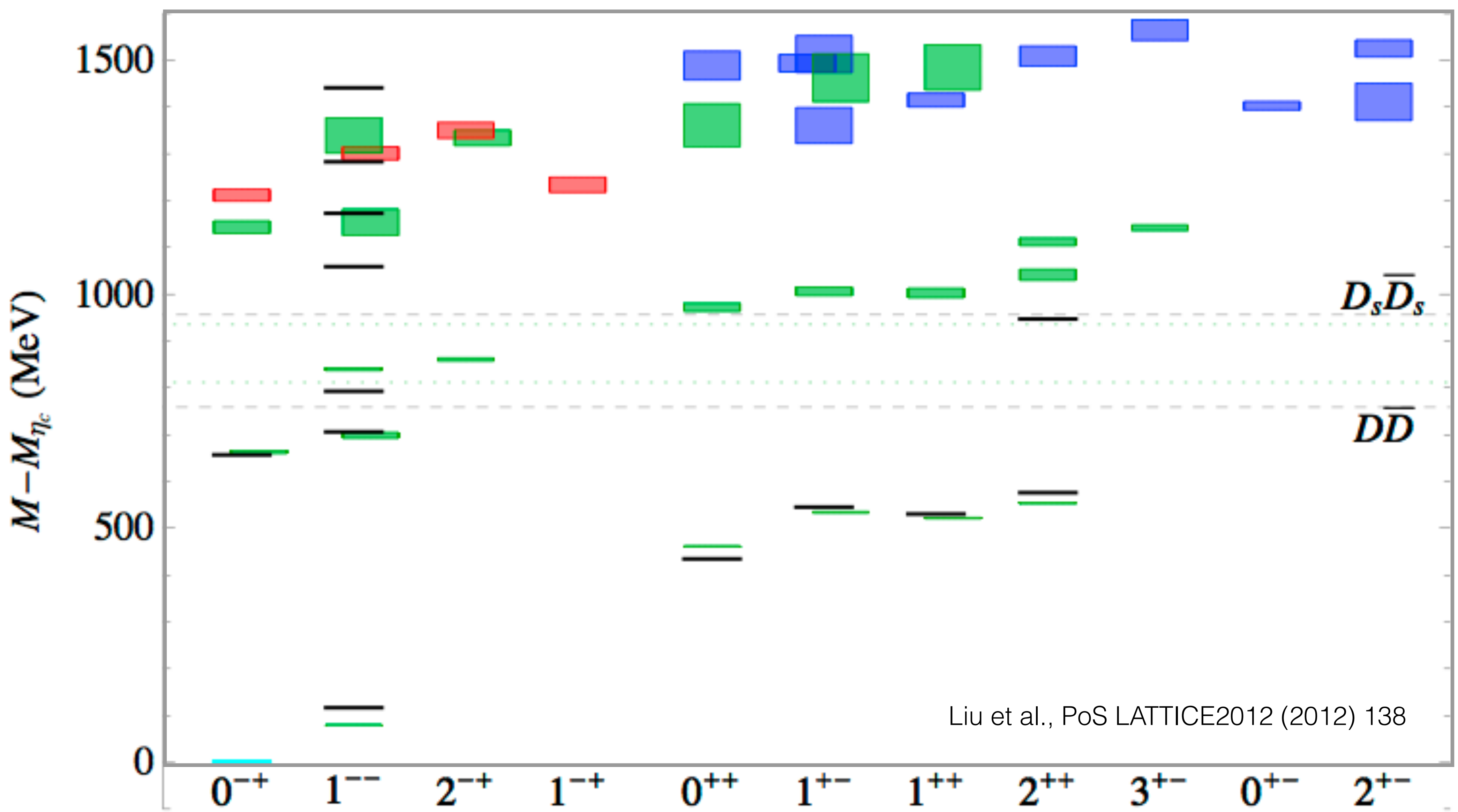
The *number* of *hadrons* is (slowly) increasing in Lattice QCD calculations

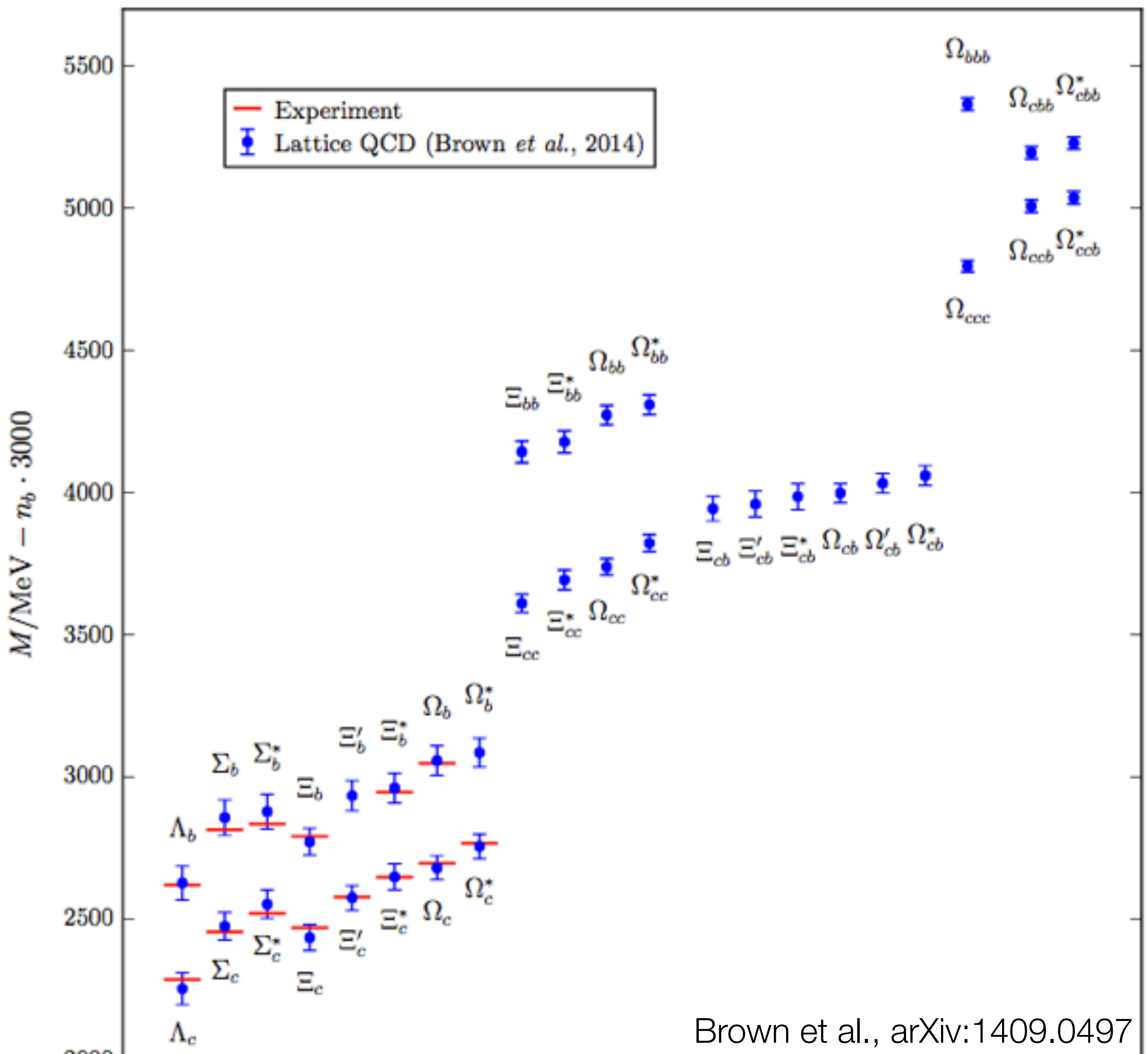


Already we've heard lots of impressive work on exotic mesons



see e.g., talks by **J. Bulava** and **S. Prelovsek**

. . . as well as some recent work on exotic baryons

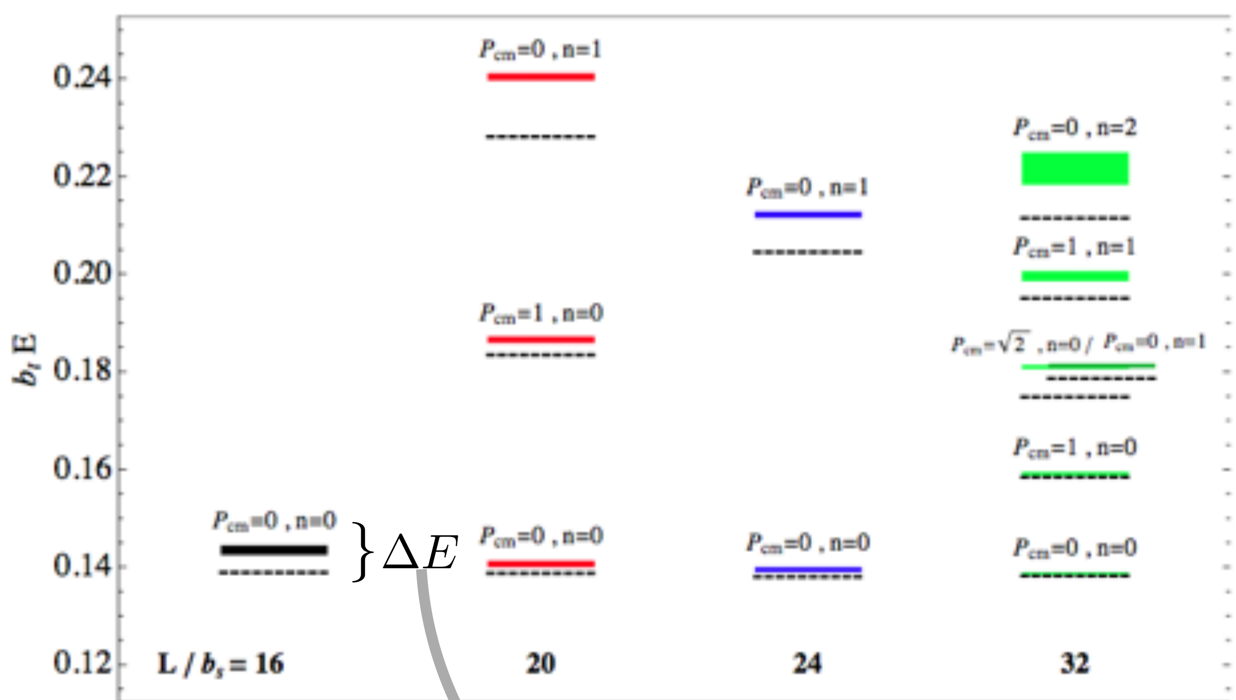


see e.g., talks by **J. Green** and **N. Mathur**

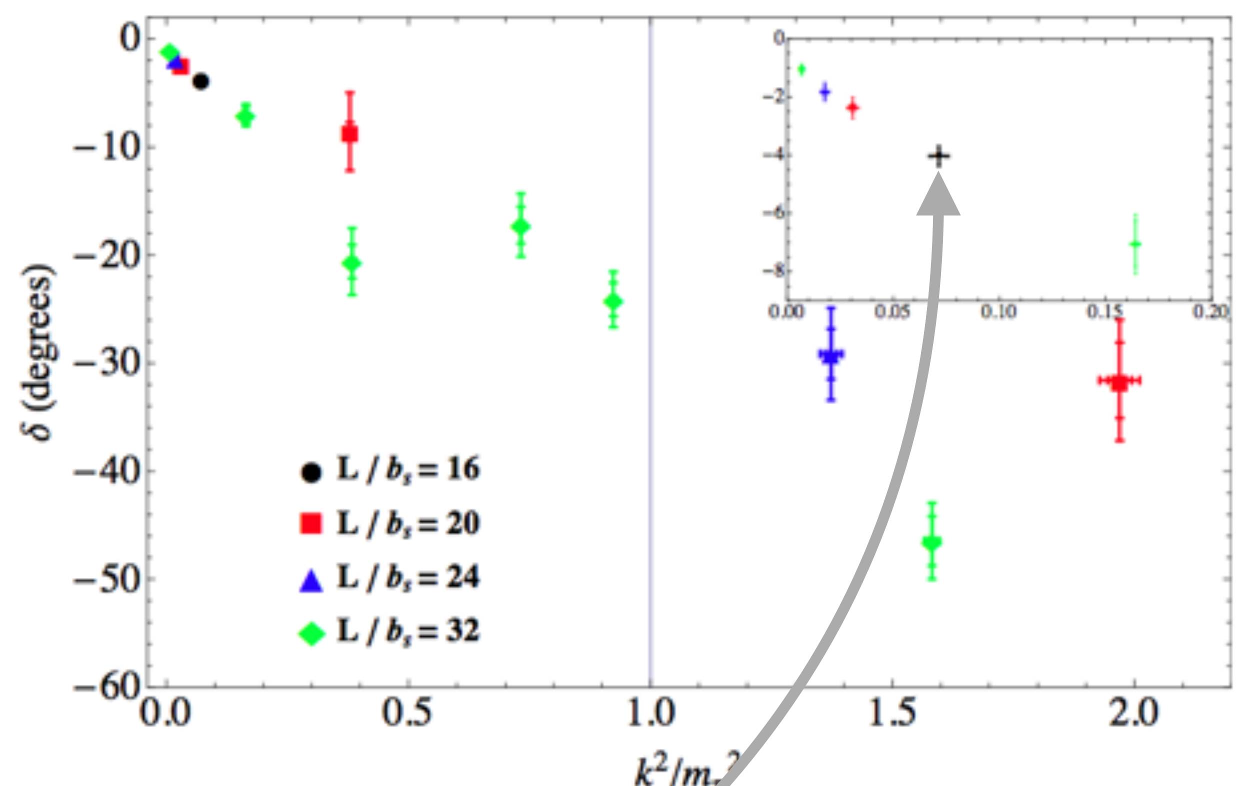
Scattering information from meson-meson readily obtained

pi-pi energy levels in a box

(from LQCD)

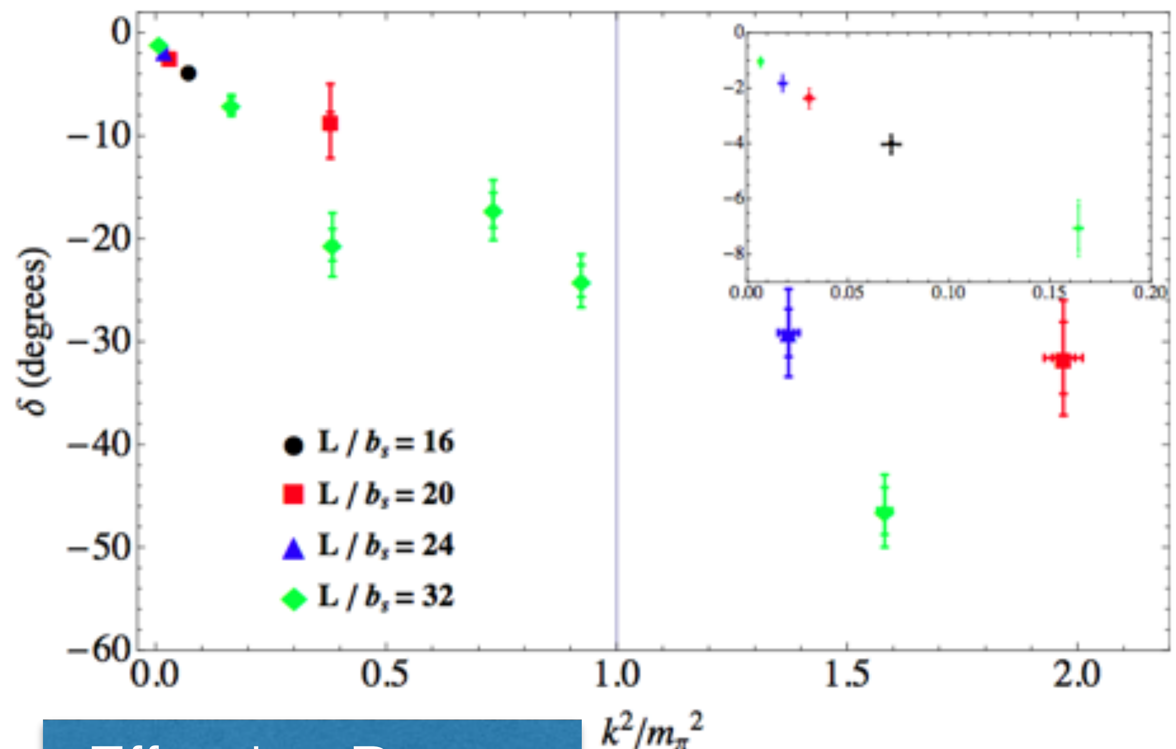


pi-pi scattering phase shifts



Lüscher's Formula

Extracting Low-Energy Constants (LECs)



see e.g., talk by **J. Bijnens**

Finite-volume ChPT

Effective Range Expansion

$$p\cot(\delta(p)) = -\frac{1}{a} + \frac{r}{2}|p|^2 + O(p^3)$$

Effective range

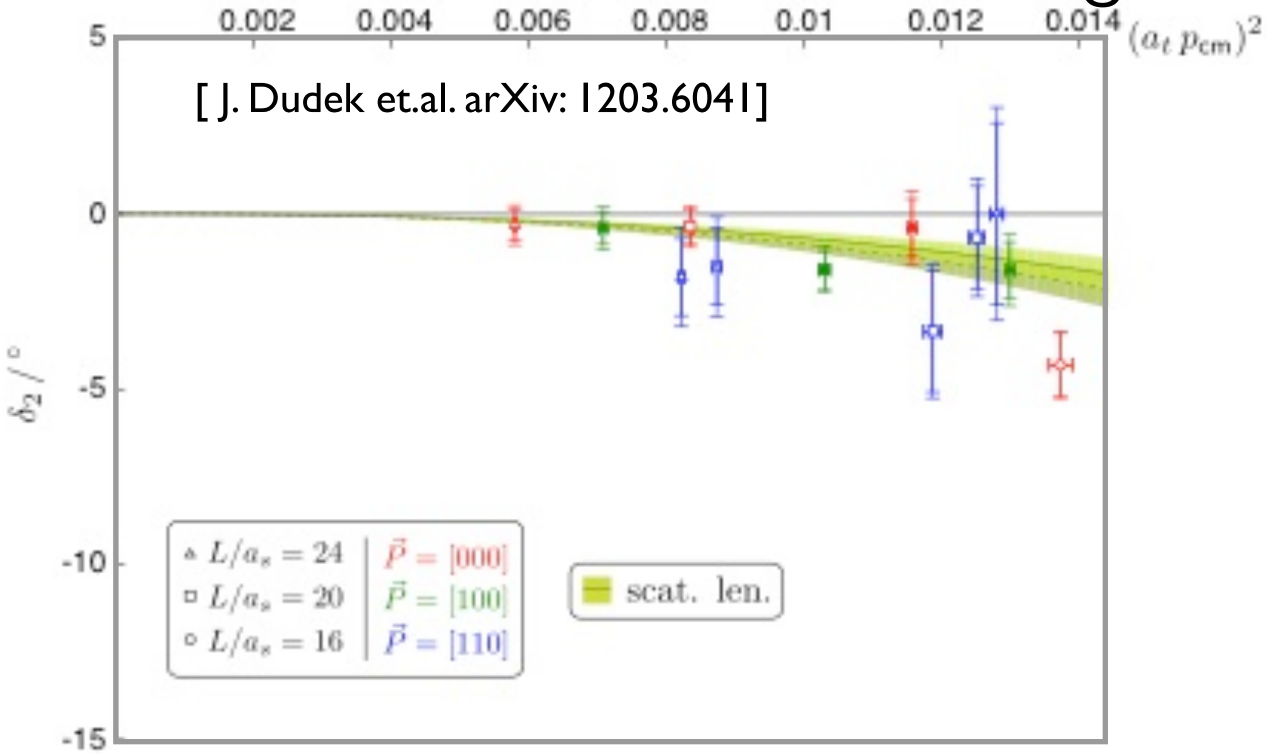
Scattering length

$$m_\pi a = 0.0417(07)(02)(16) \quad , \quad m_\pi r = 72.0(5.3)(5.3)(2.7)$$
$$P/r^3 = -2.022(58)(12)(76) \times 10^{-4} \; ,$$

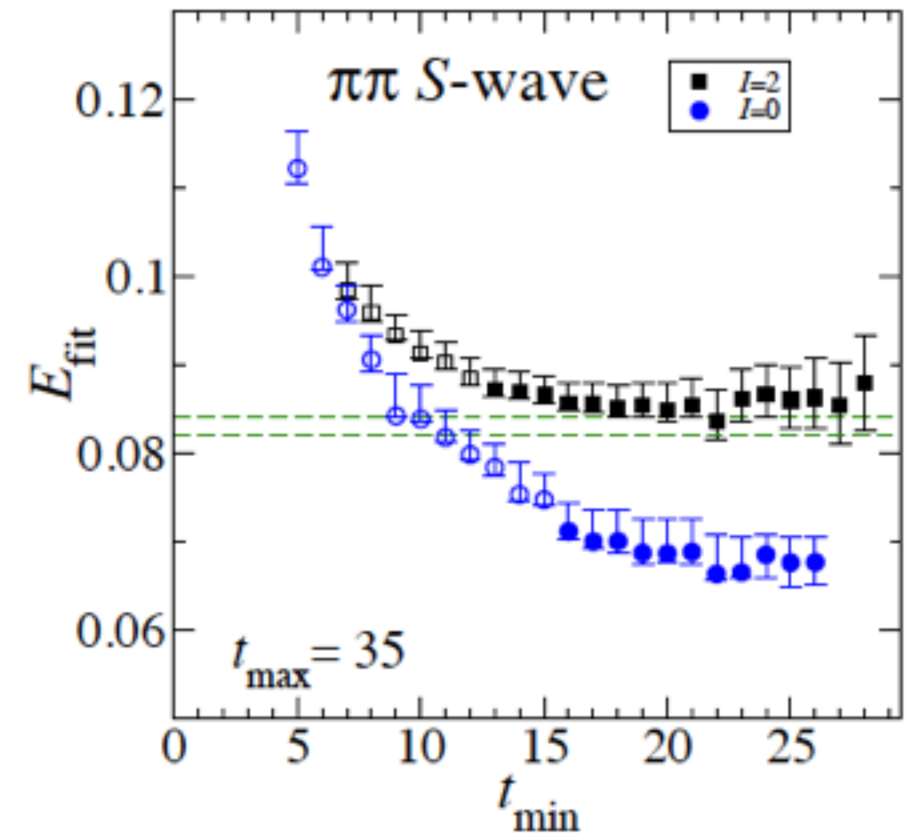
$$\bar{l}_3 - 4\bar{l}_4 = -29(27) \quad , \quad \bar{l}_1 - 6\bar{l}_4 = -32(25)$$
$$2\bar{l}_1 - 3\bar{l}_3 = 28(29) \quad , \quad \bar{l}_1 + 4\bar{l}_2 = 15.8(6.7) \; ,$$

NPLQCD, Phys.Rev. D85 (2012) 034505

π-π I=2 D-wave scattering

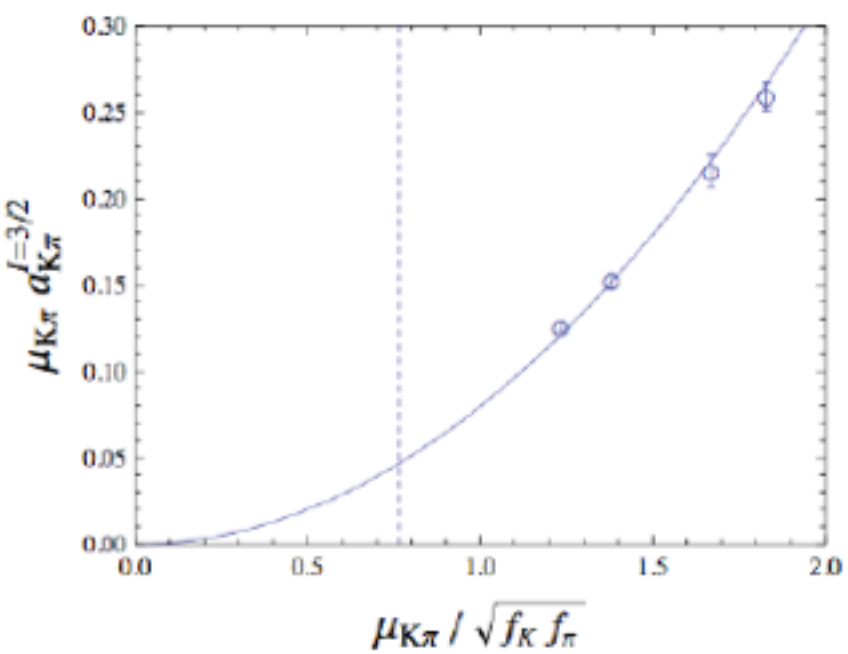


π-π I=0 S-wave scattering

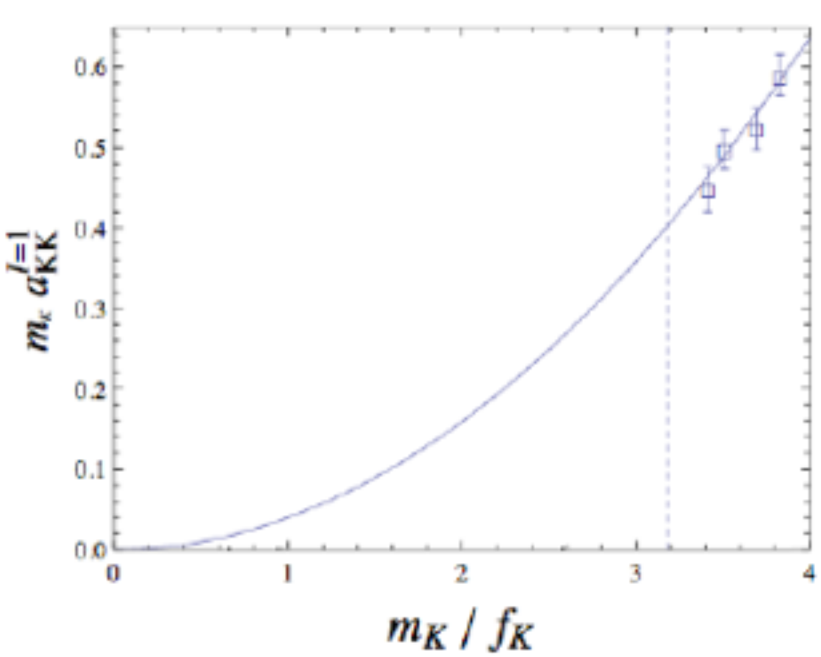


Morningstar et al., arXiv:1104.3870

NPLQCD hep/lat 0709.1169



$K^+ \pi^+$ *system*

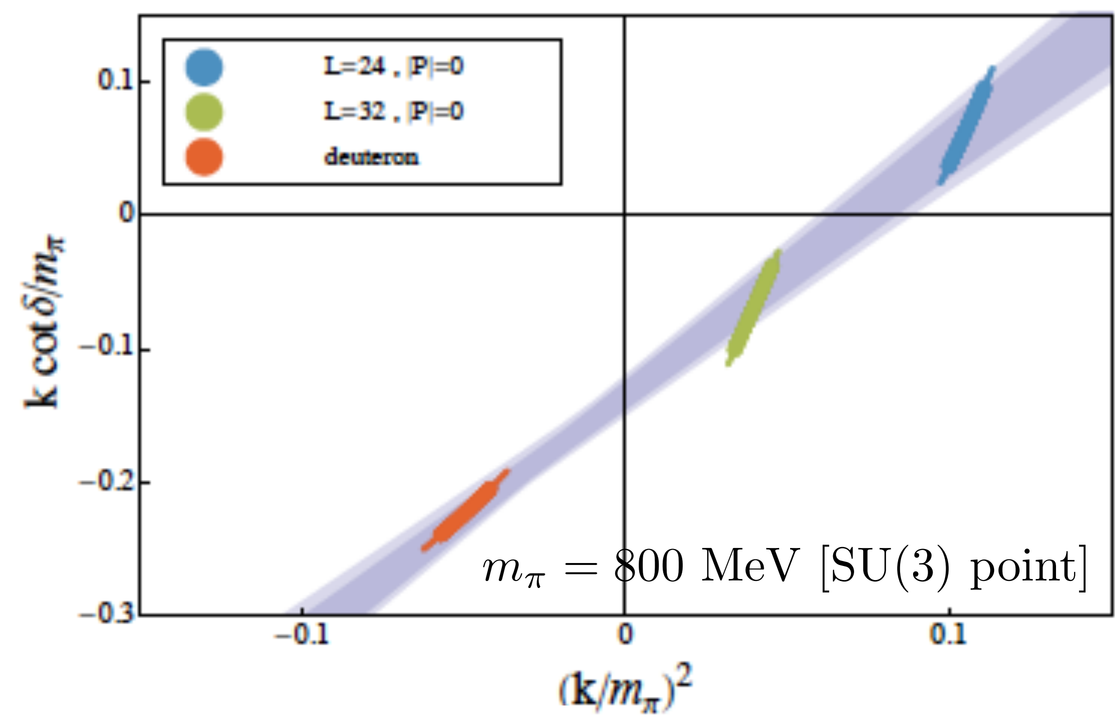


$K^+ K^+$ *system*

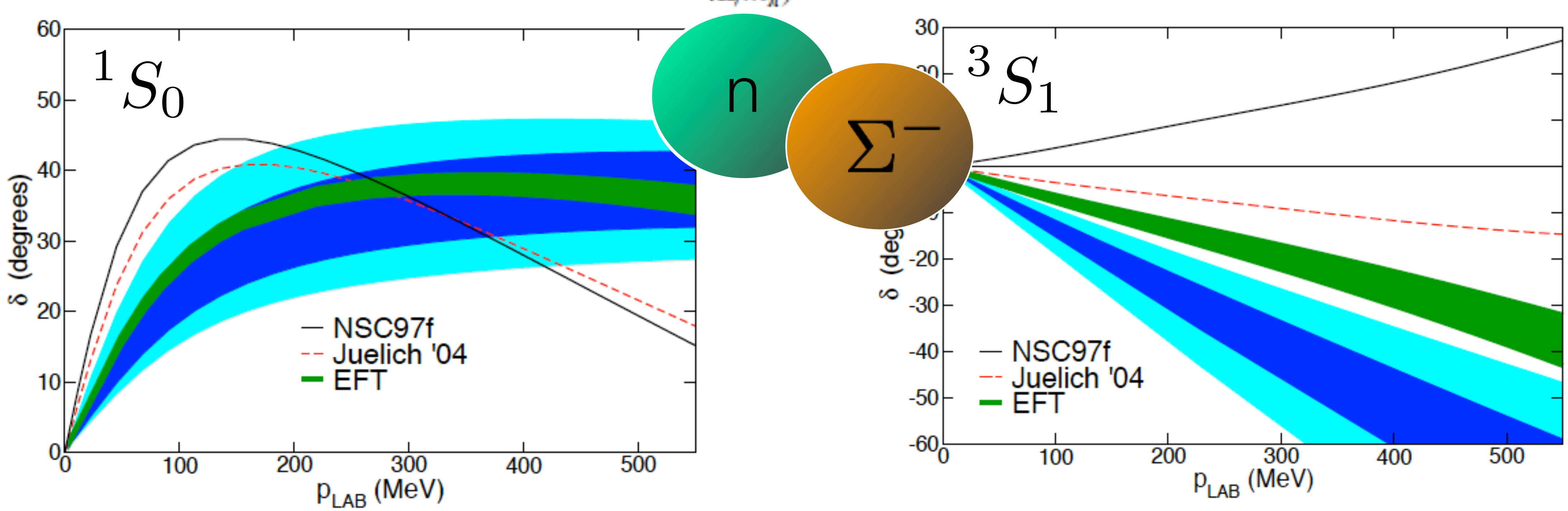
For meson-meson physics, LQCD is becoming an indispensable tool

Progress has been made in the two-baryon sector as well, albeit at unphysical pion mass

Deuteron Phase Shift



NPLQCD, Phys.Rev. C88 (2013) 2, 024003

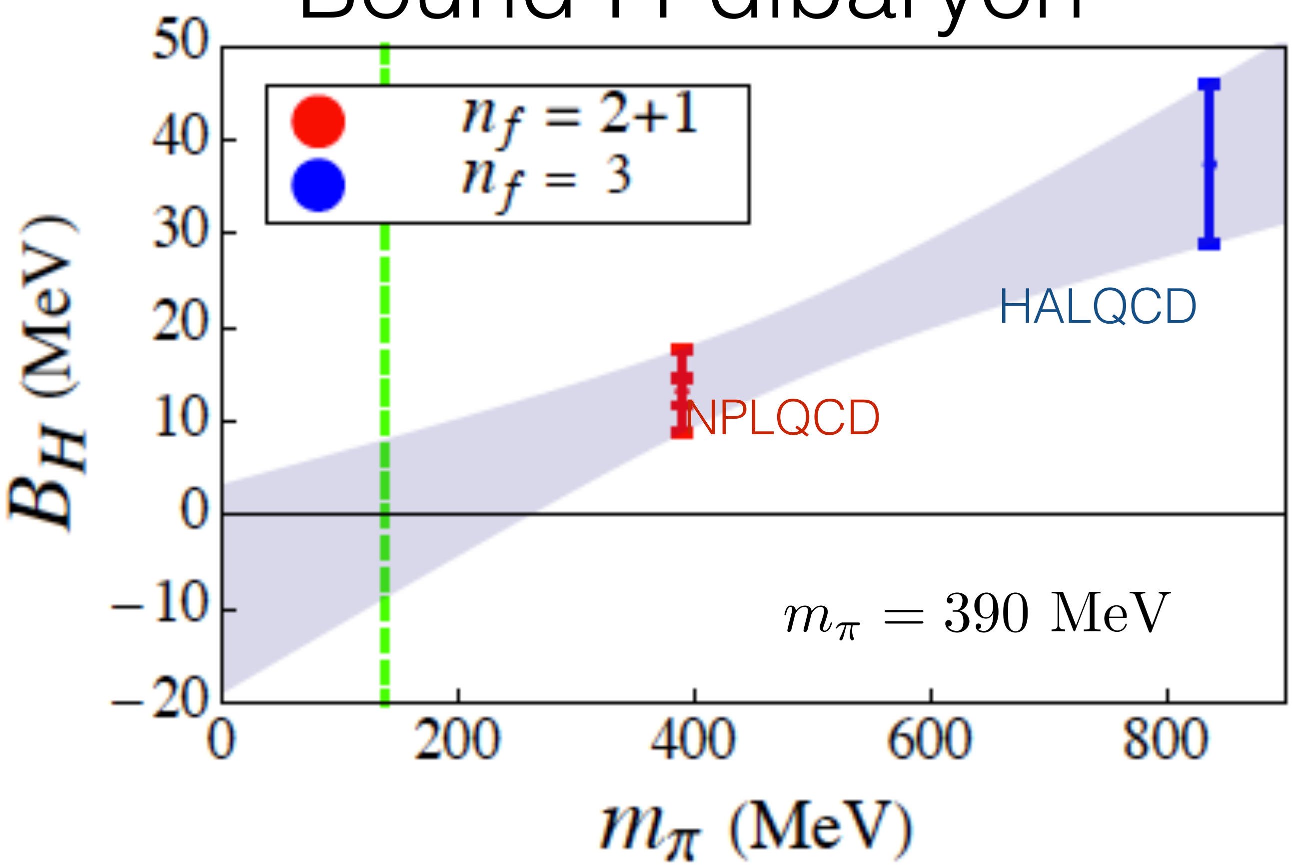


NPLQCD, Phys.Rev.Lett. 109 (2012) 172001

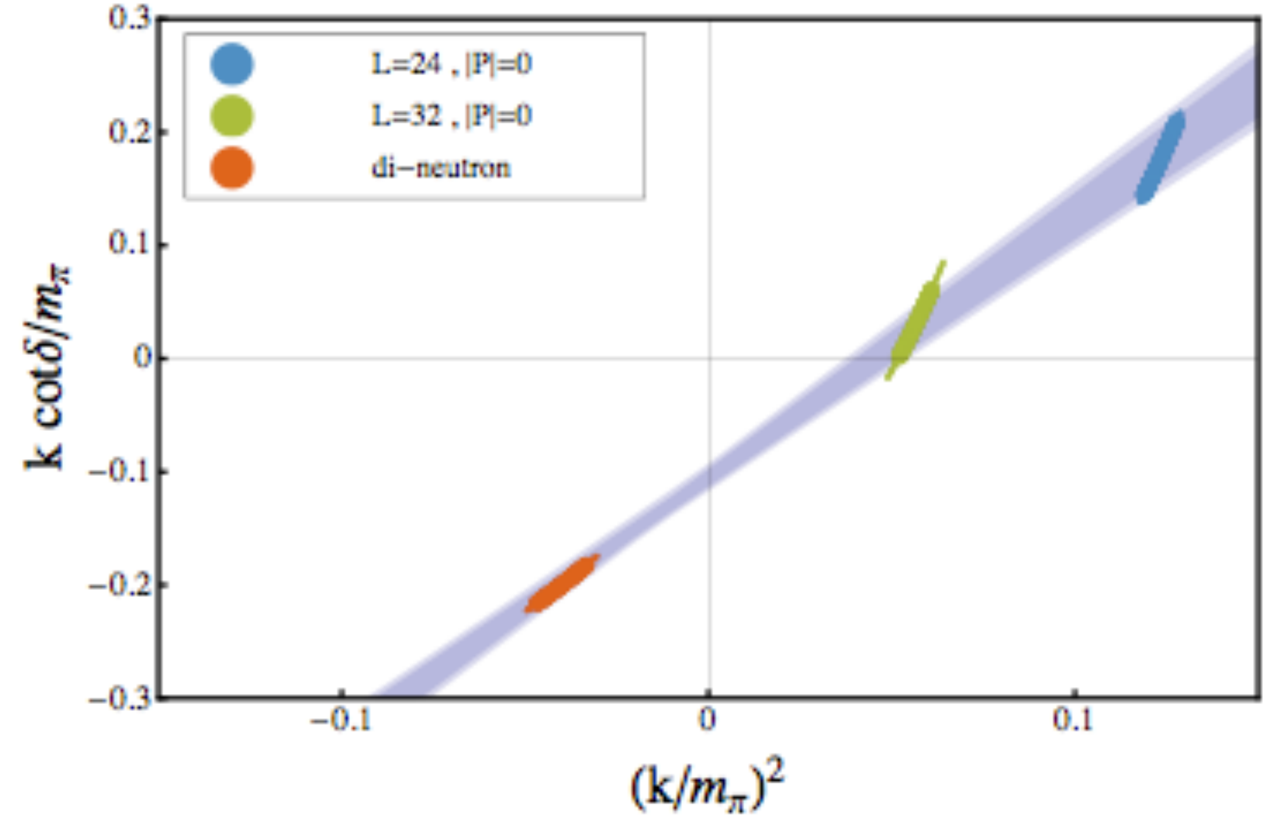
see e.g., talk by **Y. Ikeda**

These two-baryon studies have given us deeper insight into QCD away from the physical point

Bound H-dibaryon



Di-Neutron Phase Shift



NPLQCD, Phys.Rev. C88 (2013) 2, 024003

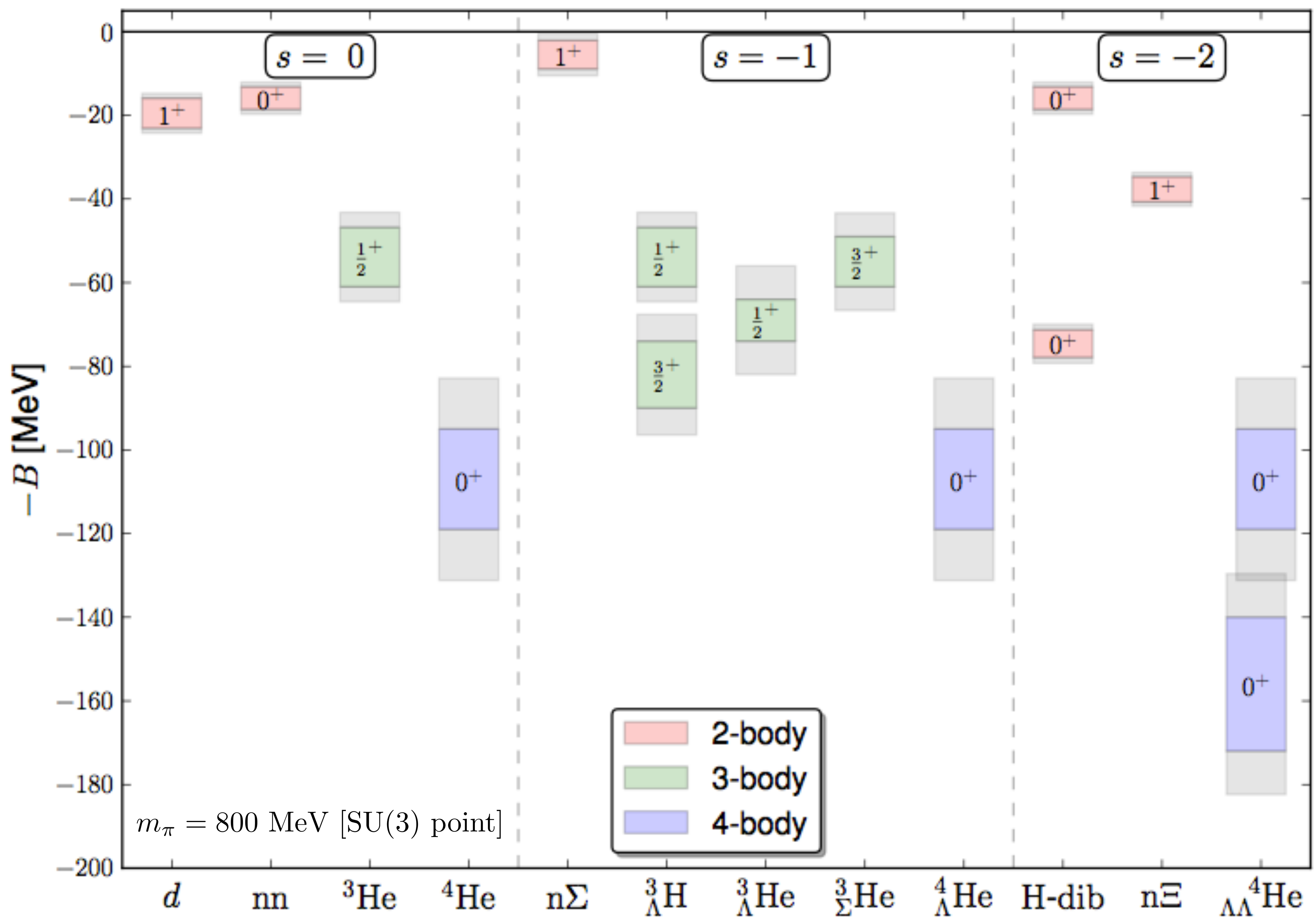
Also bound: $\Xi^- \Xi^-$

$N\Omega^-$

NPLQCD, Mod.Phys.Lett. A26 (2011) 2587-2595
NPLQCD, Phys.Rev.Lett. 106 (2011) 162001
HALQCD, Phys. Rev. Lett. 106, 162002 (2011), 1012.5928
HALQCD, arXiv:1403.7284

see e.g., talk by **K. Sasaki**

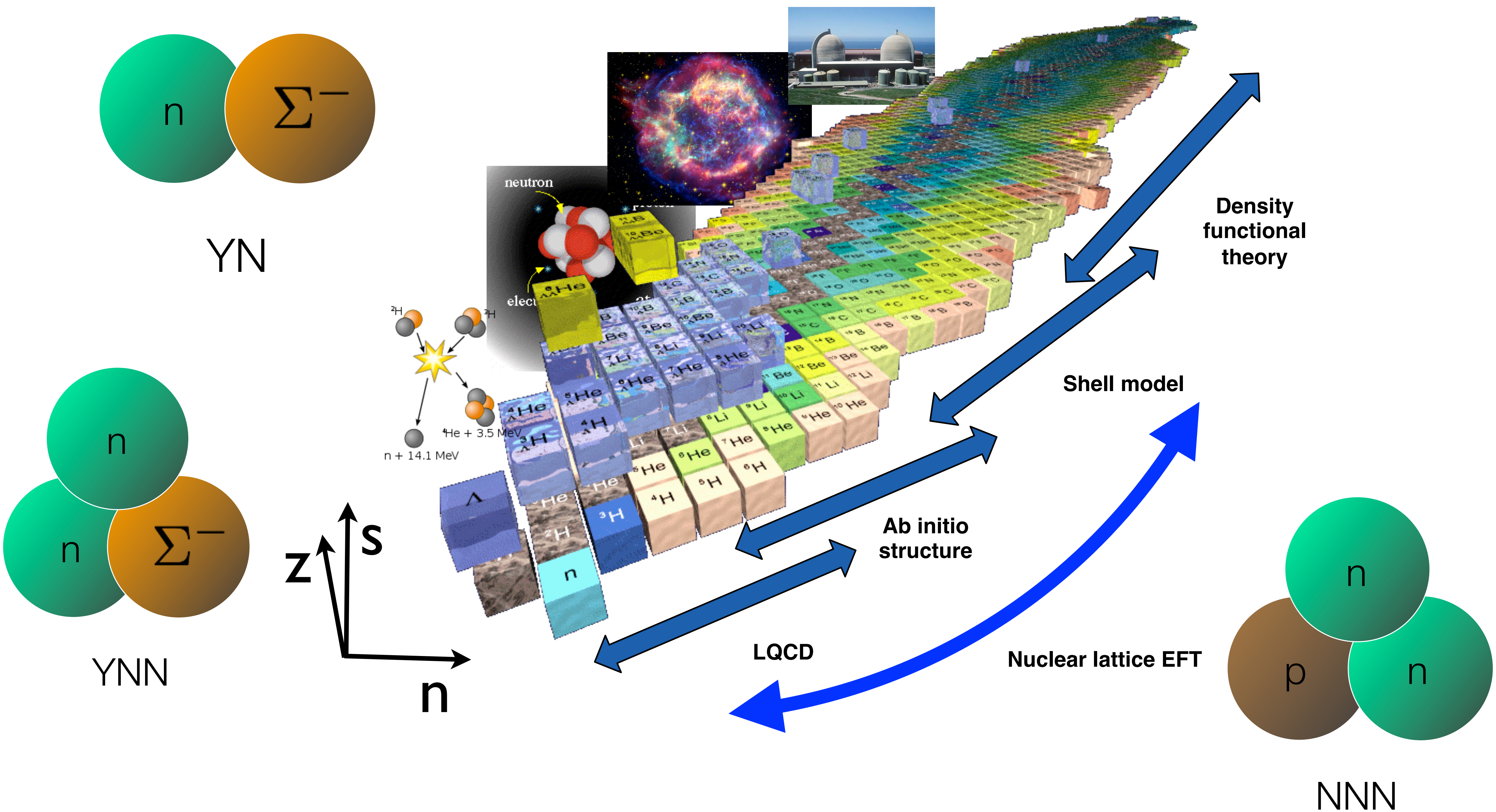
Indeed, the spectrum of A ≥ 2 hadrons is very rich away from the physical point



NPLQCD, Phys.Rev. D87 (2013) 3, 034506

“What do we learn from studying few-body systems in LQCD?”

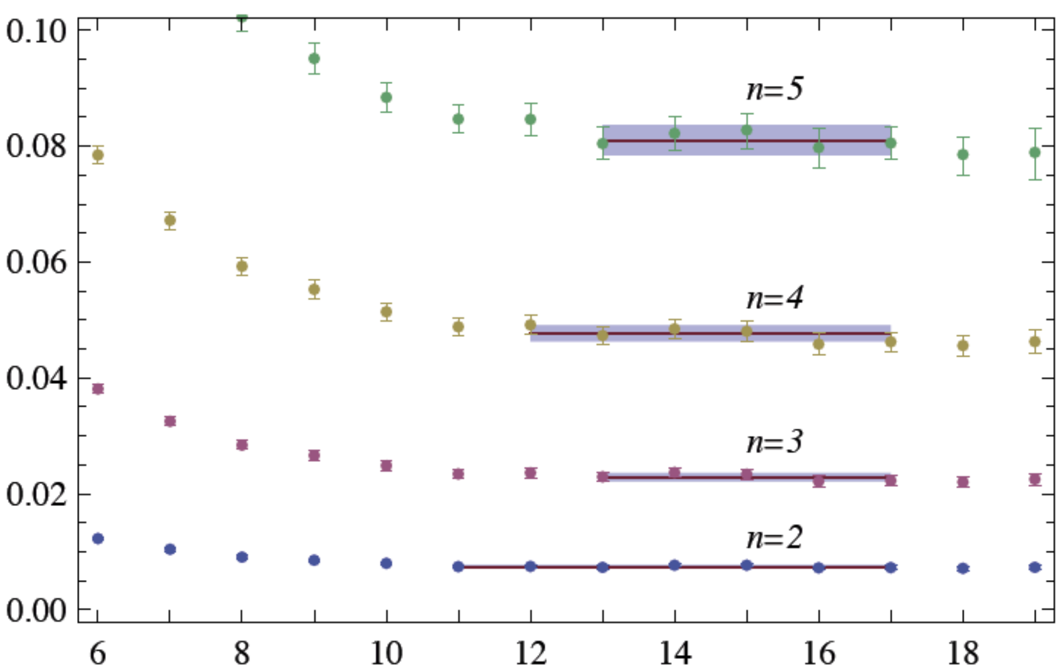
Compared to two-nucleon interactions, we know next to nothing about exotic- and few-hadron interactions

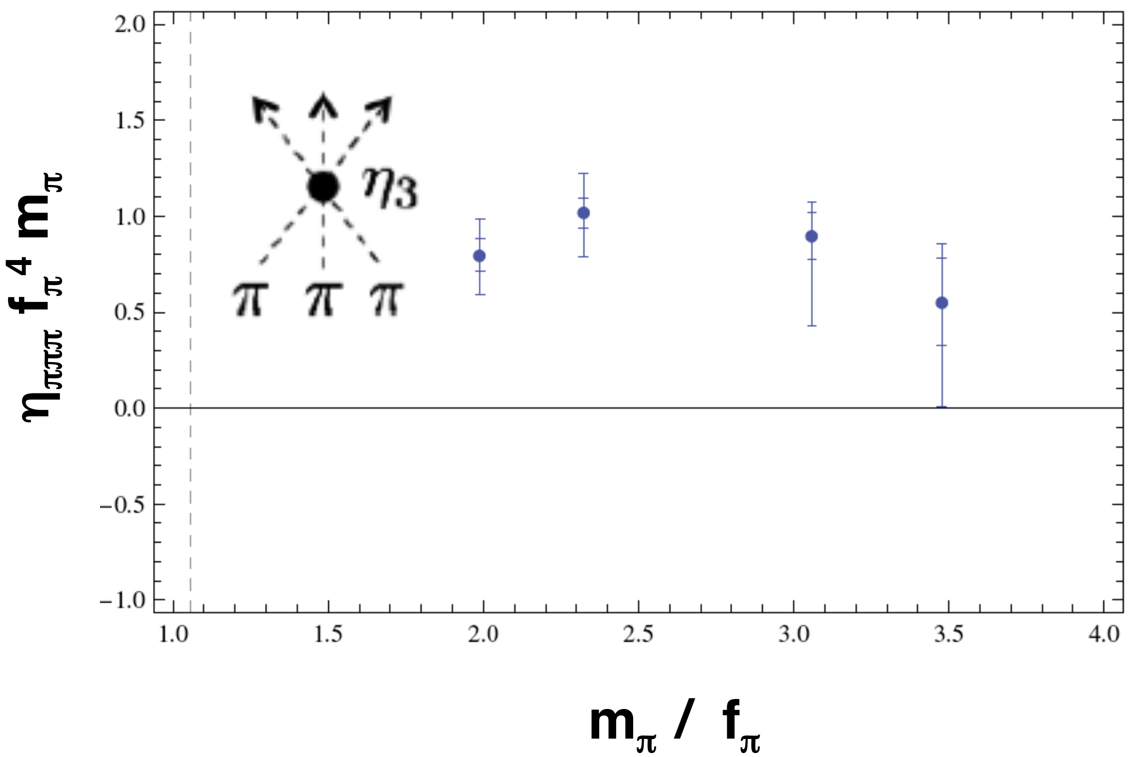


see e.g., talks by **S. Petschauer** and **E. Epelbaum**

Because hadrons are not fundamental particles, we have 3-body interactions

The 3-pion interaction

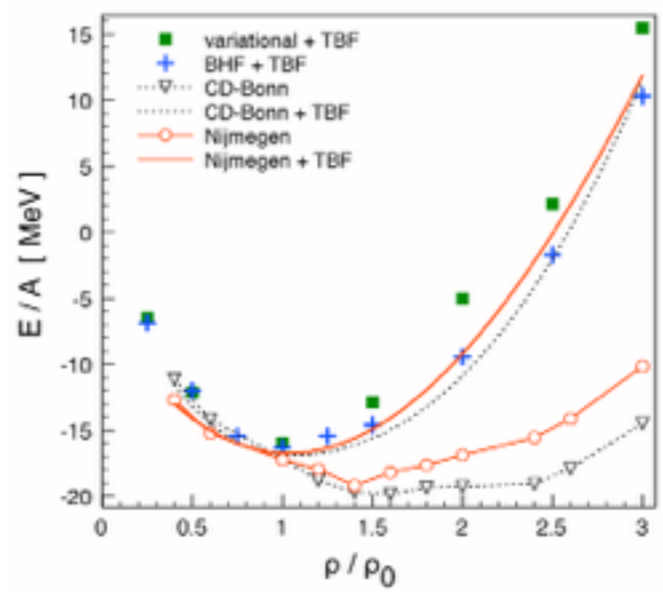




NPLQCD
Phys.Rev.Lett. 100 (2008) 082004
Phys.Rev. D78 (2008) 014507

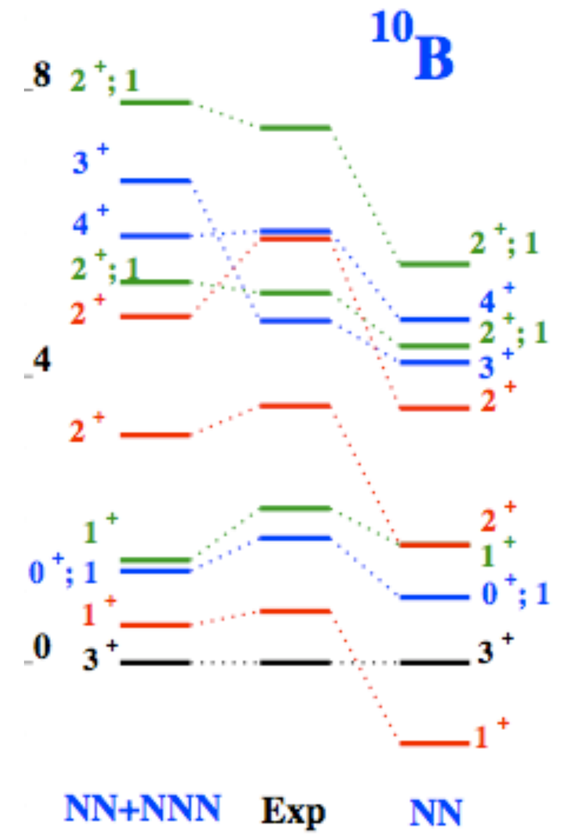
Three-baryon interactions play an important role in nuclear physics

Cannot get nuclear saturation density correct w/o it



V. Soma et al., arXiv:0808.2929 [nucl-th]

Spectrum of light-nuclei depends on it

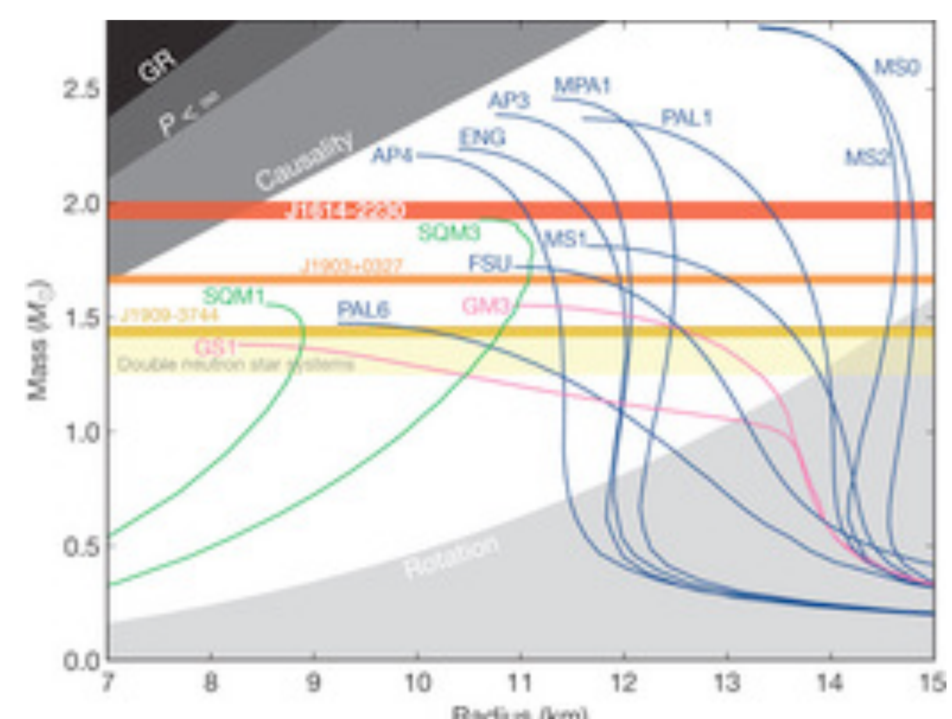


Navratil et al., Phys.Rev.Lett.99:042501,2007

NNY and NYY interactions?

- Observation of a 1.98 solar mass neutron star forces us to look again at the role of strangeness in dense nuclear matter

Nature 467 (2010) 1081-1083



Current and future accelerators promise more data on hyper-nuclei

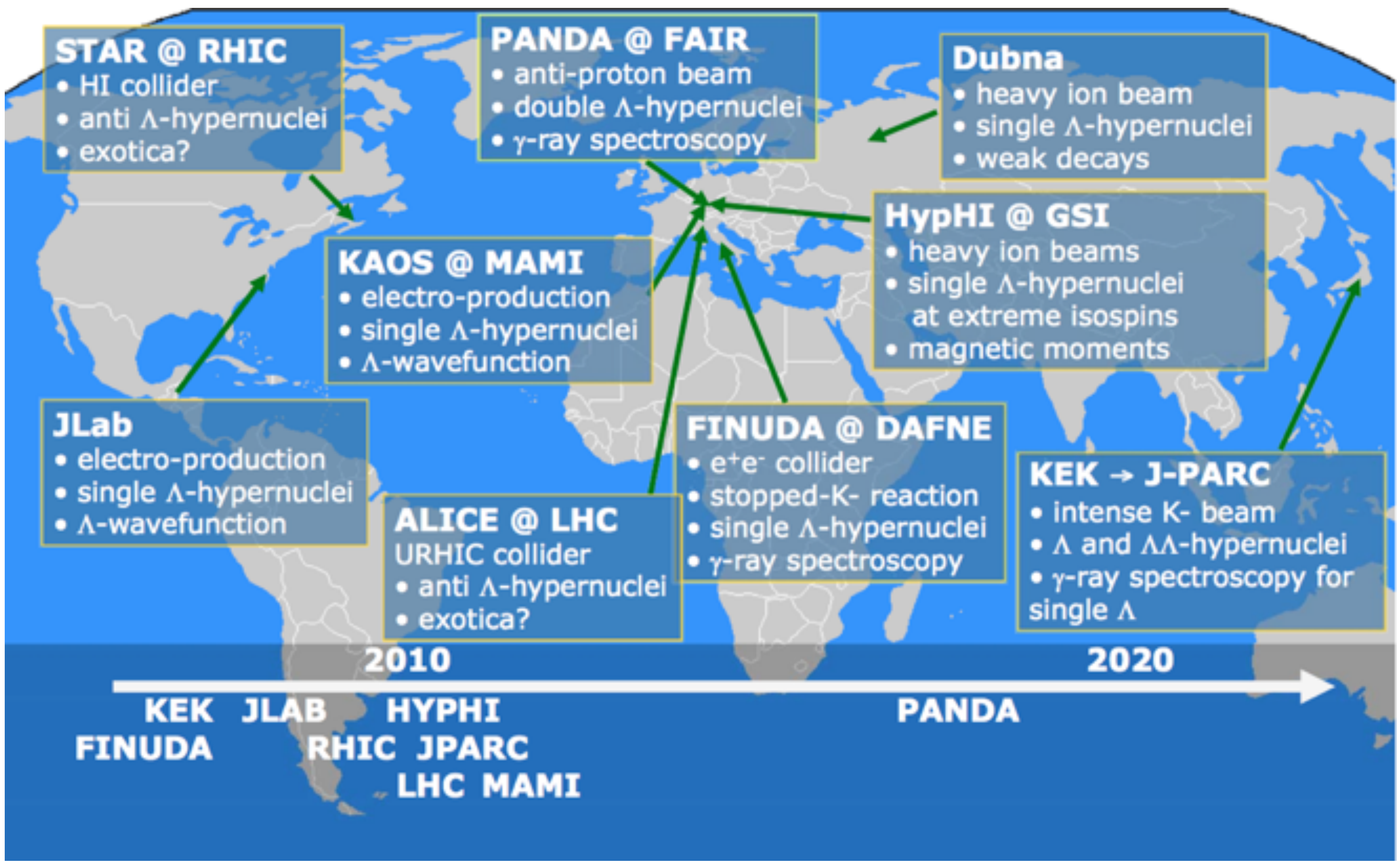
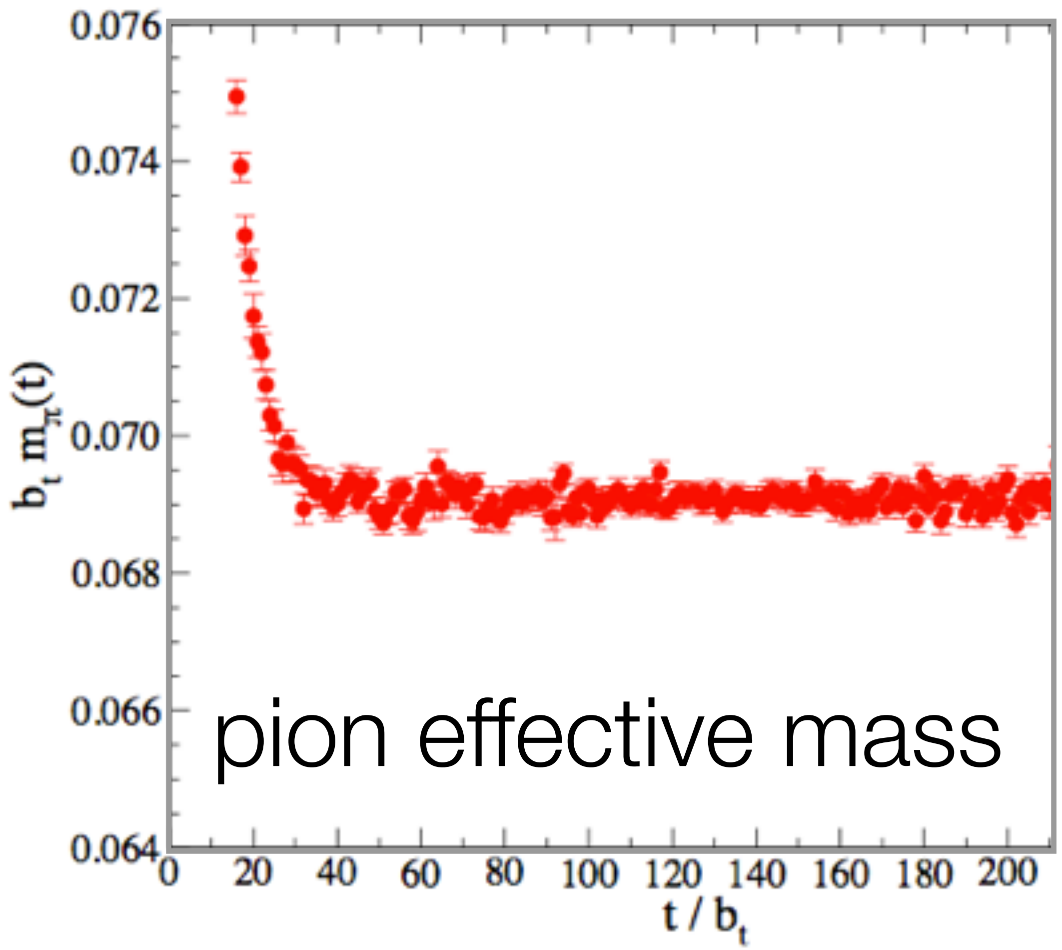


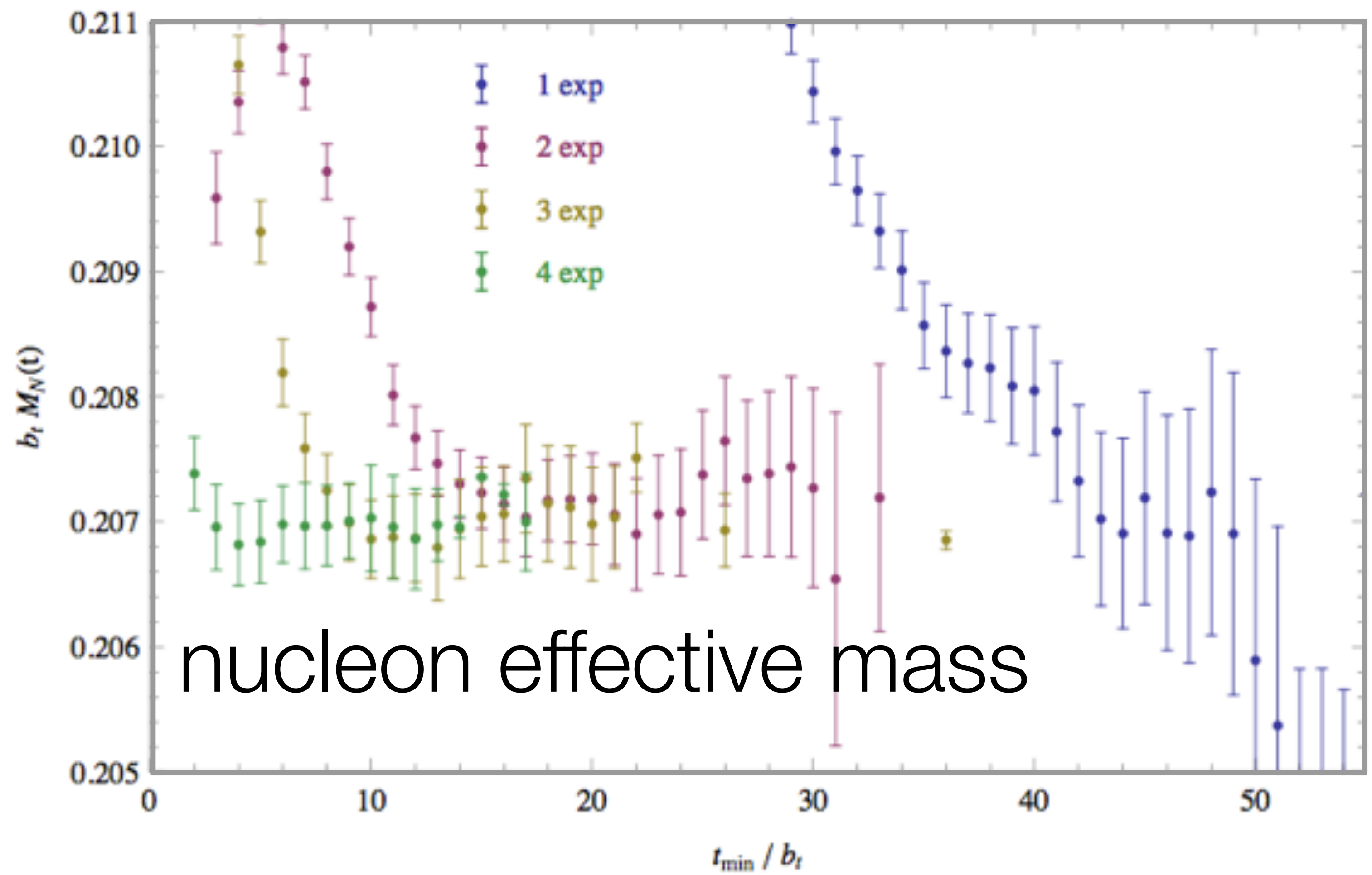
Figure courtesy of M. Savage

“What makes few-body systems so challenging for Lattice QCD?”

The sign problem

$$\left\langle O^{\{A\}}(t)\right\rangle = \frac{\int d[U]O^{\{A\}}(t)\det(M[U])e^{-S_g[U]}}{\int d[U]\det(M[U])e^{-S_g[U]}} = \sum_i C_i e^{-E_i^{\{A\}}t}$$





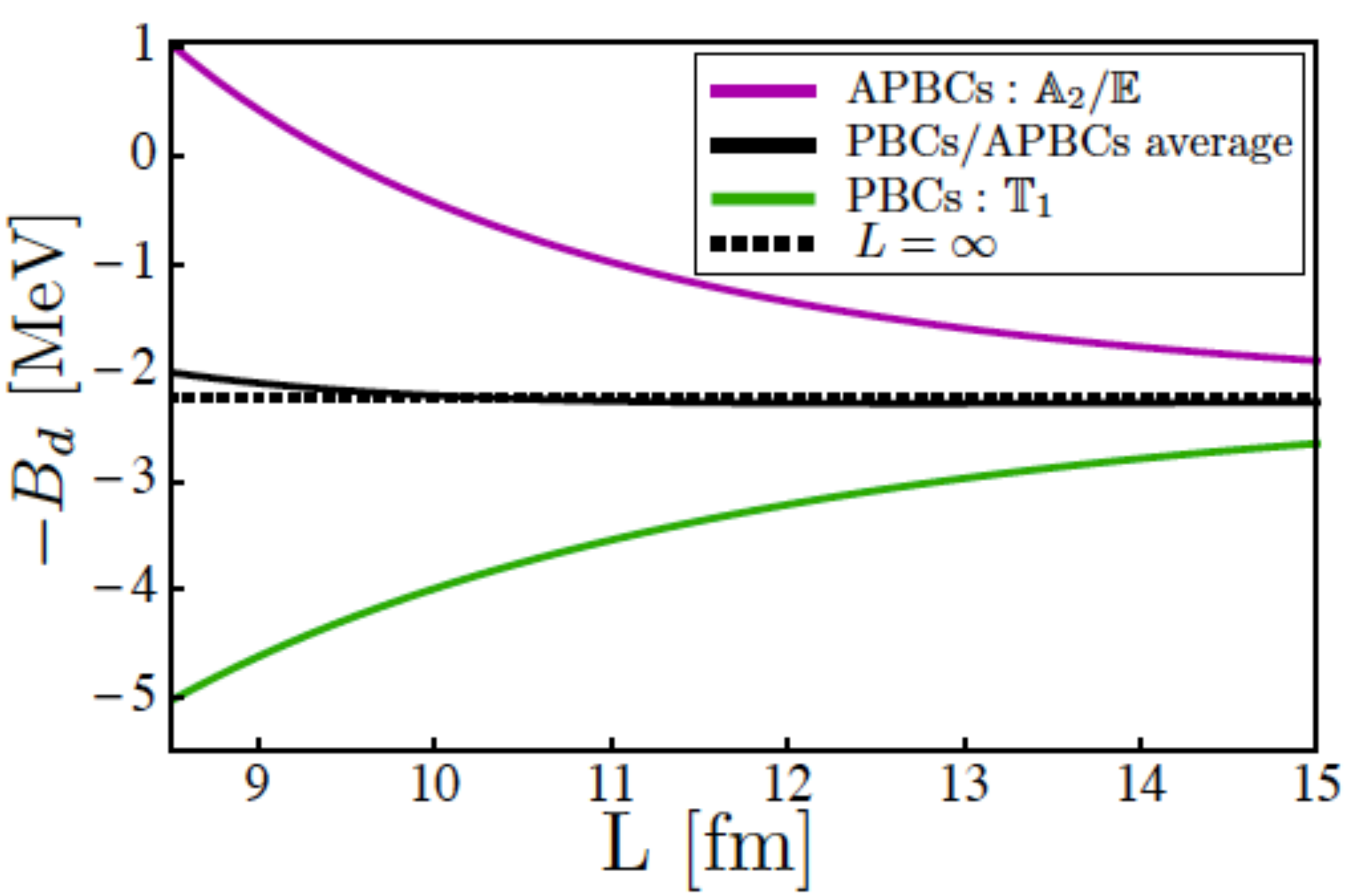
Noise-to-Signal Ratio

$$\frac{\sigma}{\bar{x}} \sim \frac{1}{\sqrt{\mathrm{N}}}\, e^{A\left(M_N - \frac{3}{2}m_\pi\right)t}$$

NPLQCD, arXiv:0903.2990

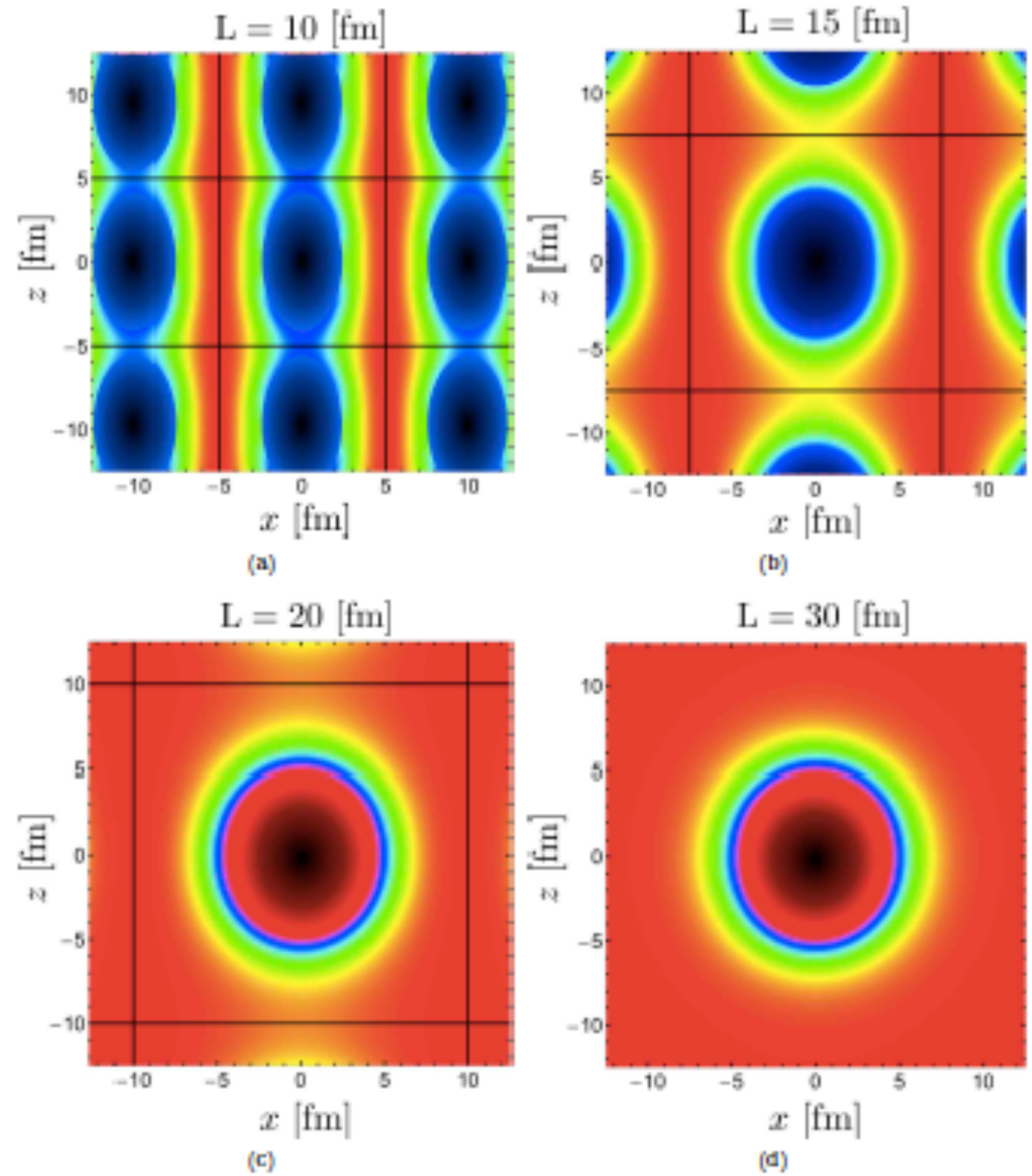
Volumes of configurations need to be larger

Deuteron Binding Energy as a function of Lattice Size



R. Briceno, Z. Davoudi, (TL), M. Savage, arXiv:1311.7686

Boosted deuteron density in the z-direction



Phys.Rev. D88 (2013) 114507

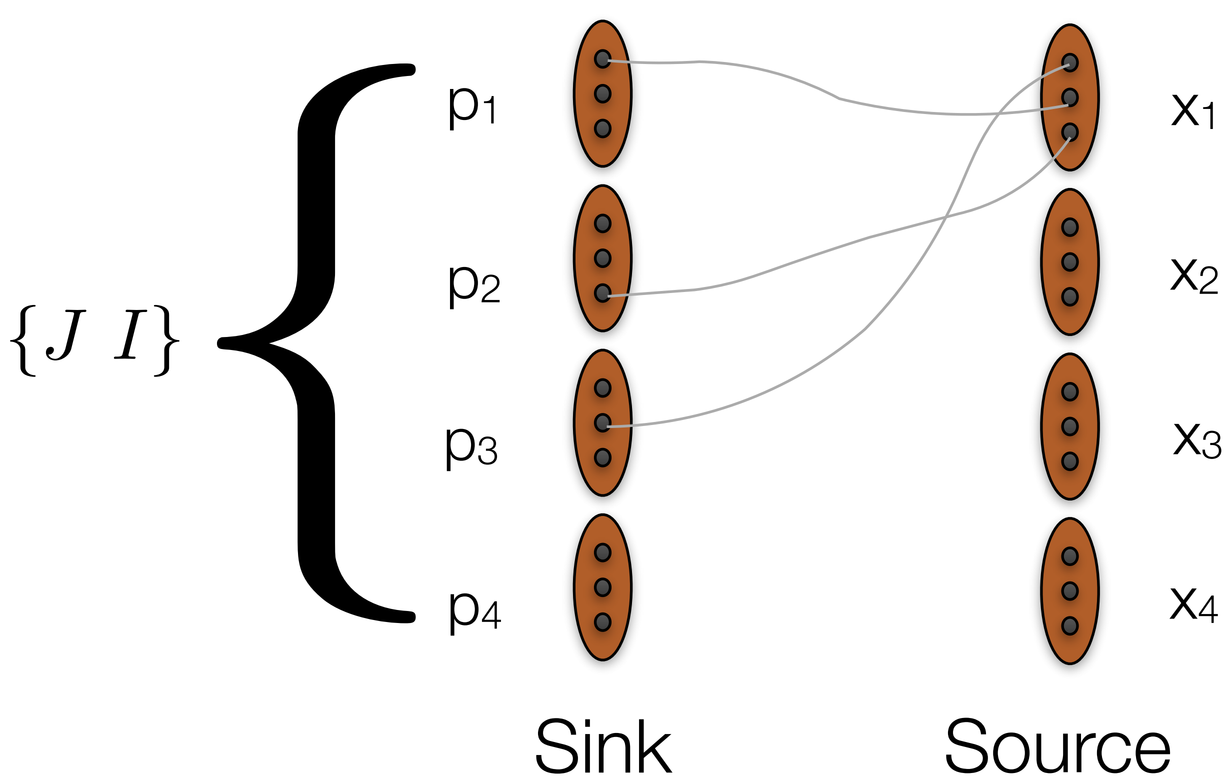
Why so hard?

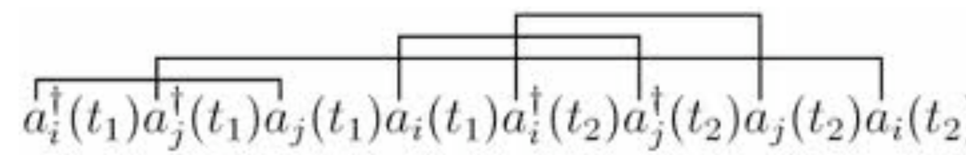
Contractions! Lots of them!

$$N_{\text{cont.}} = u!d!s! \quad (\text{Naive})$$

Proton : $N^{\text{cont}} = 2$

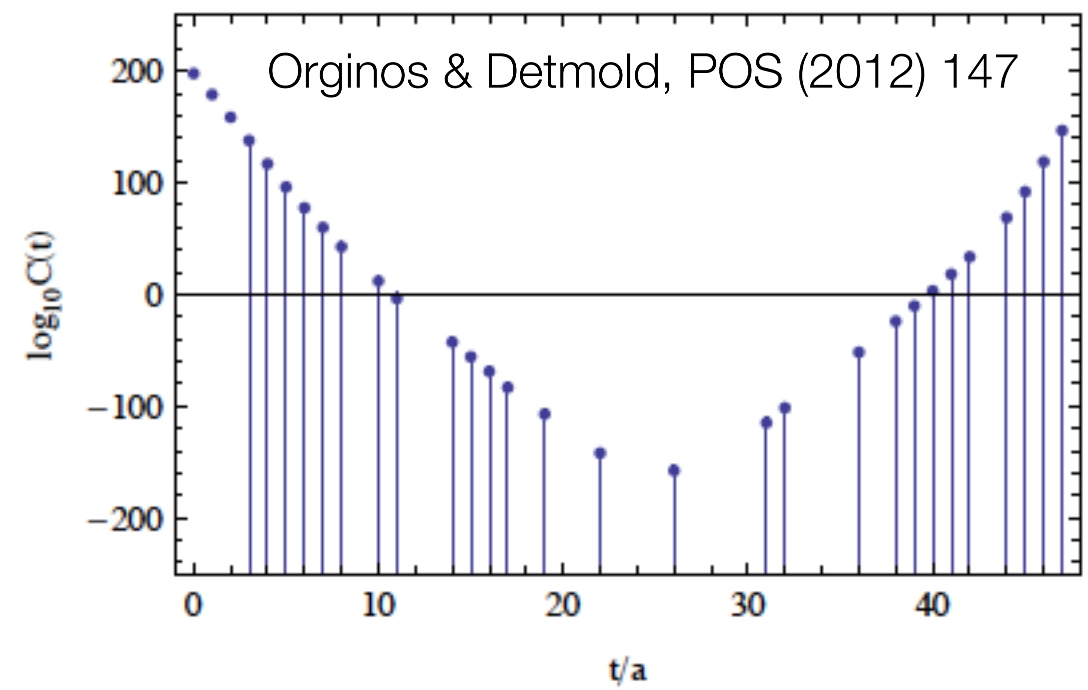
^{235}U : $N^{\text{cont}} = 10^{1494}$





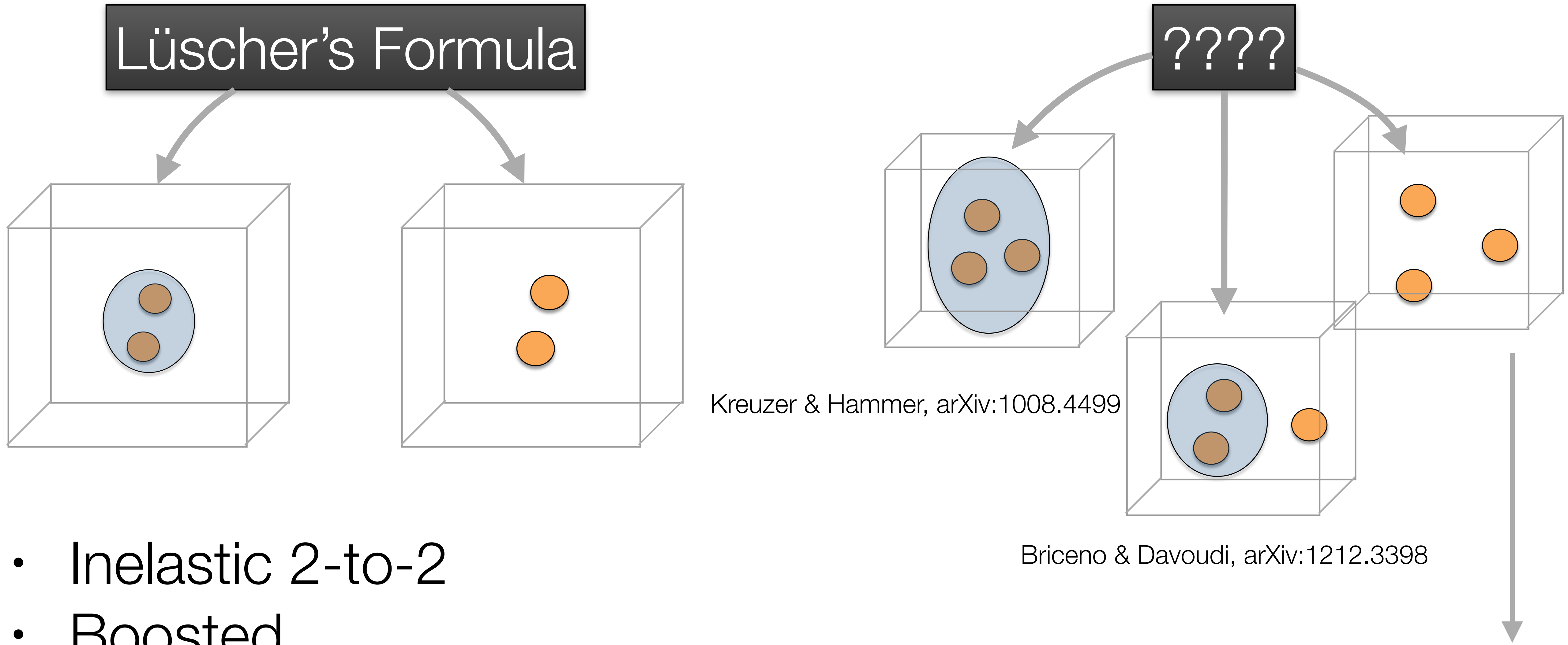
Symmetries and clever recursive relations greatly alleviate this problem

Silicon correlation function



Detmold & Orginos, arXiv:1207.1452
Doi & Endres, arXiv:1205.0585

We are quite restricted in the types of systems we look at. . .



Hansen & Sharpe, arXiv:1311.4848
Hansen & Sharpe, arXiv:1408.5933

- Inelastic 2-to-2
- Boosted
- General partial waves

e.g. see J. Bulava's talk

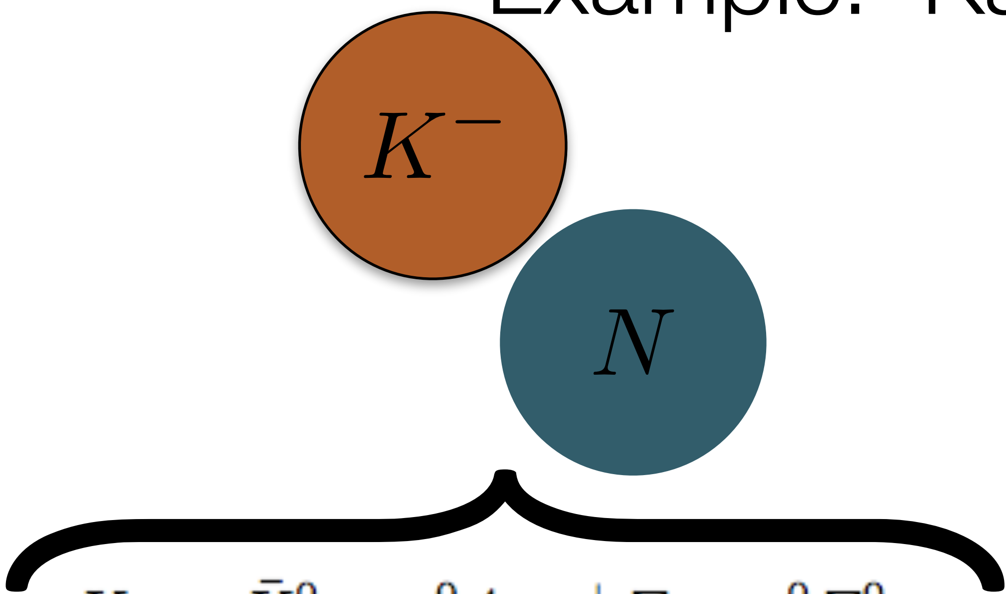
Other notable works: Polejaeva & Rusetsky, arXiv:1203.1241
Bour et al., arXiv:1107.1272

“What can we expect in the near future?”

In the future. . .

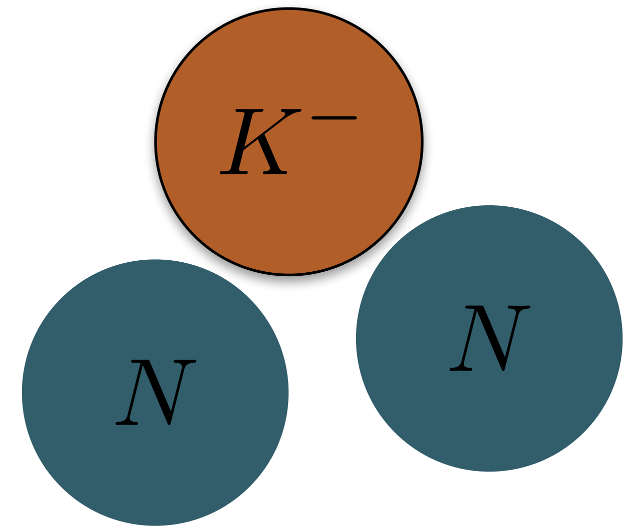
Stochastic estimates of disconnected diagrams open up many more possibilities

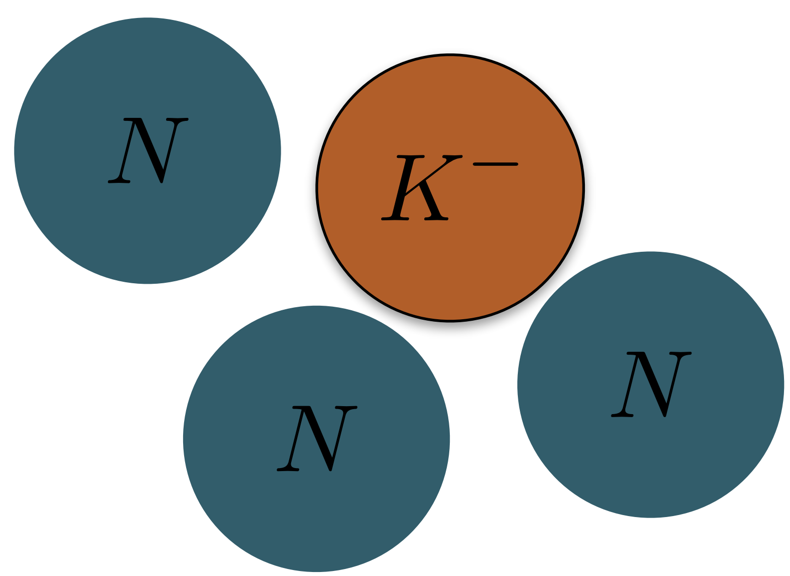
Example: Kaon in-medium interaction



$K^-p,\ \bar{K}^0 n,\ \pi^0\Lambda,\ \pi^+\Sigma^-,\ \pi^0\Sigma^0,$

$\pi^-\Sigma^+,\ \eta\Lambda,\ \eta\Sigma^0,\ K^+\Xi^-$ and $K^0\Xi^0$

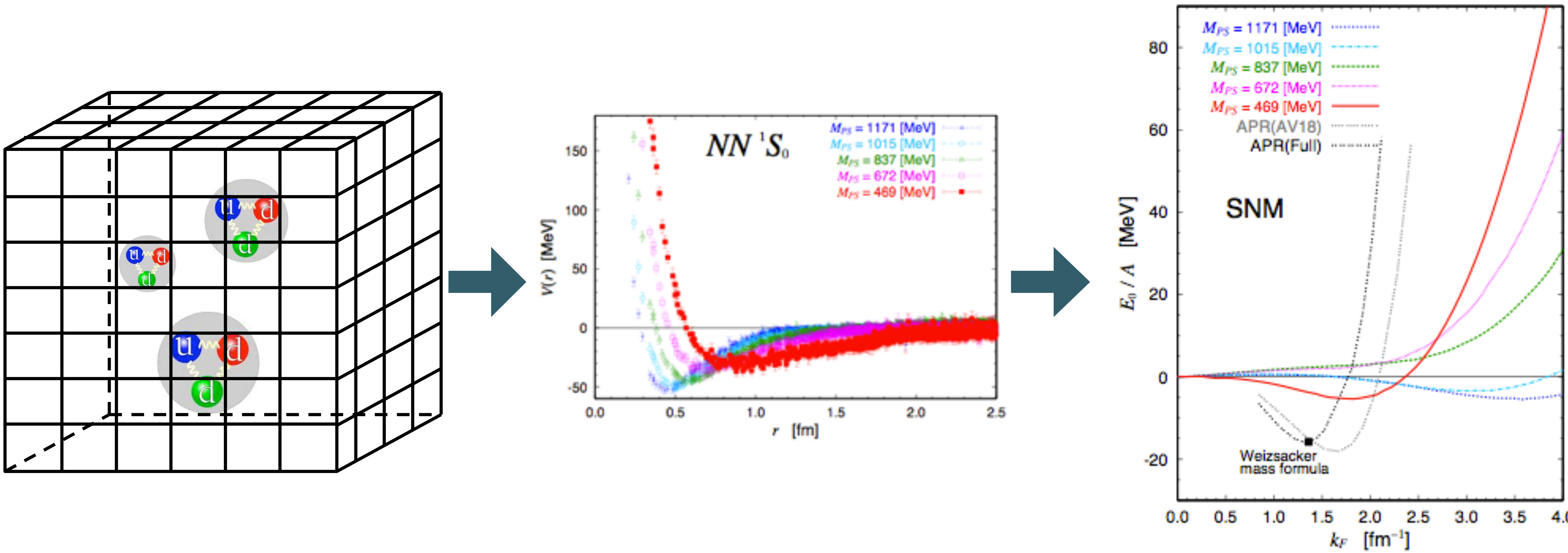




Kaon condensation in nuclear matter?

see e.g., talk by **S. Kolevatov**

Taking LQCD ‘outside-of-the-box’ . . .



The future will see LQCD results of few-body systems applied to broader areas of hadron physics

HALQCD, arXiv:1311.6223

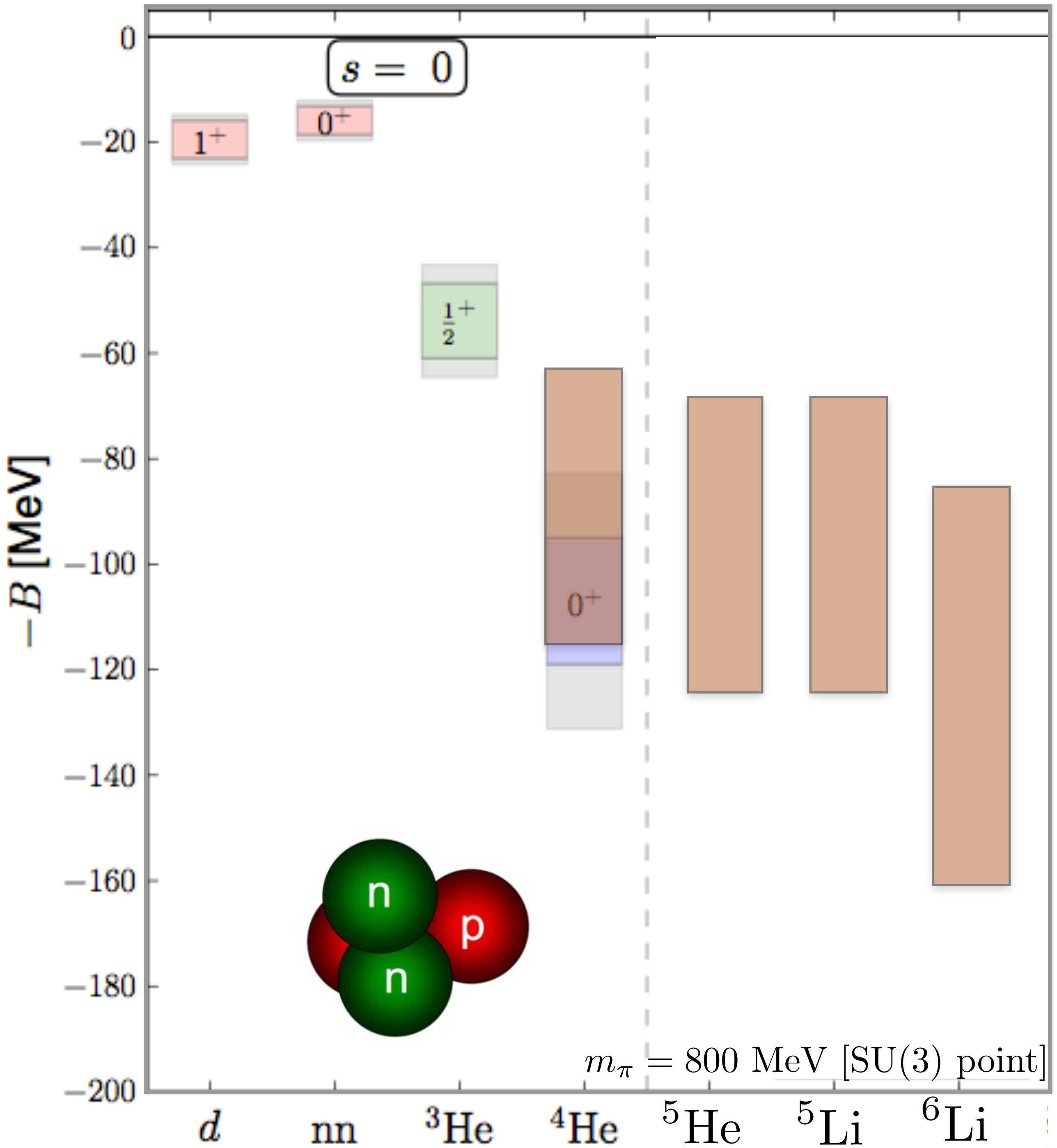
see e.g., talk by **Y. Ikeda**

. . . and into the real world. . .

Input from LQCD

m_π	140	510	805	805
Nucleus	[Nature]	[5]	[6]	[This work]
n	939.6	1320.0	1634.0	1634.0
p	938.3	1320.0	1634.0	1634.0
nn	-	7.4 ± 1.4	15.9 ± 3.8	15.9 ± 3.8 *
D	2.224	11.5 ± 1.3	19.5 ± 4.8	19.5 ± 4.8 *
^{3}n	-			-
^{3}H	8.482	20.3 ± 4.5	53.9 ± 10.7	53.9 ± 10.7 *
^{3}He	7.718	20.3 ± 4.5	53.9 ± 10.7	53.9 ± 10.7
^{4}He	28.30	43.0 ± 14.4	107.0 ± 24.2	89 ± 36
^{5}He	27.50			98 ± 39
^{5}Li	26.61			98 ± 39
^{6}Li	32.00			122 ± 50

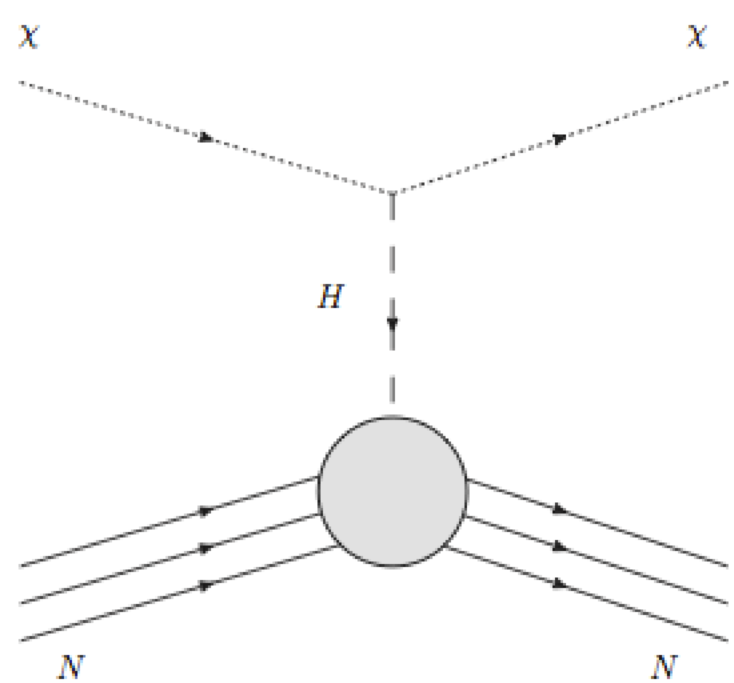
Prediction



Barnia et al., arXiv:1311.4966

. . .and going 'BSM' (Beyond-the-standard-model)

- The neutron and proton EDM from the strong θ-term
- Calculate hadronic component related to quark- and chromo-electric EDM matrix elements
  - i.e. $\mathcal{L} = -\frac{i}{2}\bar{q}\left(d_0 + d_3\tau_3\right)\sigma_{\mu\nu}\gamma_5 q\, F_{\mu\nu} - \frac{i}{2}\bar{q}\left(\tilde{d}_0 + \tilde{d}_3\tau_3\right)\sigma_{\mu\nu}\gamma_5\lambda^a q\, G^a_{\mu\nu}$
- Determine EDM of light-nuclei
- Hadronic matrix elements that couple to Dark Matter
  - e.g. $\langle N | q_f \bar{q}_f | N \rangle$
- Key to all these calculations is the use of the Gradient Flow Algorithm
- See **A. Shindler**'s talk from Tuesday and **J. de Vries**' talk later today



see e.g., talks by **S. Gardner** and **T. Blum**

Conclusion

- Much progress in hadron-hadron scattering
- Spectrum of light nuclei and hyper-nuclei very rich away from physical point
- Issues related to contractions, disconnected diagrams, and extensions of Lüscher's formalism to few-body systems being addressed
- Questions related to three-hadron interactions and hyper-nuclei interactions are starting to be tackled
- Potential for broad impact of LQCD few-body investigations