Partial twisting for scalar mesons

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1. Motivation

▶ The study of the scalar mesons $a_0(980)$ and $f_0(980)$ contributes to a better inderstanding of the low-energy structure of QCD.

▶Twisted boundary conditions facilitate the extraction of the $a_0(980)$, $f_0(980)$ resonances on the lattice [2,3].

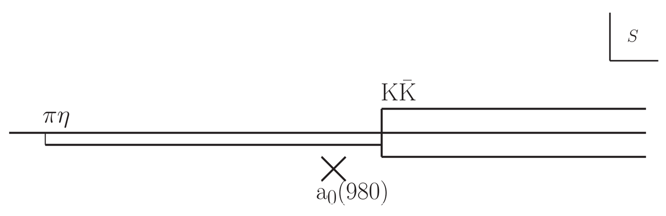
▶ Partially twisted boundary conditions, unlike full twisting, are relatively affordable.

2. Current status

▶ The Lüscher approach has been generalized to the case of partially twisted boundary conditions for systems, where no annihilation channels can occur [5,6].

3. Background

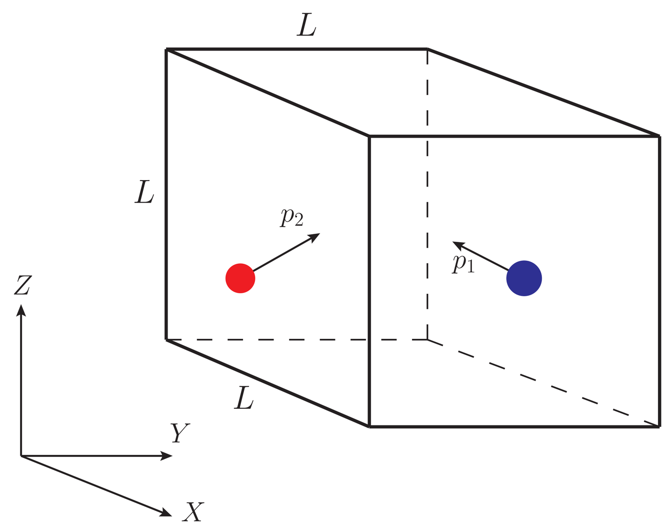
▶ Example: $a_0(980)$ - isospin I=1, S-wave scattering, coupled-channels $\pi\eta$ - $K\bar{K}$



▶ There is no model-independent conclusion about the nature of $a_0(980)$ and $f_0(980)$: $K\bar{K}$-molecules, $q\bar{q}$-states, tetraquark states,...

▶ Lattice QCD: ***ab initio*** non-perturbative calculations in QCD.

▶ Scalar mesons are resonances ⇒ scattering on a finite lattice ⇒ **Lüscher approach** to extract resonance mass and width: two particle scattering matrix in the *infinite volume* is related to the two-particle energy levels, in a *finite volume* :



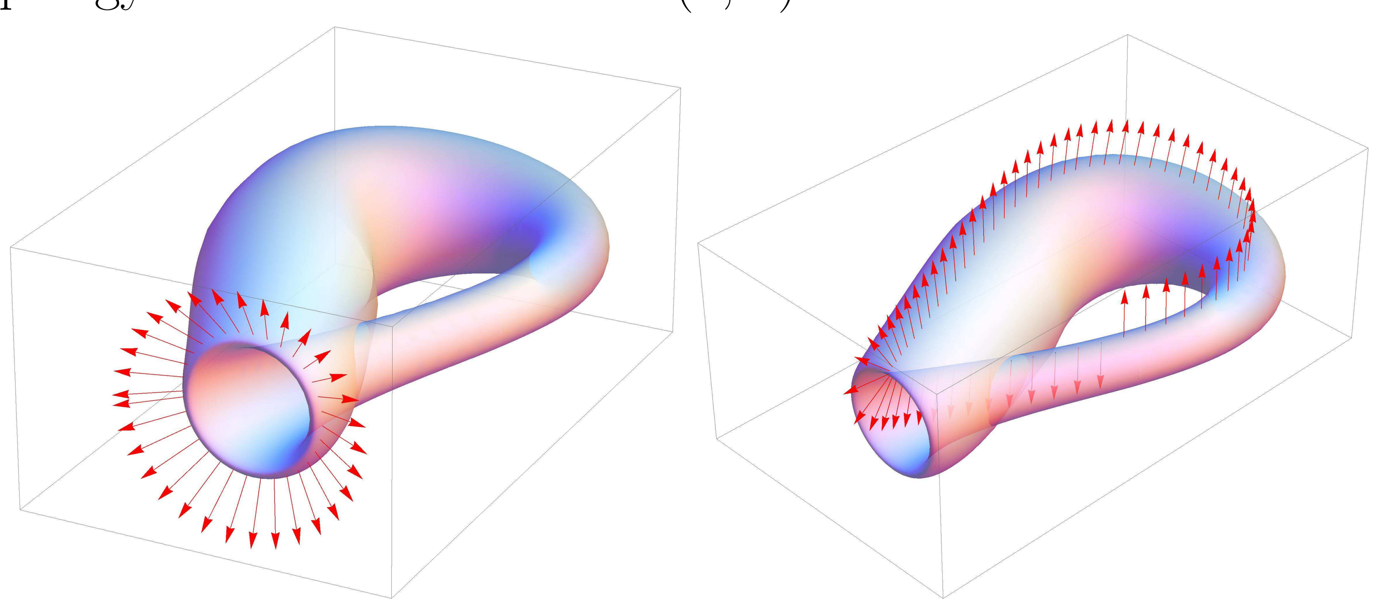
Lattice data ⇒ E(L) ⇒ scattering T-matrix.

▶ Studing the finite-volume spectrum E(L) with different type of **boundary conditions** may provide additional insight to the structure of the $a_0(980)$ and $f_0(980)$ [2,3].

▶ Twisted boundary conditions (B.C.) are imposed on quark fields $\Psi(x)$ in lattice simulations [4]:

$\Psi(\mathbf{x}+\hat{e}_i L) = e^{i\theta_i}\Psi(\mathbf{x})$ ⇒ lattice hadronic momentum $\mathbf{q}=\frac{2\pi}{L}(\mathbf{n}+\boldsymbol{\theta}/2\pi)$, $\mathbf{n}\in\mathbb{Z}^3$. Varying the twisting angle $\boldsymbol{\theta}$ substitutes varying the lattice size L ⇒ more data for one lattice size.

2D-topology of twisted B.C. for $\boldsymbol{\theta}=(0,\pi)$:

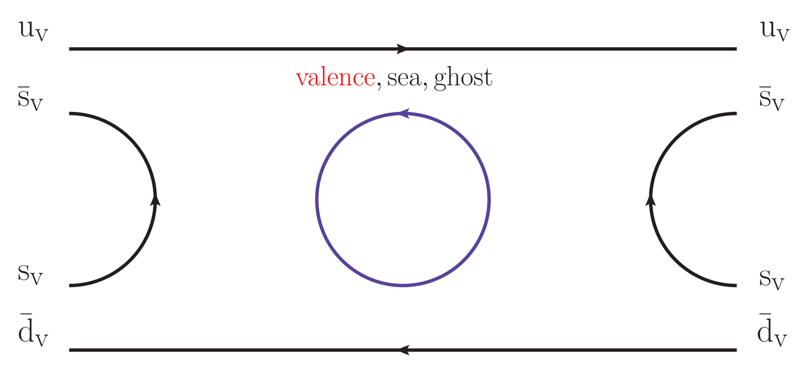


▶ Application of twisted B.C. is limited due to high computational cost: new gauge configuration for each choice of $\boldsymbol{\theta}$.

▶ **Partially twisted B.C.**: impose twisted B.C. on *valence* and periodic B.C. on *sea* quarks ⇒ no new gauge configuration is needed for each $\boldsymbol{\theta}$.

4. Techniques

▶ An example of an annihilation quark line diagram in isospin I=1, S-wave in the mesonic sector: $K\bar{K}$-scattering:



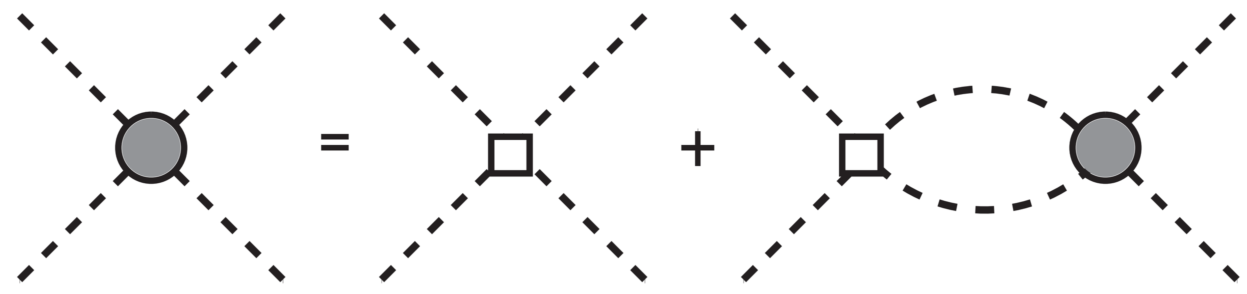
▶ PQQCD - enlarged QCD constructed from *valence*, *sea* and *ghost* (spin-1/2 commuting) quark fields. In our case:

$$\bar{Q}=(u_v, d_v, s_v, u_s, d_s, s_s, u_g, d_g, s_g), \quad m_u=m_d\neq m_s$$

▶ PQChPT - low-energy EFT of PQQCD [7]. In the mesonic sector it incorporates physical ($u_v\bar{s}_v$, etc.) and non-physical ($u_s\bar{s}_s$, $u_v\bar{s}_g$, $u_s\bar{s}_s$, etc.) mesons.

PQChPT is matched to nonrelativistic EFT (NREFT) ⇒ Lipmann-Schwinger (LS) equation.

▶ LS equation in the *infinite* volume limit:



$$\mathbf{T}=\mathbf{V}+\mathbf{VGT},$$

V- unknown potential, **G**-loop function, both **11** × **11** matrices in channel space (*valence*, *sea*, *ghost* mesons).

▶ $m_{valence} = m_{sea} = m_{ghost}$ ⇒ symmetry relations between matrix elements of **V**.

▶ Transition to a *finite* volume limit:

$$\int\frac{d^3\mathbf{q}}{(2\pi)^3}f(\mathbf{q})\to\frac{1}{L^3}\sum_{\mathbf{n}}f(\mathbf{q})$$

$$\mathbf{V}\to\mathbf{V},\quad \mathbf{G}\to\widetilde{\mathbf{G}}^{\theta}.$$

Energy levels $E(L)$ in a finite box is given by secular equation:

$$\mathrm{Det}[\mathbb{1}-\mathbf{V}\widetilde{\mathbf{G}}^{\theta}]=0$$

5. Results and outlook

▶ Partially twisted Lüscher equation: $\mathrm{Det}[\mathbb{1}-\mathbf{V}\widetilde{\mathbf{G}}^{\theta}]=0$

▶ Two particular cases of partial twisting: in case of $a_0(980)$

1. *s*-quark twist

   In the resulting Lüscher equation no-dependence on $\boldsymbol{\theta}$ remains. This obviously *does not coincide* with the result with full twisting.

2. *u*-quark twist

   The obtained result coincides with the Lüscher equation with full twisting *in the moving frame.*

▶ Future: study DK bound states, XYZ states,...

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