Holographic Glueball Decay Rates in the Witten-Sakai-Sugimoto Model

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September 8, 2014

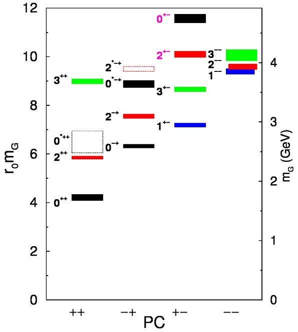
FAIR physics question

in particular for PANDA@FAIR:

What are the characteristic properties of glueballs?

Spectrum of bare glueballs (prior to mixing with $q\bar{q}$ states) more or less known from lattice:

Morningstar & Peardon hep-lat/9901004:



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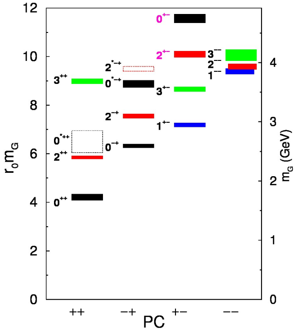

Need to know coupling to $\bar{q}q$ to identify glueball content of isoscalar mesons – not available from lattice

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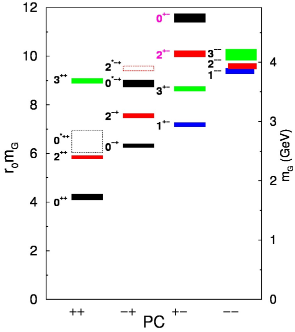


Gauge/gravity duality a new tool to study glueball properties from first principles

Outline

- Introduction to **holographic QCD**
  top-down: **Witten**[1998]-**Sakai-Sugimoto**[2004] **model**
  = supergravity limit of conjectured full string-theoretic dual of QCD
  (2 parameters only: 1 coupling + 1 mass scale!:-)
- Glueballs from anti-de Sitter supergravity revisited

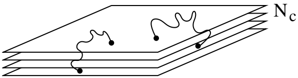
New results on: TU WIEN

- Glueball decay into pions from Sakai-Sugimoto model
  (work with Frederic Brünner & Denis Parganlija)
  [paper in preparation]

Original AdS/CFT correspondence

J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998)

"pedestrian's guide": S. S. Gubser and A. Karch, Ann. Rev. Nucl. Part . Sci. 59, 145 (2009)



(type IIB) string theory
on 5D anti-de Sitter space ($\times S_5$)

$\Leftrightarrow$

$\mathcal{N}=4$ SU(∞) super-YM theory
on 4D boundary of AdS$_5$

$$\frac{(\text{curvature radius})^4}{(\text{string length})^4} = \frac{R^4}{\ell_s^4} \qquad = \qquad g^2_{\rm YM} N_c \equiv \lambda \quad \text{'t Hooft coupling}$$

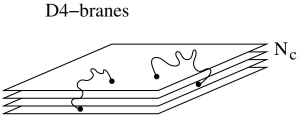
supergravity limit $\ell_s \ll R$
relatively easy

$\Leftrightarrow$

strong coupling limit $\lambda \gg 1$
impossibly difficult

Witten model: Holographic nonsupersymmetric QCD

E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998):



Type-IIA string theory with $N_c \to \infty$ $D4$ branes
dual to $4+1$-dimensional super-Yang-Mills theory

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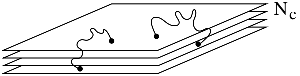


Type-IIA string theory with $N_c \to \infty$ $D4$ branes
dual to $4+1$-dimensional super-Yang-Mills theory

supersymmetry completely broken by compactification
on "thermal-like" circle $x_4 \equiv x_4 + 2\pi/M_{\mathrm{KK}}$ $_{\mathrm{(Kaluza-Klein)}}$

- antisymmetric b.c. for adjoint fermions: masses $\sim M_{\mathrm{KK}}$
- adjoint scalars not protected by gauge symmetry: also masses $\sim M_{\mathrm{KK}}$

$\to$ dual to pure-glue YM theory
3+1-dimensional at scales $\ll M_{\mathrm{KK}}$

but supergravity approximation needs weak curvature,
cannot take limit $M_{\mathrm{KK}} \to \infty$

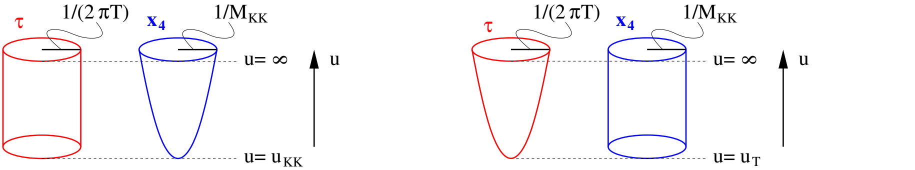
Deconfinement phase transition

Thermal circle in Euclidean time τ in addition to compactified x_4
Hawking-Page transition when $2\pi T = M_{\rm KK}$ (thus $\sim$ 1 GeV ?)

Confined phase	**Deconfined phase**

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left[d\tau^2 + d\mathbf{x}^2 + f(u)dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right]$$

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left[\tilde{f}(u)d\tau^2 + \delta_{ij}d\mathbf{x}^2 + dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{\tilde{f}(u)} + u^2 d\Omega_4^2\right]$$



$$M_{\rm KK} = \frac{3}{2}\frac{u_{\rm KK}^{1/2}}{R^{3/2}} \qquad f(u) \equiv 1 - \frac{u_{\rm KK}^3}{u^3}$$

$$T = \frac{3}{4\pi}\frac{u_T^{1/2}}{R^{3/2}} \qquad \tilde{f}(u) \equiv 1 - \frac{u_T^3}{u^3}$$

Cigar topology in x_4-u subspace $\longrightarrow$ Cigar in τ-u = **Euclidean black hole**

Glueballs in confined phase

$\exists$ scalar and tensor glueballs corresponding to 5D dilaton Φ and graviton G_{ij}

Csaki, Ooguri, Oz & Terning 1999

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Type-IIA supergravity compactified on x_4-circle many more modes:

Constable & Myers 1999; Brower, Mathur & Tan 2000

Mode	S_4	T_4	V_4	N_4	M_4	L_4
Sugra fields	G_{44}	Φ, G_{ij}	C_1	B_{ij}	C_{ij4}	G_α^α
J^{PC}	0^{++}	$0^{++}/2^{++}$	0^{-+}	1^{+-}	1^{--}	0^{++}
n=0	7.30835	22.0966	31.9853	53.3758	83.0449	115.002
n=1	46.9855	55.5833	72.4793	109.446	143.581	189.632
n=2	94.4816	102.452	126.144	177.231	217.397	277.283
n=3	154.963	162.699	193.133	257.959	304.531	378.099
n=4	228.709	236.328	273.482	351.895	405.011	492.171

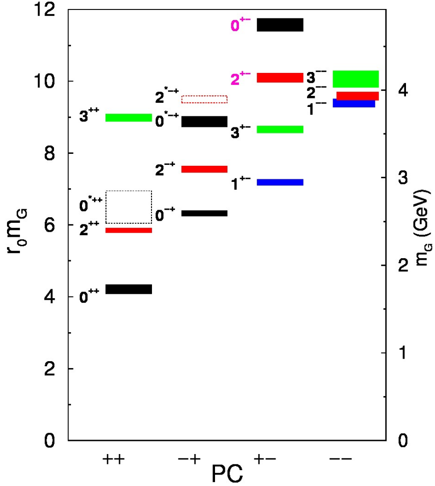
Lowest mode not from dilaton, but from "exotic polarization" – in 11D notation:

$$\underline{\delta g_{44}} = -\frac{r^2}{L^2} f\, H(r) G(x), \quad \delta g_{\mu\nu} = \frac{r^2}{L^2}\left[\frac{1}{4} H(r)\eta_{\mu\nu} - \left(\frac{1}{4} + \frac{3R^6}{5r^6 - 2R^6}\right) H(r)\frac{\partial_\mu \partial_\nu}{M^2}\right] G(x)$$

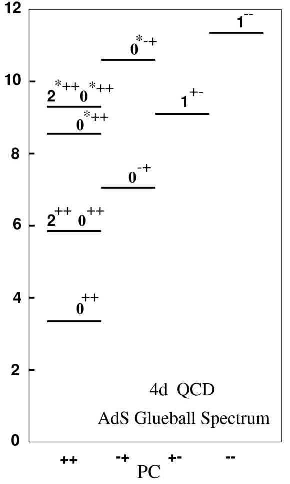
$$\delta g_{11,11} = \frac{r^2}{L^2}\frac{1}{4} H(r) G(x), \quad \delta g_{rr} = -\frac{L^2}{r^2} f^{-1}\frac{3R^6 H(r) G(r)}{5r^6 - 2R^6}, \quad \delta g_{r\mu} = \frac{90 r^7 R^6 H(r)\partial_\mu G(x)}{M^2 L^2 (5r^6 - 2R^6)^2}$$

Lattice glueballs vs. supergravity glueballs

Morningstar & Peardon hep-lat/9901004:



Brower, Mathur & Tan 2000:



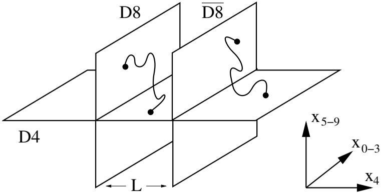
(mass scales matched on 2^{++}) → seemingly good qualitative agreement!

Sakai-Sugimoto model: Adding chiral quarks

T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)

add N_f $\mathrm{D}8$- and $\overline{\mathrm{D}8}$-branes, separated in x_4, $N_f \ll N_c$ (probe branes)

	0	1	2	3	4	5	6	7	8	9
D4	x	x	x	x	x					
D8/$\overline{\mathrm{D8}}$	x	x	x	x		x	x	x	x	x



4-8, 4-$\bar{8}$ strings
→ fundamental, massless chiral fermions

flavor symmetry
$U(N_f)_L \times U(N_f)_R$

for now: maximal separation in x_4 (antipodal on x_4 circle): $L = \pi/M_{\mathrm{KK}}$

Massless pions, massive vector mesons, massive η'

D8 brane action:

$$S_{\rm D8} = -T_{\rm D8}{\rm Tr}\int d^9x e^{-\Phi}\sqrt{-\det(\tilde{g}_{MN}+(2\pi\alpha')F_{MN})} + S_{\rm CS}$$

$$= \frac{g_{\rm YM}^2 N_c^2}{216\pi^3}\int d^4x\, dz\, {\rm Tr}\left[\frac{1}{2}(1+z^2)^{-1/3}F_{\mu\nu}^2 + (1+z^2)F_{\mu z}^2\right] + \dots$$

- massless pions in $A_z = \phi_0(z)\pi(x^\nu)$, rho meson in $A_\mu^{(1)} = \psi_1(z)\rho_\mu(x^\nu)$,
- more massive vector mesons and axial vector mesons in tower of $A_\mu^{(n)}$ modes

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eigenvalue of ψ_1 implies $m_\rho = \sqrt{0.669314}M_{\rm KK}$

$\Rightarrow$ matching $m_\rho \approx 776$ MeV fixes $M_{\rm KK} = 949$ MeV ($\Rightarrow T_{deconf} = 151$ MeV)

matching $f_\pi = \frac{\lambda N_c}{54\pi^4}M_{\rm KK}^2$ gives $\lambda = g_{\rm YM}^2 N_c \approx 16.6$

yields e.g.

- $m_{a_1}^2/m_\rho^2 \approx 2.4$ (versus 2.5 from experiment!)
- nonzero η' mass from anomaly inflow: Witten-Veneziano formula with $m_{\eta'} = \frac{\sqrt{N_f/N_c}}{3\sqrt{3}\pi}\lambda M_{\rm KK} \approx 967$MeV for $N_f = 3$ (exp.: 958 MeV !)

ρ meson decay rate

D8 mode corresponding to ρ-meson stable,
but can calculate effective action for mesons, in particular:

$$\mathcal{L}_{\rho\pi\pi} = -g_{\rho\pi\pi}\epsilon_{abc}(\partial_\mu \pi^a)\rho^{b\mu}\pi^c$$

$$g_{\rho\pi\pi} = \sqrt{2}\int dz \frac{1}{\pi(1+z^2)}\psi_1(z) = 33.98\,\lambda^{-\frac{1}{2}} N_c^{-\frac{1}{2}}$$

gives

$$\Gamma_\rho/m_\rho = \frac{g_{\rho\pi\pi}^2}{48\pi} \approx 0.1535 \qquad \text{(exp.: 0.191(1))}$$

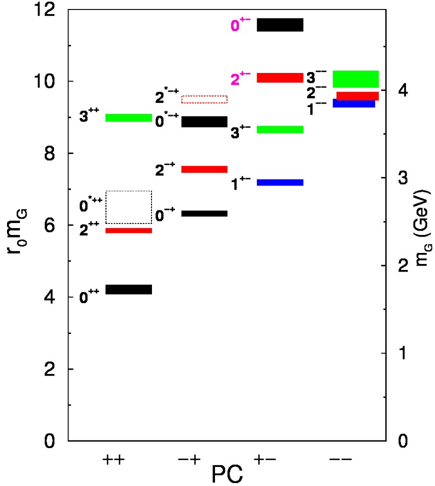
encourages **calculation of gluon decay rates**
which could not be easily obtained from (Euclidean!) lattice QCD

Lattice vs. supergravity glueballs

seemingly good qualitative agreement by matchup up 2^{++}

(but AdS spectrum somewhat stretched. . .)

Morningstar & Peardon hep-lat/9901004:



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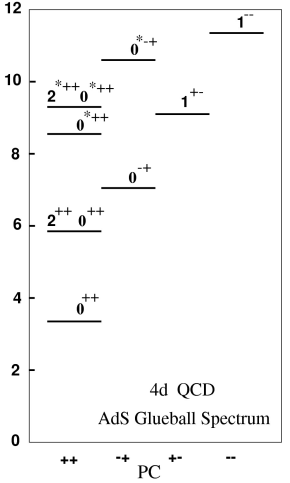

Lattice vs. supergravity glueballs in Sakai-Sugimoto model

Sakai-Sugimoto model: glueball masses $\propto M_{\rm KK} = 949$ MeV fixed by m_ρ

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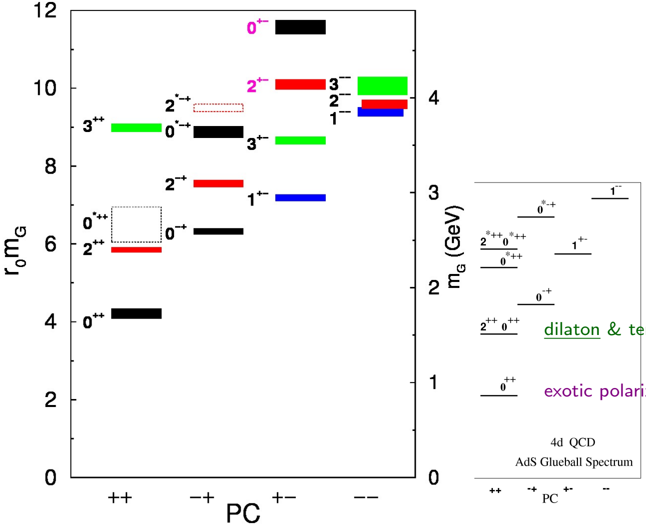

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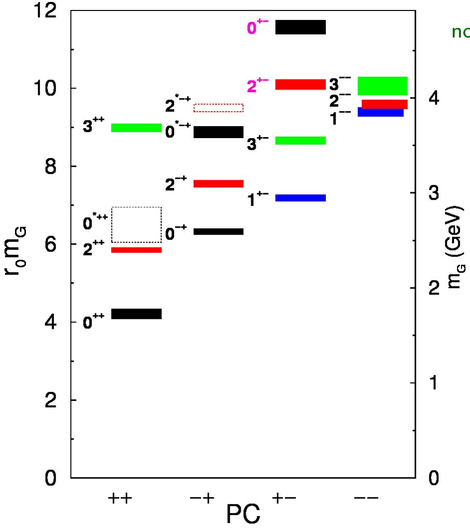
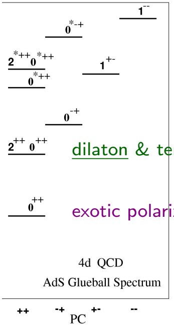


now good match lowest $0^{++} \leftrightarrow$ dilaton



dilaton & tensor 1487 MeV

exotic polarization 855 MeV

Lattice vs. supergravity glueballs in Sakai-Sugimoto model

Should exotic polarization (δG_{44} with x_4 the compactified direction of SYM$_{4+1}$) be excluded as lowest glueball mode?

- possibly not part of spectrum of holographic QCD in limit $M_{\mathrm{KK}} \to \infty, \lambda \to 0$
  (already asked by Constable & Myers)
- simpler bottom-up AdS/QCD have dilaton mode as dual for lowest glueball

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- simpler bottom-up AdS/QCD have dilaton mode as dual for lowest glueball
- next lowest scalar mode ~ 1487 MeV is (predominantly) dilaton mode (induces metric perturbations other than δG_{44})

Glueball-$\bar{q}q$ couplings in Sakai-Sugimoto model

Gravitational modes stable in confined background, but
can calculate *effective action for glueball-$\bar{q}q$ interactions*

done for lowest (exotic) mode by
Hashimoto, Tan & Terashima, Phys.Rev. D77 (2008) 086001, arXiv:0709.2208

revisited, corrected, and extended to other modes by
Brünner, Parganlija & AR, Acta Phys. Polon. Supp. 7 (2014) 533 and in prep.

"Exotic" mode:

$$S_{G\pi\pi} = \mathrm{Tr} \int d^4x \frac{1}{2} \partial_\mu \pi \, \partial_\nu \pi \left(\breve{c}_1 \eta^{\mu\nu} - c_1 \frac{\partial^\mu \partial^\nu}{M_G^2} \right) G$$

"Dilatonic" mode:

$$S_{D\pi\pi} = \mathrm{Tr} \int d^4x \frac{1}{2} \partial_\mu \pi \, \partial_\nu \pi \; \tilde{c}_1 \left(\eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{M_D^2} \right) D$$

Tensor glueball:

$$S_{T\pi\pi} = \mathrm{Tr} \int d^4x \, \partial_\mu \pi \, \partial_\nu \pi \, \bar{c}_1 T^{\mu\nu}$$

with $\{c_1, \breve{c}_1, \tilde{c}_1, \bar{c}_1\} = \{62.66, 16.39, 17.23, 21.10\} \times \lambda^{-1} N_c^{-1/2} M_{\mathrm{KK}}^{-1}$
(and many more vertices, involving also ρ_μ at this order)

Glueball decay rates in Sakai-Sugimoto model

Results for decay into two pions:

Exotic mode: $\Gamma_{G\to\pi\pi}/M_G \approx \frac{13.79}{\lambda N_c^2} \approx 0.092 \quad (M_G \approx 855\mathrm{MeV})$

Dilaton mode: $\Gamma_{D\to\pi\pi}/M_D \approx \frac{1.359}{\lambda N_c^2} \approx 0.009 \quad (M_D \approx 1487\mathrm{MeV})$

Tensor mode: $\Gamma_{T\to\pi\pi}/M_T \approx \frac{2.174}{\lambda N_c^2} \approx 0.0145 \quad (M_T \approx 1487\mathrm{MeV})$

NB: relative width of lowest (exotic) scalar mode much larger than next ones!?
another hint that it should be discarded?

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Most likely *experimental candidates* for meson with dominant scalar glueball content: $f_0(1500)$ or $f_0(1710)$

$$\Gamma^{(\text{ex})}(f_0(1500) \to \pi\pi)/(1505\text{MeV}) = 0.025(3)$$
$$\Gamma^{(\text{ex})}(f_0(1710) \to \pi\pi)/(1722\text{MeV}) = 0.017(4)$$ ← favored by arXiv:1408.4921

(the latter follows from BES data with large backgrounds;

older WA102 data would give 0.009(2)!)

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Experimental candidate for tensor glueball:
$\Gamma^{(\text{ex})}(f_J(2200))/(2231\text{MeV}) = 0.010(4)$

Glueball decay rates in Sakai-Sugimoto model (cont'd)

Branching ratios:

General pattern

- Narrow widths

$$\Gamma_{Glueball \to \pi\pi} \propto \lambda^{-1} N^{-2}$$

- Strong suppression for

$$\Gamma_{Glueball \to 4\pi} \propto \lambda^{-3} N^{-4}$$

  not really suppressed in $f_0(1500)$, $f_J(2200)$, but perhaps in $f_0(1710)$
  can be due to mixing in of $\bar{q}q$

- Even stronger suppression for $Glueball \to 4\pi^0$

  (direct $\Gamma_{GB \to 4\pi^0} \propto \lambda^{-7} N^{-4}$ from F^4 terms in DBI action
  also $\Gamma_{GB \to GB + 2\pi^0 \to 4\pi^0} \propto \lambda^{-6} N^{-3}$, but kinematically suppressed)

Summary – Glueballs in Witten-Sakai-Sugimoto model

After fitting just m_ρ to fix $M_{\rm KK} = 949$ MeV

- good prediction of higher vector and axial vector mesons masses,
- good prediction of deconfinement/chiral transition temperature,
- good prediction of glueball masses if “exotic mode” discarded;

after fitting f_π to also fix 't Hooft coupling at $\lambda = 16.6$

- good prediction of rho decay rates
- good prediction of anomalous $m'_\rho \propto N_c^{-\frac{1}{2}} \lambda M_{\rm KK}$
- narrow partial width $glueball \to \pi\pi$, quite compatible with experimental data
- strong suppression of $glueball \to 4\pi$, in particular $\to 4\pi^0$

Warrants further studies!

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Plans:

- inclusion of nonzero mass for strange quark
- mixing with quarkonia (suppressed by $N_c^{-\frac{1}{2}}$)
- medium effects (finite baryon density)

Comments on $f_0(1710)$ and gluon condensate

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but needs very large gluon condensate to have narrow glueball width:
$C^4 \equiv \langle \frac{\alpha_s}{\pi} F_{\mu\nu}^2 \rangle \sim (1.8\,\mathrm{GeV})^4$
whereas sum rules and lattice give $C \sim 0.3 \dots 0.6\,\mathrm{GeV}$

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Gluon condensate in Witten-Sakai-Sugimoto model calculated in [Kanitscheider, Skenderis & Taylor JHEP 0809] as
$C^4 \equiv \langle \frac{\alpha_s}{\pi} F_{\mu\nu}^2 \rangle = \frac{2}{3^7 \pi^4} N_c \lambda M_{\mathrm{KK}}^4 \simeq (0.28\,\mathrm{GeV})^4$
while we find very narrow glueball widths $\Gamma/M \propto \lambda^{-1} N_c^{-2}$

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[S. Janowski, F. Giacosa & D. Rischke, arXiv:1408.4921]
extended Linear Sigma Model (eLSM) with dilaton as effective glueball field identifies now the narrow state $f_0(1710)$ as almost pure glueball

but needs very large gluon condensate to have narrow glueball width:
$C^4 \equiv \langle \frac{\alpha_s}{\pi} F_{\mu\nu}^2 \rangle \sim (1.8\,\mathrm{GeV})^4$
whereas sum rules and lattice give $C \sim 0.3 \ldots 0.6\,\mathrm{GeV}$

Gluon condensate in Witten-Sakai-Sugimoto model calculated in [Kanitscheider, Skenderis & Taylor JHEP 0809] as
$C^4 \equiv \langle \frac{\alpha_s}{\pi} F_{\mu\nu}^2 \rangle = \frac{2}{3^7 \pi^4} N_c \lambda M_{\mathrm{KK}}^4 \simeq (0.28\,\mathrm{GeV})^4$
while we find very narrow glueball widths $\Gamma/M \propto \lambda^{-1} N_c^{-2}$

suggests that gluon condensate not so simply related to parameters of eLSM