On transport properties of charged drop in external electric field

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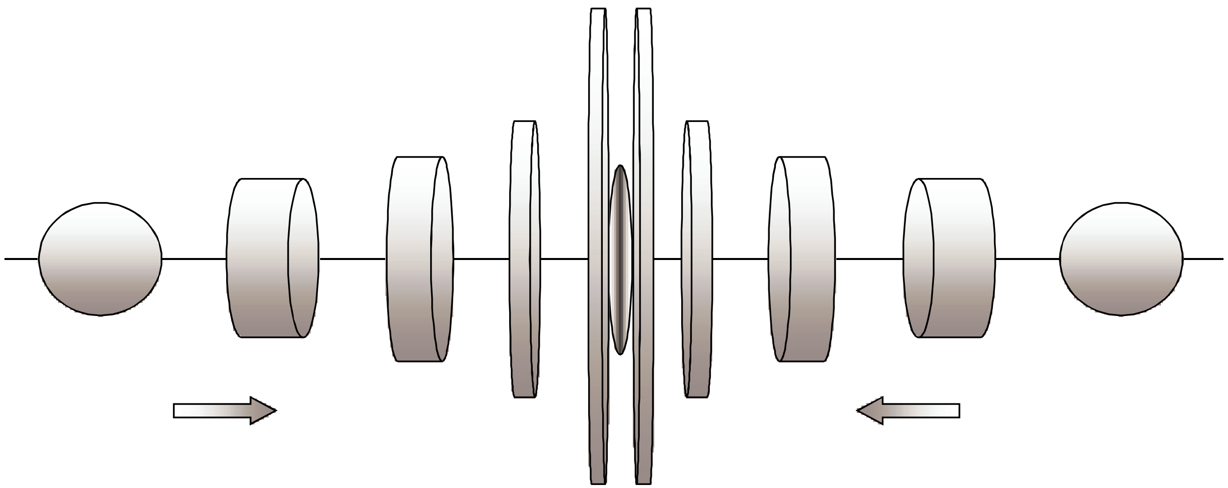

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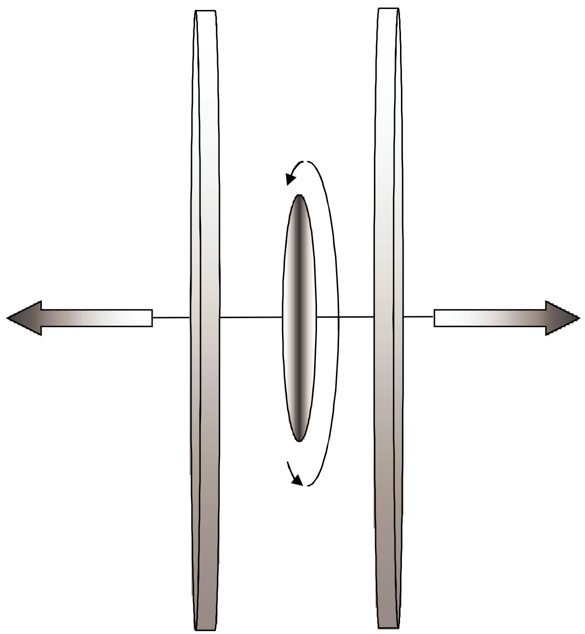

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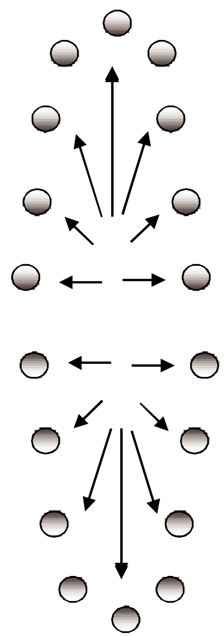

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- The drop is unstable and we consider a process of the drop's expansion/compression basing on the microscopic description of the process;
- Conclusion: we need to solve Vlasov equation for the time-dependent distribution function.

Plasma parameters

- Coulomb coupling parameter with a as some characteristic length:

$$\Gamma = \frac{U_p}{\mathcal{E}_{kin}} \propto \frac{q^2}{a\, k_B\, T}; \quad a \propto \left(\frac{3}{4\pi n}\right)^{1/3},$$

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- Our parameters:

$$m \propto 1/a : \frac{q^2}{amc^2} \propto \frac{q^2}{c^2} \propto \Gamma \frac{a\,k_B\,T}{c^2} < 1;$$

$$\frac{Q_{Ext}q}{amc^2} \approx \frac{U_p^{Ext}}{\mathscr{E}_{kin0}} < 1$$

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External field configuration

- In the rest frame of the drop:

$$\Delta\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial^2 t} - \frac{\varphi}{r^2_{DExt}} = -4\pi\, q\, n\, dV_{Ext}\,\delta(\vec{r} - \vec{r}_s(t))$$

- Solution for the external field:

$$\varphi(\vec{r},t) = \frac{Q_{Ext}}{2\pi^2}\int d^2k_\perp\, e^{k_\perp(r_\perp - b)}\int_{-\infty}^{\infty}\frac{e^{k_z(z - v\,t)}}{k_\perp^2 + k_z^2/\gamma + 1/r^2_{DExt}}$$

where $\vec{r} = (r_\perp, z)$ is position of the hot drop and the position of the incident matter is given by $\vec{r_s} = (b, vt)$ with $r_\perp = (r_x, r_y)\,,\; b = (b_x, b_y)$.

External field configuration

- At the relativistic limit when $v \approx c$:

$$
\begin{aligned}
A_z(\vec{r},t) &= \varphi(\vec{r},t) = 2\,Q_{Ext}\,\delta(z - ct\,)\,K_0(|r_\perp - b|/r_D) \\
A_x(\vec{r},t) &= A_y(\vec{r},t) = 0\,.
\end{aligned}
$$

- After a gauge transform

$$
\begin{aligned}
f &= -2\,Q_{Ext}\,\theta(z - ct)\,K_0(|r_\perp - b|/r_D) \\
\varphi(\vec{r},t) &= A_z(\vec{r},t) = 0 \\
A_x(\vec{r},t) &= -2\,Q_{Ext}\,\theta(z - ct)\,\partial_x\,K_0(|r_\perp - b|/r_D) \\
A_y(\vec{r},t) &= -2\,Q_{Ext}\,\theta(z - ct)\,\partial_y\,K_0(|r_\perp - b|/r_D)
\end{aligned}
$$

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External field configuration

- Cylindrical coordinates

$$A_r(\vec{r},t) = A_x(\vec{r},t)\cos\theta + A_y(\vec{r},t)\sin\theta$$

$$A_r(\vec{r},t) = -2Q_{Ext}\theta(\tau)\frac{\partial K_0(|r-b|/r_{0D})}{\partial r} = -2\frac{Q_{Ext}}{r_{0D}}\frac{\partial K_0(\xi_0)}{\partial \xi_0}\theta(\tau)$$

with $\tau = ct$ and $\xi_0 = \frac{r}{r_{0D}}$.

- Electrical field:

$$E_{rext}(r,\,\tau) = -\,2\,\frac{Q_{Ext}}{r_{0D}}\,\frac{\partial K_0(\xi_0)}{\partial \xi_0}\,\delta(\tau) = 2\,\frac{Q_{Ext}}{r_{0D}}\,K_1(\xi_0)\,\delta(\tau)$$

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Vlasov equation

- Usual form:

$$\frac{1}{\zeta_r}\,\frac{\partial\, f_s}{\partial\,\tau} + \frac{\partial\, f_s}{\partial r} = -\,\frac{q}{c\,\zeta_r}\,(\,E_{rs} - E_{rext}\,)\,\frac{\partial\, f_s}{\partial p_r}\,, \quad \zeta_r = v_r/c$$

- Integral form

$$f_s(r,\vec{\zeta},\tau) = -\frac{q}{c\zeta_r}\int dr^{'}d\tau^{'}\mathscr{E}(\tau-\tau^{'},r-r^{'})\,\cdot$$

$$\cdot\left(E_{rs}(r^{'},\tau^{'}) - E_{rext}(r^{'},\tau^{'})\right)\frac{\partial\, f_s(r^{'},\vec{\zeta},\tau^{'})}{\partial p_r(\zeta_r)} + f_0(r-\zeta_r\tau,\vec{\zeta})\,,$$

$$with\;\; \mathscr{E} = \zeta_r\,\theta(\tau)\,\delta(\,r - \zeta_r\,\tau\,)$$

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Vlasov equation

- Final expression

$$f_s(r,\vec{\zeta},\tau) = -\frac{4\,\pi\,q^2\,n}{rc}\int_0^{\tau} d\tau^{'}\frac{\partial f_s(r-\vec{\zeta}(\tau-\tau^{'}),\vec{\zeta},\tau^{'})}{\partial p_r(\zeta_r)}\cdot$$

$$\cdot\int^{r-\zeta_r(\tau-\tau^{'})} dzz\int f_s(z,v_r^{'},v_\theta^{'},\tau^{'})d^2v^{'} +$$

$$+\frac{2\,q\,Q_{Ext}}{c\,r_{0D}}\,K_1(\xi_0\,-\,\zeta_r\frac{\tau}{r_{0D}})\,\frac{\partial\,f_{s0}(r-\zeta_r\tau,\vec{\zeta})}{\partial p_r(\zeta_r)} + f_0(r-\zeta_r\tau,\vec{\zeta})$$

- At $\tau = 0$ we have

$$f_{s0}(r,p_r)\;-\;\frac{2\,q\,Q_{Ext}}{c\,r_{0D}}\,K_1(\xi_0\,)\,\frac{\partial\,f_{s0}(r,p_r)}{\partial p_r(\zeta_r)}\;=\;f_0(r,p_r)$$

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Vlasov equation

- Initial conditions:

$$f_{s0}(r,\vec{\zeta}) = \sum_{i=0}^{\infty} F_i^{s0}\,\Lambda_0^i\,, \quad \Lambda_0 = \frac{2\,q\,Q_{Ext}}{mc^2 r_{0D}} \approx \frac{U_p^{ext}}{\mathcal{E}_{kin}}$$

and in the first two orders of the approximation:

$$f_{s0}(r,\vec{\zeta}) = f_0(r,\vec{\zeta}) + \Lambda_0\, K_1(\xi_0\,)\, \frac{\partial\, f_0(r,\vec{\zeta})}{\partial \zeta_r}$$

- Solution of the equation:

$$f_s(r,\,\vec{\zeta},\,\tau) = \sum_{i=0}^{\infty} f_{si}(r,\vec{\zeta})\,\tau^i$$

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Distribution function

- First order approximation:

$$f_{s1}(r, \vec{\zeta}) = -\zeta_r \frac{\partial f_0(r,\vec{\zeta})}{\partial r} - r^2 \Lambda \frac{\partial f_0(r,\vec{\zeta})}{\partial \zeta_r} - $$
$$- \zeta_r \, \Lambda_0 \, K_1(\xi_0) \frac{\partial^2 f_0(r,\vec{\zeta})}{\partial \zeta_r \partial r} - r^2 \, \Lambda \, \Lambda_0 \, K_1(\xi_0) \frac{\partial^2 f_0(r,\vec{\zeta})}{\partial \zeta_r^2}$$

where $\Lambda = \frac{2\pi q^2 n}{rmc^2}$.

- Applicability of this approximation scheme:

$$r^2 \, \Lambda \, \tau < 1 \;\; or \;\; U_p \, / \, mc^2 < 1$$

Initial distribution function

- Initial equilibrium distribution function

$$f_0(r,\vec{p}) = \left(\frac{m}{2\pi}\right)\delta(H_\perp - \omega_r P_\theta - kT_\perp)G(p_z)\,, \int_{-\infty}^{\infty} G(p_z)\,dp_z = 1$$

where

$$\left(\frac{m}{2\pi}\int_{-\infty}^{\infty} dv_\theta \int_{-\infty}^{\infty} dv_r \delta(H_\perp - \omega_r P_\theta - kT_\perp)\right)_{r=0} = 1\,,$$

$$H_\perp = \frac{1}{2\,m}\left(p_r^2 + p_\theta^2\right) + q\,\Phi_{s0}\,,$$

$$P_\theta = r\,(p_\theta - m\,\omega_c\,r\,/\,2)\,,\ \omega_c = \frac{|q|\,B_0}{m\,c}\,,\ s_q = \frac{8\,\pi\,q^2}{m\,\omega_c^2}\,,$$

$$r_b^2 = \frac{2kT_\perp/m}{(\omega_r^+ - \omega_r)\,(\omega_r - \omega_r^-)}\,,\ \omega_r^\pm = \frac{\omega_c}{2}\{1 \pm (1 - n\,s_q)^{1/2}\}\,.$$

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Transport properties

- Radial velocity:

$$\langle v_r \rangle = \frac{\int d^2v\, v_r\, f_s}{\int d^2v\, f_s} = -c\,\Lambda_0 K_1(\xi_0) + \tau\,\frac{c\,r}{r_b^2}\left(\frac{2\,k\,T_\perp}{mc^2} + \frac{2\pi q^2 n\, r_b^2}{mc^2}\right)$$

- Azimuthal flow velocity:

$$\langle v_\theta \rangle = \frac{\int d^2v\, v_\theta\, f_s}{\int d^2v\, f_s} = \omega_r\, r + \tau\,\Lambda_0\, K_1\, \omega_r$$

- Transverse shear Viscosity coefficient:

$$\sigma_{r,\theta} = n\,m \int d^2v\,(v_r - \langle v_r \rangle)\,(v_\theta - \langle v_\theta \rangle) = n\,m\,\tau\,c\,(\Lambda_0\, K_1)^2$$

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Shear viscosity

- We have an anomalous viscosity, see references, where τ is not a mean free path:

$$\tau \propto \hbar / m\, c\,, \quad \eta^{max}_{r\theta} \propto \hbar\, n\, (\Lambda_0\, K_1)^2 \approx \hbar\, n \left(\frac{U^{ext}_p}{\mathscr{E}_{kin}}\right)^2$$

- The entropy of the process remains constant, $s = s_0 = const$, therefore the ratio

$$\eta / s = \eta / s_0$$

changes only because a change of the viscosity coefficient η and overall ratio remain small.

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- Shear viscosity depends on the external field value and changes from zero to some maximum value but remaining small anyway;
- The ratio of viscosity to the entropy also changes from zero to some small value, the entropy remains constant during all process of drop's expansion/compression.
- For 3-D case of drop's expansion/compression with magnetic field included (in calculation) results must be similar.

References

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