Pair double heavy diquarks production in high energy proton–proton collisions

Alexei P. Martynenko[1,2] and Anton M. Trunin[2,3]

[1]Samara State University

[2]Korolyov Samara State Aerospace University

[3]Bogoliubov Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research

Pair charmonium production

e^+e^- annihilation:

- K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. **89**, 142001 (2002)

$$\sigma[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}_{\geq 4} = 33^{+7}_{-6} \pm 9 \text{ fb}$$

Theoretical predictions:

- E. Braaten and J. Lee, Phys. Rev. D 67, 054007 (2003); **72**, 099901(E) (2005)
- K.Y. Liu, Z. G. He, and K. T. Chao, Phys. Lett. B **557**, 45 (2003)

$$\sigma[e^+e^- \to J/\psi + \eta_c] = 3.78 \pm 1.26 \text{ fb}$$
$$\sigma[e^+e^- \to J/\psi + \eta_c] = 5.5 \text{ fb}$$

New experiments:

- K. Abe et al. (Belle Collaboration), Phys. Rev. D **70**, 071102 (2004)
- B. Aubert et al. (BABAR Collaboration), Phys. Rev. D **72**, 031101 (2005)

$$\sigma[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}_{>2} = 25.6 \pm 2.8 \pm 3.4 \text{ fb}$$
$$\sigma[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}_{>2} = 17.6 \pm 2.8 \pm 2.1 \text{ fb}$$

Pair charmonium production

Further theoretical efforts:

- Y.J. Zhang, Y.J. Gao, K.T. Chao, Phys. Rev. Lett. **96**, 092001 (2006)
  G.T. Bodwin, D. Kang, T. Kim, J.Lee, C. Yu, AIP Conf. Proc. **892**, 315 (2007)
  Z. G. He, Y. Fan, and K. T. Chao, Phys. Rev. D **75**, 074011 (2007)
- J.P. Ma, Z.G. Si, Phys. Rev. D **70**, 074007 (2004)
  A.E. Bondar, V.L. Chernyak, Phys. Lett. B **612**, 215 (2005)
  V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys. Rev. D **72**, 074019 (2005)
- D. Ebert, A.P. Martynenko, Phys. Rev. D **74**, 054008 (2006)
  D. Ebert, R.N. Faustov, V.O. Galkin, A.P. Martynenko,
  Phys. Lett. B **672**, 264 (2009)
  E.N. Elekina, A.P. Martynenko, Phys. Rev. D **81**, 054006 (2010)
  A.P. Martynenko, A.M. Trunin, arXiv:1106.2741
- G.T. Bodwin, J. Lee, C. Yu, Phys. Rev. D **77**, 094018 (2008)

$$\sigma_{[Bodwin,Lee,Yu]} = \mathbf{17.6}^{+8.1}_{-6.7}\ \text{fb}$$

nonrelativistic result, (fb)	relativistic corrections	QED	NLO α_s (+QED)	correlations of relativistic & NLO α_s
5.4	2.9	1.0	6.9	1.4

Pair diquarks production

In this work we consider the process $pp \to \mathcal{D} + \bar{\mathcal{D}} + X$:

- nonrelativistic cross sections
- relativistic corrections
  - perturbative corrections to the production amplitude
    - expansion of propagators
    - WF transformation law
  - wave functions of the bound states
    - 'effective' relativistic Hamiltonian (Breit potential)
  - non-zero bound energy effects

Diquarks — compact pairs of (anti)quark–quarks (like cc or $\bar{b}\bar{c}$) in antisymmetric triplet color states, which manifest themselves like antiquarks (quarks) in meson.

$$\text{Diquark production} \xRightarrow{\text{double–heavy}} \text{Baryon production}$$

- an addition of light quark can be considered with the probability ≈ 1

Pair diquarks production

e^+e^- annihilation:

- V.V. Braguta, V.V. Kiselev, A.E. Chalov, Phys. At. Nucl. **65**, 1537 (2002)
- A.P. Martynenko, A.M. Trunin, Physical Review D **89**, 014004 (2014)

Still no experimental confirmation on double–heavy baryons

The 'discovery' of Ξ_{cc}^+ in decays to $\Lambda_c^+ K^- \pi^+$ and pD^+K^-:

- M. Mattson *et al.* (SELEX Collaboration), Phys. Rev. Lett. **89**, 112001 (2002)
- A. Ocherashvili *et al.* (SELEX Collaboration), Phys. Lett. B **628**, 18 (2005)

Contradictions with theory and subsequent experiments:

- V.V. Kiselev, A.K. Likhoded, hep-ph/0208231 (2002).
- R. Chistov *et al.* (Belle Collaboration), Phys. Rev. Lett. **97**, 162001 (2006).
- B. Aubert *et al.* (BABAR Collaboration), Phys. Rev. D **74**, 011103 (2006).

No positive identification so far:

- Y. Kato *et al.* (Belle Collaboration), arXiv:1312.1026 (2013)
- R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. **1312**, 090 (2013)

Quasipotential approach to relativistic quark model

Our calculation of relativistic corrections is based on the **quasipotential approach** (or the single–time formulation of the quantum field theory)

Bethe–Salpeter equation:

$$(\not{p}_1 - m_1)(\not{p}_2 - m_2)\psi_P(p) = i\int \frac{d^4q}{(2\pi)^4} K_{12}(p,q;P)\psi_P(q),$$

$\psi_P(x_1, x_2) = \langle 0|T\{\psi_1(x_1)\psi_2(x_2)\}|P\rangle$ — Bethe–Salpeter amplitude or wave function, $x_1^0 \neq x_2^0$:

«. . . a proton today and an electron yesterday do not constitute a hydrogen atom»
A. Eddington

Logunov–Tavkhelidze equation:

- A.A. Logunov, A.N. Tavkhelidze, Nuovo Cimento **29**, 380 (1963)
- V.G. Kadyshevsky, Nucl. Phys. **B** 6, 125 (1968)
- C. Itzykson, V.G. Kadyshevsky, I.T. Todorov, Phys. Rev. D **1**, 2823 (1970)
- R.N. Faustov, Teor. Mat. Fiz **3**, 240 (1970)

$$\left[M - \sqrt{\mathbf{p}^2 + m_1^2} - \sqrt{\mathbf{p}^2 + m_1^2}\right]\psi^{(+)}(\mathbf{p}) = \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\,\psi^{(+)}(\mathbf{q})$$

Quasipotential approach to relativistic quark model

Quasipotential equation in Schrödinger-like form:

- I.T. Todorov, Phys. Rev. D **3**, 2351 (1971)
- R.N. Faustov and A.P. Martynenko, Teor. Mat. Fiz **64**, 179 (1985)

$$\left[\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right]\psi^{(+)}(\mathbf{p}) = \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\,\psi^{(+)}(\mathbf{q}),$$

$$b^2(M) = \mathbf{p^2}\big|_{\text{on shell}} = \frac{1}{4M^2}\left[M^2-(m_1+m_2)^2\right]\left[M^2-(m_1-m_2)^2\right],$$

$\mu_R = \frac{1}{4M^3}\left[M^4-(m_1^2-m_2^2)^2\right]$ — relativistic reduced mass.

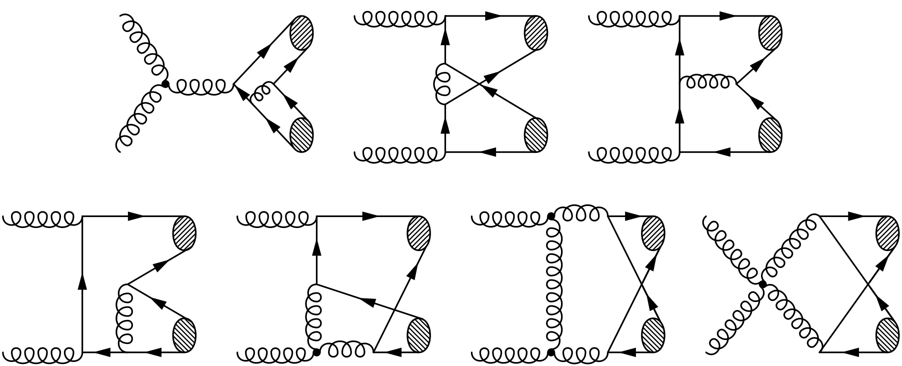
Quasipotential construction:

- R.N. Faustov, Fiz. El. Chast. Atom. Yad. **3**, 238 (1972)
- D. Ebert, V.O. Galkin, R.N. Faustov, Phys. Rev. D **57**, 5663 (1998)
- D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Rev. D **72**, 034026 (2005)
- V.A. Matveev, V.I. Savrin, A.N. Sissakian, A.N. Tavkhelidze, Teor. Mat. Fiz **132**, 267 (2002)

35 LO α_s SPS gluon fusion diagrams

$$\mathcal{M}[gg \to D_{bc} + \bar{D}_{\bar{b}\bar{c}}] = \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3} \bar{\Psi}(p, P)\bar{\Psi}(q, Q) \otimes \mathcal{T}(p_1, p_2; q_1, q_2),$$

$$d\sigma[pp \to D_{bc} + \bar{D}_{\bar{b}\bar{c}} + X] = \int dx_1 dx_2\, f_{g/p}(x_1, \mu) f_{g/p}(x_2, \mu)\, d\sigma[gg \to D_{bc} + \bar{D}_{\bar{b}\bar{c}}]$$



Production amplitude

$$\mathcal{M}[gg \to D_{bc} + \bar{D}_{\bar{b}\bar{c}}](k_1, k_2, P, Q) = M\pi^2\alpha_s^2 \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3} \mathrm{Tr}\,\mathfrak{M},$$

$$\begin{aligned}
\mathfrak{M} = \bar{\Psi}^{bc}_{P,p}\gamma_\beta \bar{\Psi}^{cb}_{Q,q}\gamma_\omega \Gamma_1^{\beta\omega} + \bar{\Psi}^{cb}_{P,-p}\gamma_\beta\, \Gamma_2^{\beta\omega\theta}\gamma_\omega \bar{\Psi}^{bc}_{Q,-q}\gamma_\theta + \bar{\Psi}^{bc}_{P,p}\gamma_\beta\, \Gamma_3^{\beta\omega\theta}\gamma_\omega \bar{\Psi}^{cb}_{Q,q}\gamma_\theta + \\
+\bar{\Psi}^{bc}_{P,p}\,\hat{\varepsilon}_1 \frac{m_c - \hat{k}_1 + \hat{p}_1}{(k_1 - p_1)^2 - m_c^2}\gamma_\beta(\bar{\Psi}^{cb}_{Q,q}\gamma_\omega\Gamma_4^{\beta\omega} + \Gamma_5^{\beta\omega}\bar{\Psi}^{cb}_{Q,q}\gamma_\omega) + \\
+\bar{\Psi}^{cb}_{P,-p}\,\hat{\varepsilon}_1 \frac{m_b - \hat{k}_1 + \hat{p}_2}{(k_1 - p_2)^2 - m_b^2}\gamma_\beta(\bar{\Psi}^{bc}_{Q,-q}\gamma_\omega\Gamma_6^{\beta\omega} + \Gamma_7^{\beta\omega}\bar{\Psi}^{bc}_{Q,-q}\gamma_\omega) + \\
+\bar{\Psi}^{bc}_{P,p}\,\hat{\varepsilon}_2 \frac{m_c - \hat{k}_2 + \hat{p}_1}{(k_2 - p_1)^2 - m_c^2}\gamma_\beta\Gamma_8^{\beta\omega}\bar{\Psi}^{cb}_{Q,q}\gamma_\omega + \bar{\Psi}^{cb}_{P,-p}\,\hat{\varepsilon}_2 \frac{m_b - \hat{k}_2 + \hat{p}_2}{(k_2 - p_2)^2 - m_b^2}\gamma_\beta\Gamma_9^{\beta\omega}\bar{\Psi}^{bc}_{Q,-q}\gamma_\omega + \\
+\bar{\Psi}^{cb}_{P,-p}\gamma_\beta \frac{m_b + \hat{k}_1 - \hat{q}_2}{(k_1 - q_2)^2 - m_b^2}\hat{\varepsilon}_1\bar{\Psi}^{bc}_{Q,-q}\gamma_\omega\Gamma_{10}^{\beta\omega} + \bar{\Psi}^{bc}_{P,p}\gamma_\beta \frac{m_c + \hat{k}_1 - \hat{q}_1}{(k_1 - q_1)^2 - m_c^2}\hat{\varepsilon}_1\bar{\Psi}^{cb}_{Q,q}\gamma_\omega\Gamma_{11}^{\beta\omega},
\end{aligned} \tag{7}$$

$k_{1,2} = x_{1,2}\sqrt{S}/2\,(1, 0, 0, \pm 1)$ — the initial gluon four-momenta;
$P,\ Q$ — the total four-momenta of outcoming diquarks;
$p = L_P(0, \mathbf{p}),\ q = L_Q(0, \mathbf{q})$ — the relative four-momenta of (anti)quarks.
$\varepsilon_{1,2}$ — the polarization vectors of initial gluons.

Vertex functions

$$\Gamma_1^{\beta\omega} = \mathcal{K}_1 D_\mu{}^\beta(p_1+q_1) D_\nu{}^\omega(p_2+q_2)\big(\varepsilon_1^\nu\varepsilon_2^\mu + \varepsilon_1^\mu\varepsilon_2^\nu - 2g^{\mu\nu}(\varepsilon_1\varepsilon_2) -$$

$$-D_{\lambda\kappa}(k_1-p_1-q_1)\mathfrak{E}_1^{\lambda\mu}(p_1+q_1)\mathfrak{E}_2^{\kappa\nu}(p_2+q_2) - D_{\kappa\lambda}(k_1-p_2-q_2)\mathfrak{E}_1^{\kappa\nu}(p_2+q_2)\mathfrak{E}_2^{\lambda\mu}(p_1+q_1)\big),$$

$$\Gamma_2^{\beta\omega\theta} = \mathcal{K}_2\mathfrak{E}_2^\mu(-k_1)D_\mu{}^\beta(k_1+k_2)D^{\theta\omega}(p_1+q_1)\frac{m_b - \hat{p}_1 - \hat{q}_1 - \hat{q}_2}{(p_1+q_1+q_2)^2 - m_b^2} +$$
$$+\mathcal{K}_5\varepsilon_2^\omega\mathfrak{E}_1^{\mu\nu}(p_1+q_1)D_\mu{}^\beta(k_1-p_1-q_1)D_\nu{}^\theta(p_1+q_1)\frac{m_b + k_2 - q_2}{(k_2-q_2)^2 - m_b^2} +$$
$$+D^{\theta\beta}(p_1+q_1)\frac{m_b + \hat{p}_1 + \hat{p}_2 + \hat{q}_1}{(p_1+p_2+q_1)^2 - m_b^2}\Big(\mathcal{K}_2\mathfrak{E}_1^\mu(-k_2)D_\mu{}^\omega(k_1+k_2) +$$
$$+\mathcal{K}_9\varepsilon_1^\omega\hat{\varepsilon}_2\frac{m_b + \hat{k}_1 - \hat{q}_2}{(k_1-q_2)^2 - m_b^2} + \mathcal{K}_7\varepsilon_2^\omega\hat{\varepsilon}_1\frac{m_b + \hat{k}_2 - \hat{q}_2}{(k_2-q_2)^2 - m_b^2}\Big),$$

Auxiliary functions:

$$\begin{gathered}\mathfrak{E}_{1,2}^{\mu\nu}(x) = g^{\mu\nu}(k_{1,2} - 2x)\varepsilon_{1,2} + \varepsilon_{1,2}^\mu(2k_{1,2}^\nu - x^\nu) + \varepsilon_{1,2}^\nu(k_{1,2}^\mu + x^\mu),\\ \mathfrak{E}_{1,2}^\mu(x) = \varepsilon_{2,1}^\nu\mathfrak{E}_{1,2}^{\mu\nu}(x).\end{gathered} \tag{8}$$

The gluon propagators are taken in Feynman gauge:

$D_{\mu\nu}^F(k) = -ig^{\mu\nu}/k^2$

Color structure of the amplitude

11 different color factors:

$$\mathcal{K}_1 = -3\mathcal{C}_0 - 3\mathcal{C}_1 + 4\mathcal{C}_3, \quad \mathcal{K}_2 = \frac{4}{3}\mathcal{C}_1, \quad \mathcal{K}_3 = \frac{2i}{3}(\mathcal{C}_0 + 2\mathcal{C}_1 - 4\mathcal{C}_2),$$

$$\mathcal{K}_4 = \frac{i}{3}(\mathcal{C}_0 - \mathcal{C}_1 - \mathcal{C}_2), \quad \mathcal{K}_5 = \frac{3}{2}\mathcal{C}_0 + \mathcal{C}_1 - 2\mathcal{C}_3, \quad \mathcal{K}_6 = -\frac{i}{3}(\mathcal{C}_0 + 3\mathcal{C}_1 - 5\mathcal{C}_2),$$

$$\mathcal{K}_7 = \frac{2i}{3}(\mathcal{C}_0 - 2\mathcal{C}_2), \quad \mathcal{K}_8 = -\frac{i}{3}(\mathcal{C}_0 + 2\mathcal{C}_1 - 5\mathcal{C}_2), \quad \mathcal{K}_9 = \frac{2i}{3}(\mathcal{C}_0 + 2\mathcal{C}_1 - 2\mathcal{C}_2),$$

$$\mathcal{K}_{10} = \frac{3}{2}\mathcal{C}_0 + 2\mathcal{C}_1 - 2\mathcal{C}_3, \quad \mathcal{K}_{11} = -\frac{i}{3}(\mathcal{C}_0 + 2\mathcal{C}_1 - \mathcal{C}_2),$$

$$\mathcal{C}_0 = \delta^{g_1 g_2}\delta_{AB}, \quad \mathcal{C}_1 = i\, f^{g_1 g_2 a}(T^a)_{BA},$$
$$\mathcal{C}_2 = (T^{g_1} T^{g_2})_{BA}, \quad \mathcal{C}_3 = f^{g_1 ea} f^{g_2 eb}(T^a T^b)_{BA}.$$

$e^{c_1 c_2 A}/\sqrt{2}$ and $e^{c_3 c_4 B}/\sqrt{2}$, $c_i = 1, 2, 3$ — color part of the wave functions of D_{bc} and $\bar{D}_{\bar{b}\bar{c}}$ diquarks, respectively

Transformation of relativistic wave functions

Quasipotential wave functions are calculated in the diquark rest frame and then transformed to the reference frames moving with the four-momenta $P(Q)$:

$$\bar{\Psi}^{bc}_{P,p} = \frac{\bar{\Psi}^0_{D_{bc}}(\mathbf{p})}{\sqrt{\frac{\epsilon_c(p)}{m_c}\frac{\epsilon_c(p)+m_c}{2m_c}\frac{\epsilon_b(p)}{m_b}\frac{\epsilon_b(p)+m_b}{2m_b}}}\left[\frac{\hat{v}_1-1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m_b(\epsilon_b(p)+m_b)}-\frac{\hat{p}}{2m_b}\right]$$
$$\times\Sigma^P(1+\hat{v}_1)\left[\frac{\hat{v}_1+1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m_c(\epsilon_c(p)+m_c)}+\frac{\hat{p}}{2m_c}\right],$$
$$\bar{\Psi}^{cb}_{Q,q} = \frac{\bar{\Psi}^0_{\bar{D}_{\bar{b}\bar{c}}}(\mathbf{q})}{\sqrt{\frac{\epsilon_c(q)}{m_c}\frac{\epsilon_c(q)+m_c}{2m_c}\frac{\epsilon_b(q)}{m_b}\frac{\epsilon_b(q)+m_b}{2m_b}}}\left[\frac{\hat{v}_2-1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m_c(\epsilon_c(q)+m_c)}+\frac{\hat{q}}{2m_c}\right]$$
$$\times\Sigma^Q(1+\hat{v}_2)\left[\frac{\hat{v}_2+1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m_b(\epsilon_b(q)+m_b)}-\frac{\hat{q}}{2m_b}\right].$$

$v_1 = \frac{P}{M}$, $v_2 = \frac{Q}{M}$;
$\epsilon(p) = \sqrt{m^2+\mathbf{p}^2}$;
$m_{b,c}$ — quark mass.

$\Sigma_{P,Q} = \gamma_5$ or $\varepsilon_{P,Q}$ — for scalar or axial–vector diquarks, respectively

Expansion of quark and gluon propagators

$$\frac{1}{(p_{1,2}+q_{1,2})^2} = \frac{1}{s\,\eta_{1,2}^2}\Big[1 \mp \frac{2(pQ+qP)}{s\,\eta_{1,2}} - \frac{p^2+2pq+q^2}{s\,\eta_{1,2}^2} + \dots\Big],$$
$$\frac{1}{(p_1+q_1+q_2)^2 - m_b^2} = \frac{1}{Z_1}\Big[1 - \frac{2pQ+p^2}{Z_1} + \frac{4(pQ)^2}{Z_1^2} + \dots\Big], \qquad (9)$$
$$\frac{1}{(k_2-q_1)^2 - m_c^2} = \frac{1}{Z_2}\Big[1 + \frac{2k_2q-q^2}{Z_2} + \frac{4(k_2Q)^2}{Z_2^2} + \dots\Big],$$

where $s = x_1x_2S$ and $t = (P-k_1)^2 = (Q-k_2)^2$ — the Mandelstam variables for the gluonic subprocess $gg \to D_{bc} + \bar{D}_{\bar{b}\bar{c}}$.

Leading order denominators:

$$Z_1 = s\,\eta_1 + \eta_2^2 M^2 - m_b^2 \qquad Z_2 = t\,\eta_1 - \eta_1\eta_2 M^2 - m_c^2 \qquad (10)$$

Neglecting bound state energy:

$$s\,\eta_{1,2} \qquad s\,\eta_{1,2}^2 \qquad \eta_{1,2}(M^2-t) \qquad \eta_{1,2}(M^2-s-t) \qquad (11)$$

In the case of the most unfavourable values of the variables $x_{1,2}$ and t the expansion parameters in (9) can be roughly assessed as $4p^2/M^2$ and $4q^2/M^2$ (for bc diquarks).

Expansion of the amplitude

Example from $gg \to 2J/\psi$ amplitude:

$$\mathcal{M}_1^{ab} = \frac{32\alpha_s^2\delta^{ab}}{9\,m\,s^4}\int \frac{m+\epsilon(p)}{2\epsilon(p)}R(p)p^2dp\int \frac{m+\epsilon(q)}{2\epsilon(q)}R(q)q^2dq\Big\{3s^2\big[\varepsilon_1\cdot\varepsilon_2(s\,\varepsilon_P^*\cdot\varepsilon_Q^*-$$
$$-2\,\varepsilon_P^*\cdot Q\,\varepsilon_Q^*\cdot P)-2\,\varepsilon_P^*\cdot\varepsilon_Q^*(\varepsilon_1\cdot P\,\varepsilon_2\cdot Q+\varepsilon_1\cdot Q\,\varepsilon_2\cdot P)+2\,\varepsilon_P^*\cdot Q(\varepsilon_1\cdot P\,\varepsilon_2\cdot\varepsilon_Q^*+$$
$$+\varepsilon_1\cdot\varepsilon_Q^*\,\varepsilon_2\cdot P)-\varepsilon_1\cdot\varepsilon_P^*\times(s\,\varepsilon_2\cdot\varepsilon_Q^*-2\,\varepsilon_2\cdot Q\,\varepsilon_Q^*\cdot P)-\varepsilon_2\cdot\varepsilon_P^*(s\,\varepsilon_1\cdot\varepsilon_Q^*-$$
$$-2\,\varepsilon_1\cdot Q\,\varepsilon_Q^*\cdot P)\big]\Big(3(1-c_p-c_q-c_p^2-c_q^2)+c_pc_q(67+3c_p+3c_q)+3c_p^2c_q^2\Big)+\ldots\Big\}$$

$$c_p=\frac{m-\epsilon(p)}{m+\epsilon(p)}\qquad c_q=\frac{m-\epsilon(q)}{m+\epsilon(q)}$$

For D_{bc} and $\bar{D}_{\bar{b}\bar{c}}$ diquarks — additional complication due to the unequal quark masses m_b and m_c, e.g.:

$$\frac{m+\epsilon(p)}{2\epsilon(p)}\to\sqrt{\frac{(\epsilon_c(p)+m_c)(\epsilon_b(p)+m_b)}{2\epsilon_c(p)\,2\epsilon_b(p)}}\qquad(12)$$

The principal structure of the relativstic corrections to the amplitude remains the same

Effective relativistic Hamiltonian

$$H = H_0 + \Delta U_1 + \Delta U_2 + \Delta U_3,$$

$$H_0 = \sqrt{\mathbf{p}^2 + m_1^2} + \sqrt{\mathbf{p}^2 + m_2^2} - m_1 - m_2 - \frac{2\tilde{\alpha}_s}{3r} + \frac{1}{2}Ar + \frac{1}{2}B,$$

$$\Delta U_1(r) = -\frac{\alpha_s^2}{6\pi r}\left[2\beta_0 \ln(\mu r) + a_1 + 2\gamma_E\beta_0\right],$$

$$\Delta U_2(r) = -\frac{\alpha_s}{3m_1m_2r}\left[\mathbf{p}^2 + \frac{\mathbf{r}(\mathbf{r}\mathbf{p})\mathbf{p}}{r^2}\right] + \frac{1}{3}\pi\alpha_s\left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right)\delta(\mathbf{r}) -$$

$$-\frac{\alpha_s}{3m_1m_2}\left[\frac{\mathbf{S}^2}{r^3} - 3\frac{(\mathbf{S}\mathbf{r})^2}{r^5} - \frac{4\pi}{3}(2\mathbf{S}^2 - 3)\delta(\mathbf{r})\right] - \frac{(m_1 + m_2)\alpha_s^2}{2m_1m_2r^2},$$

$$\Delta U_3(r) = f_V\left[\frac{A}{8r}\left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right) + \frac{2A}{3m_1m_2r}\mathbf{S}_1\mathbf{S}_2 + \frac{A}{6m_1m_2r}\left(\frac{3}{r^2}(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r}) - \mathbf{S}_1\mathbf{S}_2\right)\right]. \tag{13}$$

'Rationalization' of the kinetic energy term:

$$T = \sqrt{\mathbf{p}^2 + m_1^2} + \sqrt{\mathbf{p}^2 + m_2^2} \approx 2 \times \frac{\mathbf{p^2}}{2\tilde{\mu}} + \frac{1}{2}\left(\frac{m_1^2}{\tilde{m}_1} + \frac{m_2^2}{\tilde{m}_2}\right),$$
$$\tilde{\mu} = \frac{2\tilde{m}_1\tilde{m}_2}{\tilde{m}_1 + \tilde{m}_2}, \qquad \tilde{m}_{1,2} = \sqrt{\mathbf{p}_{\mathrm{eff}}^2 + m_{1,2}^2}. \tag{14}$$

Effective relativistic Hamiltonian

Diquark, $n^{2S+1}L_J$	f_V	b, GeV	$\mathbf{p}^2_{eff}$, GeV^2	M^{num}, GeV	M [1], GeV	M [2], GeV	M [3], GeV
SD_{bc}, 1^1S_0	0.9	1.5	0.32	6.517	6.48	6.558	6.519
AVD_{bc}, 1^3S_1	0.9	1.5	0.32	6.526	6.48	6.562	6.526
AVD_{cc}, 1^3S_1	0.9	1.5	0.26	3.224	3.16	3.238	3.226

[1] S.S. Gershtein, V.V. Kiselev, A.K. Likhoded, A.I. Onishchenko, Phys. Rev. D **62**, 054021 (2000).
[2] F. Giannuzzi, Phys. Rev. D **79**, 094002 (2009).
[3] D. Ebert, R.N. Faustov, V.O. Galkin, W. Lucha, Phys. Rev. D **76**, 114015 (2007).

Diquark	M^{num}, GeV	$R(0)$, GeV$^{3/2}$	$\tilde{R}(0)$, GeV$^{3/2}$	ω_{10}	ω_{01}	ω_{11}	ω_{02}	ω_{20}
SD_{bc}	6.517	0.67	0.50	0.0383	0.0045	0.00039	0.00005	0.00314
AVD_{bc}	6.526	0.67	0.48	0.0384	0.0045	0.00038	0.00005	0.00308
AVD_{cc}	3.224	0.53	0.38	0.0323		0.0023		

Nonrelativistic result

$$d\sigma^{\rm NR}[gg\to D_{cc}+\bar D_{\bar c\bar c}](s,t)=\frac{\pi\alpha_s^4|R_{D_{cc}}(0)|^4}{54M_c^2s^8\left(M_c^2-t\right)^4\left(M_c^2-s-t\right)^4}\big[27648M_c^{24}-$$
$$-72M_c^{22}(1595s+4596t)+3M_c^{20}(67687s^2+437088st+605232t^2)-8M_c^{18}\times$$
$$\times(28007s^3+278328s^2t+849501st^2+754920t^3)+4M_c^{16}(48546s^4+575480s^3t+$$
$$+2731629s^2t^2+5276664st^3+3390660t^4)-2M_c^{14}(66854s^5+867710s^4t+$$
$$+5237453s^3t^2+15810492s^2t^3+21825720st^4+10831968t^5)+M_c^{12}(64025s^6+$$
$$+980113s^5t+6934011s^4t^2+27679700s^3t^3+59798910s^2t^4+63129024st^5+$$
$$+25238304t^6)-2M_c^{10}(9796s^7+190998s^6t+1629993s^5t^2+8003124s^4t^3+$$
$$+23392115s^3t^4+38627220s^2t^5+32576040st^6+10803456t^7)+M_c^8(4006s^8+$$
$$+94606s^7t+1029199s^6t^2+6247798s^5t^3+23171033s^4t^4+52444016s^3t^5+$$
$$+69078684s^2t^6+47988288st^7+13491360t^8)-2M_c^6(322s^9+7064s^8t+99306s^7t^2+$$
$$+779460s^6t^3+3657884s^5t^4+10718238s^4t^5+19496435s^3t^6+21114948s^2t^7+$$
$$+12361788st^8+2995920t^9)+M_c^4(68s^{10}+1153s^9t+19692s^8t^2+217805s^7t^3+$$
$$+1362129s^6t^4+5166549s^5t^5+12342213s^4t^6+18546596s^3t^7+16897269s^2t^8+$$
$$+8485920st^9+1796688t^{10})-2M_c^2t(s+t)^2(16s^8+243s^7t+5526s^6t^2+49040s^5t^3+$$
$$+215626s^4t^4+530597s^3t^5+741924s^2t^6+546660st^7+163296t^8)+t^2(s+t)^2\times$$
$$\times(8s^8+25s^7t+2536s^6t^2+21366s^5t^3+78759s^4t^4+157896s^3t^5+179640s^2t^6+$$
$$+108864st^7+27216t^8)\big]. \qquad (15)$$

Relativistic cross section

$$d\sigma[gg \to D_{bc} + \bar{D}_{\bar{b}\bar{c}}](s,t) = \frac{\pi M^2 \alpha_s^4}{65\,536\, s^2} |\tilde{R}(0)|^4 \times$$
$$\Big[\boldsymbol{F^{(1)}(s,t)} - 4(\omega_{01} + \omega_{10} - \omega_{11}) F^{(1)}(s,t) - 4 m_c^{-1} m_b^{-1} (m_c^2 \omega_{\frac{1}{2}\frac{3}{2}} + m_b^2 \omega_{\frac{3}{2}\frac{1}{2}}) F^{(1)}(s,t)$$
$$+6(\omega_{01} + \omega_{10})^2 F^{(1)}(s,t) + \omega_{\frac{1}{2}\frac{1}{2}} (1 - 3\omega_{01} - 3\omega_{10}) F^{(2)}(s,t) + \omega^2_{\frac{1}{2}\frac{1}{2}} F^{(3)}(s,t)\Big].$$

Relativistic generalization of $R(0) = \sqrt{\frac{2}{\pi}} \int R(p) p^2 \, dp$:

$$\tilde{R}(0) = \sqrt{\frac{2}{\pi}} \int_0^\infty \sqrt{\frac{(\epsilon_c(p) + m_c)(\epsilon_b(p) + m_b)}{2\epsilon_c(p)\, 2\epsilon_b(p)}} R(p) p^2 \, dp, \tag{16}$$

$$I_{nk} = \int_0^{m_c} p^2 R(p) \sqrt{\frac{(\epsilon_c(p) + m_c)(\epsilon_b(p) + m_b)}{2\epsilon_c(p)\, 2\epsilon_b(p)}} \left(\frac{\epsilon_c(p) - m_c}{\epsilon_c(p) + m_c}\right)^n \left(\frac{\epsilon_b(p) - m_b}{\epsilon_b(p) + m_b}\right)^k dp,$$
$$\omega_{nk} = \sqrt{\frac{2}{\pi}} \frac{I_{nk}}{\tilde{R}(0)}, \quad 0 < n + k \le 2. \tag{17}$$

Numerical results

Energy $\sqrt{S}$	Diquarks pair	CTEQ5L		CTEQ6L1	
		$\sigma_{\text{nonrel.}}$, nb	$\sigma_{\text{rel.}}$, nb	$\sigma_{\text{nonrel.}}$, nb	$\sigma_{\text{rel.}}$, nb
$\sqrt{S} = 7$ TeV	$SD_{bc} + S\bar{D}_{\bar{b}\bar{c}}$	0.063	0.018	0.057	0.016
	$AVD_{bc} + AV\bar{D}_{\bar{b}\bar{c}}$	0.25	0.053	0.23	0.049
	$AVD_{cc} + AV\bar{D}_{\bar{c}\bar{c}}$	1.39	0.28	1.07	0.22
$\sqrt{S} = 14$ TeV	$SD_{bc} + S\bar{D}_{\bar{b}\bar{c}}$	0.14	0.039	0.12	0.034
	$AVD_{bc} + AV\bar{D}_{\bar{b}\bar{c}}$	0.55	0.12	0.48	0.10
	$AVD_{cc} + AV\bar{D}_{\bar{c}\bar{c}}$	2.51	0.51	1.94	0.40

$$\sigma[pp \to 2J/\psi + X] = 23.0 \text{ (nonrel.)} \qquad 9.6 \text{ (rel.)} \quad \text{nb} \tag{18}$$

The upper bound estimate on the pair double–heavy *ccq* baryons production cross section appears to be more than order of magnitude smaller in comparison with the appropriate charmonium result

Thank you for attention!