Gravitational waves from spinning neutron stars

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Context: Gravitational wave searches

- LIGO, Virgo & GEO600 detectors have collected large amounts of data at/near initial design sensitivity.
- $\sim$ 100 papers published, all upper limits.
- Advanced LIGO and Virgo will improve astrophysical reach; aLIGO expected to take data late 2015.
- Motivation behind *gravitational wave astronomy* is two-fold:
  1. To confirm a key prediction of General Relativity
  2. To probe physics in extreme regimes
- I will talk about GWs from **spinning neutron stars**

Gravitational wave emission form ‘mountains’

- A neutron star rotating steadily with spin $f_{\rm spin}$ at distance r radiates GWs:

$$h = 3 \times 10^{-29} \left(\frac{\epsilon}{10^{-7}} \right) \left(\frac{f_{\rm spin}}{10\,{\rm Hz}} \right)^2 \left(\frac{1\,{\rm kpc}}{r} \right),$$

  where the ellipticity $\epsilon = (I_{yy} - I_{xx})/I_{zz}$ may be non-zero due to:
  1. Strains in solid crust, or possibly core, or
  2. Magnetic forces.
- Emission is at $2f_{\rm spin}$ (although can get harmonic at f_{spin} if superfluid pinning occurs and is misaligned with body-axes (DIJ 2010)).
- **Maximum/likely values of ϵ depends upon physics of high density interior.**

Possible astronomical targets

Possible targets include:

- **Known isolated pulsars**
- Accreting neutron stars
- Central Compact Objects
- ‘Gravitars’

The Crab nebula (HST)

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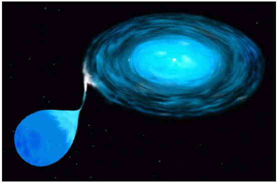

Artist’s impression!

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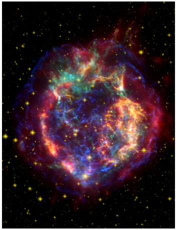

Cassiopeia A (‘Cas A’)

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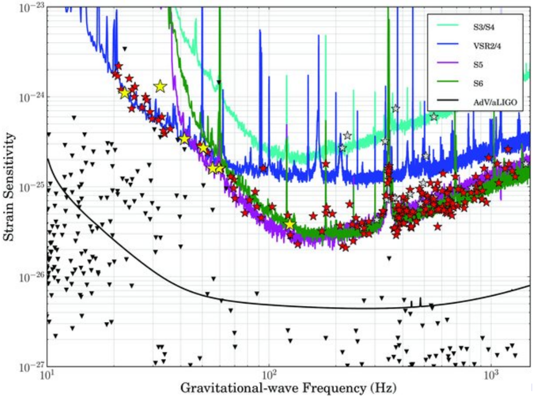
Not seen electromagnetically!

Direct upper limits

- Direct upper limits already obtained, from non-detection of GWs by LIGO/Virgo.
- ‘Spin-down limit’ beaten for two pulsars (Aasi et al (2014); see Figure).
- For Crab, no more than $\sim$ 1% of spin-down energy going into gravitational wave channel, $\epsilon \lesssim 10^{-4}$.
- For Vela, no more than $\sim$ 10% of spin-down energy going into gravitational wave channel, $\epsilon \lesssim 6 \times 10^{-4}$.
- Need theoretical modelling to say when upper limits start to get interesting.

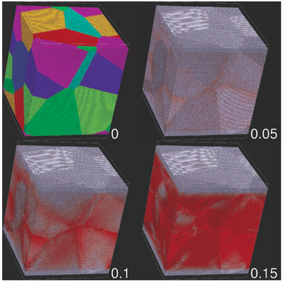


Elastic mountains: ‘normal’ neutron stars

- Maximum elastic mountain size determined by balance between gravitational and elastic forces:

$$\epsilon \approx \frac{\mu V_{\rm crust}}{GM^2/R} \times u_{\rm break} \approx 10^{-6} \left(\frac{u_{\rm break}}{10^{-1}}\right) .$$

- Shear modulus has long been known to be $\lesssim 10^{29}$ erg cm^{-3}.
- Recent large-scale molecular dynamics of Horowitz & Kadau (2009) indicate very high breaking strain, $\theta_{\rm max} \sim 0.1$ (see Figure)
- Plastic flow may relax crust on longer timescales (Chugunov & Horowitz 2010).



Elastic mountains: more exotic scenarios

- Exotic states of matter *might* lead to solid cores giving larger maximum allowed ellipticites.
- $\epsilon_{\max} \sim 10^{-1}$ possible for solid quark stars, 10^{-3} for hybrid stars (Johnson-McDaniel & Owen 2013).
- Crystalline colour superconducting quark matter also relevant (Mannarelli et al 2007) leading to similarly large maximum ellipticities (Haskell et al 2007 and Lin 2007)
- Lack of detection of such a large mountain *does not* rule out such exotic states of matter ...
- ... need estimates of *likely* ellipticities, not just upper bounds!

Magnetic mountains

- Magnetic field lines have an effective tension, and deform star (Chandrasekhar & Fermi 1953). Roughly,

$$\epsilon \sim \frac{\int B^2 \, dV}{GM^2/R} \sim 10^{-12} \left(\frac{B}{10^{12}\,\mathrm{G}} \right)^2 .$$

- If protons form type II superconductor, magnetic field confirmed to fluxtubes. Effect of this is to increase tension by a factor of H_c/B, where $H_c \sim 10^{15}$ G, increasing ellipticity:

$$\epsilon \sim 10^{-9} \frac{B}{10^{12}\,\mathrm{G}} .$$

- Either way, ellipticities are small, GWs undetectable.

‘Exotic’ magnetic mountains

- If CFL or 2SC phases occur in neutron star cores, can get *colour-magnetic flux tubes* (Iida & Baym 2002, Iida 2005, Alford & Sedrakian 2010).
- This leads to flux tube tension $\sim 10^3$ larger than in protonic superconductivity case. Glampedakis, DIJ & Samuelsson (2012) estimate ellipticity:

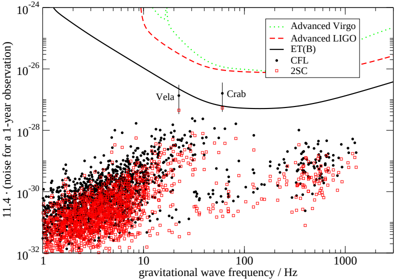
  $$\epsilon_{\rm CFL} \sim 10^{-7} \left(\frac{f_{\rm vol}}{1/2}\right) \left(\frac{B_{\rm int}}{10^{12}\,{\rm G}}\right) \left(\frac{\mu_{\rm q}}{400\,{\rm MeV}}\right)^2 ,$$

  where
  - $f_{\rm vol}$ = fraction of stellar volume in deconfined state,
  - $B_{\rm int}$ = *internal* magnetic field strength,
  - $\mu_{\rm q}$ = quark chemical potential.
- Can allow for internal field to be some multiple of external field:

  $$B_{\rm int} = \alpha B_{\rm ext}.$$

‘Exotic’ magnetic mountains cont . . .

- For given stellar parameters $f_{\rm vol}$, α and $\mu_{\rm q}$ can then balance observed spin-down of pulsars against combined GW & EM torque to estimate $B_{\rm int}$ and hence h.
- GW amplitudes scale as $h \sim f_{\rm vol}\alpha\mu_{\rm q}^2$; for sensible values ($f_{\rm vol} = 0.5$, $\alpha = 2$, $\mu_{\rm q} = 400$ MeV) obtain:



Clearly of interest for Crab and Vela pulsars.

Summary

- Search for GWs from spinning neutron stars ongoing.
- Maximum/likely levels of emission sensitive to high density equation of state.
- Key outstanding issues:
    1. What determine *realistic* level of ellipticity of solid phase(s)?
    2. What is strength and geometry of internal magnetic field?
    3. In the event of a detection, how can we distinguish between the various deformation mechanisms?
- New data late $\sim$ 2015. Watch this space!