Chiral Gap Effect in Curved Space

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Based on PRL113, 091102 (2014) with Nino Flachi

Challenging Opportunities

QCD phase transitions

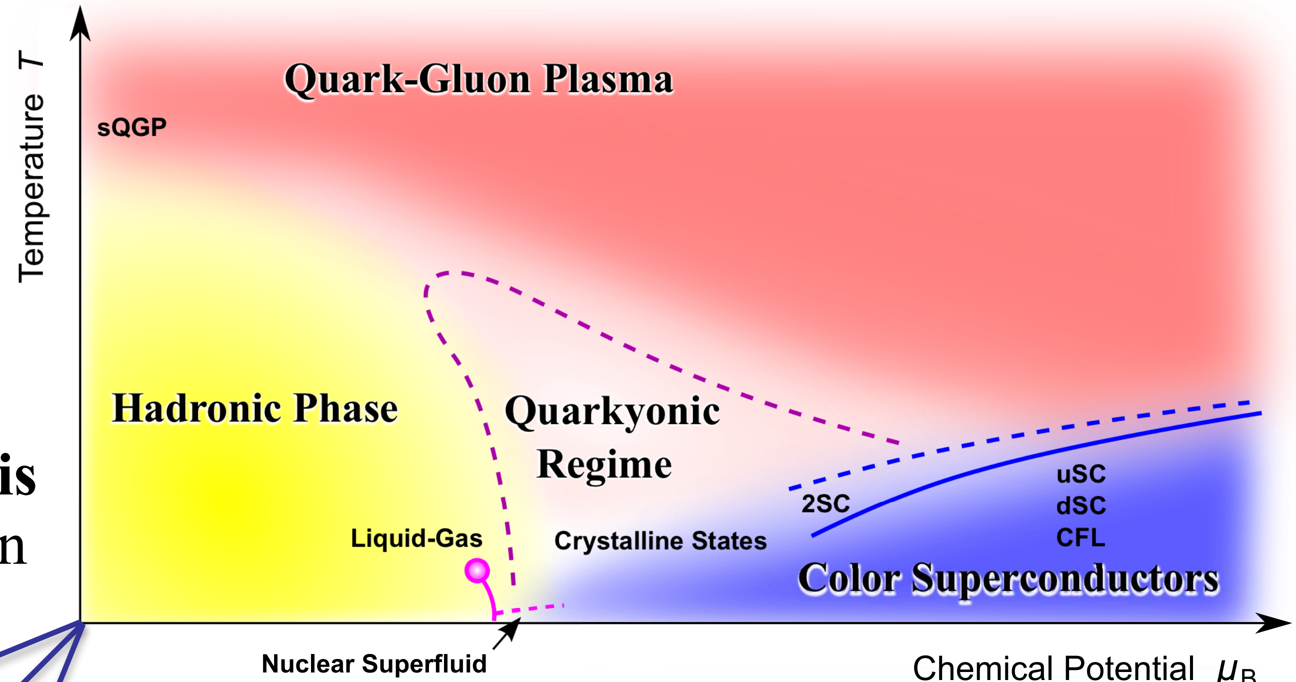
- □ External parameters analogous to T, μ_B, $\boldsymbol{B}$, $\boldsymbol{E}$, etc…
- □ ***B***-field like effects on the chiral properties
- □ No sign problem in the lattice-QCD simulation

QCD-based vacuum structures induced by *R*

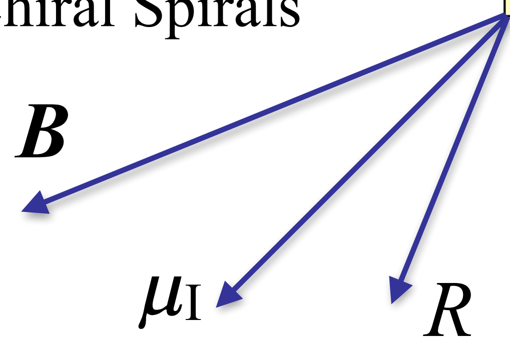
- □ Quantum phase transition of chiral restoration
- □ Deconfinement??? (too difficult to analyze so far)

"Simplest" real-time physics problem

- □ ***E***-field like effects on the particle production
- □ Early Universe and Black Holes (future extension)
- □ Application to the thermalization in HIC



Magnetic Catalysis
Magnetic Inhibition
Chiral Spirals

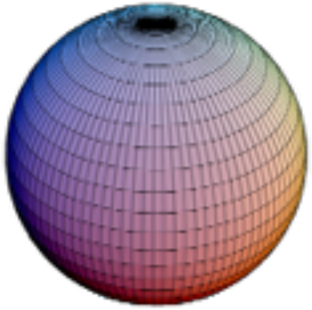


Chiral Gap Effect (this talk)
Many other unknowns (future problems)

Pion Condensation
FFLO States (with mismatched Fermi surfaces)

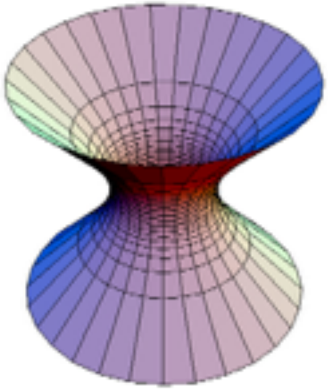
"Solvable" Studies

Inagaki-Muta-Odintsov (1997)
and many references therein



de Sitter space

Maximally symmetric
Constant scalar curvature ($R > 0$)



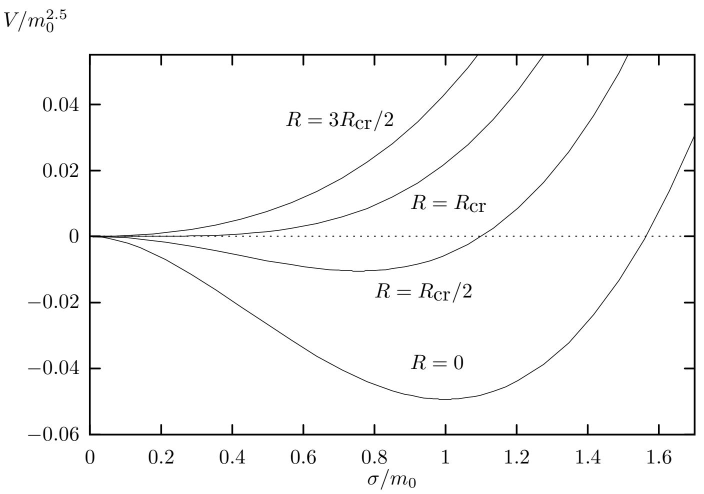
Anti de Sitter space

Maximally symmetric
Constant scalar curvature ($R < 0$)

Fermion sector can be exactly treatable
Chiral models in the MFA can be "exactly solvable"

“Solvable” Studies

Inagaki-Muta-Odintsov (1997)
and many references therein



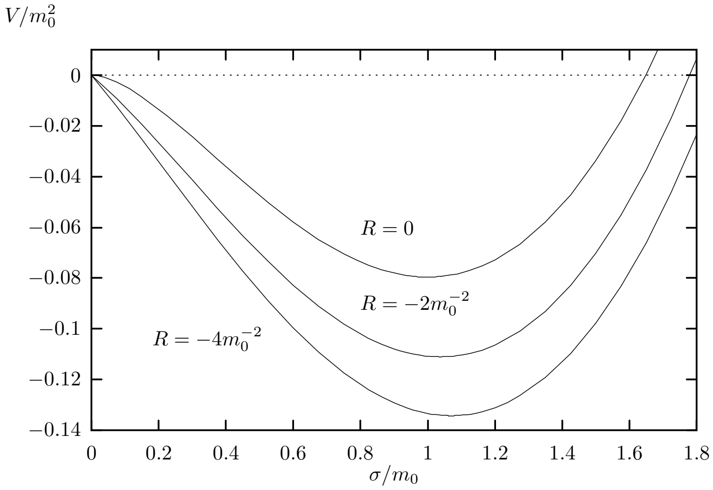
NJL model results in the MFA

de Sitter space

Symmetry breaking weakened — restored like T

Anti de Sitter space

Symmetry breaking strengthened — like B



Effective (1+1)D: Gorbar (1999)

Barriers for QCD research

Too complicated calculations

- □ "Exactly solvable" problems often come with (unnecessarily) too complicated expressions (with special functions having complex arguments etc etc).
- □ Is there any "intuitive" way to understand?

QCD-unfriendly textbook knowledge

- □ Some "standard" techniques established but… there is no introductory guide for QCD physicists.
- □ For example: One-loop effective action is known but not for the QCD physics like chiral phase transition.

For Example

Parker-Toms

$$W = -\frac{i\hbar}{2}\ln\det\left[\ell^2\left(\Box + \frac{1}{4}R + m^2\right)\right]$$

$$\mathbf{divp}W = \frac{\hbar}{\epsilon}(4\pi)^{-n/2}\int dv_x \mathrm{tr}E_{n/2}(x)$$

$$\mathrm{tr}E_2 = \frac{1}{30}\Box R + \frac{1}{72}R^2 - \frac{1}{45}R_{\mu\nu}R^{\mu\nu} - \frac{7}{360}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \frac{1}{3}m^2R + 2m^4.$$

$$2(m^2 + R/12)^2$$

Is this a general result? (The answer is Yes)

If so, how can it be consistent with Chiral Symmetry?

(Some) Technical Details

$$\mathrm{Det}[i\gamma^a e_a^\mu \nabla_\mu - M] = \mathrm{Det}\left[-\underline{\Box} + M^2 + \underline{\frac{R}{4}}\right]$$

Still depends on *R* etc

R-resummed form of the heat kernel (Parker et al)

$$\mathrm{Tr}\, e^{-t[-\partial_\tau^2 - \Delta + M^2 + R/4]}$$

Requiring $g_{tt}=1$

$$= \frac{1}{(4\pi t)^2} e^{-t[-\partial_\tau^2 + M^2 + \underline{R/4 - R/6}]}(1 + \cdots)$$

$R/12$

$R_{\mu\nu}, R_{\mu\nu\sigma\rho}$

Remarks

Scalar curvature dominance for large dimensions

$$\frac{R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}}{R^2} = \frac{2}{D(D-1)}$$

$$\frac{R_{\mu\nu}R^{\mu\nu}}{R^2} = \frac{1}{D}$$

Rigorous for (anti) de Sitter
Generally true for large D

T independently introduced from the geometry

Conformal trans. to g_{tt} =1 in Minkowski
Introducing T in Euclidean

Schwarzchild BH is *R*-flat but after conformal trans. *R*>0

Chiral Invariant "Mass Gap"

$$M^2 \;\to\; M^2 + \frac{R}{12}$$

Consistent with QCD chiral symmetry?

Curvature should not break chiral symmetry…?
Analogous to the thermal mass:

$$iS^{-1} \sim p_0\gamma^0 - \frac{m_T^2}{p_0}\gamma^0 - M$$

Thermal self-energy

Chiral Gap Effect

For concreteness, using the NJL notation:

$$\rho = \underbrace{G^2 [\langle\bar{\psi}\psi\rangle^2}_{M^2} + \langle\bar{\psi}\gamma^5\boldsymbol{\tau}\psi\rangle^2]$$

$$M^2 \;\to\; M^2 + \frac{R}{12}$$ **Chiral variant**

$$\rho \;\to\; \rho + \frac{R}{12G^2}$$ **Chiral invariant**

Effective Potential

In flat space:

$$V = a(T - T_c)\rho + \lambda\rho^2 + \cdots$$

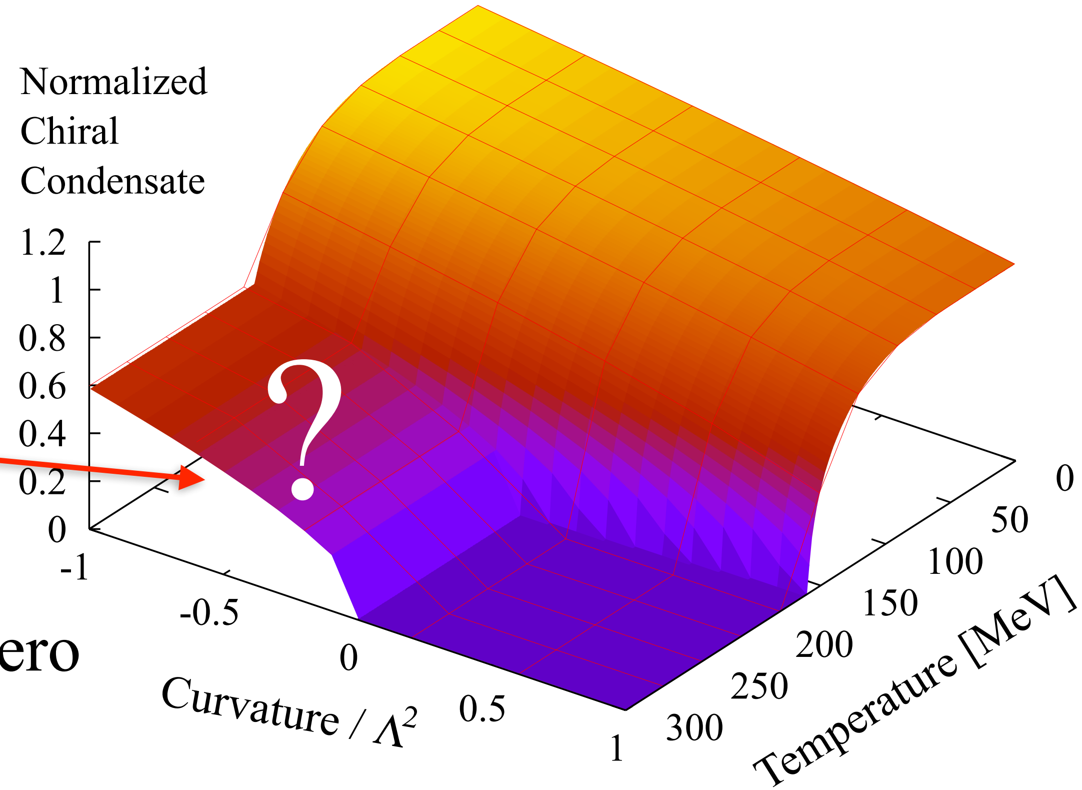
In curved space:

$$V = \left[a(T - T_c) + \frac{\lambda R}{6G^2}\right]\rho + \lambda\rho^2 + \cdots$$

$$T_c^* = T_c - \frac{\lambda R}{6G^2 a}$$

Qualitatively explains "solvable" calculations for dS and AdS

Puzzle?



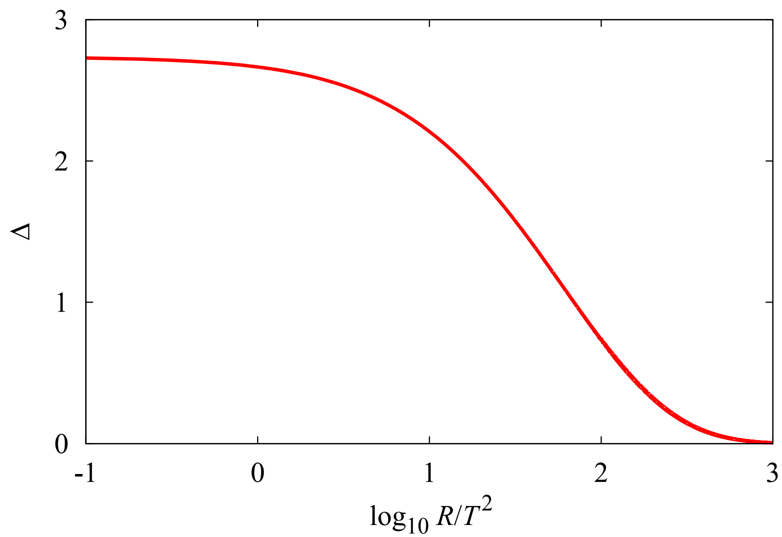
Tachyonic?
Needs some new "condensate"?
Stabilized by non-zero thermal mass?

Implication to Deconfinement

Explicit center symmetry breaking:

$$\beta\Omega^T_{\rm loop} = -2N_{\rm f}\,V\int\frac{d^3p}{(2\pi)^3}\,{\rm Tr}\Big[\ln\big(1+L\,e^{-\beta(\varepsilon_p-\mu)}\big)+\ln\big(1+L^\dagger\,e^{-\beta(\varepsilon_p+\mu)}\big)\Big]$$

$$\Delta \equiv (\beta^4/V)\big(\Omega^T_{\rm loop}[\Phi=1]-\Omega^T_{\rm loop}[\Phi=-1]\big)$$



Mass-gap suppresses center-symmetry breaking

If R is large, QCD approaches more like a pure YM theory

1st order?
(Needs lattice simulation)

Particle Production

Parker (1965)
Hawking radiation (1974)
Unruh (1976)

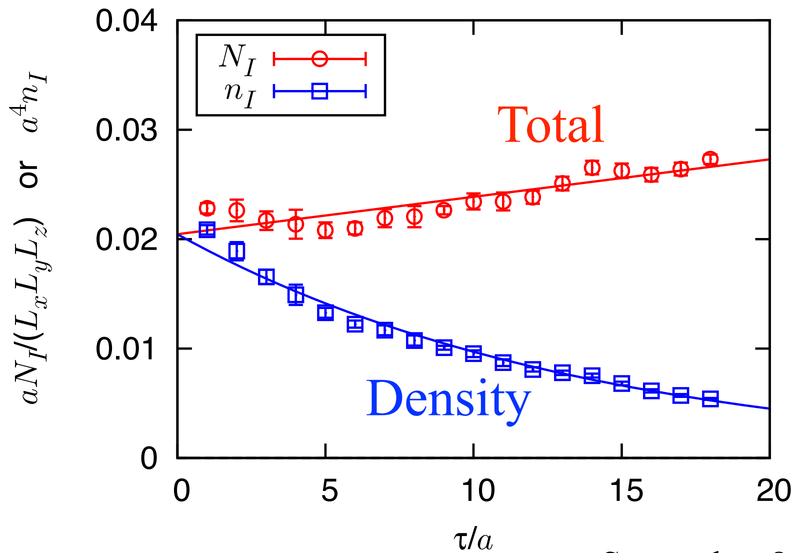
$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$$

$$a(\pm\infty) \sim a_< \ \leftrightarrow \ a_>$$ (time scale ~s)

Produced particle spectrum: $n_k \sim 1/(e^{4\pi s a_<^2 k} - 1)$

(massless scalar)

A. Yamamoto: 1405.6665



Not easy to do the simulation to see the QCD phase transitions

Chiral properties control the mass that affects the production rate

Work in progress (w Morales)

Difficulties in Lattice

- **How to avoid coordinate singularities?**
  - For a simple problem of homogeneous R or the BH problem, the choice of the polar coordinates is useful, but it has a singularity at the origin…
- **How to formulate light fermions?**
  - Staggered fermion is the simplest way, but cannot be justified in curved space (due to spin connections).
- **How to perform renormalization?**
  - Lattice spacing depends on the geometry just like the anisotropic lattice at finite T. Negligible for weakly curved cases, but indispensable for phase transitions.

Summary and Extensions

- **Chiral symmetric mass-gap is possible in curved space — Chiral Gap Effect.**

- **Where gravity is strong, QCD approaches more like a pure YM theory (which is good?)**

- **Applications to HIC, astrophysics, and condensed matter physics coming soon!**
  - *QCD-wall near the BH horizon* (Flachi)
  - *QCD origin of Dark Energy* (Zhitnitsky)

- **Lattice QCD in gravity is a new direction to go.**

New Era of: