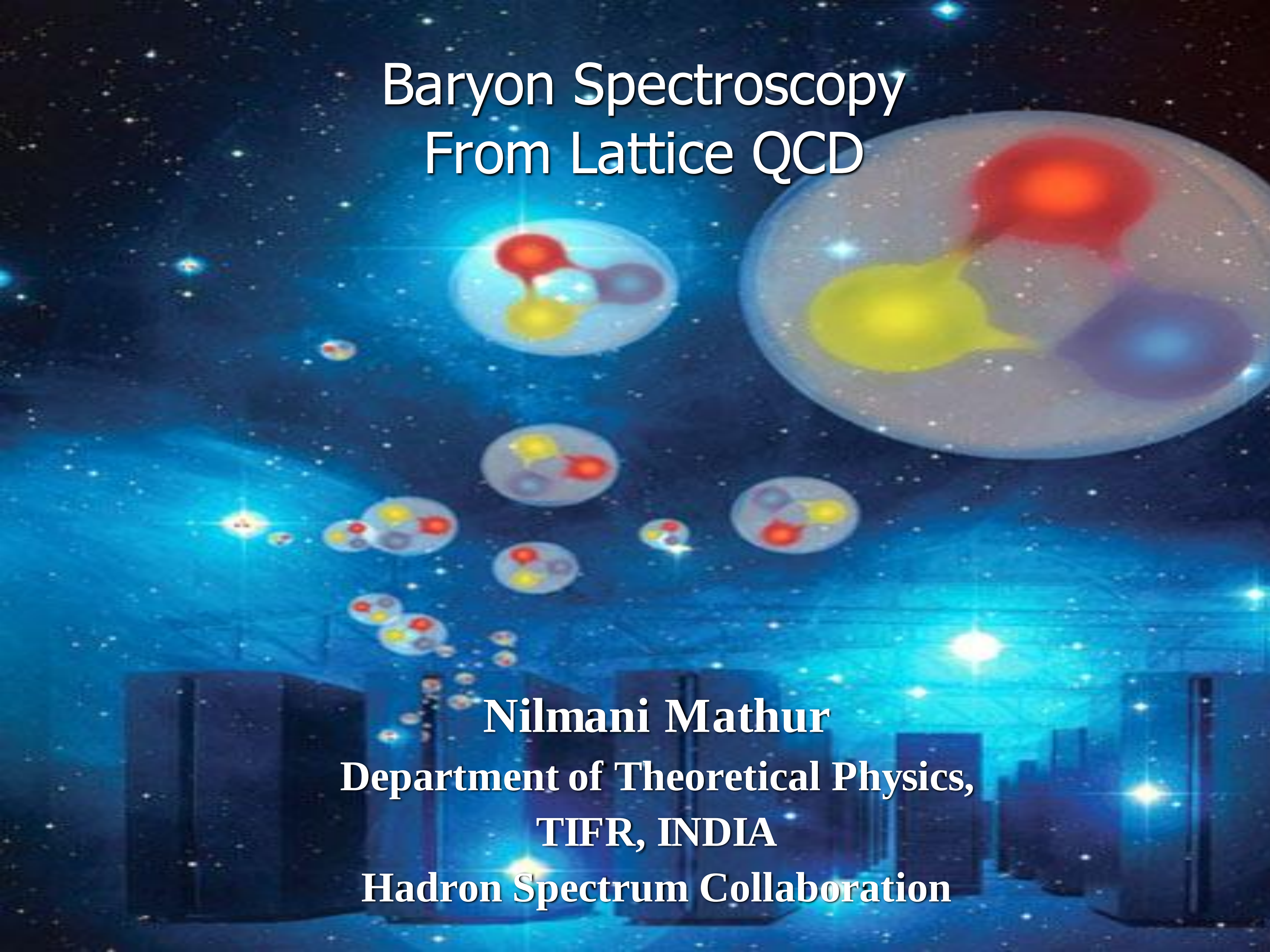

Baryon Spectroscopy From Lattice QCD

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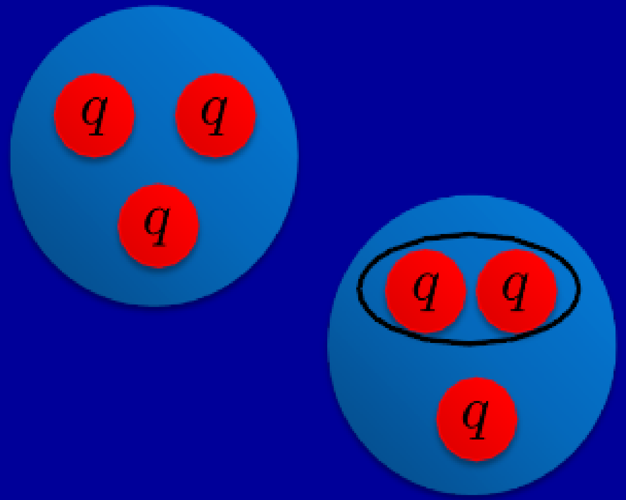
Hadron Spectrum Collaboration

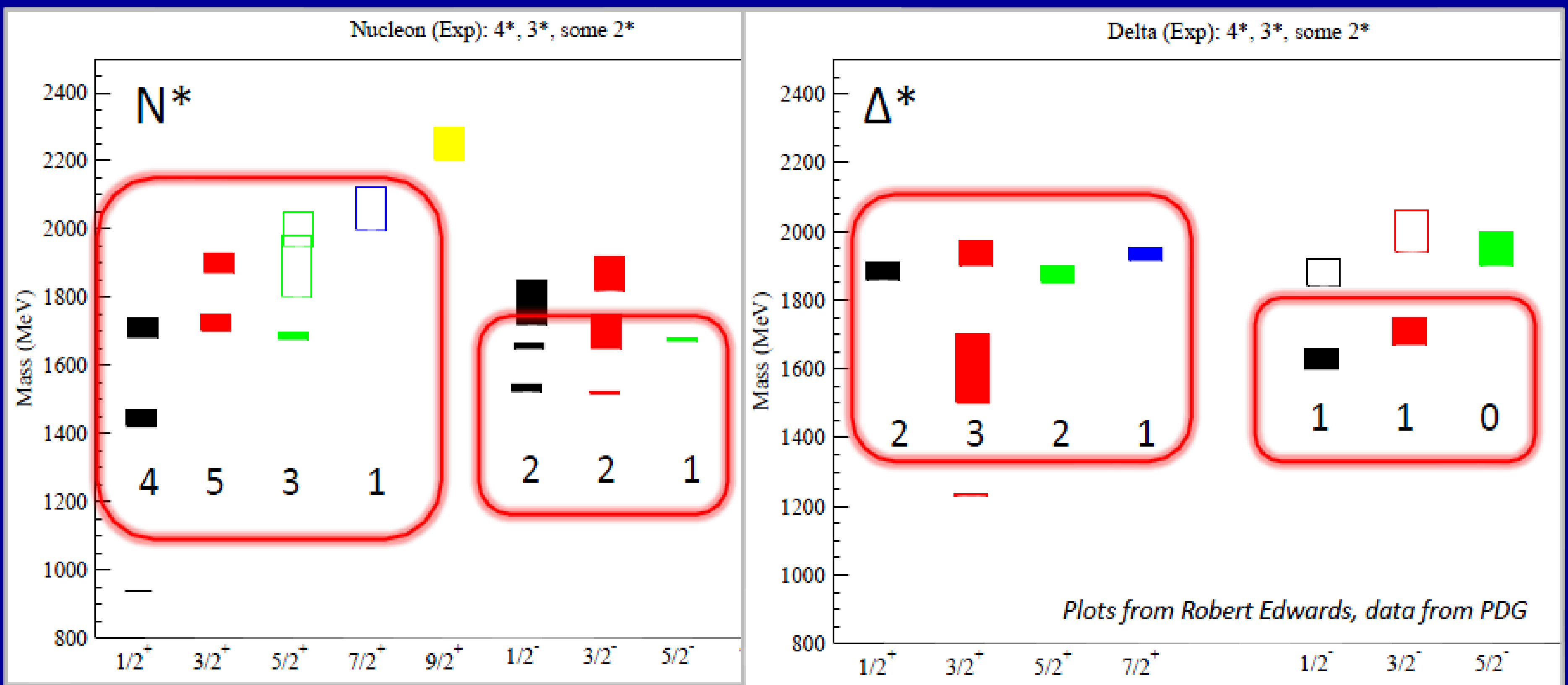
Baryons

- Light (nucleon, delta,…)
- Strange (Cascade, Lambda,…)
- Heavy (Charm, Bottom)

Hadron Spectroscopy – Baryons

- Missing states?
- ‘Freezing’ of degrees of freedom?
- Gluonic excitations?
- Flavour structure

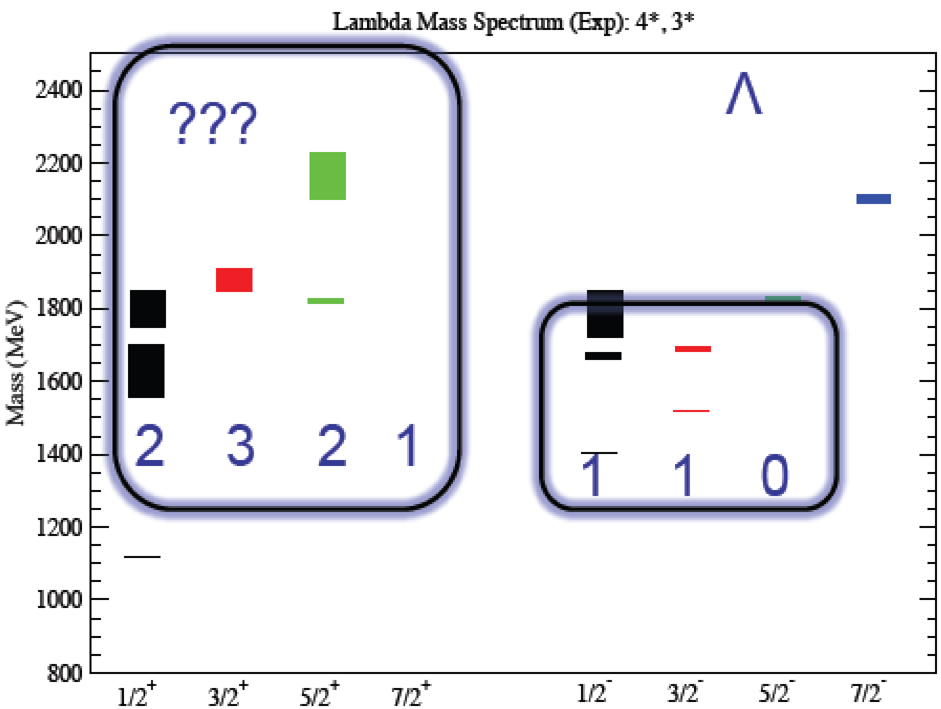


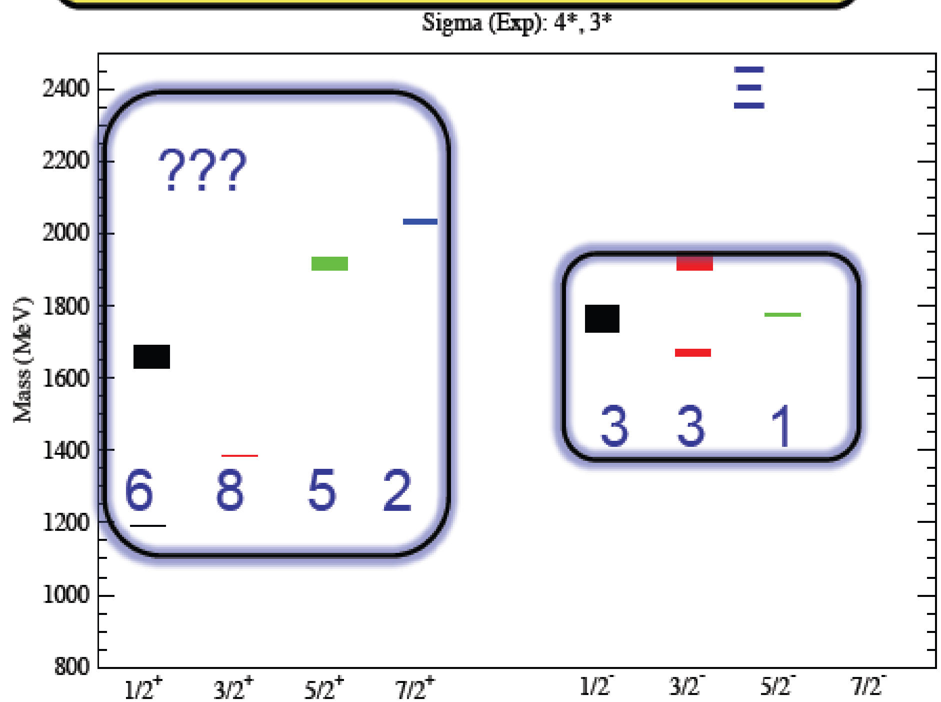


Strange Quark Baryon Spectrum

Strange quark baryon spectrum even sparser

Since SU(3) flavor symmetry broken, expect mixing of $\mathbf{8}_F$ & $\mathbf{10}_F$

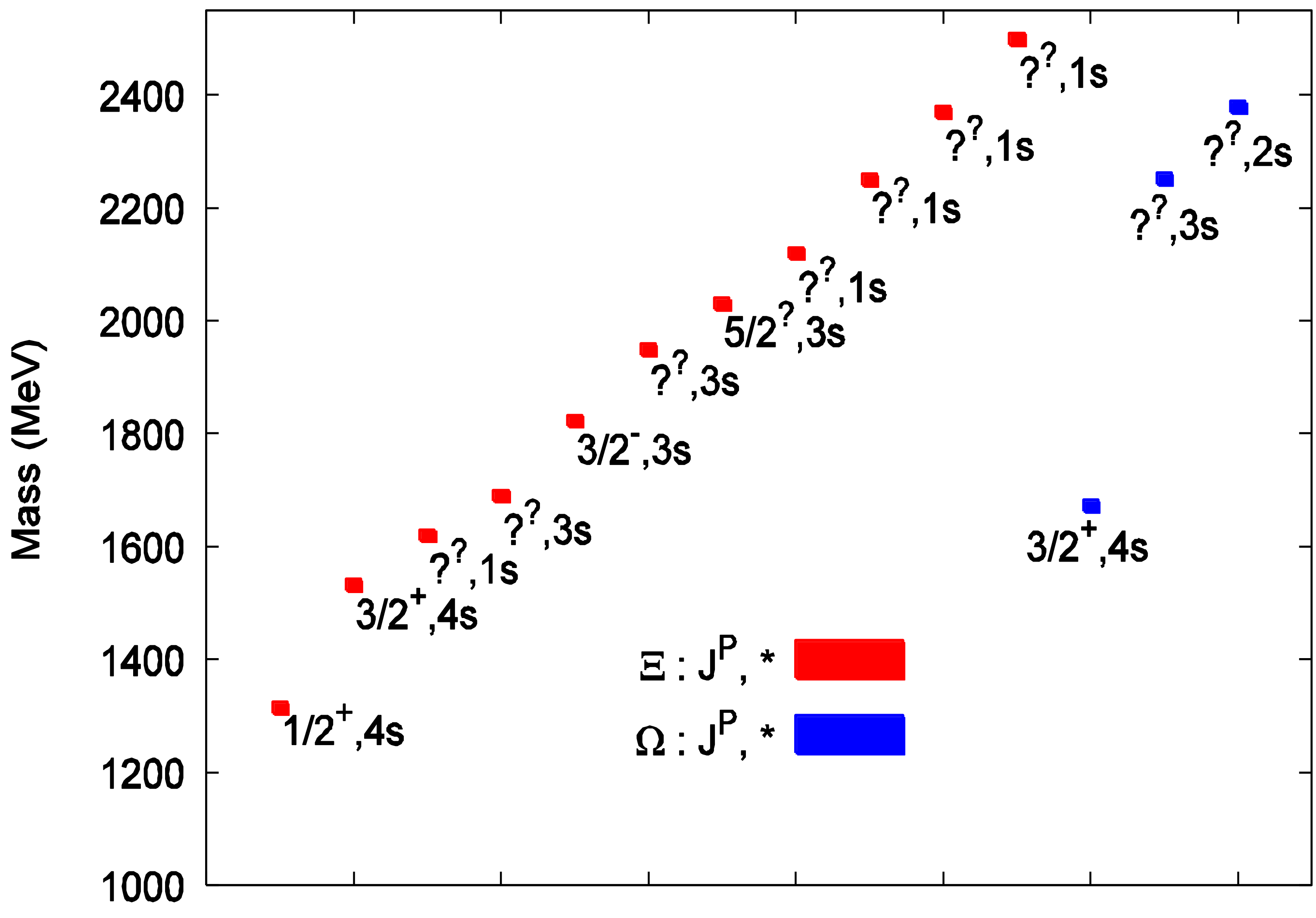


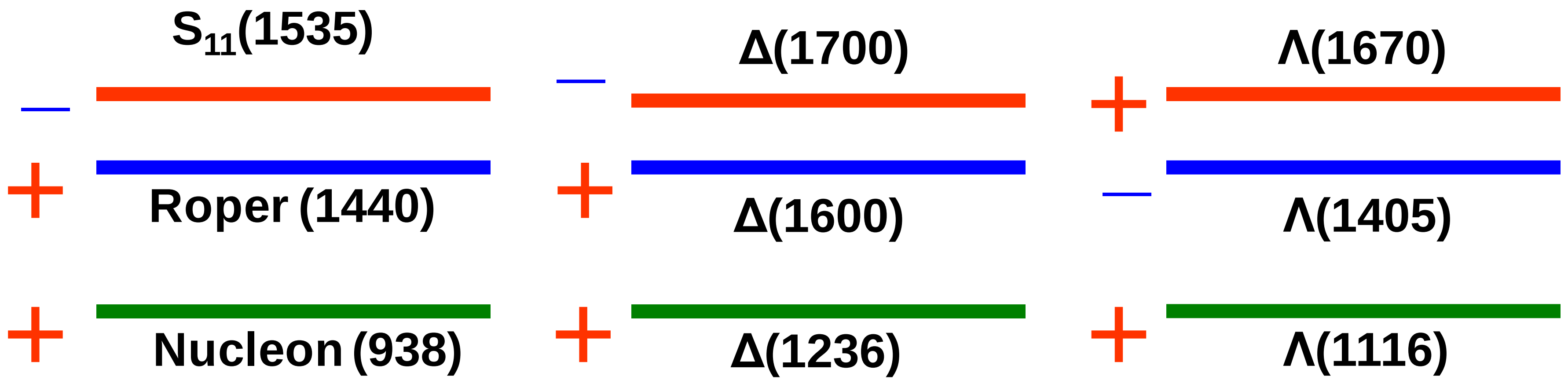


Even less known states in Ξ & Ω

@Edwards

CASCADE Spectra





Hyperfine Interaction of quarks in Baryons

$$\lambda_c^1 \cdot \lambda_c^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

..Isgur

Color-Spin Interaction

Excited positive > Negative

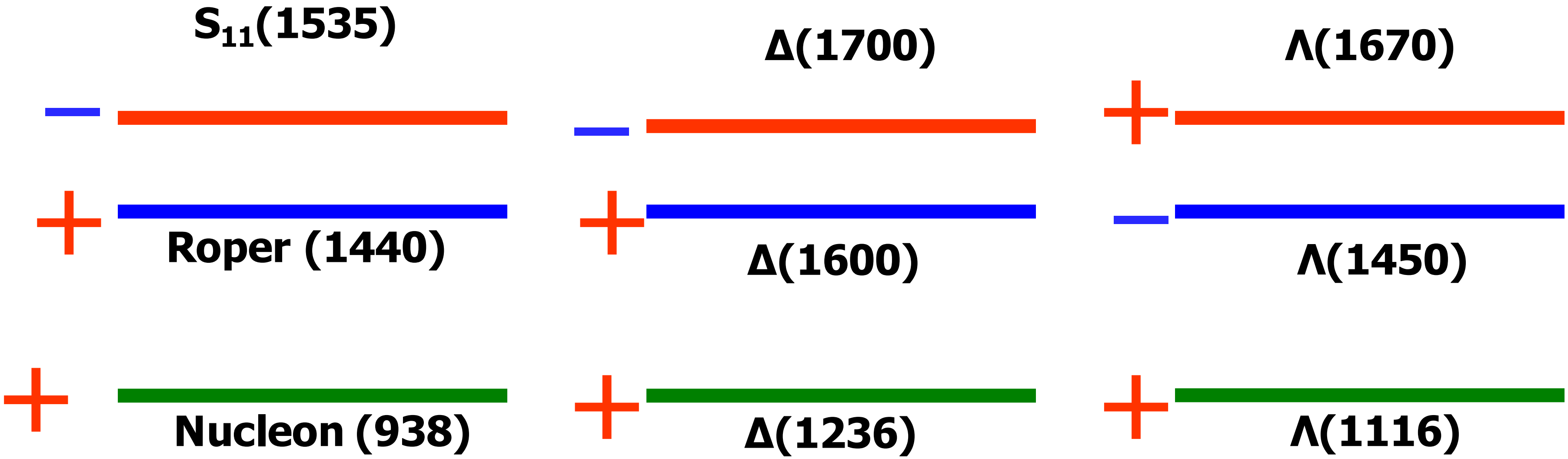
$$\lambda_F^1 \cdot \lambda_F^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

Glozman & Riska
Phys. Rep. 268,263 (1996)

Flavor-Spin interaction

Chiral symmetry plays major role

Negative > Excited positive



What is the structure of these resonance states, for example,

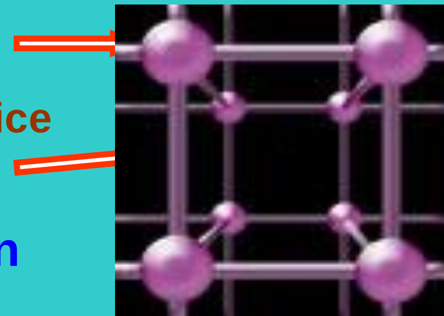
Roper ((1440) $1/2^+$) resonance?

Radial excitation? $q^4\underline{q}$ state?

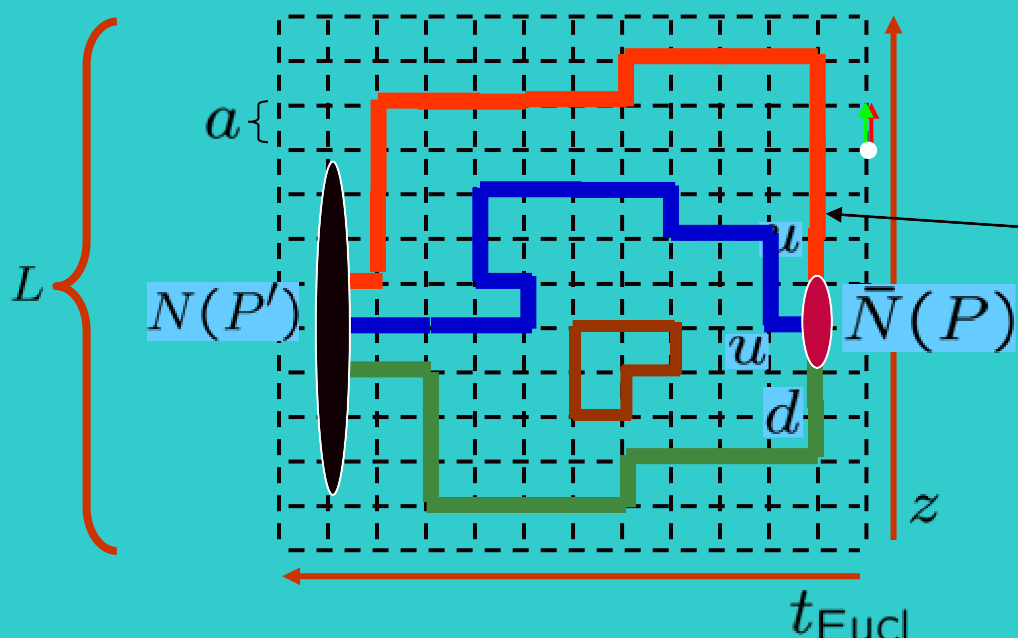
- **Hybrid state (qqq*g*)?**
- **Dynamical meson-baryon state?**

What is the structure of Λ (1405)?

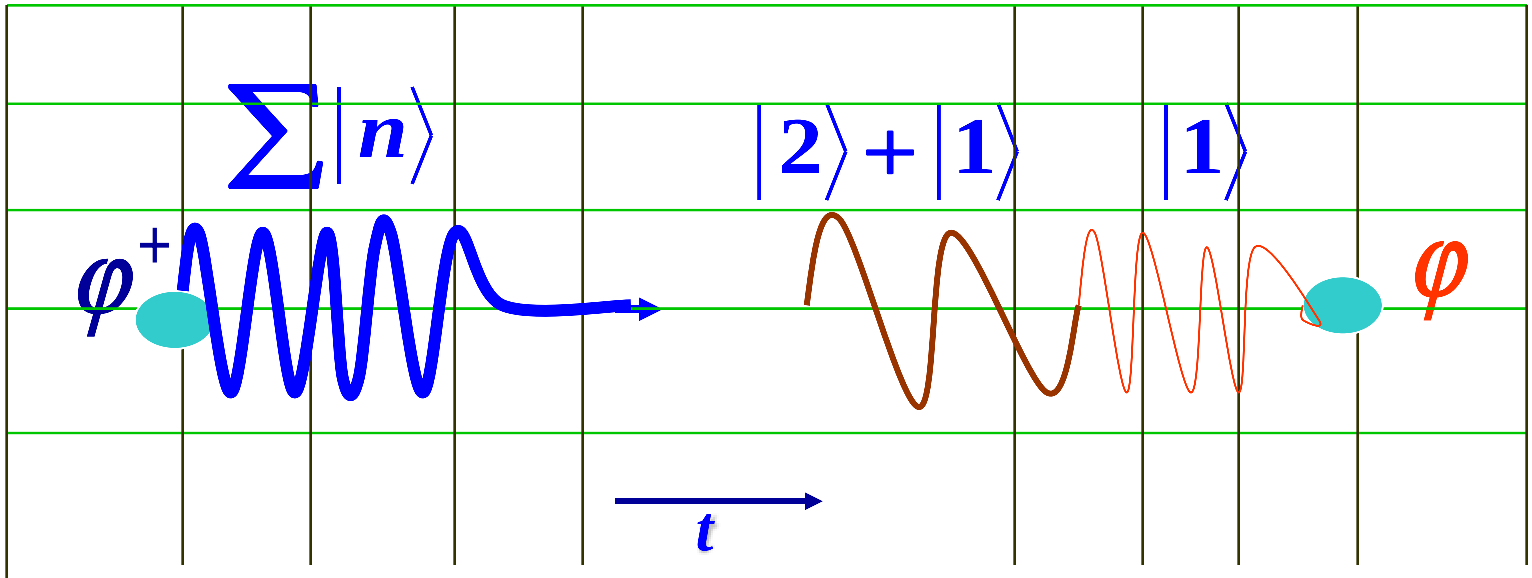
Dynamical meson-baryon state ? Fivequark state?



$$<\hat{O}> = \frac{\int DU \{\det D\}^{n_f}\, O[U, D^{-1}]\, e^{-S_g[U]}}{\int DU\, \{\det D\}^{n_f}\, e^{-S_g[U]}} = \prod_n \int dU_n \frac{1}{Z} \{\det D(U)\}^{n_f}\, e^{-S_g[U]}\, O[U, D^{-1}]$$



quark propagators :
Inverse of very large matrix of space-time, spin and color



$$\varphi(t) = e^{Ht}\varphi(0)e^{-Ht}$$

$$G(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}.(\vec{x}-\vec{x}_0)} \sum_{n,\vec{q}} \langle 0|\varphi(x)|n,\vec{q}\rangle\langle n,\vec{q}|\varphi(x_0)|0\rangle$$

$$= \sum_{\vec{x}} e^{-i\vec{p}.(\vec{x}-\vec{x}_0)} \sum_{n,\vec{q}} \langle 0|e^{H(t-t_0)-i\vec{p}'.(\vec{x}-\vec{x}_0)}\varphi(x_0)e^{-H(t-t_0)+i\vec{p}'.(\vec{x}-\vec{x}_0)}|n,\vec{q}\rangle\langle n,\vec{q}|\varphi(x_0)|0\rangle$$

$$= \sum_{\vec{x}} e^{-i\vec{p}.(\vec{x}-\vec{x}_0)} \sum_{n,\vec{q}} e^{i\vec{q}.(\vec{x}-\vec{x}_0)-E_q^n(t-t_0)} \langle 0|\varphi(x_0)|n,\vec{q}\rangle\langle n,\vec{q}|\varphi(x_0)|0\rangle$$

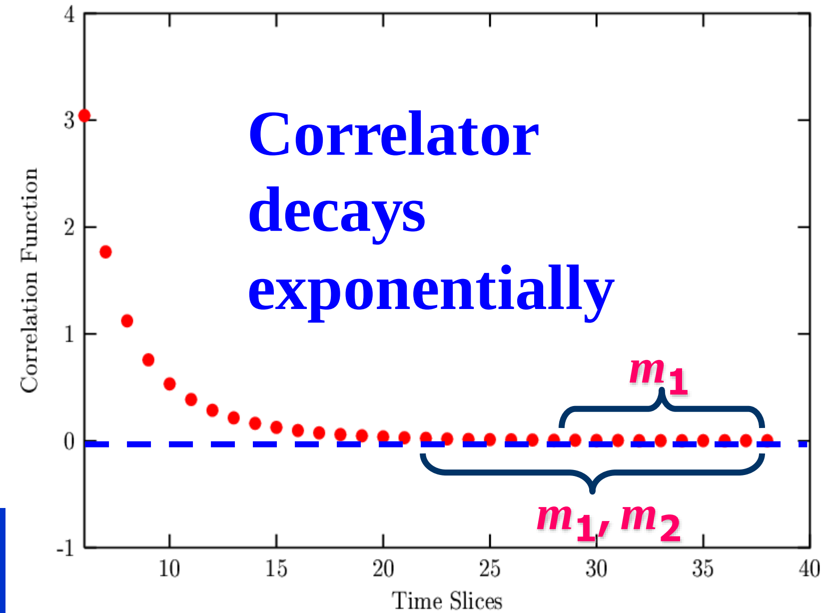
$$\approx \sum_{n,\vec{q}} \delta(\vec{p}-\vec{q}) e^{i(\vec{p}-\vec{q}).\vec{x}_0-E_q^n(t-t_0)} \langle 0|\varphi(x_0)|n,\vec{q}\rangle\langle n,\vec{q}|\varphi(x_0)|0\rangle$$

$$= \sum_{n} e^{-E_p^n(t-t_0)} \left|\langle 0|\varphi(x_0)|n,\vec{p}\rangle\right|^2$$

$$= \sum_{n} W_n e^{-E_p^n(t-t_0)} \xrightarrow[t\to\infty]{} W_1 e^{-E_1^n(t-t_0)}$$

Analysis (Extraction of Mass)

$$G(\tau) = \sum_{i=1}^{N} W_i\, e^{-m_i \tau} \underset{\tau\to\infty}{\approx} W_1\, e^{-m_1 \tau}$$



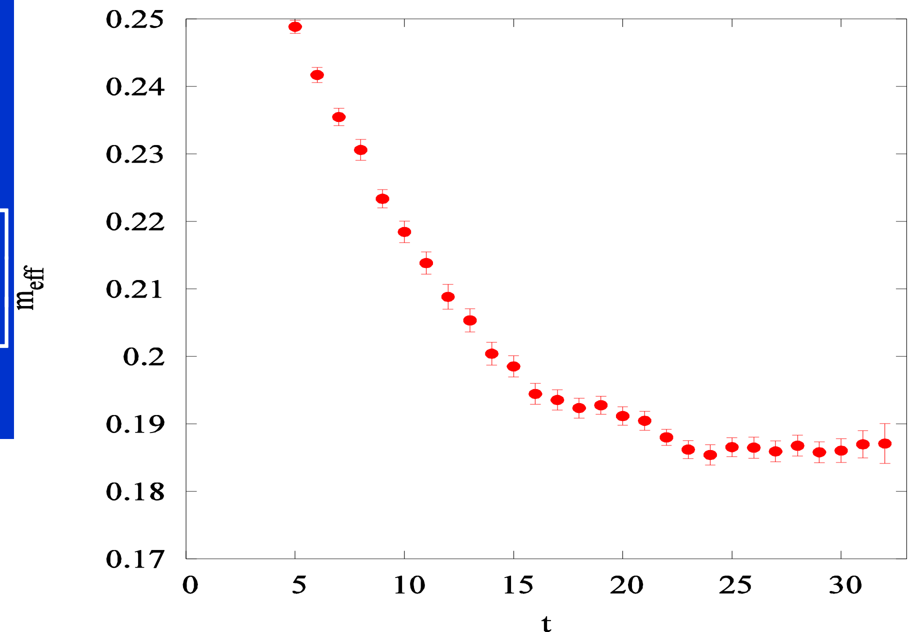
Effective mass :

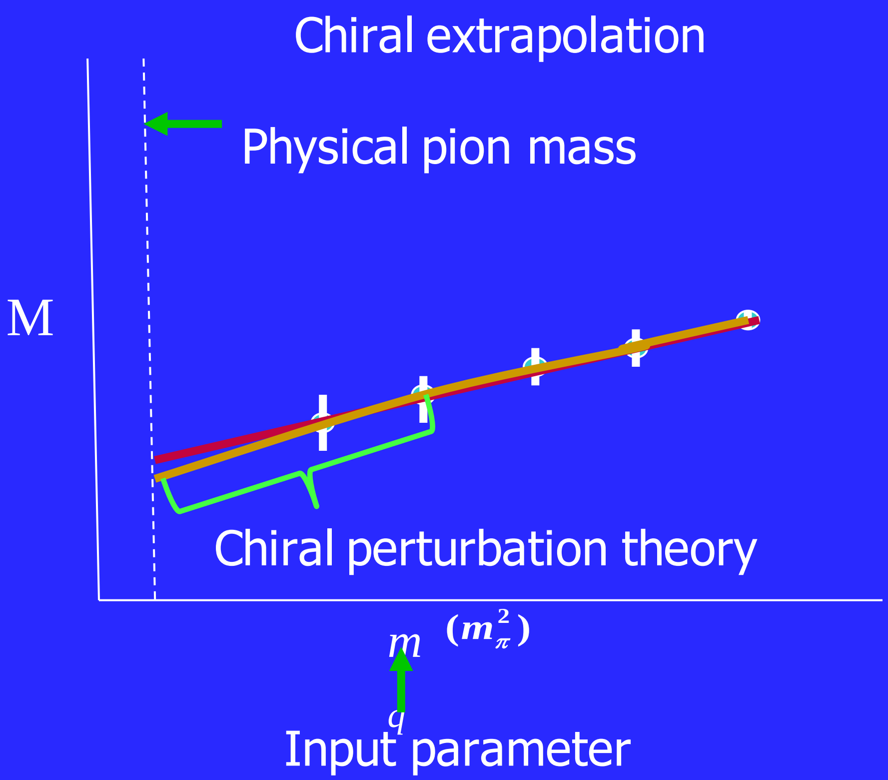
$$\frac{G(\tau)}{G(\tau+1)} = e^{-m_1\tau + m_1(\tau+1)}$$

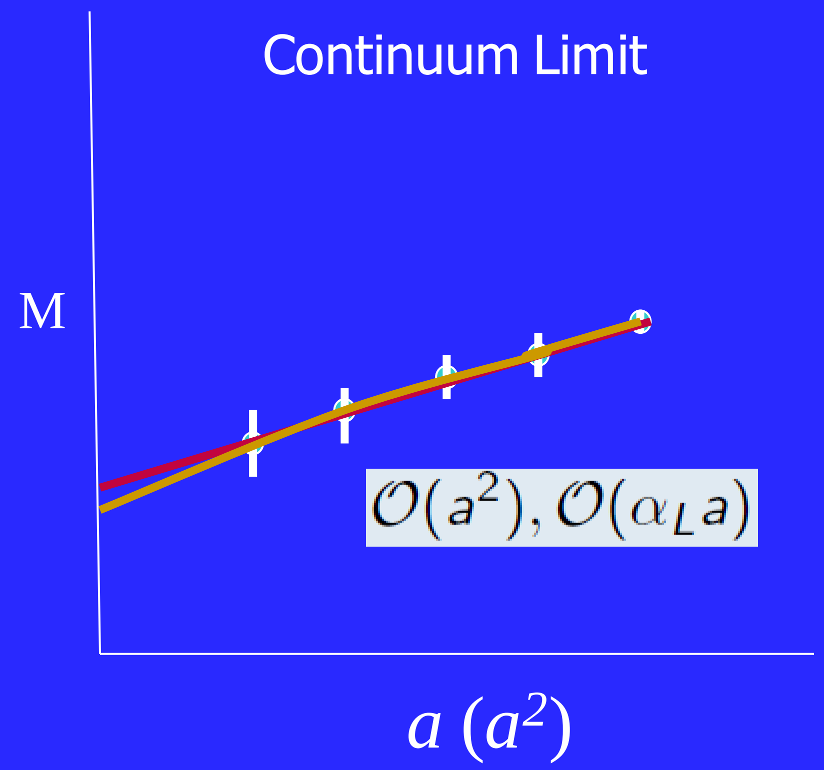
$$m(\tau) = \ln\left[\frac{G(\tau)}{G(\tau+1)}\right]$$

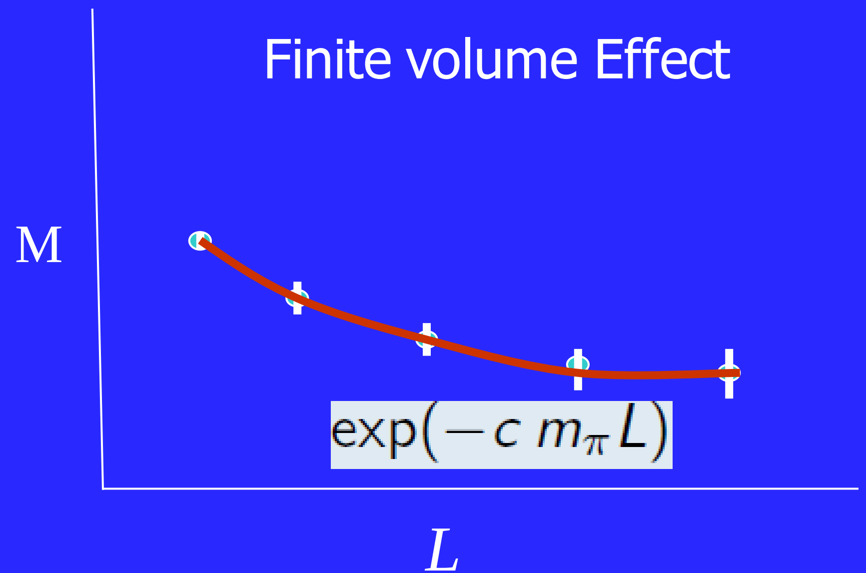
$$= \ln\left[\frac{|w_1|^2\, e^{-E_1\tau} + |w_2|^2\, e^{-E_2\tau} + \ldots}{|w_1|^2\, e^{-E_1(\tau+a_\tau)} + |w_2|^2\, e^{-E_2(\tau+a_\tau)} + \ldots}\right]$$

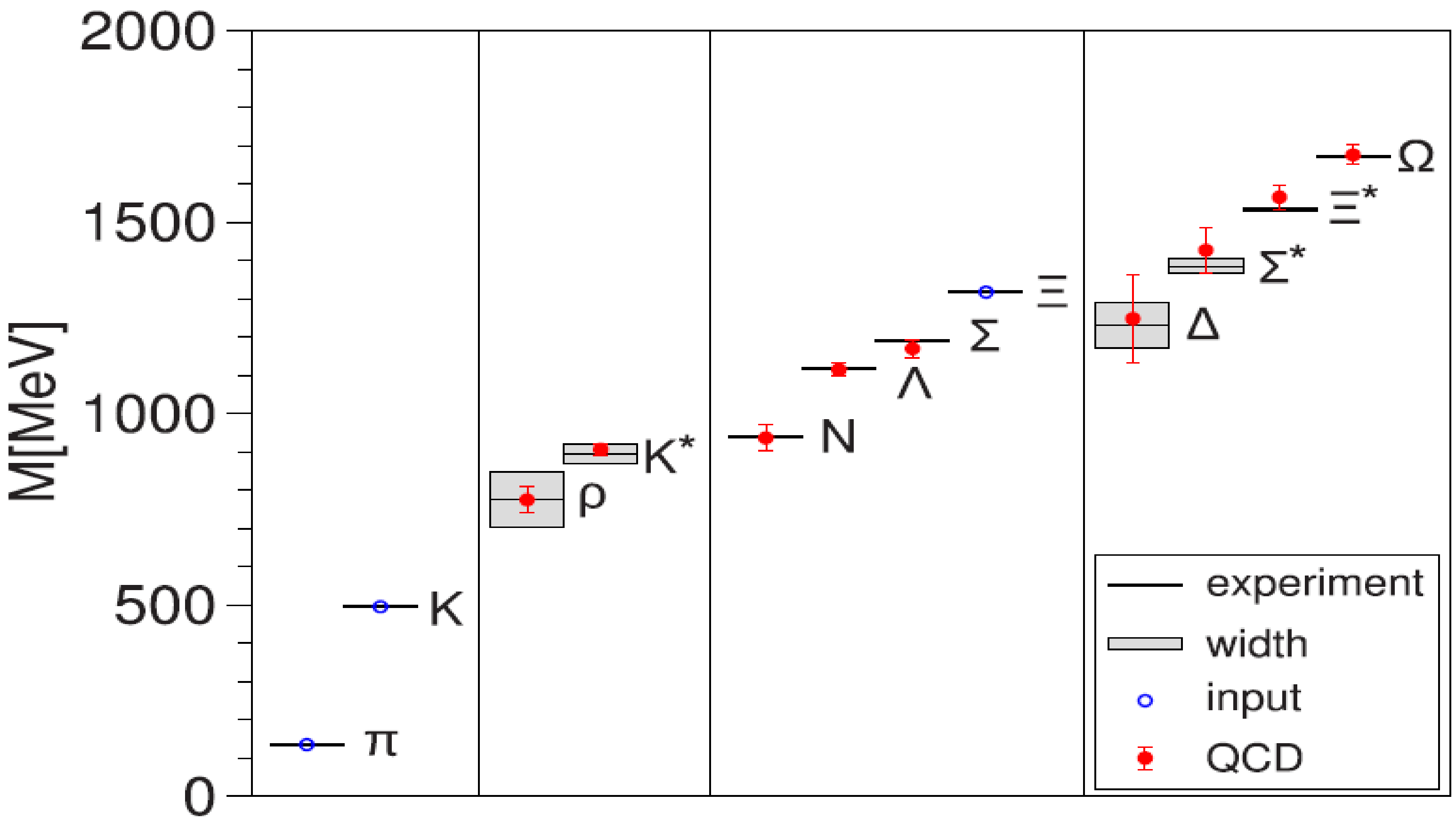
$$\approx a_\tau E_1 [1 + \vartheta(|w_2|^2 / |w_1|^2\, e^{(E_2-E_1)\tau/a_\tau})$$




Chiral extrapolation
Physical pion mass
M
Chiral perturbation theory
m (m_π^2)
q
Input parameter


Continuum Limit
M
O(a^2), O(α_L a)
a (a^2)


Finite volume Effect
M
exp(−c m_π L)
L

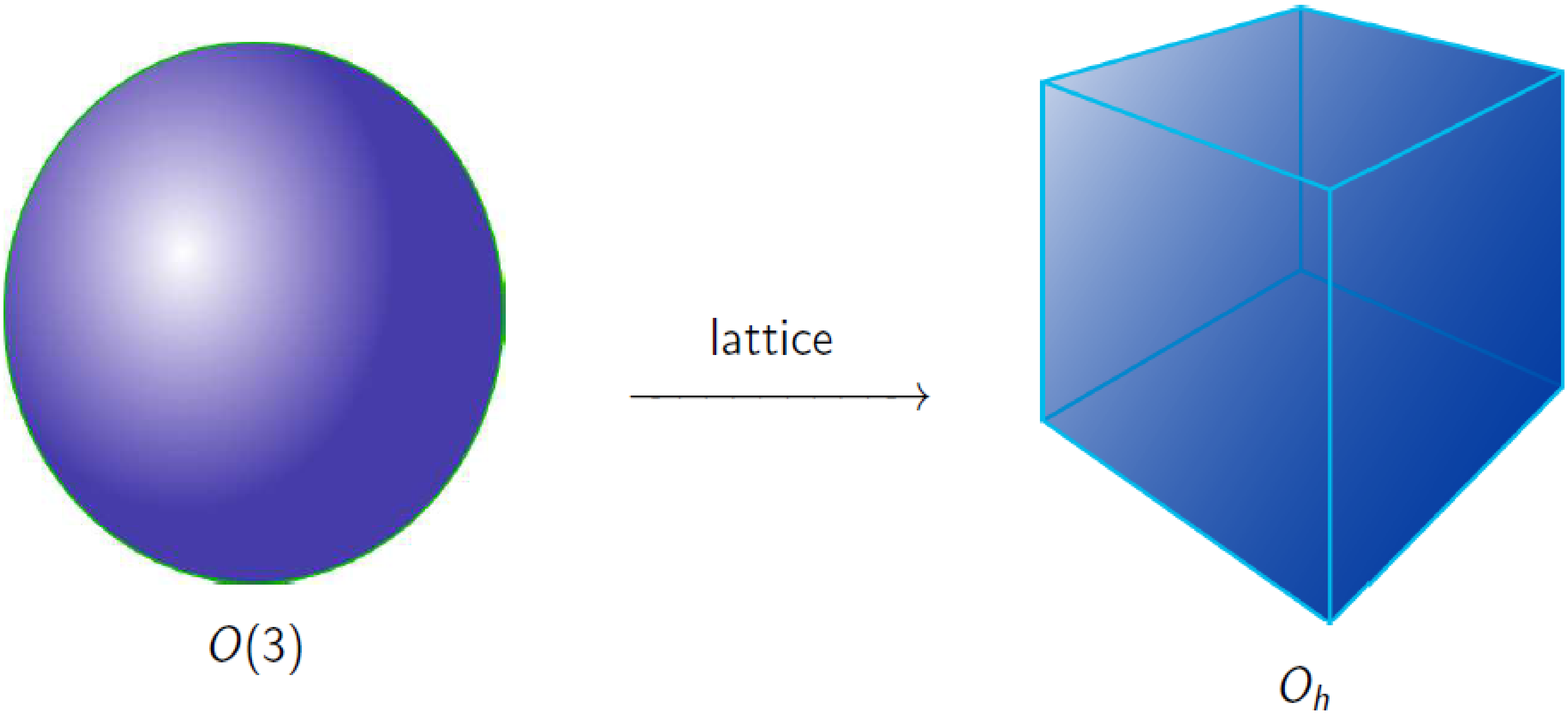


S.Durr et.al, Science 322, 1224 (2008)

Hadron Spectrum Collaboration

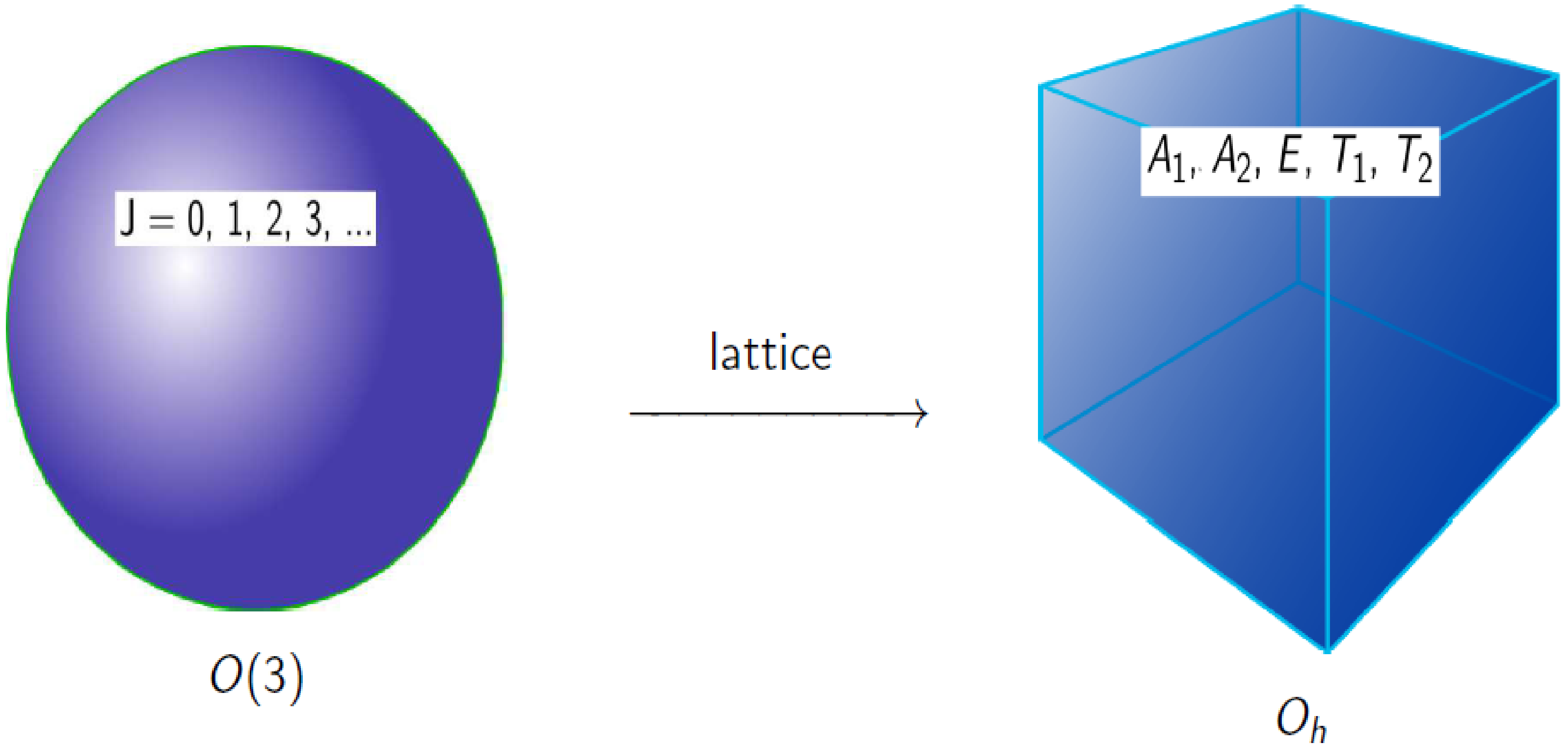
Jefferson Lab, Univ. of Cambridge, Maryland, CMU, Tata Institute, Trinity College

Continuum → Lattice : Symmetries



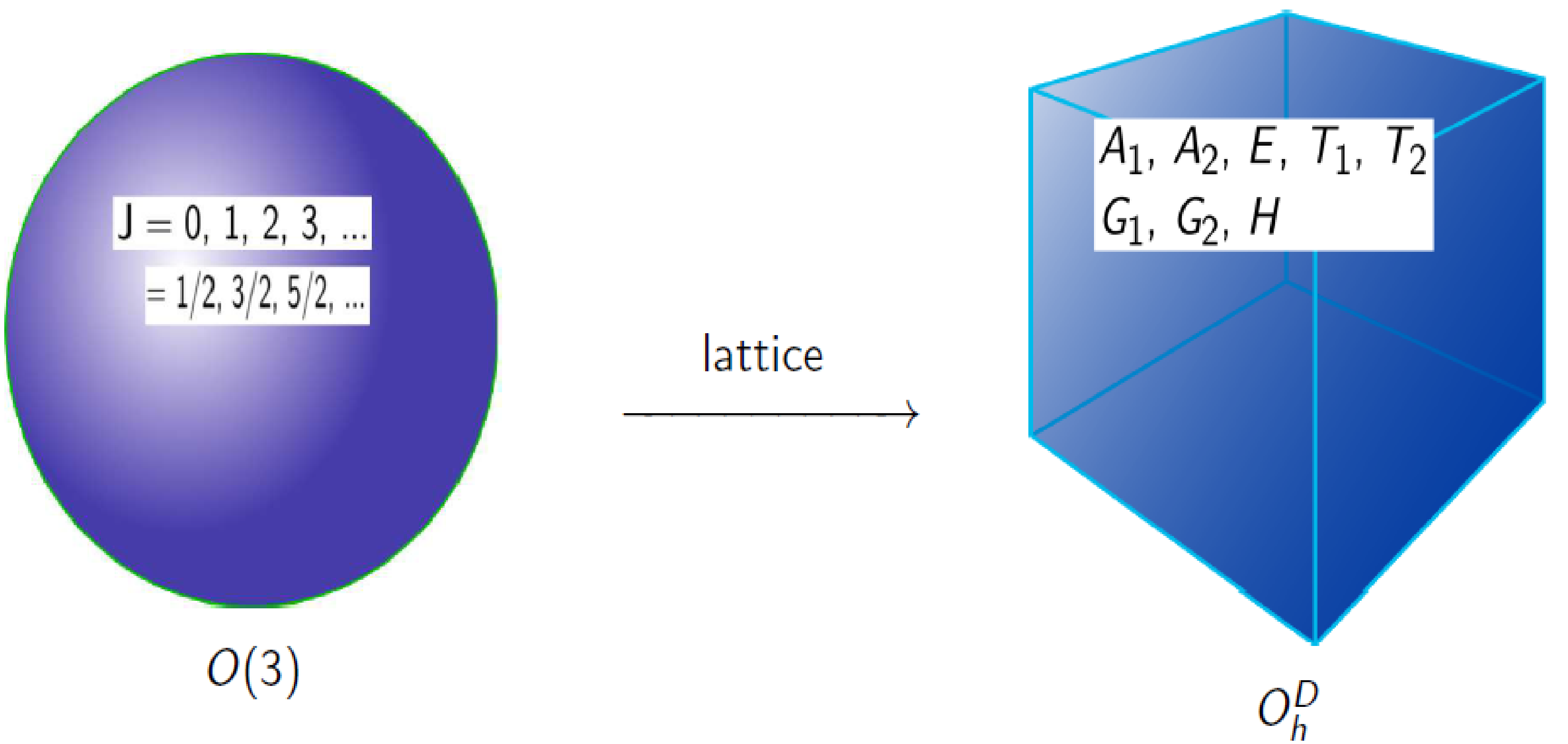
- Eigenstates of lattice Hamiltonian transform under irreps, Λ^n, of O_h.
- Continuum states with same J^P but different J_z : seperated across different lattice irreps.
- Subduce the continuum operators into the irreps of O_h.

Continuum → Lattice : Irreps (1)



- Integer spin objects see an O_h symmetry on lattice.

Continuum → Lattice : Irreps (2)



- Half-integer spin objects see an O_h^D symmetry on lattice.

Octahedral group and lattice operators

Construct operator which transform irreducibly under the symmetries of the lattice

Λ	J
A_1	0⊕4⊕6⊕8 …
A_2	3⊕6⊕7⊕9 …
E	2⊕4⊕5⊕6 …
T_1	1⊕3⊕4⊕5 …
T_2	2⊕3⊕4⊕5 …

	A_1	A_2	E	T_1	T_2
$J=0$	1				
$J=1$				1	
$J=2$			1		1
$J=3$		1		1	1
$J=4$	1		1	1	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Meson

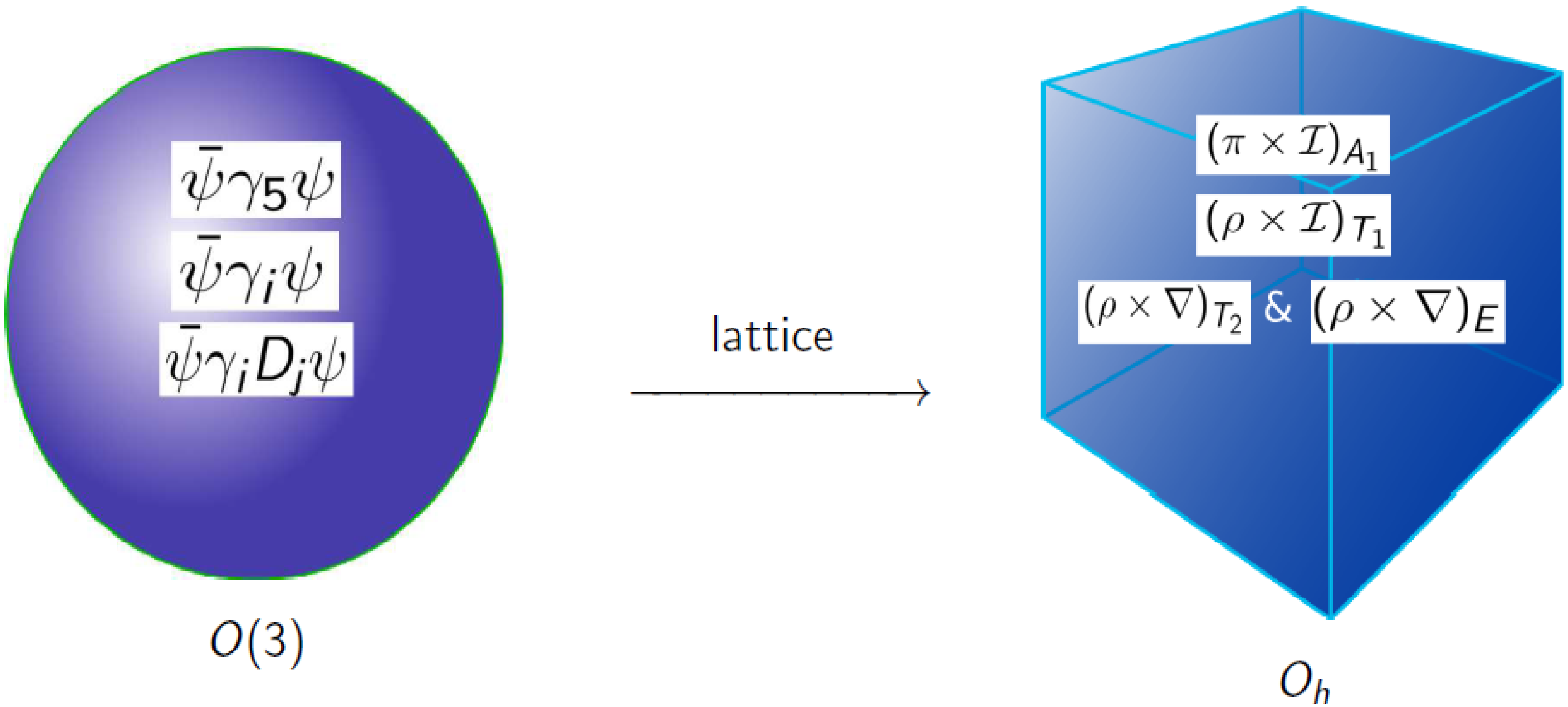
…R.C. Johnson, Phys. Lett.B 113, 147(1982)

Λ	J
G_1	1/2⊕7/2⊕9/2⊕11/2 …
G_2	5/2⊕7/2⊕11/2⊕13/2 …
H	3/2⊕5/2⊕7/2⊕9/2 …

J	G_1	G_2	H
$\frac{1}{2}$	1	0	0
$\frac{3}{2}$	0	0	1
$\frac{5}{2}$	0	1	1
$\frac{7}{2}$	1	1	1
$\frac{9}{2}$	1	0	2
$\vdots$	$\vdots$	$\vdots$	$\vdots$

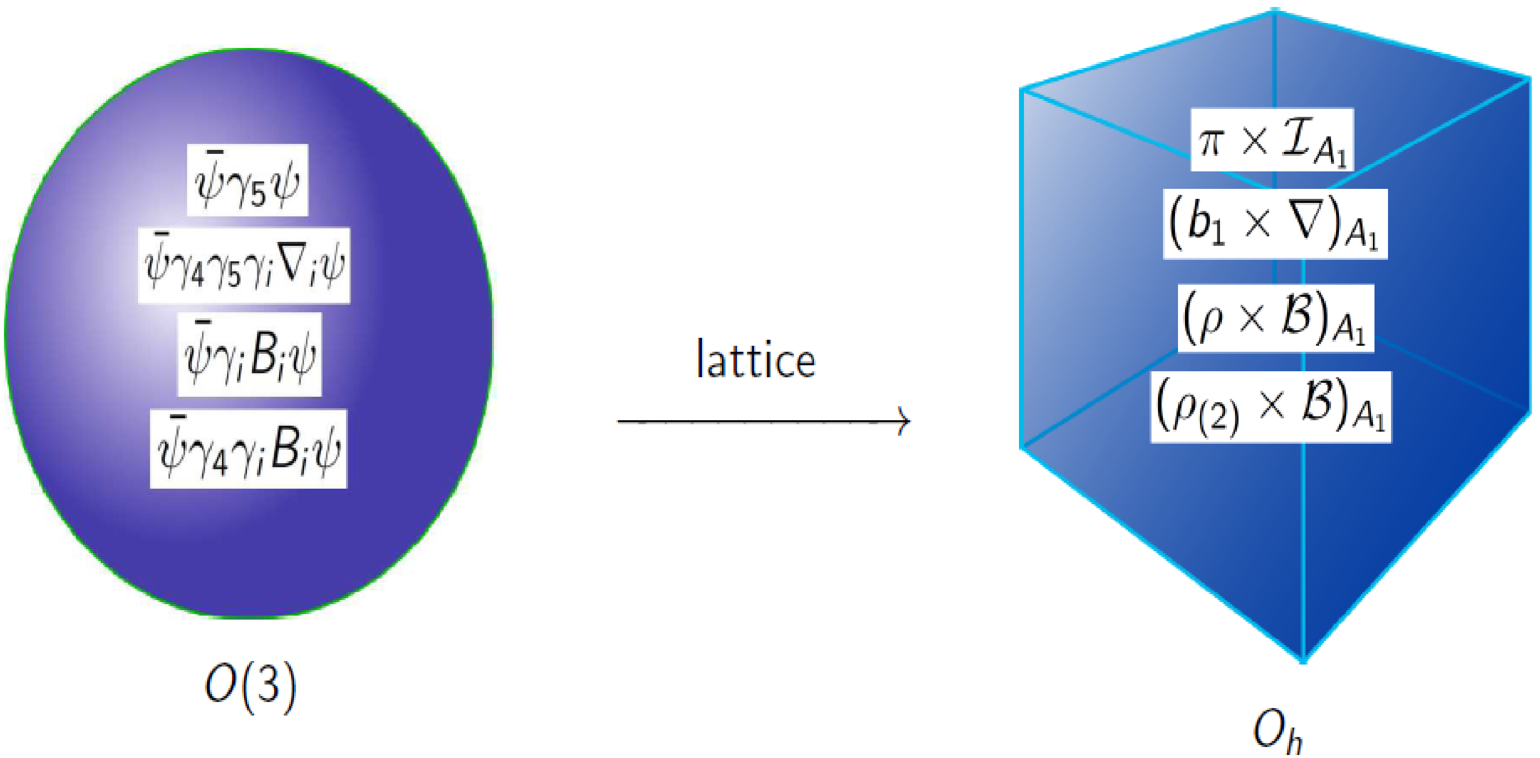
Baryon

Continuum → Lattice : Operators (1)



- Operators in the continuum get distributed over the lattice irreps.

Continuum → Lattice : Operators (2)



- Multiple continuum operators with various spin-spatial structures reducing onto same lattice irreps with varying lattice extensions : Excited states.

Spectroscopy : baryon operator construction

- Aim : Extraction of highly excited states.
  Local operators → low lying states.
  Extended operators → States with radial and orbital excitations.

- Proceeds in two steps
  Construct continuum operators with well defined quantum nos.
  Reduce/subduce into the irreps of the reduced symmetry.

- Used set of baryon continuum operators of the form
  $\Gamma^{\alpha\beta\gamma} q^\alpha q^\beta q^\gamma$, $\Gamma^{\alpha\beta\gamma} q^\alpha q^\beta (D_i q^\gamma)$ and $\Gamma^{\alpha\beta\gamma} q^\alpha q^\beta (D_i D_j q^\gamma)$

- Excluding the color part, the flavor-spin-spatial structure

$$O^{[J^P]} = [\mathcal{F}_{\Sigma_F} \otimes \mathcal{S}_{\Sigma_S} \otimes \mathcal{D}_{\Sigma_D}]^{J^P}.$$

- γ-matrix convention : $\gamma_4 = \text{diag}[1,1,-1,-1]$;
  Non-relativistic → purely based on the upper two component of q.
  Relativistic → All operators except non-relativistic ones.

- Subset of $D_i D_j$ operators that include $[D_i, D_j] \sim F_{ij}$ → hybrid.

HSC : Phys.Rev. D84, 074508 (2011), 1104.5152

NR Ops with one Deri-vative

$SU(3)_F$	S	L	J^P		
$\mathbf{8_F}$	$\frac{1}{2}$	1	$\frac{1}{2}^-$	$\frac{3}{2}^-$	
	$\frac{3}{2}$	1	$\frac{1}{2}^-$	$\frac{3}{2}^-$	$\frac{5}{2}^-$
$N_8(J)$			**2**	**2**	**1**
$\mathbf{10_F}$	$\frac{1}{2}$	1	$\frac{1}{2}^-$	$\frac{3}{2}^-$	
$N_{10}(J)$			**1**	**1**	**0**
$\mathbf{1_F}$	$\frac{1}{2}$	1	$\frac{1}{2}^-$	$\frac{3}{2}^-$	
$N_1(J)$			**1**	**1**	**0**

NR Ops with two Deri-vatives

$SU(3)_F$	S	L	J^P			
$\mathbf{8_F}$	$\frac{1}{2}$	0	$\frac{1}{2}^+$			
	$\frac{1}{2}$	0	$\frac{1}{2}^+$			
	$\frac{1}{2}$	1	$\frac{1}{2}^+$	$\frac{3}{2}^+$		
	$\frac{1}{2}$	2		$\frac{3}{2}^+$	$\frac{5}{2}^+$	
	$\frac{1}{2}$	2		$\frac{3}{2}^+$	$\frac{5}{2}^+$	
	$\frac{3}{2}$	0		$\frac{3}{2}^+$		
	$\frac{3}{2}$	2	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$	$\frac{7}{2}^+$
$N_8(J)$			**4**	**5**	**3**	**1**
$\mathbf{10_F}$	$\frac{1}{2}$	0	$\frac{1}{2}^+$			
	$\frac{1}{2}$	2		$\frac{3}{2}^+$	$\frac{5}{2}^+$	
	$\frac{3}{2}$	0		$\frac{3}{2}^+$		
	$\frac{3}{2}$	2	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$	$\frac{7}{2}^+$
$N_{10}(J)$			**2**	**3**	**2**	**1**
$\mathbf{1_F}$	$\frac{1}{2}$	0	$\frac{1}{2}^+$			
	$\frac{1}{2}$	2		$\frac{3}{2}^+$	$\frac{5}{2}^+$	
	$\frac{3}{2}$	1	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$	
$N_1(J)$			**2**	**2**	**2**	**0**

NR hybrid Ops with two Deri-vatives

$SU(3)_F$	S	L	J^P		
$\mathbf{8_F}$	$\frac{1}{2}$	1	$\frac{1}{2}^+$	$\frac{3}{2}^+$	
	$\frac{3}{2}$	1	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$
$M_8(J)$			**2**	**2**	**1**
$\mathbf{10_F}$	$\frac{1}{2}$	1	$\frac{1}{2}^+$	$\frac{3}{2}^+$	
$M_{10}(J)$			**1**	**1**	**0**
$\mathbf{1_F}$	$\frac{1}{2}$	1	$\frac{1}{2}^+$	$\frac{3}{2}^+$	
$M_1(J)$			**1**	**1**	**0**

Total number of ops up to two Deri-vatives

	$SU(3)_F$	I	S	G_1	H	G_2
N	$\mathbf{8_F}$	$\frac{1}{2}$	0	22	37	15
Δ	$\mathbf{10_F}$	$\frac{3}{2}$	0	19	31	12
Λ	$\mathbf{1_F}$	0	0	17	27	10
Λ	$\mathbf{8_F}$	0	0	22	37	15
Σ	$\mathbf{8_F}$	1	-1	22	37	15
Σ	$\mathbf{10_F}$	1	-1	19	31	12
Ξ	$\mathbf{8_F}$	$\frac{1}{2}$	-2	22	37	15
Ξ	$\mathbf{10_F}$	$\frac{1}{2}$	-2	19	31	12
Ω	$\mathbf{10_F}$	0	-3	19	31	12

Variational Analysis

ϕ_i : gauge invariant fields on a timeslice t that corresponds to Hilbert space operator ϕ_j whose quantum numbers are also carried by the states **|n>.**

Construct a matrix

$$C(t) = \begin{bmatrix} \langle 0|\phi_1(t)\phi_1^+(0)|0\rangle & \langle 0|\phi_1(t)\phi_2^+(0)|0\rangle & \dots & \dots \\ \langle 0|\phi_2(t)\phi_1^+(0)|0\rangle & \langle 0|\phi_2(t)\phi_2^+(0)|0\rangle & \dots & \dots \\ \dots & \dots & & \dots \\ \dots & \dots & & \dots \end{bmatrix}$$

- Need to find out variational coefficient $\{v_\alpha^{(m)}, \alpha = 1,2,\dots,n\}$ such that the overlap to a state is maximum

$$\Phi^{(m)}(t)|0\rangle = \sum_\alpha^N v_\alpha^{(m)}\phi_\alpha(t)|0\rangle$$

$$= (1-\varepsilon_m)e^{-\hat{H}t}|m\rangle + \sum_{n\neq m}\varepsilon_n e^{-\hat{H}t}|n\rangle \quad \text{with } \varepsilon_n << 1$$

- Variational solution → Generalized eigenvalue problem :

$$C(t)v^n(t,t_0) = \lambda_n(t,t_0)C(t_0)v^n(t,t_0)$$

"Rayleigh-Ritz method"
Diagonalize:
eigenvalues → spectrum
eigenvectors → spectral "overlaps" Z_i^n

- Eigenvalues give spectrum :

$$\lim_{t\to\infty}\lambda_n(t,t_0) = e^{-(t-t_0)E_n}(1+e^{-t\Delta E_n})$$

- Eigenvectors give the optimal operator :

$$\Phi^m(t) = v_1^m\phi_1(t) + v_2^m\phi_2(t) + \dots$$

Generalized eigenvalue problem

Solving the generalized eigenvalue problem for $C_{ij}(t)$.

$$C_{ij}(t)v_j^{(n)}(t,t_0) = \lambda^{(n)}(t,t_0)C_{ij}(t_0)v_j^{(n)}(t,t_0)$$

Solve for many t_0's.

Choice of t_0's crucial $\Rightarrow$ Determine quality of extractions.

- Principal correlators given by eigenvalues

$$\lambda_n(t,t_0) \sim (1-A_n)\exp^{-m_n(t-t_0)} + A_n \exp^{-m'_n(t-t_0)}$$

Extraction of a tower of states.

- Eigenvectors related to the overlap factors

$$Z_i^{(n)} = \langle 0|\mathcal{O}_i|n\rangle = \sqrt{2E_n}\exp^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$$

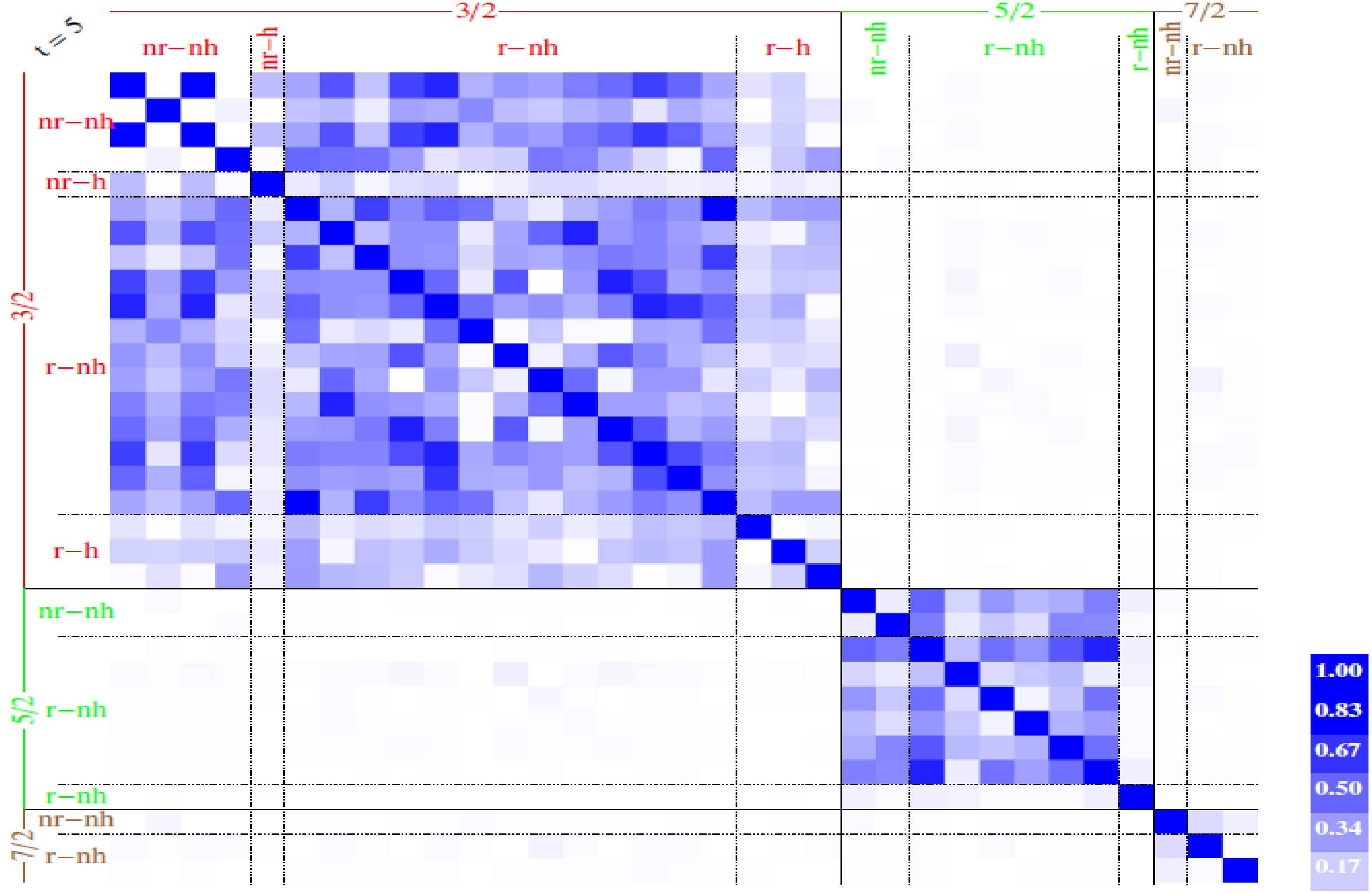
Spin identification.

C. Michael, Nucl. Phys. B 259, 58, (1985)
M. Luscher and U. Wolff, Nucl. Phys. B 339, 222 (1990)

Rotational Invariance in Spectrum

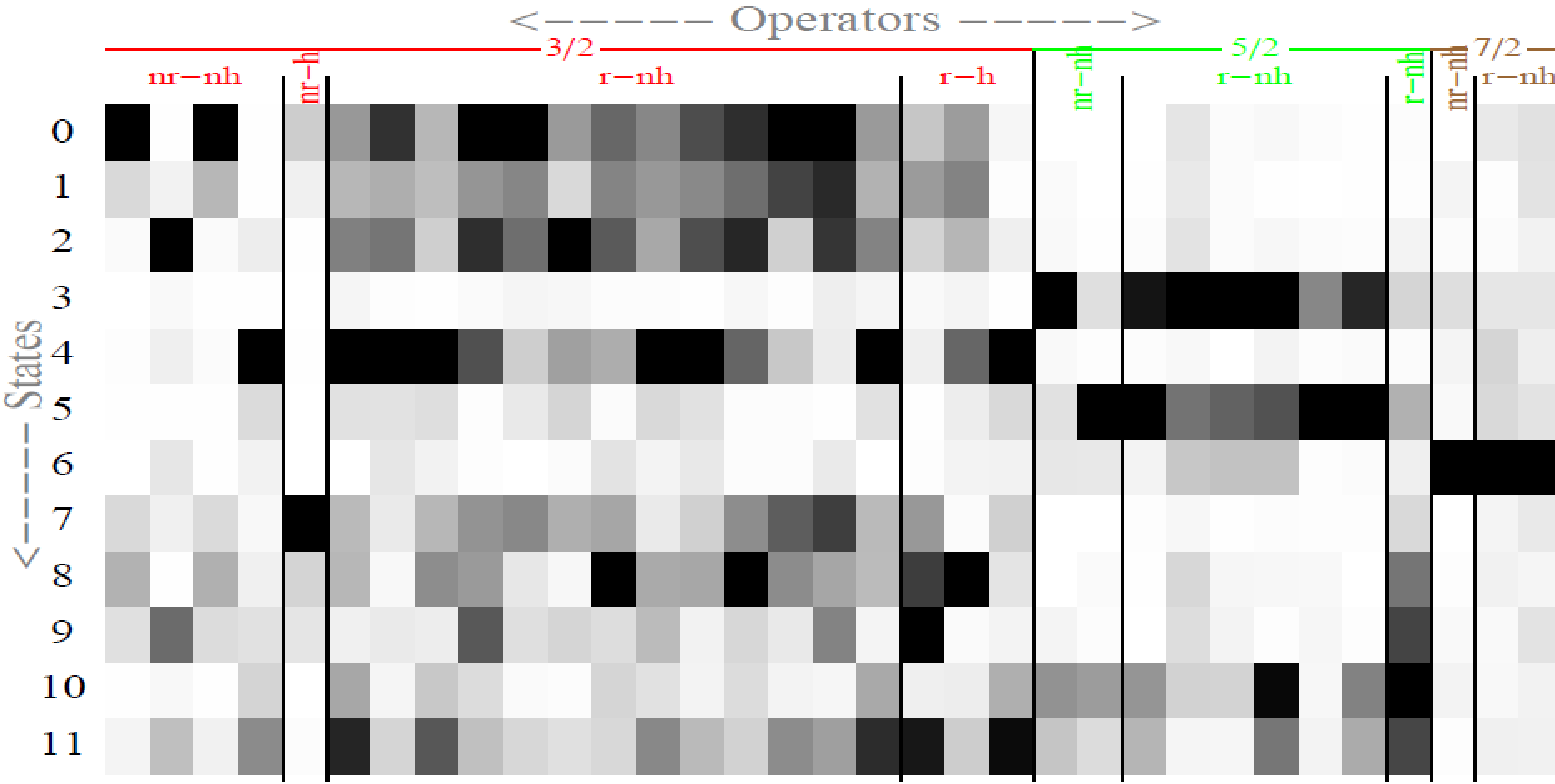
If there is rotational invariance there will be no overlap (coupling) between different J, that is the matrix $C \propto \delta_{J,J'}$

Approximate block-diagonality has been observed



Confin

Spin identification using overlap factors : (Ω_{ccc}, H_g)



$$nr - nh = non - relativistic \;\&\; non - hybrid$$

$$nr - h = non - relativistic \;\&\; hybrid$$

$$r - nh = relativistic \;\&\; non - hybrid$$

$$r - h = relativistic \;\&\; hybrid$$

Spin identification from overlap factors

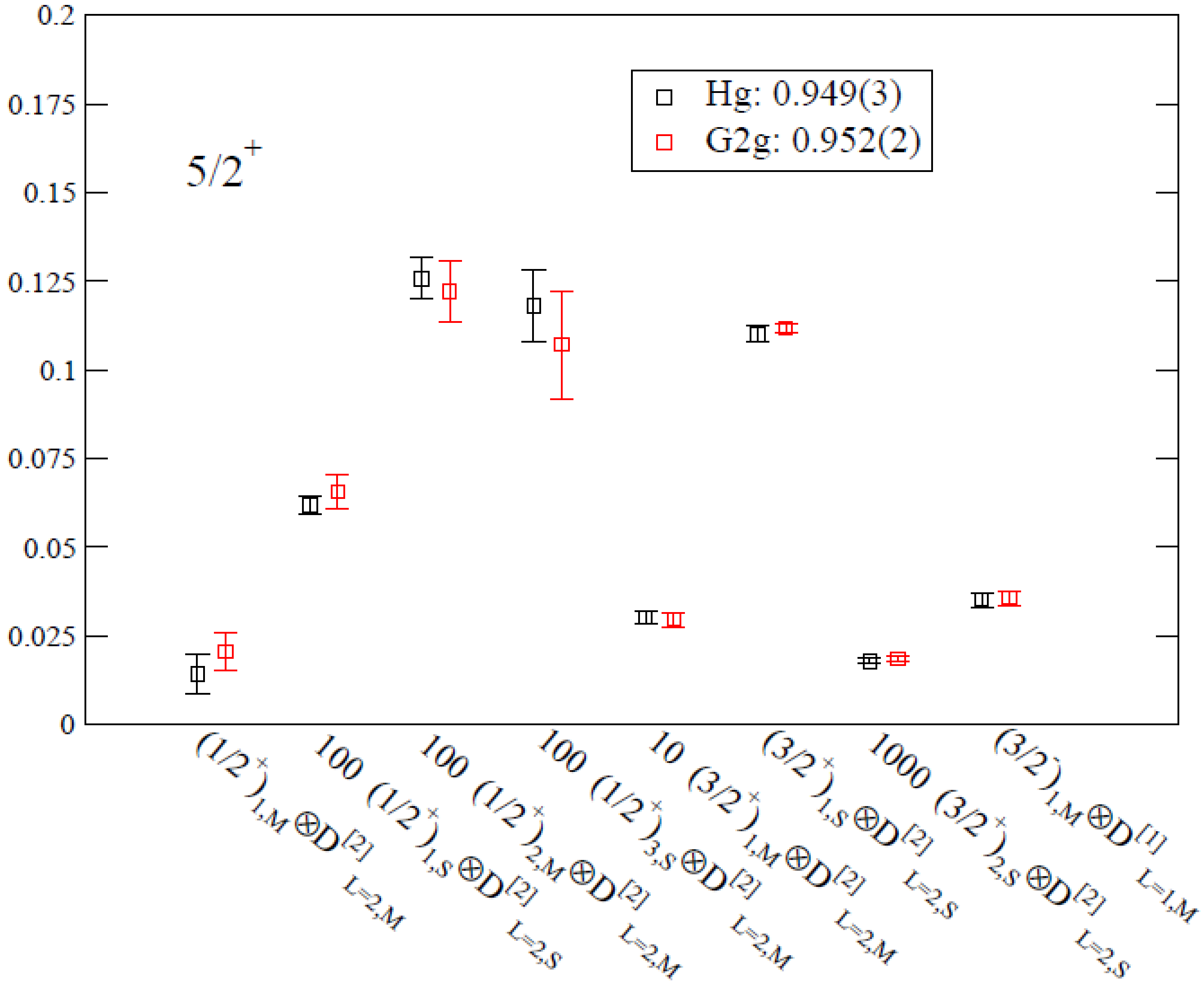
- For example, a continuum operator $O_{jk} = \bar{\psi}\gamma_j D_k \psi$.
  Projects on to 2^{++}.
- In the continuum, $\langle 0|O_{jk}|2^{++}\rangle = Z\epsilon_{jk}$.
- On lattice, O_{jk} gets subduced over two lattice irreps $(\rho\times\nabla)_{T_2}$ and $(\rho\times\nabla)_E$.
- Then

$$\langle 0|(\rho\times\nabla)^i_{T_2})|2^{++}\rangle = \alpha_{ijk}\langle 0|O_{jk}|2^{++}\rangle = Z_1\alpha_{ijk}\epsilon_{jk}$$

$$\langle 0|(\rho\times\nabla)^i_{E})|2^{++}\rangle = \beta_{ijk}\langle 0|O_{jk}|2^{++}\rangle = Z_2\beta_{ijk}\epsilon_{jk}$$

  where α_{ijk} and β_{ijk} are the Clebsch-Gordan coefficients.
- If "close" to the continnum, then $Z \sim Z_1 \sim Z_2$.

Overlap factors (Z) across multiple irreps : $5/2^+$



Lattice parameters

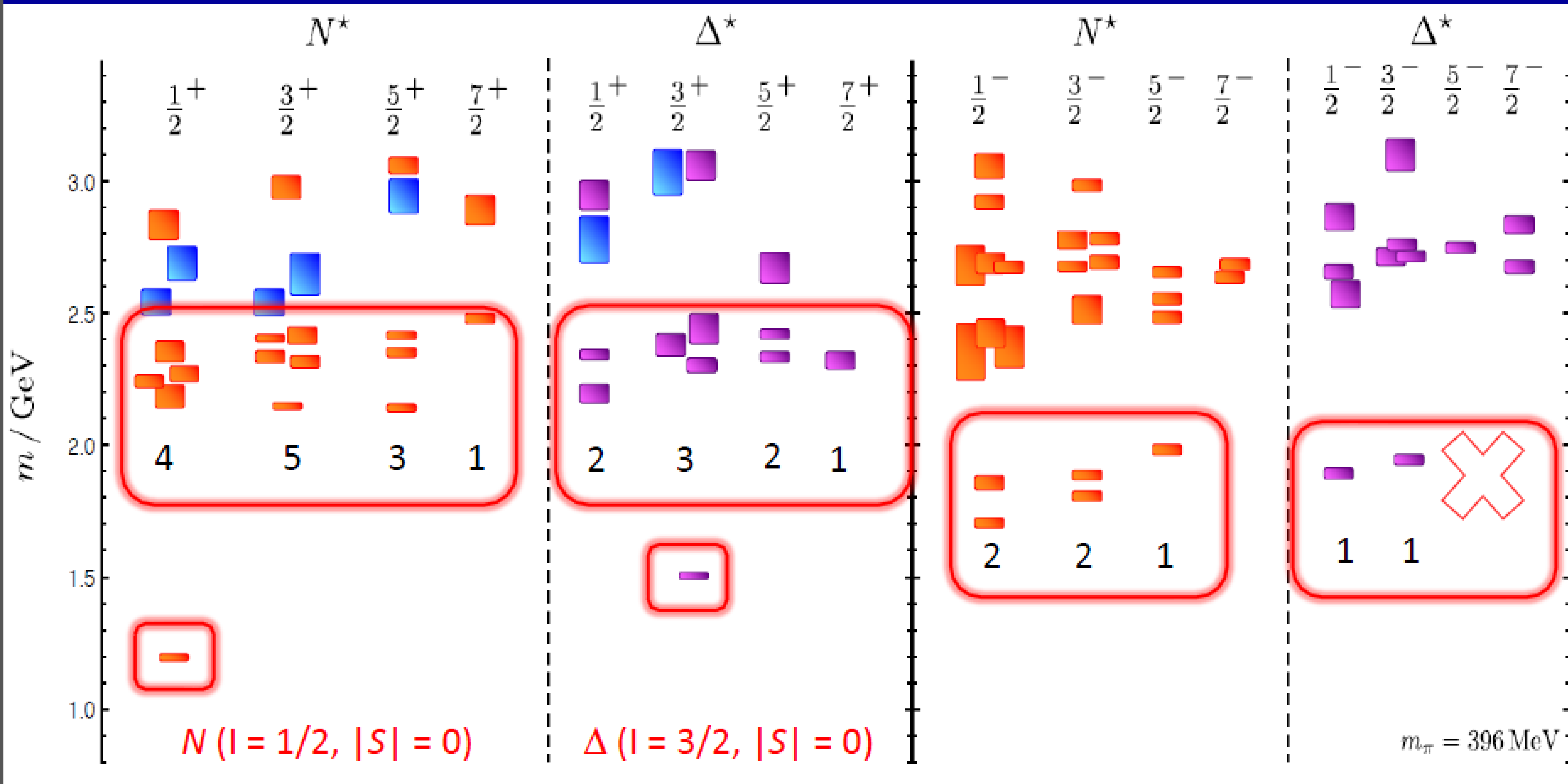
- N_f = 2+1 QCD
  - Gauge action: Symanzik-improved
  - Fermion action: Clover-improved Wilson

- Anisotropic: a_s= 0.122 fm, a_t = 0.035 fm

ensemble	1	2	3
m_ℓ	−.0840	−.0830	−.0808
m_s	−.0743	−.0743	−.0743
Volume	$16^3\times$ 128	$16^3\times$ 128	$16^3\times$ 128
Physical volume	(2 fm)^3	(2 fm)^3	(2 fm)^3
$N_{\rm cfgs}$	344	570	481
$t_{\rm sources}$	8	5	7
m_π	0.0691(6)	0.0797(6)	0.0996(6)
m_K	0.0970(5)	0.1032(5)	0.1149(6)
m_Ω	0.2951(22)	0.3040(8)	0.3200(7)
m_π (MeV)	396	444	524

N and Δ baryons

HSC : [PR D84 074508; D85 054016]



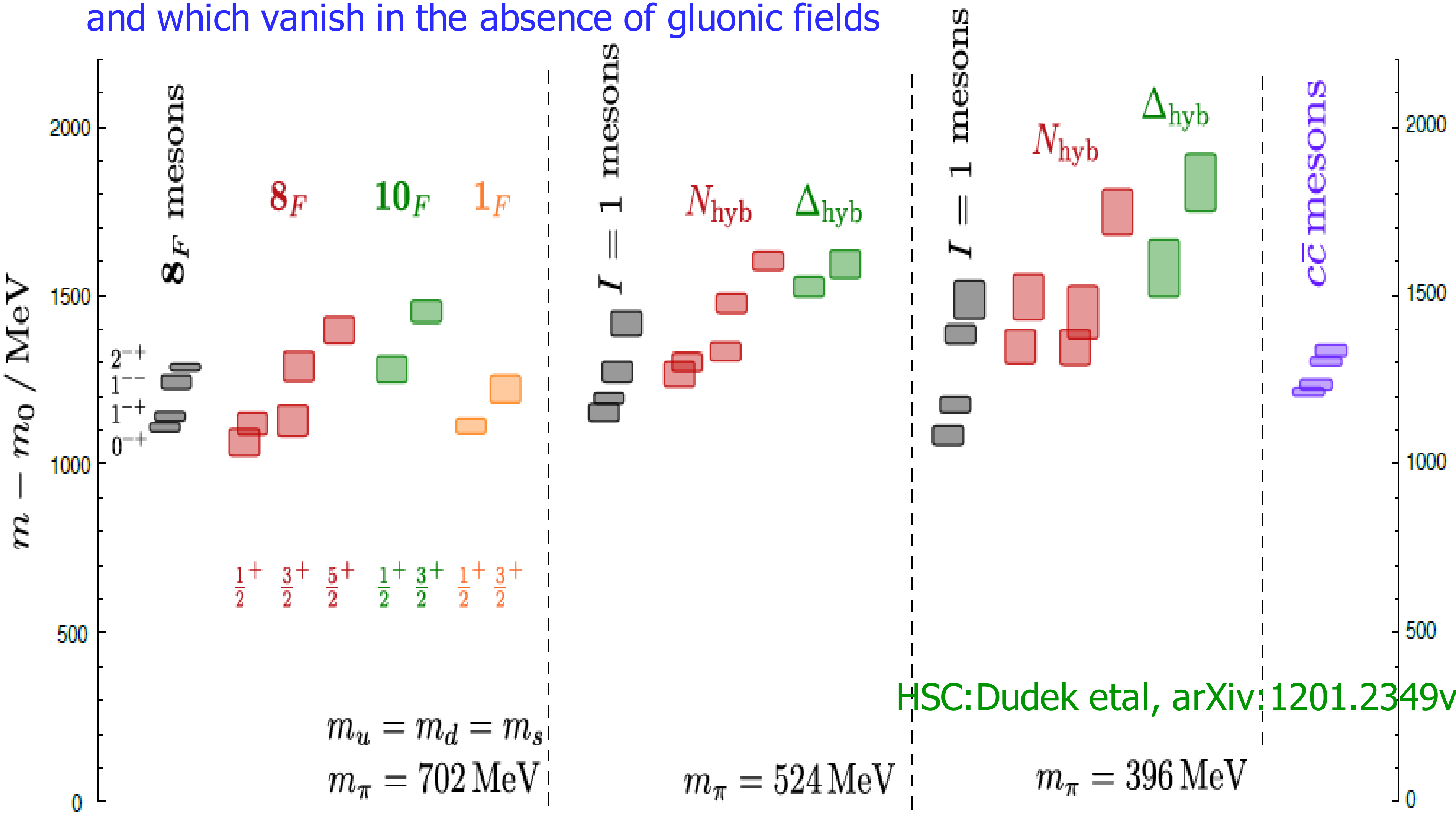
Counting expected in non. rel. quark model, SU(6) x O(3)

N_f = 2+1, $M_\pi \approx 400$ MeV

@Edwards

Hybrid Baryons

States have maximum overlap to operators constructed from chromoganetic field and which vanish in the absence of gluonic fields

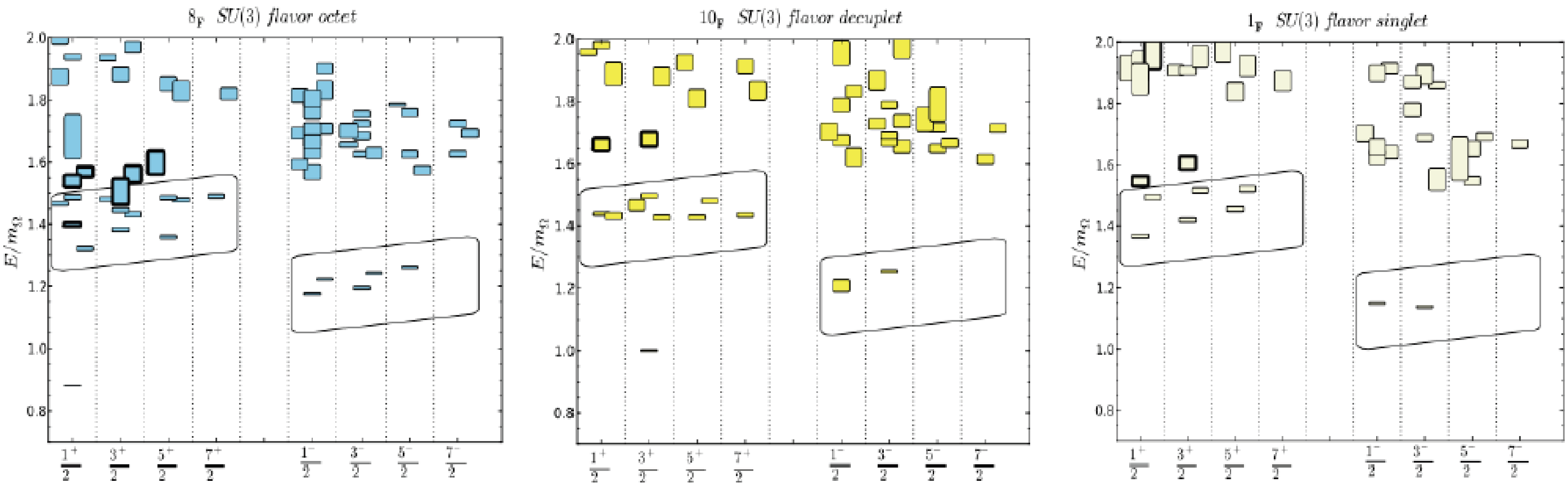


SU(3) flavor limit

In SU(3) flavor limit – have exact flavor Octet, Decuplet and Singlet representations

HSC : Phys.Rev. D87 (2013) 054506

8_F 10_F 1_F



$m_\pi \sim 700$ MeV

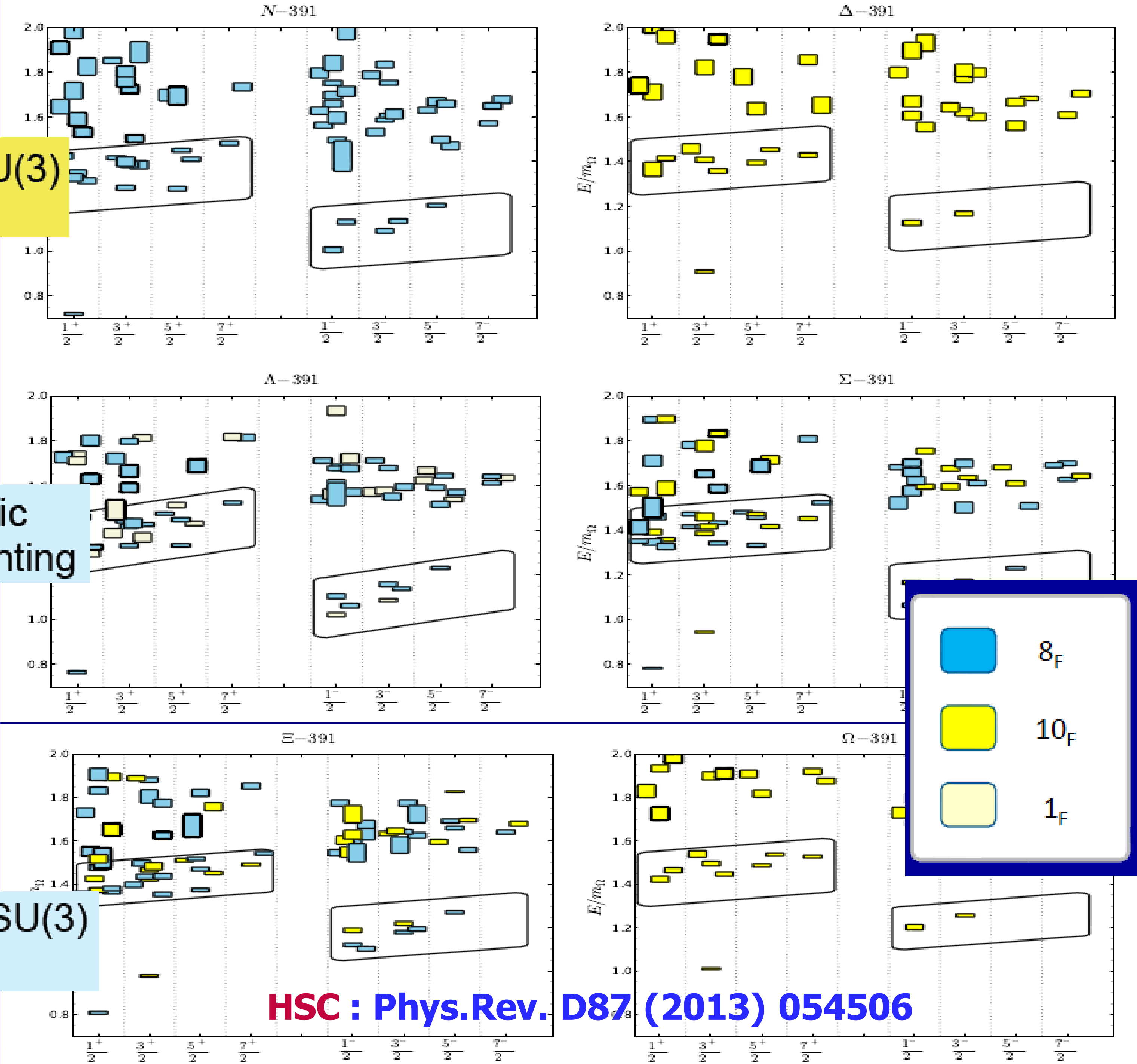
Full non-relativistic quark model counting

Additional levels with significant gluonic components

Light quarks – SU(3) flavor broken

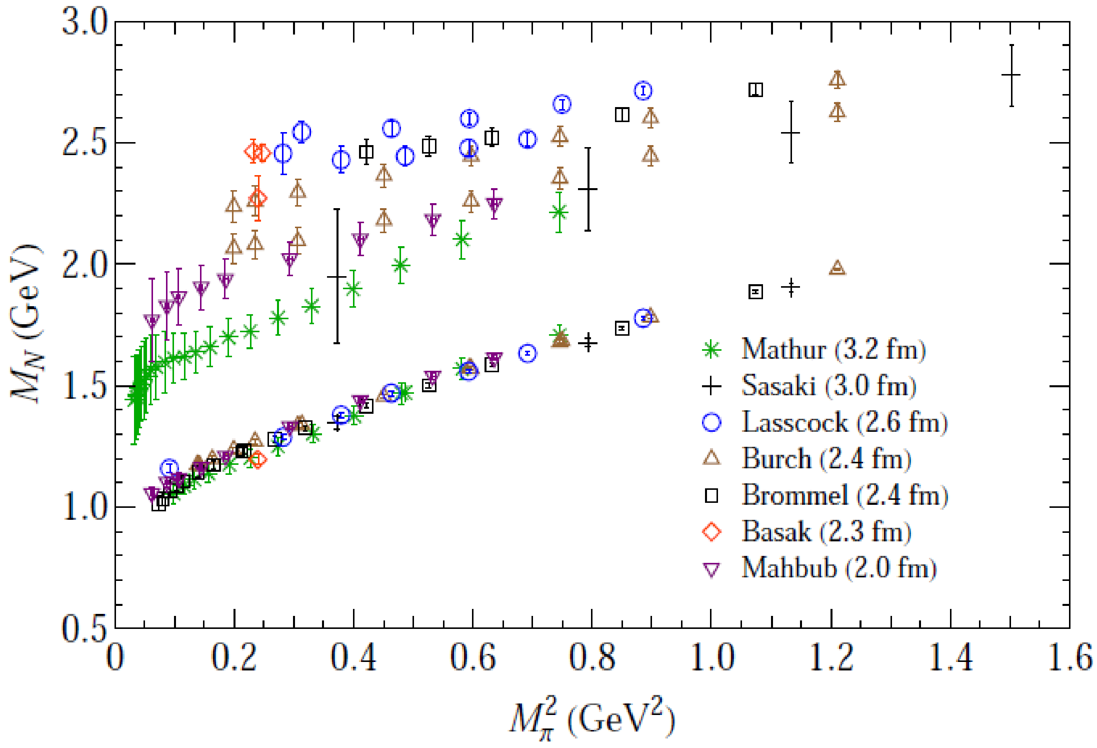
Full non-relativistic quark model counting

Some mixing of SU(3) flavor irreps

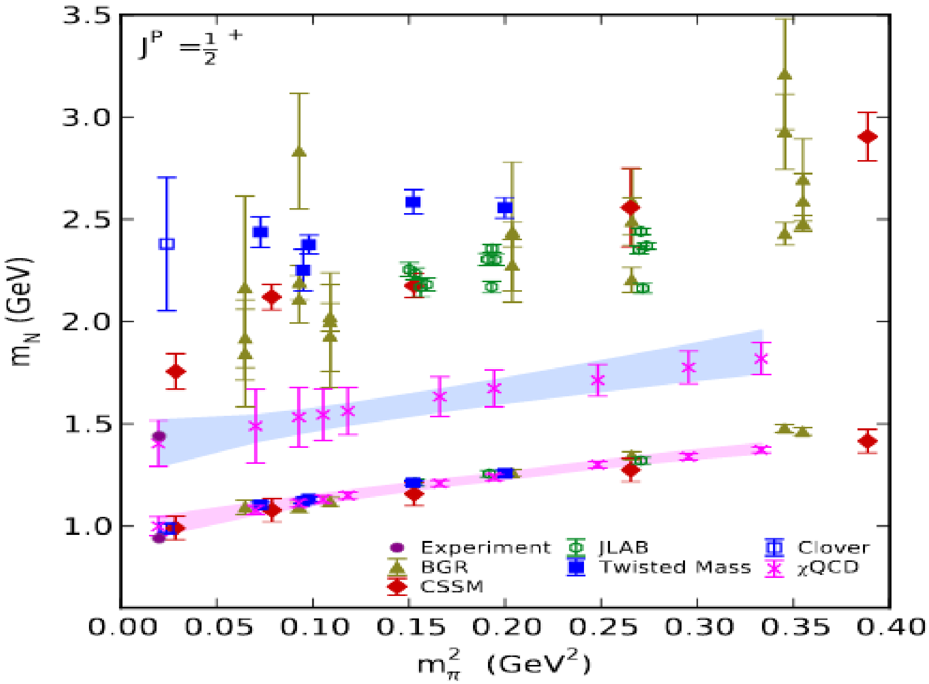


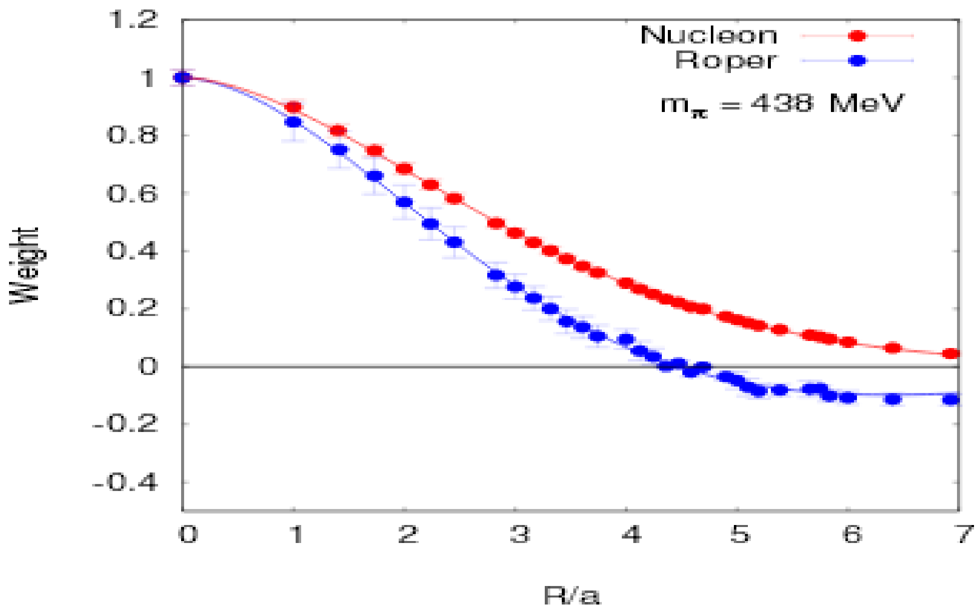
HSC : Phys.Rev. D87 (2013) 054506

The Roper Puzzle



Quenched



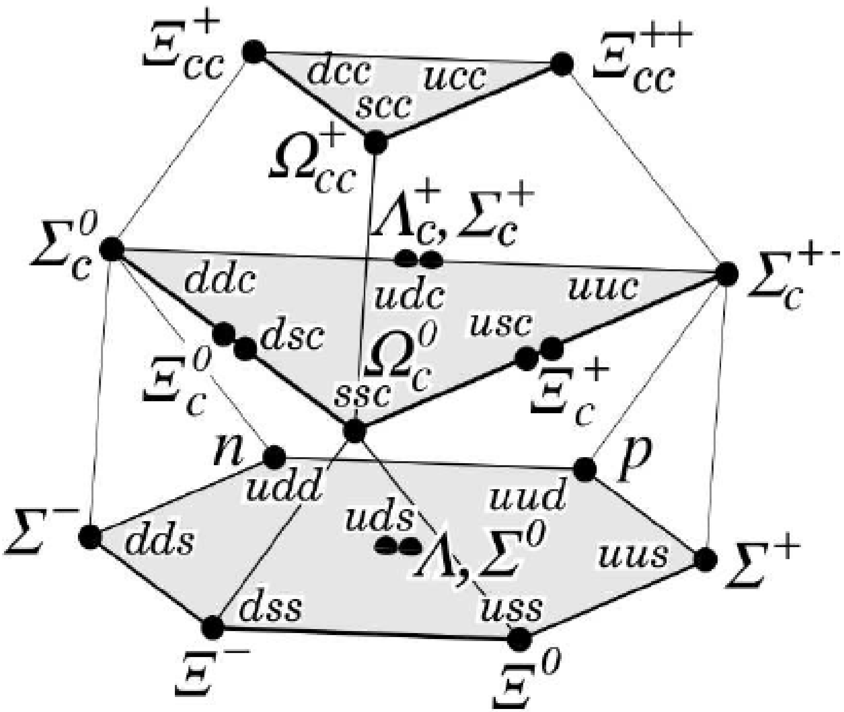


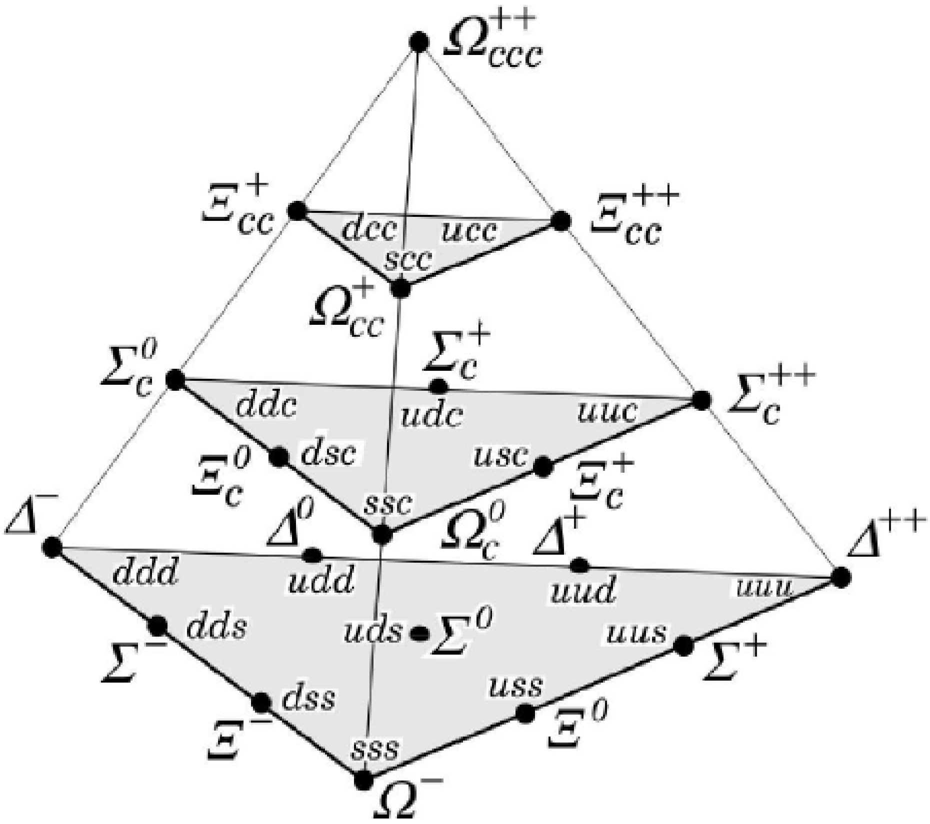
χ QCD :
arXiv:1403.6847

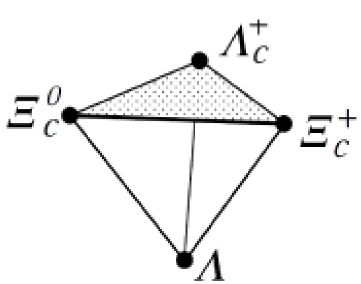
Charm baryons

- Singly charm baryons $\rightarrow$ Light quark dynamics.
  Very high production rate at e^+e^- and p-p colliders.
- Doubly charm baryons : $\bar{Q}Q$ or $\bar{Q}q$ picture?
  Controversial discovery status.
- Triply charm baryons : A charmonia analogue in baryons.
  quark-quark interactions.
- Experimental prospects : LHCb, Belle II, BES, PANDA @ FAIR.

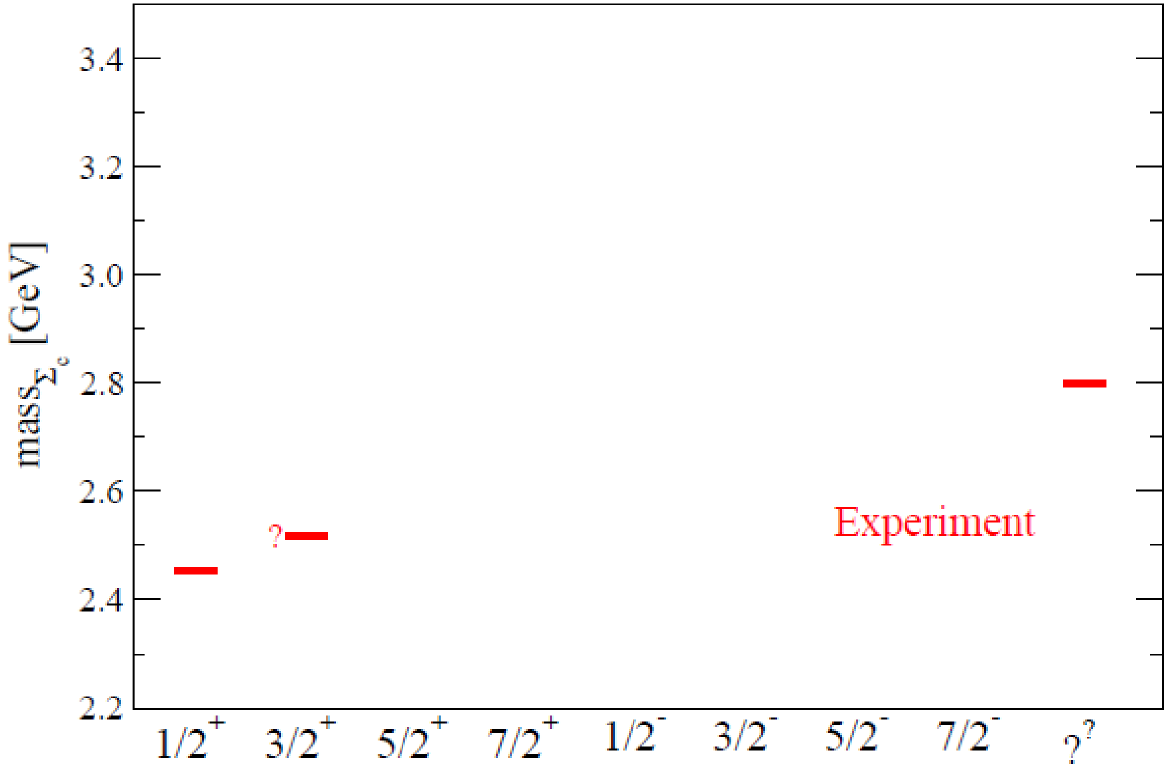
Charm baryons : Nomenclature



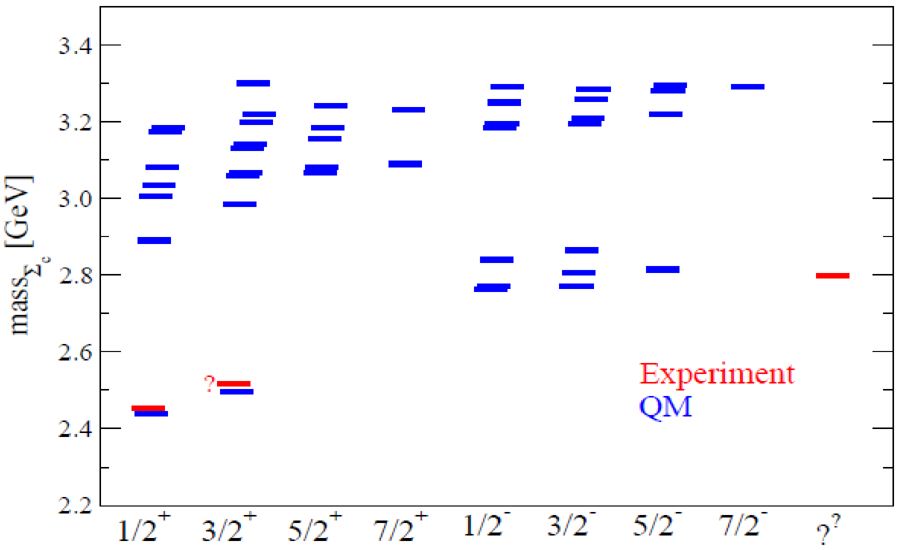




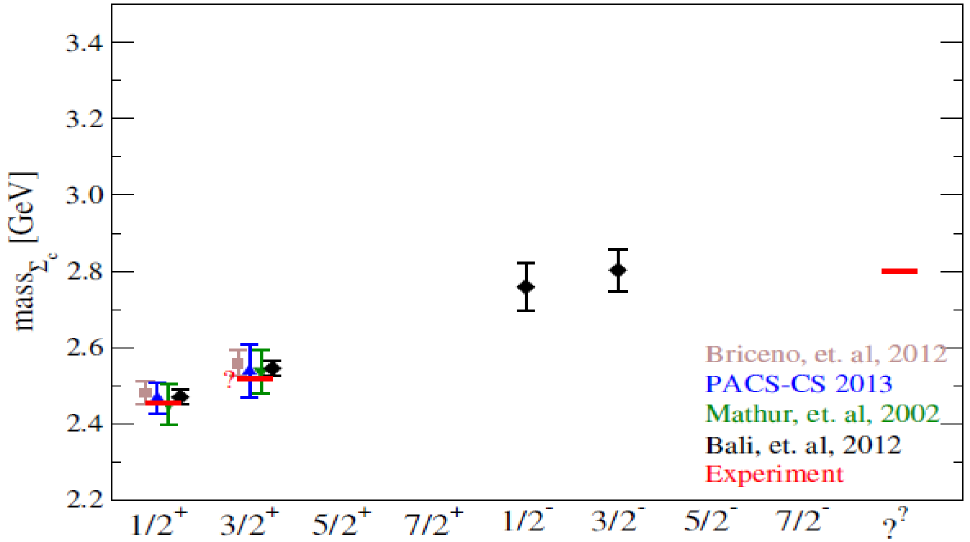
We have one heavy and 2+1 light flavor states.



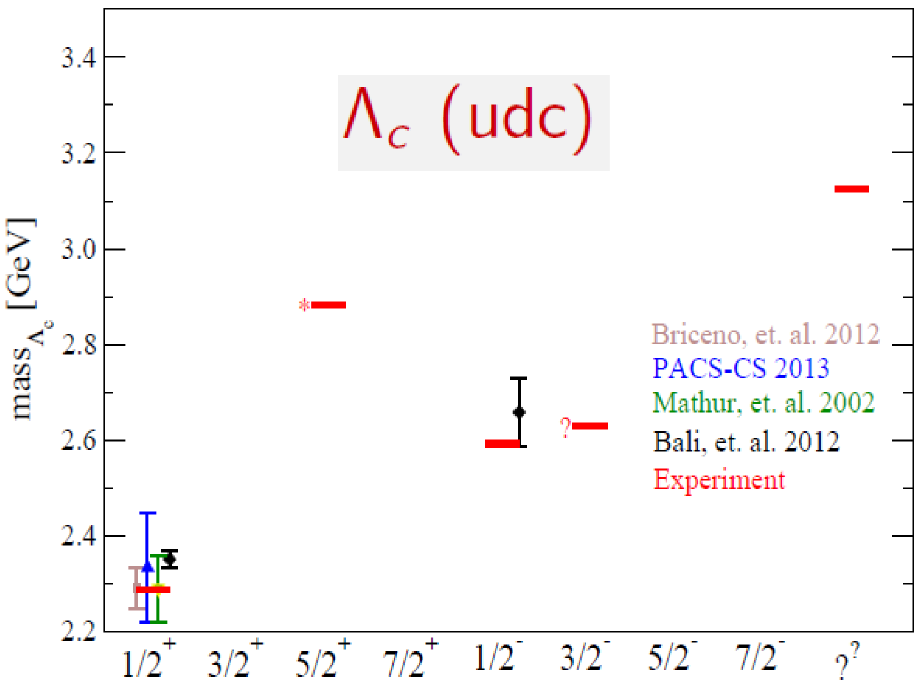
mass$_{\Sigma_c}$ [GeV]
3.4
3.2
3.0
2.8
2.6
2.4
2.2
Experiment
?
1/2$^+$
3/2$^+$
5/2$^+$
7/2$^+$
1/2$^-$
3/2$^-$
5/2$^-$
7/2$^-$
?$^?$

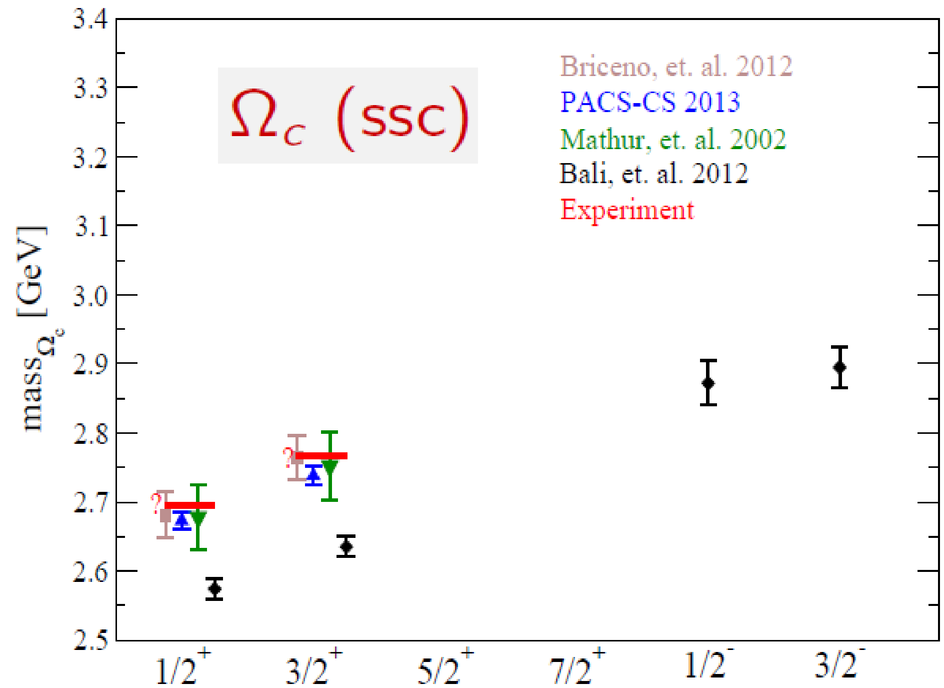


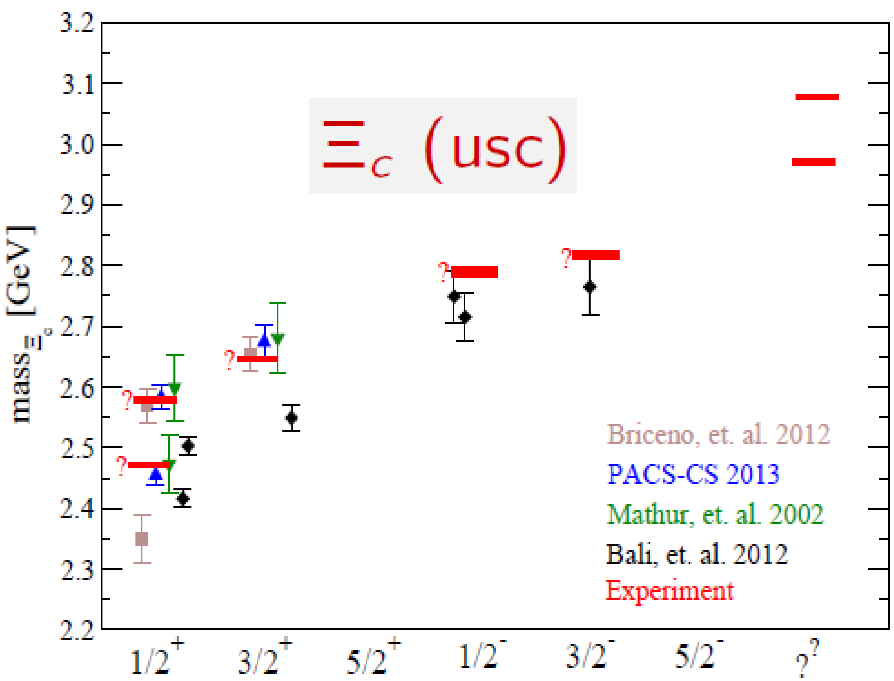
mass$_{\Sigma_c}$ [GeV]
3.4
3.2
3.0
2.8
2.6
2.4
2.2
Experiment
QM
?
1/2$^+$
3/2$^+$
5/2$^+$
7/2$^+$
1/2$^-$
3/2$^-$
5/2$^-$
7/2$^-$
?$^?$



mass$_{\Sigma_c}$ [GeV]
3.4
3.2
3.0
2.8
2.6
2.4
2.2
Briceno, et. al, 2012
PACS-CS 2013
Mathur, et. al, 2002
Bali, et. al, 2012
Experiment
?
1/2$^+$
3/2$^+$
5/2$^+$
7/2$^+$
1/2$^-$
3/2$^-$
5/2$^-$
7/2$^-$
?$^?$







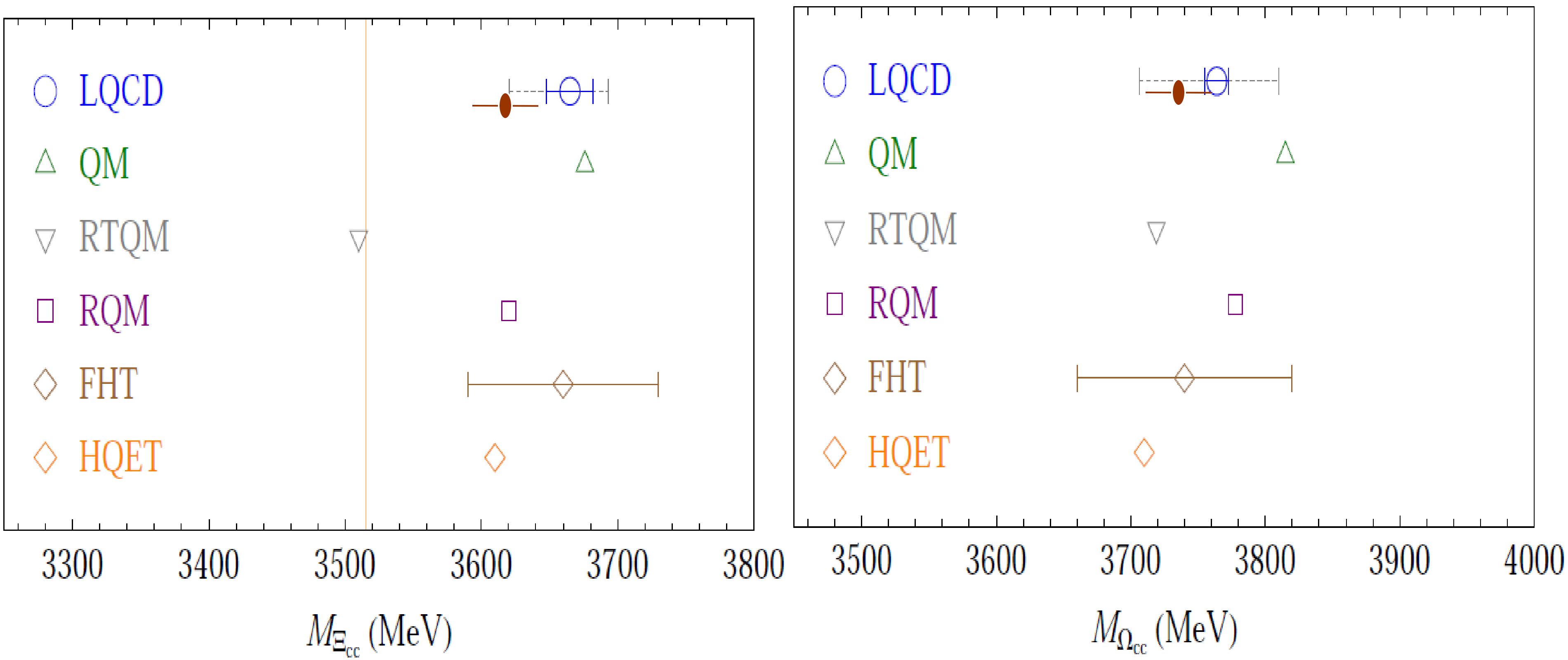
Doubly Charm (ccu, ccd, ccs):

- Discovery is controversial
- SELEX (2002) claimed to have got it (ccu)
- BELLE (2013), LHCb(2014) did not

Triply Charm (ccc):

- Nothing yet
- LHCb ?
- Super Belle (lets hope)

Doubly charmed baryons



No. of interpolating operators

Ω_{ccc}

	G_1		H		G_2	
	g	u	g	u	g	u
Total	20	20	33	33	12	12
Hybrid	4	4	5	5	1	1
NR	4	1	8	1	3	0

Λ_{cdu}

	G_1		H		G_2	
	g	u	g	u	g	u
Total	53	53	86	86	33	33
Hybrid	12	12	16	16	4	4
NR	10	3	17	4	7	1

Ω_{ccs}, Ξ_{ccu}, Ω_{css} and Σ_{cuu}.

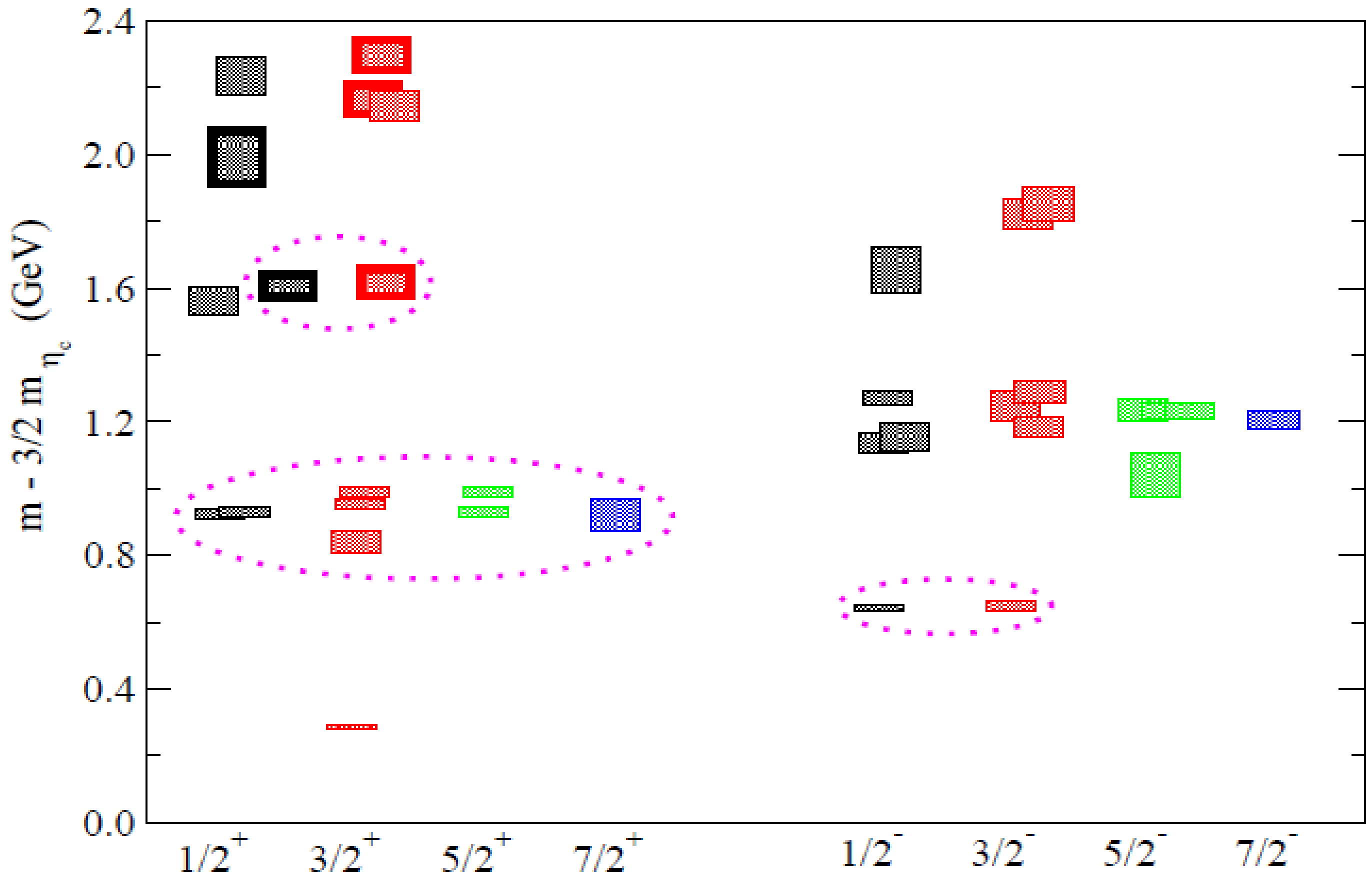
	G_1		H		G_2	
	g	u	g	u	g	u
Total	55	55	90	90	35	35
Hybrid	12	12	16	16	4	4
NR	11	3	19	4	8	1

Ξ_{csu}

	G_1		H		G_2	
	g	u	g	u	g	u
Total	116	116	180	180	68	68
Hybrid	24	24	32	32	8	8
NR	23	6	37	10	15	2

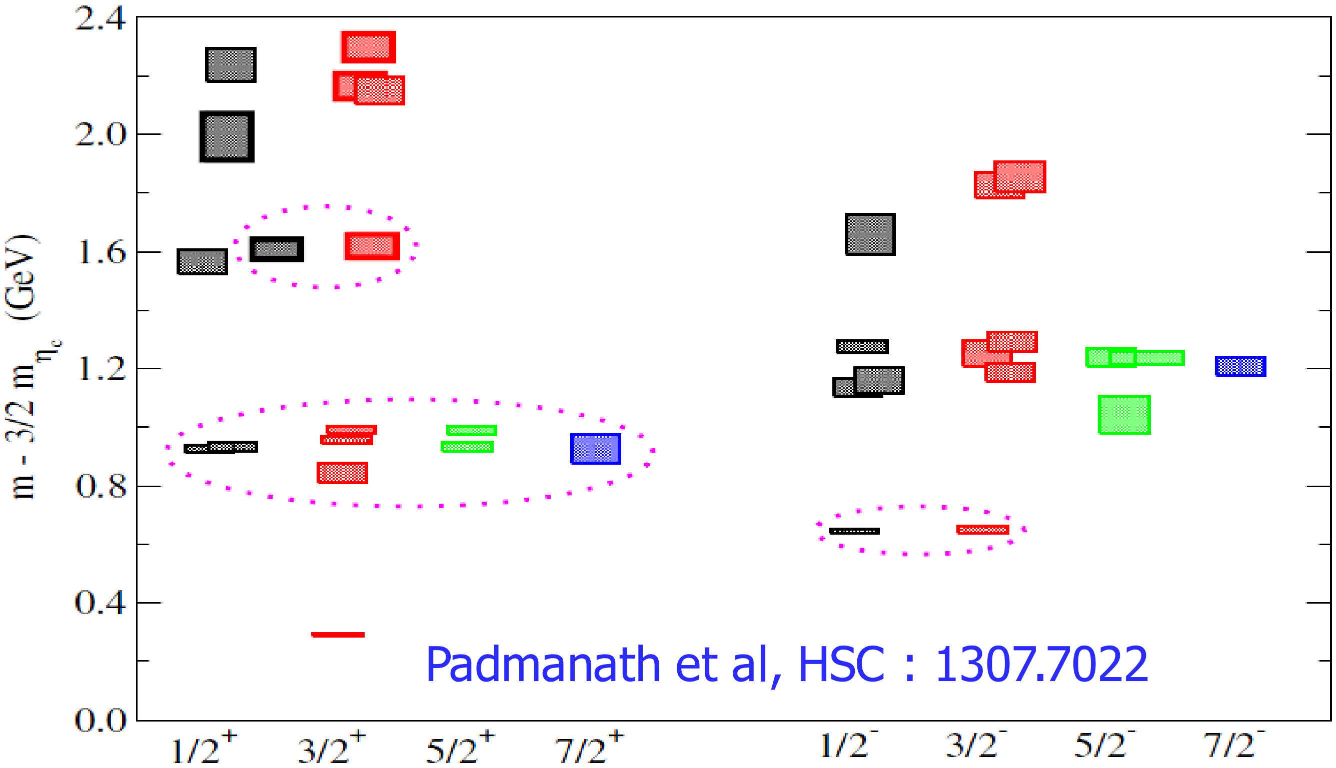
Ω_{ccc} spectrum

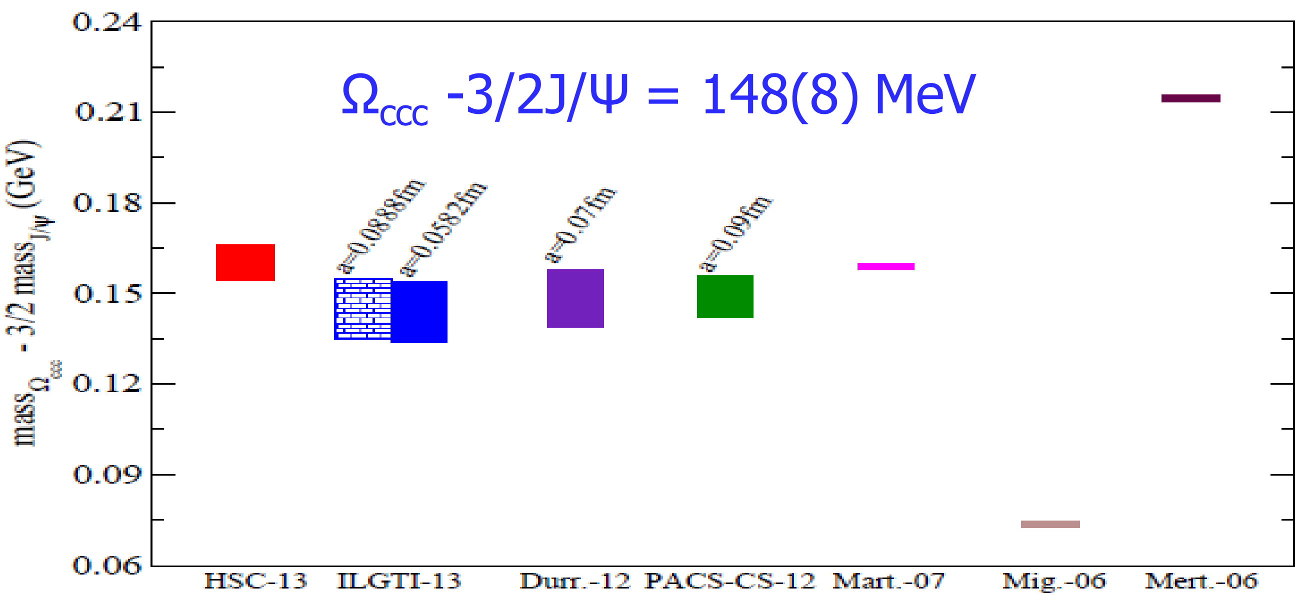
Padmanath et al, HSC : 1307.7022



Magenta ellipses : States with strong non-relativistic content.

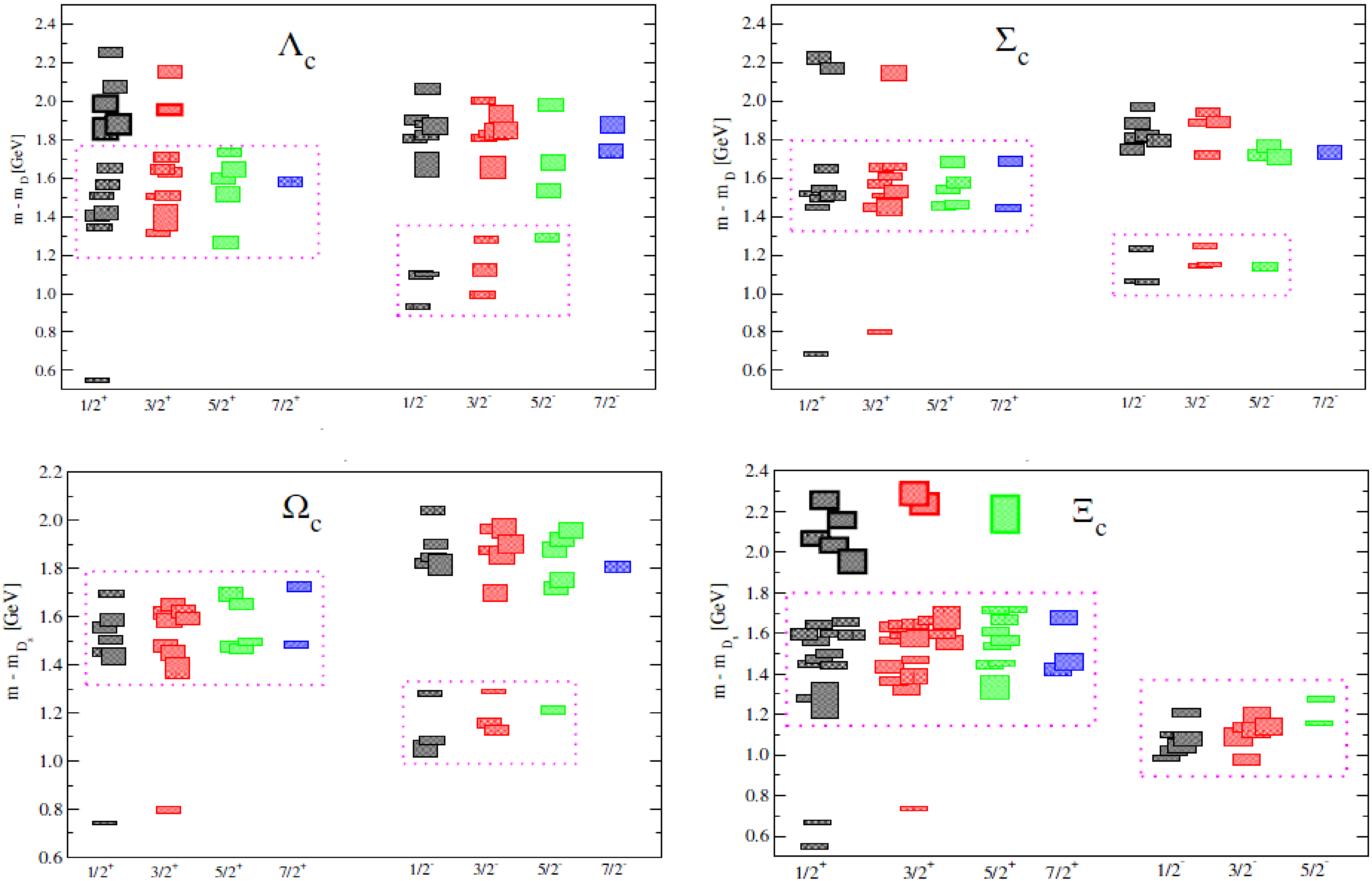
Boxes with thick border : States with strong hybrid nature.





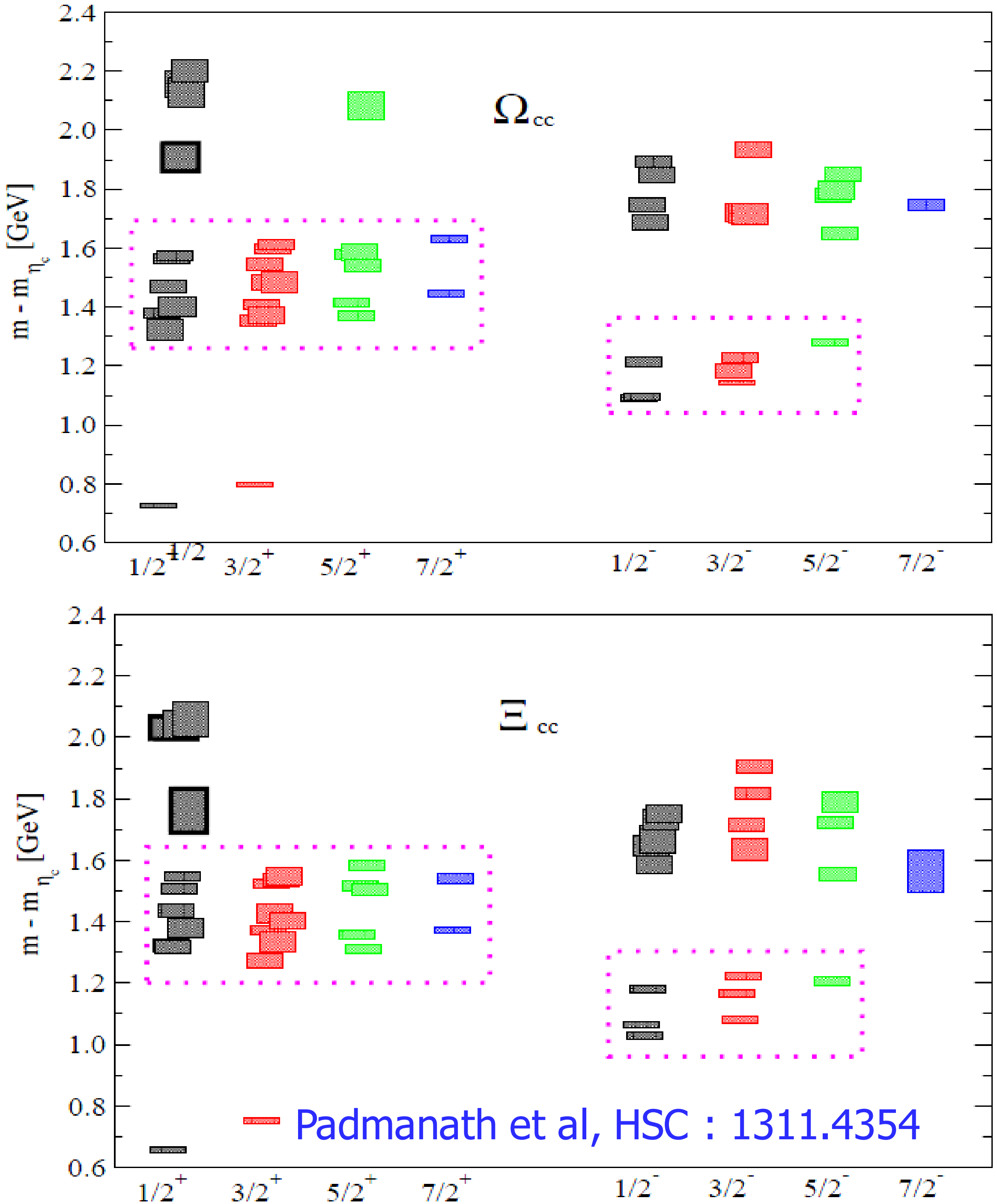
Confinement

Singly Charm baryons

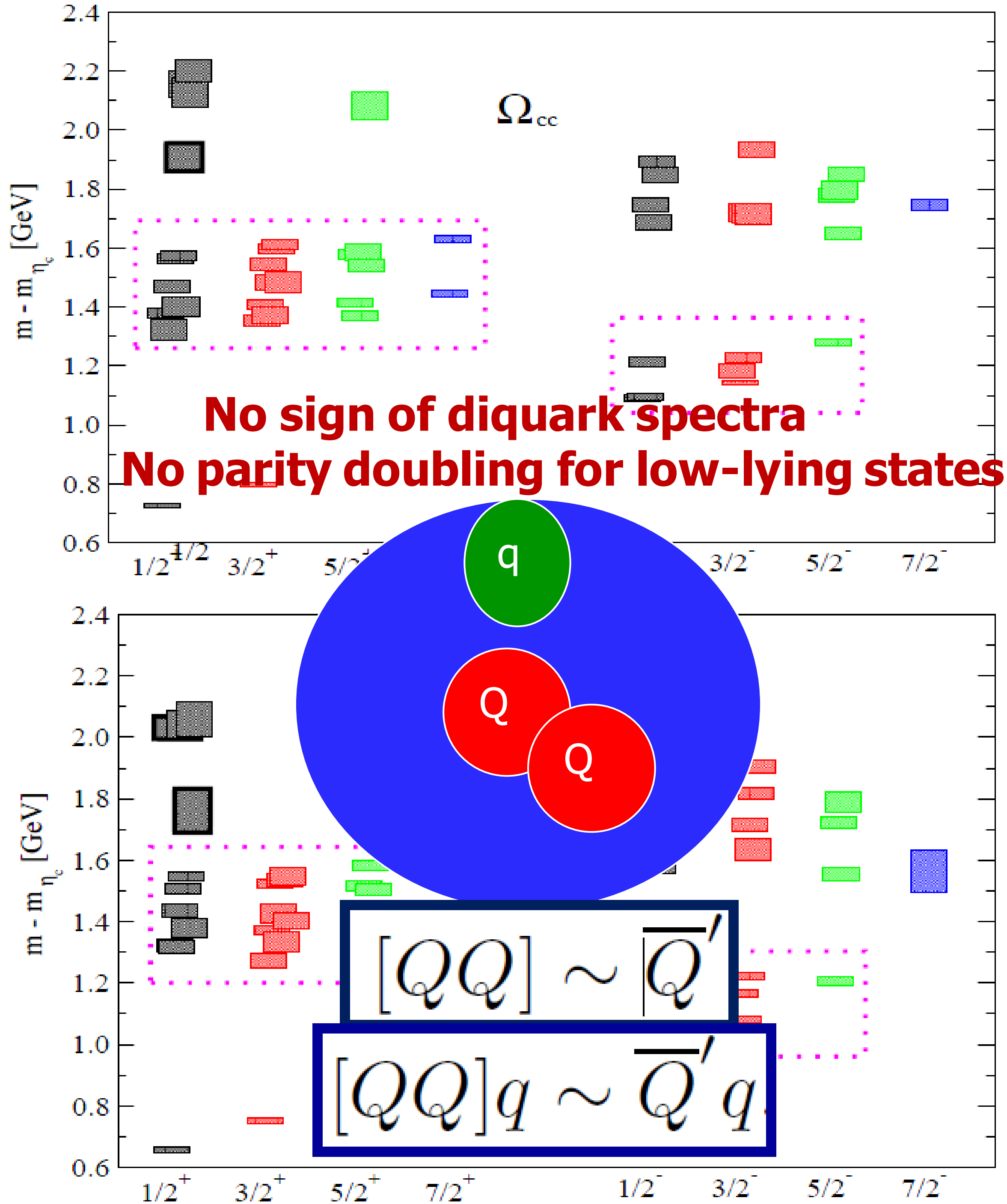


Padmanath et al, HSC : 1311.4806

DOUBLY CHARM BARYONS



DOUBLY CHARM BARYONS



No sign of diquark spectra

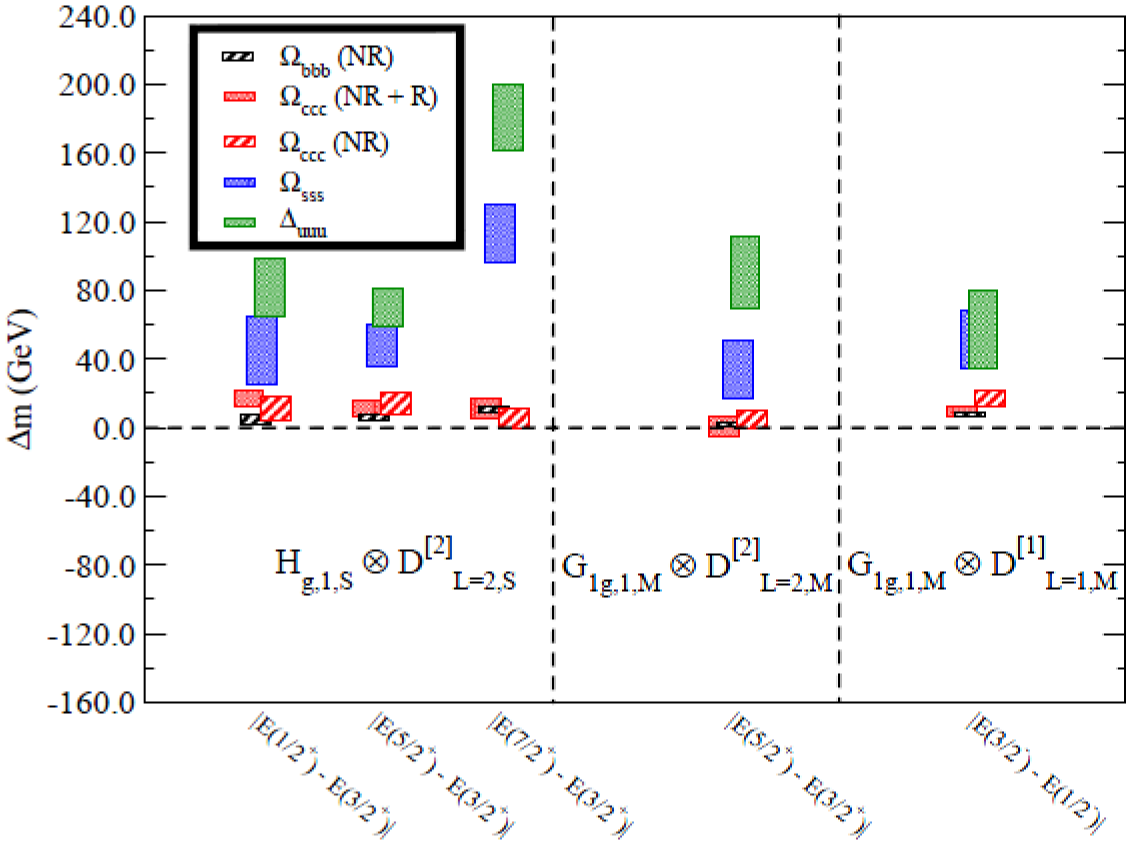
No parity doubling for low-lying states

$$[QQ] \sim \overline{Q}'$$

$$[QQ]q \sim \overline{Q}'q$$

How heavy is charm?

Can NRQCD sill work?

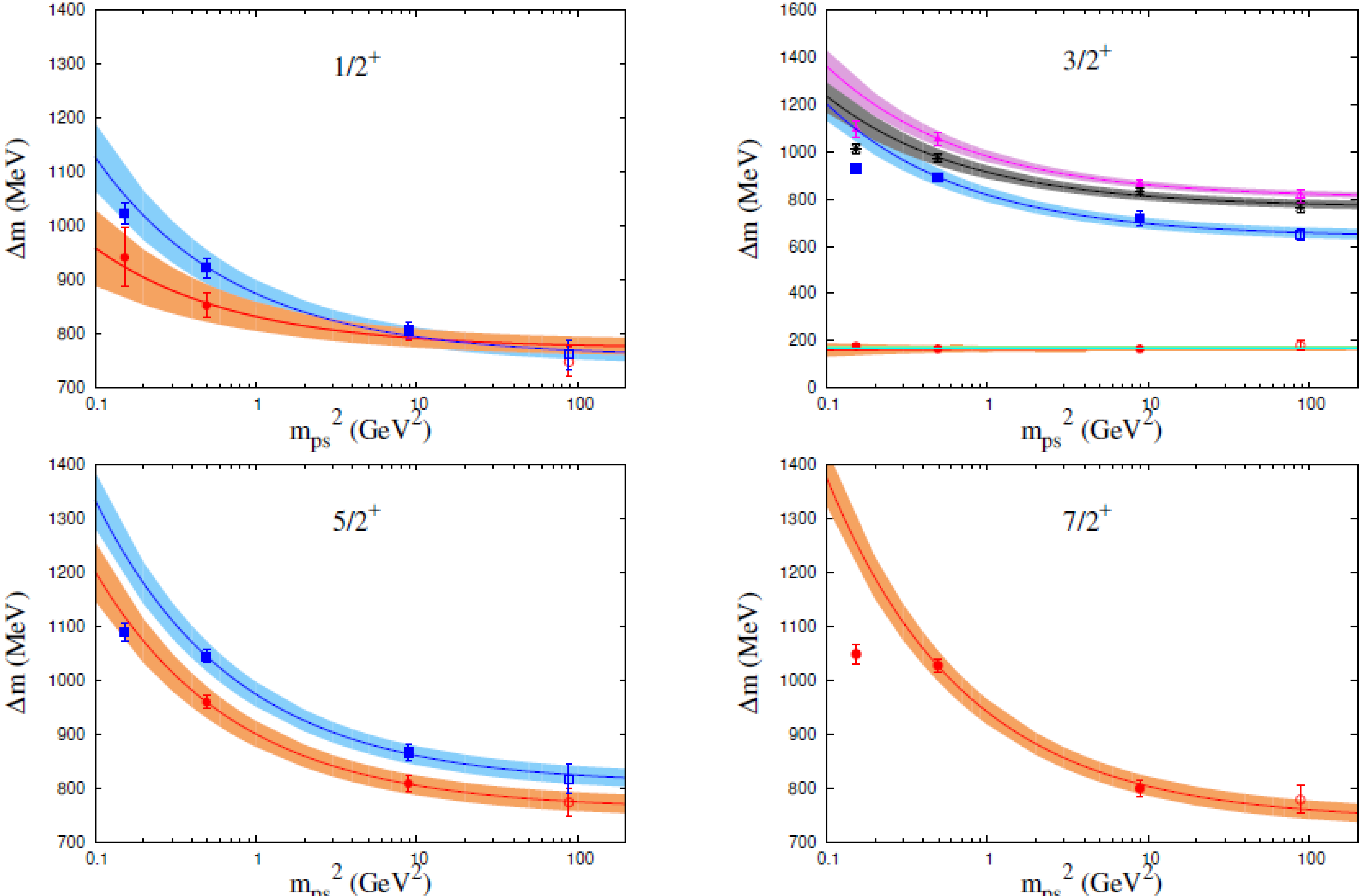


Padmanath et al, HSC : 1307.7022

HQET expansion for energy splittings

- Consider the splittings :
  $m_{\Delta_{uuu}} - \frac{3}{2}\, m_{\omega_{\bar{u}u}}$, $m_{\Omega_{sss}} - \frac{3}{2}\, m_{\phi_{\bar{s}s}}$, $m_{\Omega_{ccc}} - \frac{3}{2}\, m_{J/\psi_{\bar{c}c}}$ and $m_{\Omega_{bbb}} - \frac{3}{2}\, m_{\Upsilon_{\bar{b}b}}$.
- Valence heavy quark content subtracted by the factor 3/2.
  Mimics the binding energy.
- Heavy Quark Effective Theory (HQET) : Mass of a heavy hadron,
  $$m_{H_{n\ Q}} = n\ m_Q + A + B/m_Q + O(1/m_Q^2).$$
- Splittings : $\Delta m \ \sim\ a_1 + b_1/m_Q + O(1/m_Q^2)\ \sim\ a + b/m_{PS} + O(1/m_{PS}^2)$.
- Light quark data excluded from the fits.

Fits with HQET $(a + b/m_{PS})$: triple flavored baryons



Predictions from HQET + our results

$$m_{B_c^*} - m_{B_c} = 80 \ \pm \ 8 \ MeV$$

- Consider the energy splittings

$$(\Xi_{cc}^* - D,\ \Omega_{cc}^* - D_s,\ \Omega_{ccc}^* - \eta_c \text{ and } \Omega_{ccb}^* - B_c),$$
$$(\Xi_{cc}^* - D^*,\ \Omega_{cc}^* - D_s^*,\ \Omega_{ccc}^* - J/\psi \text{ and } \Omega_{ccb}^* - B_c^*)$$

- Extrapolation of the fit to these splittings $\rightarrow m_{B_c^*} - m_{B_c}$.

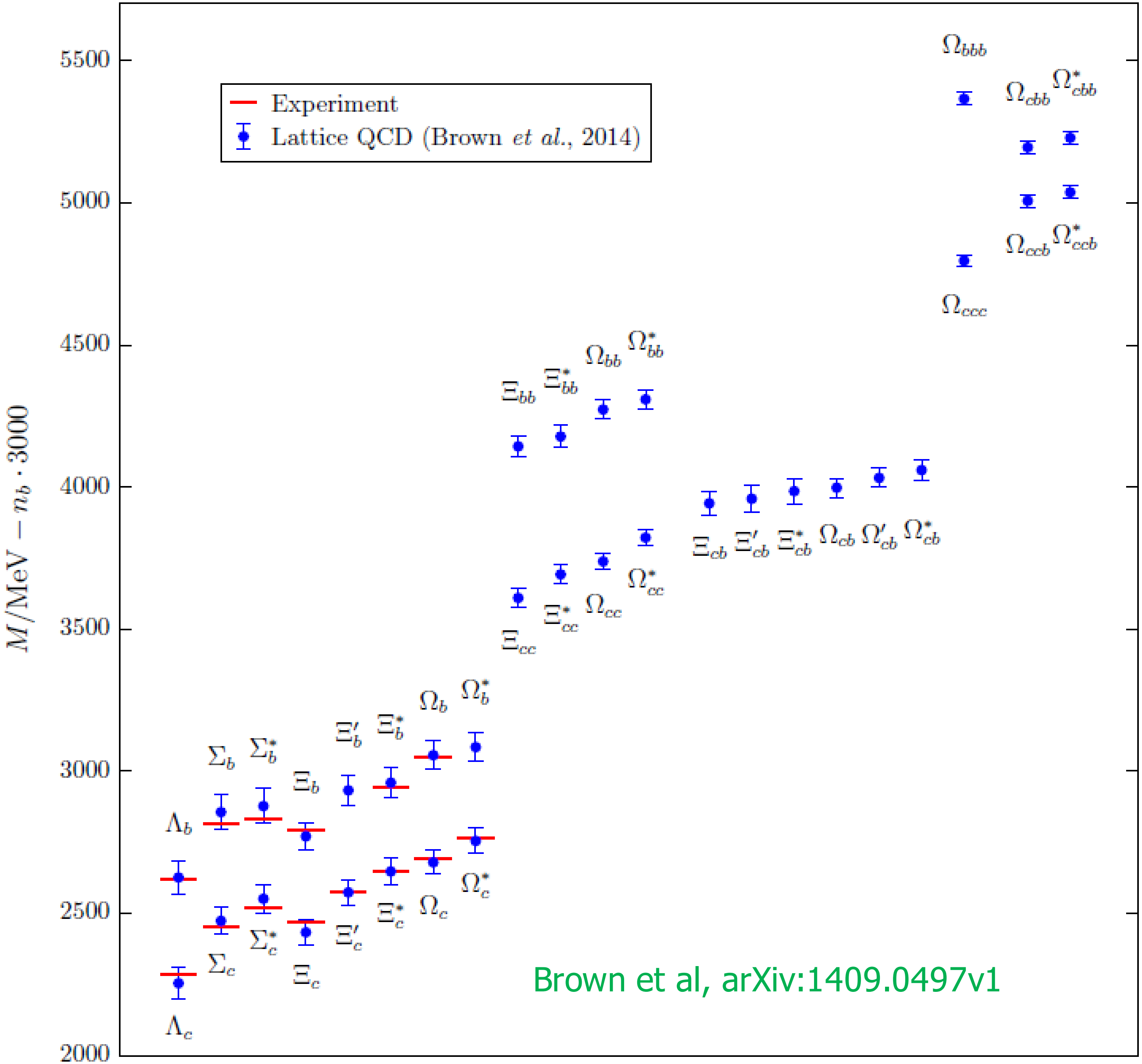
$$m_{\Omega_{ccb}^*} = 8050 \ \pm \ 10 \ MeV$$

Bottom Baryons

Baryon	Quark	J^P	mass(MeV)
Λ_b^0	udb	$\frac{1}{2}^+$	5620.2 ± 1.6
Σ_b^+	uub	$\frac{1}{2}^+$	5807.8 ± 2.7
Σ_b^-	ddb	$\frac{1}{2}^+$	5815.2 ± 2.0
Σ_b^{*+}	uub	$\frac{3}{2}^+$	5829.0 ± 3.4
Σ_b^{*-}	ddb	$\frac{3}{2}^+$	5836.4 ± 2.8
Ξ_b^-	dsb	$\frac{1}{2}^+$	5790.5 ± 2.7
Ω_b^-	ssb	$\frac{1}{2}^+$	6071 ± 40

Similar to previous charm baryon study bottom baryons need to be studied thoroughly.

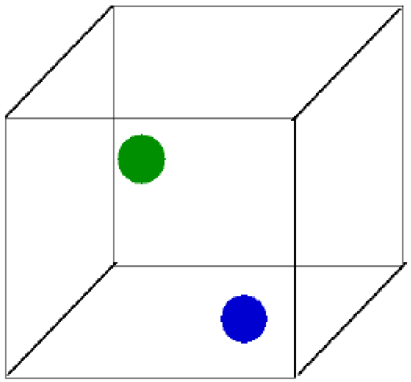
For triply bottom baryon one study has been carried out (S.Meinel :PRD85, 114510(2012))



Brown et al, arXiv:1409.0497v1

Multi-particle states

A problem for finite box lattice



- ✓ Finite box : Momenta are quantized
- ✓ Lattice Hamiltonian can have both resonance and decay channel states (scattering states)

- ✓ A → x+y, Spectra of m_A and $\sqrt{m_x^2 + p_n^2} + \sqrt{m_y^2 + p_n^2}, p_n = \frac{2\pi n}{La}$
- ✓ One needs to separate out resonance states from scattering states

What is a resonance particle?

- Resonances are simply energies at which differential cross-section of a particle reaches a maximum.
- In scattering expt. resonance ➔ dramatic increase in cross-section with a corresponding sudden variation in phase shift.
- Unstable particles but they exist long enough to be recognized as having a particular set of quantum numbers.
- They are not eigenstates of the Hamiltonian, but has a large overlap onto a single eigenstates.
- Volume dependence of spectrum in finite volume is related to the two-body scattering phase-shift in infinite volume.
- Near a resonance energy : phase shift rapidly passes through pi/2, an abrupt rearrangement of the energy levels known as avoided "level crossing" takes place.

Identifying a Resonance State

- Relate finite box energy to infinite volume phase shifts by Luscher formula
- Calculate energy spectrum for several volumes to evaluate phase shifts for various volumes
- Extract resonance parameters from phase shifts

Luscher's Method

- Relates the spectrum in a finite box to the scattering phase shift (and so resonance properties)

Lüscher's formula

$$\delta(p) = -\phi(\kappa) + \pi n$$

$$\tan\phi(\kappa) = \frac{\pi^{3/2}\kappa}{Z_{00}(1;\kappa^2)}$$

$$\kappa = \frac{pL}{2\pi}$$

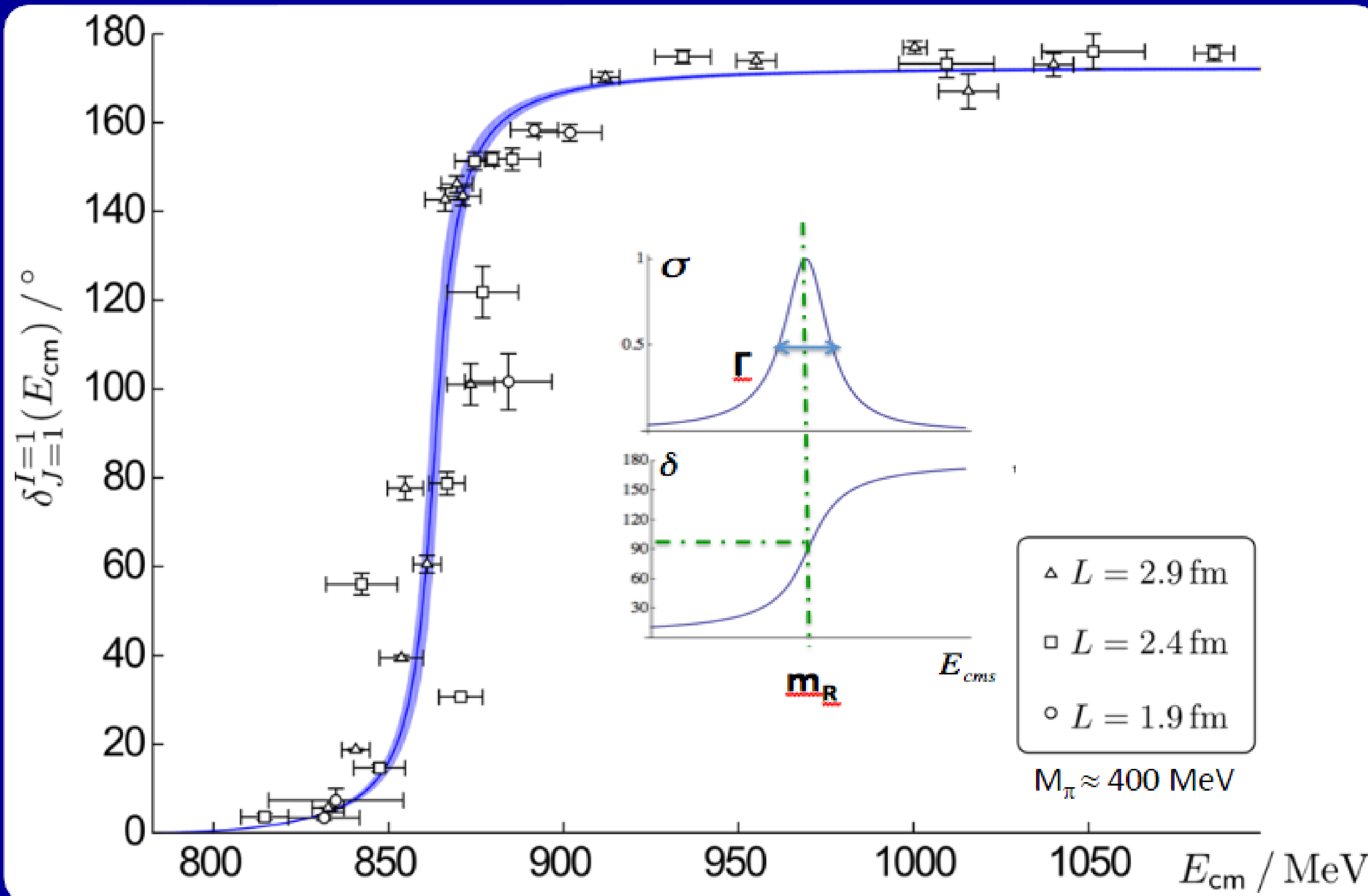
- p_n is defined for level n with energy E_n from the dispersion relation:

$$E_n = 2\sqrt{m^2 + p_n^2}$$

The ρ resonance

Rho decay

HSC : [PR D87, 034505]



Ongoing and future study

- Include multi-particle operators for baryons
- Calculate resonance parameters

Hadron Spectroscopy

Experiments

LHCb

ATLAS CMS

CLAS12 GlueX

+ others at 12 GeV JLab

BESIII KLOE2

PANDA

+ others at GSI

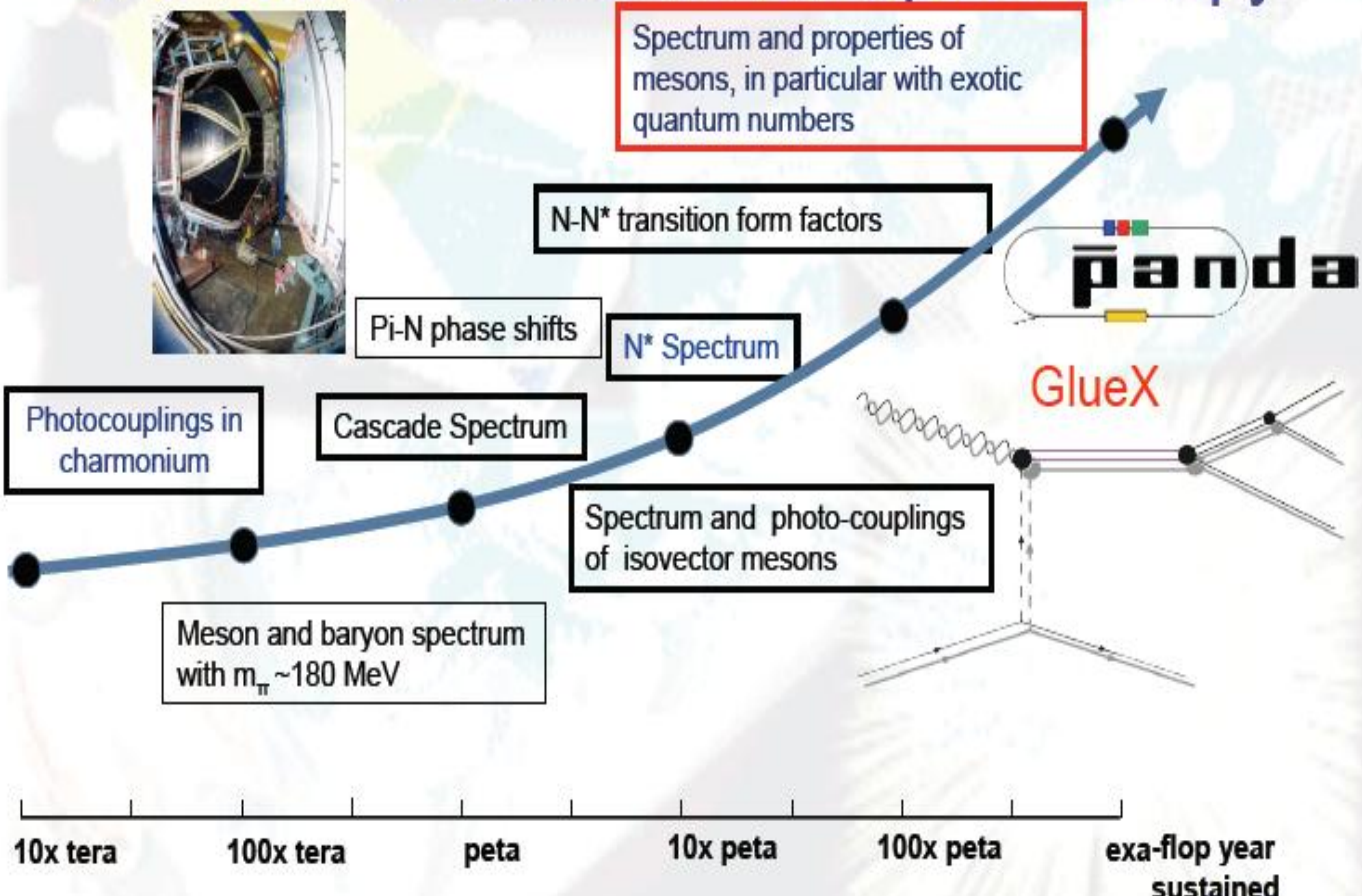
COMPASS

Belle II

ELSA MAMI J-PARC Spring-8

@C.Thomas

The road to exascale for Spectroscopy



Conclusion

Lattice QCD has entered an era where it can make significant precise contributions to nuclear and particle physics.

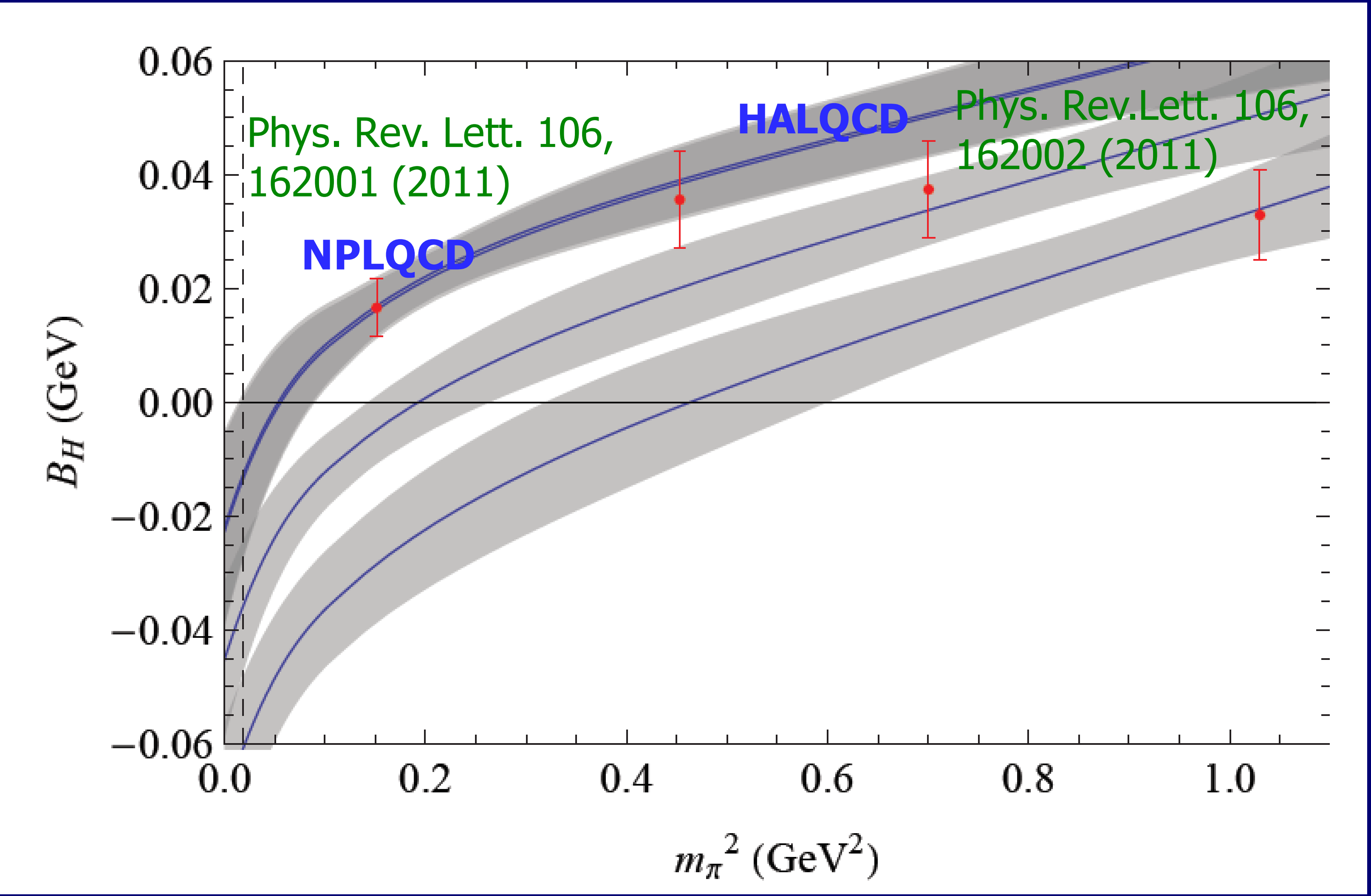
Particle Masses : Understanding the Structure and Interaction of Hadrons.

- Full QCD calculations are now accessible at physical pion mass and at reasonably large volumes. Lattice QCD is able to reproduce ground state baryons accurately for many hadrons.

- However, resonance states, including excited state masses, are still not accessible comprehensively. Data analysis becomes increasingly difficult as we go towards chiral limit due to the appearance of multi-particle states.

- A comprehensive program is ongoing at Hadron Spectrum Collaboration by using multi-operator variational method with distilation technique in order to extract resonance states.

H dibaryon (uuddss, I=0, $^1S_0$)



Shanahan et al, Phys. Rev. Lett. 107, 092004 (2011)

Mass in Euclidean space

Fourier transform in Euclidean time

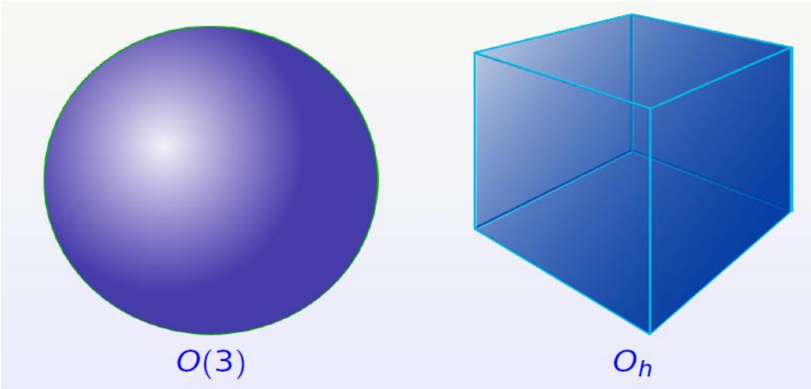
$$\int d\tau\, e^{ip_4\tau}\, \frac{e^{-M_n|\tau|}}{2M_n} = \frac{1}{2M_n(M_n - ip_4)} + \frac{1}{2M_n(M_n + ip_4)}$$

$$= \frac{1}{M_n^2 + p_4^2} \xrightarrow{p_4 \to -iE} \frac{1}{M_n^2 - E^2}$$

M_n : location of poles in the propagator of |n>.
pole masses of physical state

Symmetries of the lattice Hamiltonian

- SU(3) gauge group (colour)
- $Z_n \otimes Z_n \otimes Z_n$ cyclic translational group (momentum)
- SU(2) isospin group (flavour)
- O_h^D crystal point group (spin and parity)



Operators

Mesons: fermion bi-linears

$\bar{\psi}\Gamma\psi$ — J = 0, 1

$\bar{\psi}\Gamma\overleftrightarrow{D}\psi$ — J = 0, 1, 2 — gauge-covariant derivatives ~ 1^{--}

$\bar{\psi}\Gamma\overleftrightarrow{D}\overleftrightarrow{D}\psi$ — J = 0, 1, 2, 3 — coupling $\langle 1m_1; 1m_2 | L_{12} m_{12}\rangle \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2}$

$\bar{\psi}\Gamma\overleftrightarrow{D}\overleftrightarrow{D}\overleftrightarrow{D}\psi$ — J = 0, 1, 2, 3, 4 — 2 derivatives can give chromo B field 1^{+-}

Baryons: three quarks

$$\Phi^{J,j} = \langle 1l_1; 1l_2 | Ll\rangle\langle Ll; Ss | Jj\rangle \vec{D}_{l_1} \vec{D}_{l_2} [\psi\psi\psi]_s$$

$$\mathbf{1} \otimes \mathbf{1} \otimes \mathcal{S} \to \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$$

$J/\Psi - \eta_c$	$70(2)(^{5}_{4})$	$76(3)(^{7}_{5})$
D	$1842(28)(^{33}_{31})$	$1850(35)(^{28}_{24})$
D_s	$1980(23)(^{26}_{23})$	$1958(33)(^{23}_{21})$
$D^* - D$	$98(6)(^{5}_{3})$	$101(6)(^{6}_{5})$
$D_s^* - D_s$	$94(4)(^{4}_{3})$	$96(4)(^{4}_{5})$
B_s^0	$5380(108)(^{21}_{18})$	$5375(103)(^{20}_{21})$
$B^* - B^0$	$32(4)(^{3}_{2})$	$35(6)(^{3}_{3})$
$B_s^* - B_s^0$	$29(3)(^{2}_{2})$	$32(4)(^{3}_{2})$
Σ_c	$2407(32)(^{32}_{37})$	$2452(38)(^{38}_{36})$
Ξ_c	$2440(27)(^{28}_{26})$	$2473(34)(^{34}_{33})$
Ω_c	$2652(25)(^{27}_{31})$	$2678(33)(^{33}_{31})$
$\Sigma_c^* - \Sigma_c$	$75(20)(^{14}_{12})$	$86(18)(^{12}_{13})$
$\Xi_c^* - \Xi_c'$	$71(18)(^{12}_{9})$	$81(16)(^{11}_{10})$
$\Omega_c^* - \Omega_c$	$65(13)(^{7}_{8})$	$74(14)(^{8}_{8})$
$\Sigma_c - \Lambda_c$	$128(28)(^{39}_{28})$	$162(36)(^{33}_{26})$
$\Xi_c' - \Xi_c$	$104(19)(^{20}_{23})$	$126(21)(^{15}_{22})$
Ξ_{cc}	$3562(47)(^{27}_{25})$	$3588(66)(^{32}_{27})$
Ω_{cc}	$3681(44)(^{17}_{19})$	$3698(60)(^{26}_{23})$
$\Xi_{cc}^* - \Xi_{cc}$	$63(14)(^{9}_{7})$	$70(11)(^{7}_{7})$
$\Omega_{cc}^* - \Omega_{cc}$	$56(8)(^{7}_{6})$	$63(7)(^{5}_{5})$
Λ_b	$5664(98)(^{33}_{46})$	$5672(102)(^{35}_{41})$
Ξ_b	$5762(83)(^{29}_{38})$	$5788(86)(^{30}_{36})$
Ω_b	$6021(75)(^{27}_{34})$	$6040(77)(^{25}_{31})$
$\Sigma_b^* - \Sigma_b$	$22(10)(^{7}_{6})$	$24(11)(^{7}_{5})$
$\Xi_b^* - \Xi_b'$	$21(10)(^{7}_{6})$	$23(11)(^{7}_{5})$
$\Omega_b^* - \Omega_b$	$18(7)(^{6}_{4})$	$20(8)(^{5}_{3})$
$\Sigma_b - \Lambda_b$	$141(24)(^{30}_{22})$	$175(27)(^{26}_{24})$
$\Xi_b' - \Xi_b$	$124(22)(^{32}_{18})$	$148(25)(^{24}_{15})$
$\Xi_{bb}^* - \Xi_{bb}$	$22(6)(^{4}_{3})$	$20(6)(^{3}_{4})$
$\Omega_{bb}^* - \Omega_{bb}$	$20(4)(^{3}_{3})$	$19(4)(^{3}_{3})$
Ξ_{cb}'	$6810(150)(^{62}_{79})$	$6840(228)(^{38}_{72})$
Ω_{cb}'	$6935(135)(^{72}_{89})$	$6954(214)(^{62}_{81})$
$\Xi_{cb}^* - \Xi_{cb}'$	$46(8)(^{4}_{6})$	$43(9)(^{6}_{6})$
$\Omega_{cb}^* - \Omega_{cb}'$	$40(6)(^{4}_{5})$	$39(6)(^{5}_{5})$
$\Xi_{cb} - \Xi_{cb}'$	$11(6)(^{4}_{5})$	$9(5)(^{6}_{4})$
$\Omega_{cb} - \Omega_{cb}'$	$10(5)(^{4}_{4})$	$9(4)(^{4}_{4})$

Mathur, Lewis, Woloshyn

PRD66, 014502 (2002); PRD64, 094509 (2001)