HPQCD

DiRAC

Lattice QCD results for mesons containing b quarks from the HPQCD collaboration

Ron Horgan
DAMTP, University of Cambridge

CONFINEMENT XI

St Petersburg

Outline

- Radiative improvement of NRQCD using background field approach.
- Lattice action and most recent configurations.
- Quarkonium and B-meson spectrum; hyperfine splittings.
- *B*-meson decay constants: f_B/f_{B_s} .
- $B - \bar{B}$ mixing.
- Mass of the b quark.
- $B \longrightarrow K^* \mu^+\mu^-$ decay
- The future for heavy quarks: NRQCD and HISQ .

Radiatively Improved NRQCD

Evolve heavy quark Green's function with kernel:

$$K(\tau) = \left(1 - \frac{\delta H|_\tau}{2}\right)\left(1 - \frac{H_0|_\tau}{2n}\right)^n U_4^\dagger(\tau-1)\left(1 - \frac{H_0|_{\tau-1}}{2n}\right)^n\left(1 - \frac{\delta H|_{\tau-1}}{2}\right),$$

where

$$H_0 = \frac{\vec{\Delta}^{(2)}}{2M_0}, \qquad \delta H = -c_1\frac{(\vec{\Delta}^{(2)})^2}{8M_0^3} + c_2\frac{ig}{8M_0^2}\left(\vec{\Delta}^{(\pm)}\cdot\vec{E} - \vec{E}\cdot\vec{\Delta}^{(\pm)}\right)$$
$$-c_3\frac{g}{8M_0^2}\vec{\sigma}\cdot\left(\vec{\Delta}^{(\pm)}\times\vec{E} - \vec{E}\times\vec{\Delta}^{(\pm)}\right) - c_4\frac{g}{2M_0}\vec{\sigma}\cdot\vec{B} + c_5\frac{a^2\vec{\Delta}^{(4)}}{24M_0} - c_6\frac{a(\vec{\Delta}^{(2)})^2}{16nM_0^2} + \mathcal{O}(v^6)$$

- At tree level $c_i = 1$
- Radiatively improve c_1, c_2, c_4, c_5, c_6 to 1-loop using Background Field Method
- Also include certain four-fermion operators in NRQCD action:

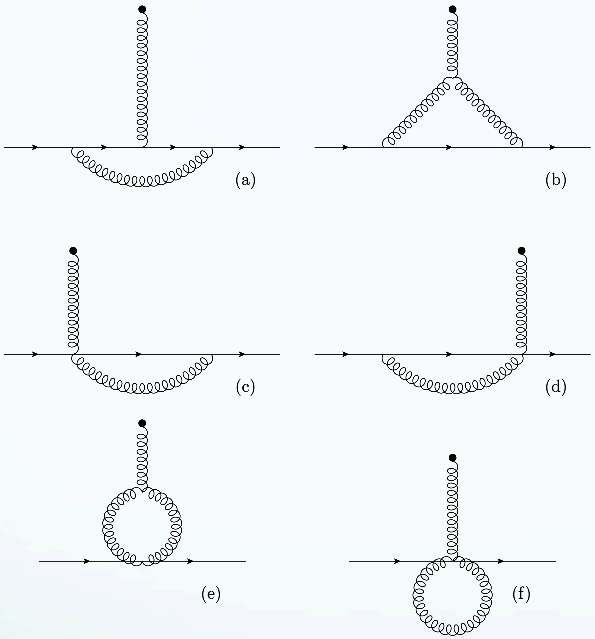
$$S_{4f} = d_1\frac{\alpha^2}{M^2}(\psi^\dagger\chi^*)(\chi^T\psi) + d_2\frac{\alpha^2}{M^2}(\psi^\dagger\vec{\sigma}\chi^*)\cdot(\chi^T\vec{\sigma}\psi).$$

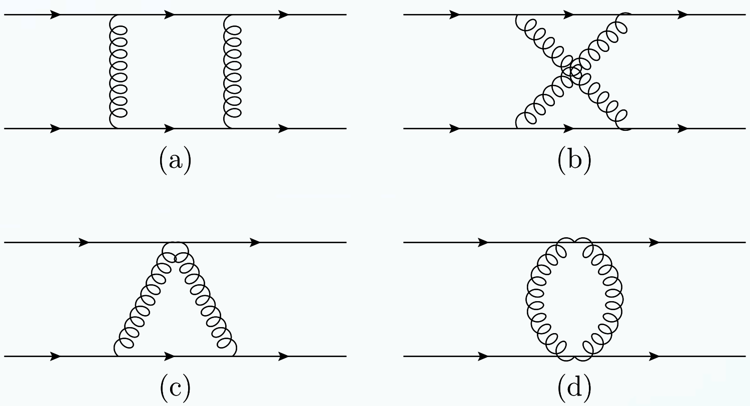
Matching with Background Field Method

- NRQCD is an effective theory containing operators with $D > 4$ which, at tree level, can be restricted to be gauge-covariant.
- Vital to use formulation where no non-covariant operators are generated by radiative processes.
- Gauge invariance is retained by the method of background field gauge, and ensures gauge invariance of the effective action.
- All counter-terms are FINITE in BFG => can compute ALL matching, both continuum relativistic and non-relativistic, using lattice regularization: QED-like Ward Identities.
- Derive 1PI gauge-invariant effective potential. Match (on-shell) S-matrix.
- Implemented in HiPPY and HPsrc for automated lattice perturbation theory.

Matching with Background Field Method

In particular, evaluate the spin-dependent diagrams vital for accurate evaluation of hyperfine structure:





Improve also:

- Currents for Υ, B decays
- Wilson operators for $B - \bar{B}$ mixing

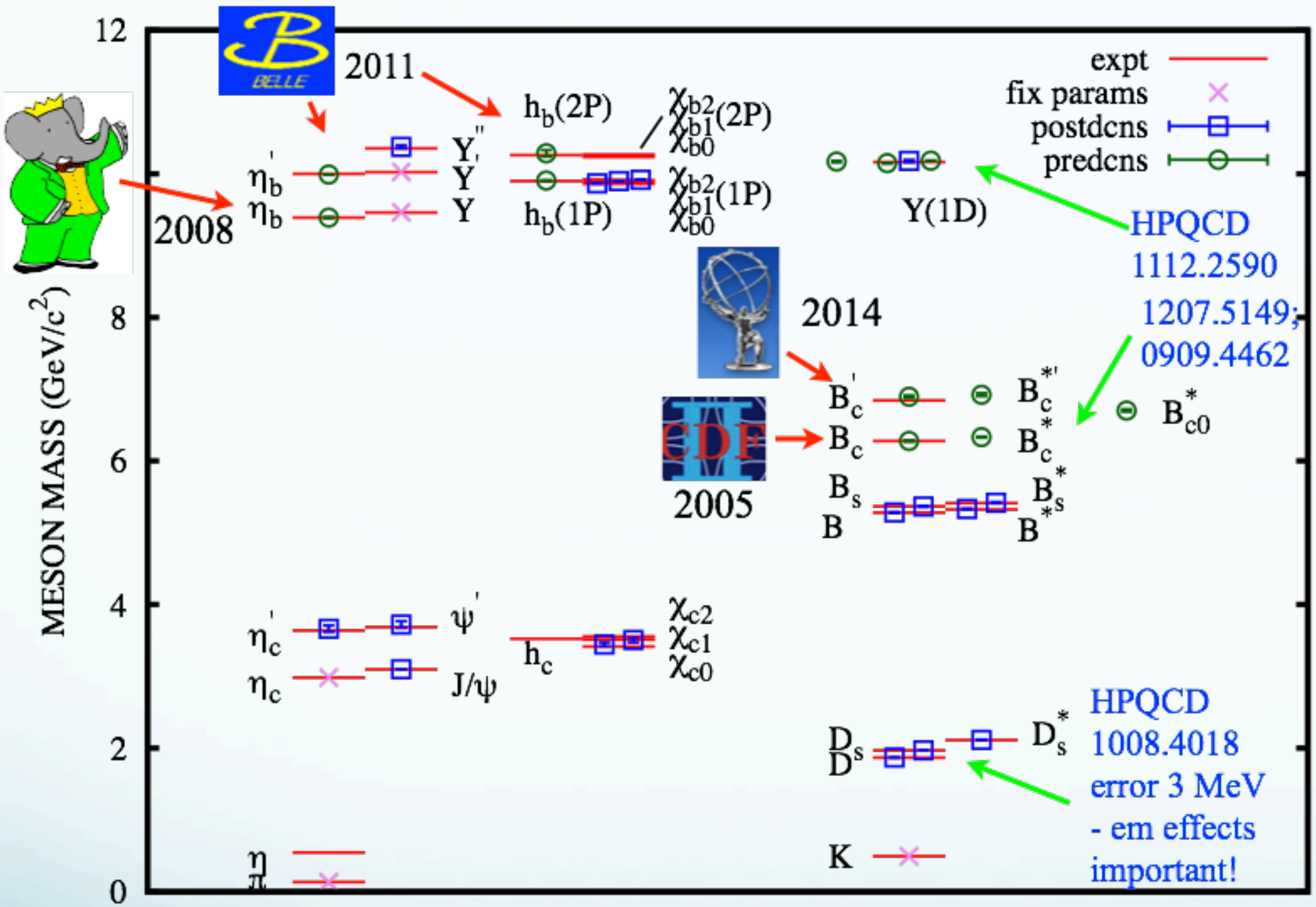
Most Recent Configurations

- HISQ staggered 2+1+1 (u,d,s,c sea quark) configurations generated by the MILC collaboration with radiatively improved gluon action.
- Lattice spacing generally determined by $\Upsilon(2S-1S)$ splitting. Errors are statistics, NRQCD systematics, experiment.
- Sets 3,6,8 are at the physical point; no chiral extrapolation necessary.

Set	β	a_Υ (fm)	am_l	am_s	am_c	$L\times T$	$n_{\rm cfg}$	m_π(MeV)
1	5.8	0.1474(5)(14)(2)	0.013	0.065	0.838	16×48	1020	307
2	5.8	0.1463(3)(14)(2)	0.0064	0.064	0.828	24×48	1000	215
3	5.8	0.1450(3)(14)(2)	0.00235	0.0647	0.831	32×48	1000	131
4	6.0	0.1219(2)(9)(2)	0.0102	0.0509	0.635	24×64	1052	305
5	6.0	0.1195(3)(9)(2)	0.00507	0.0507	0.628	32×64	1000	218
6	6.0	0.1189(2)(9)(2)	0.00184	0.0507	0.628	48×64	1000	132
7	6.3	0.0884(3)(5)(1)	0.0074	0.037	0.440	32×96	1008	314
8	6.3	0.0873(2)(5)(1)	0.0012	0.0363	0.432	64×96	621	128

Results presented use these and also MILC 2+1 Asqtad configurations.

The gold-plated spectrum - HPQCD



Use 2nd generation, HISQ 2+1+1 sea: sets 1,2,4,5,7 (not at physical point)

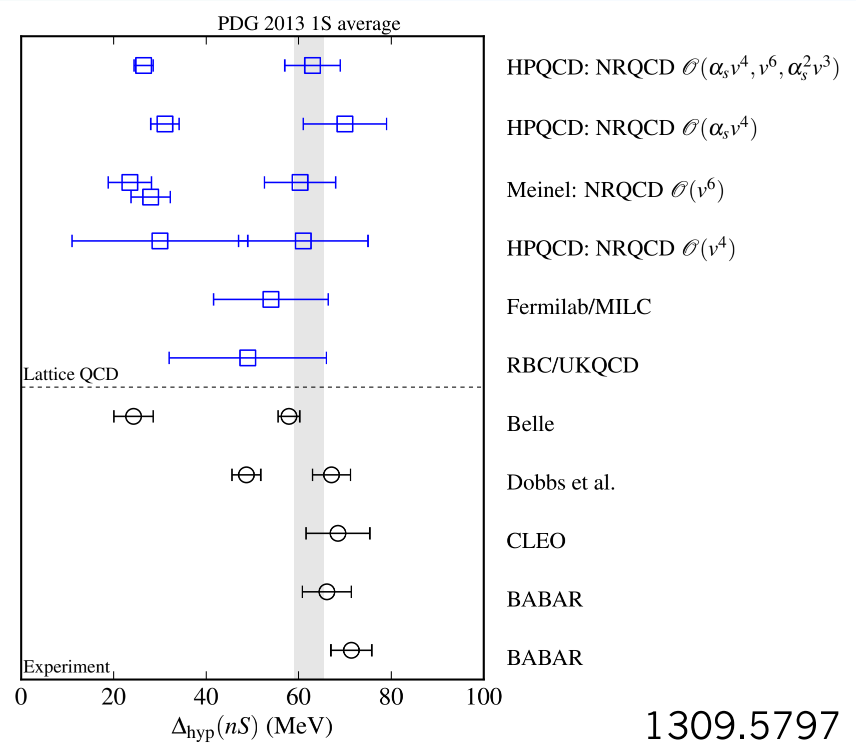
Radiatively improved NRQCD for b quark with HISQ u,s,d,c valence quarks.

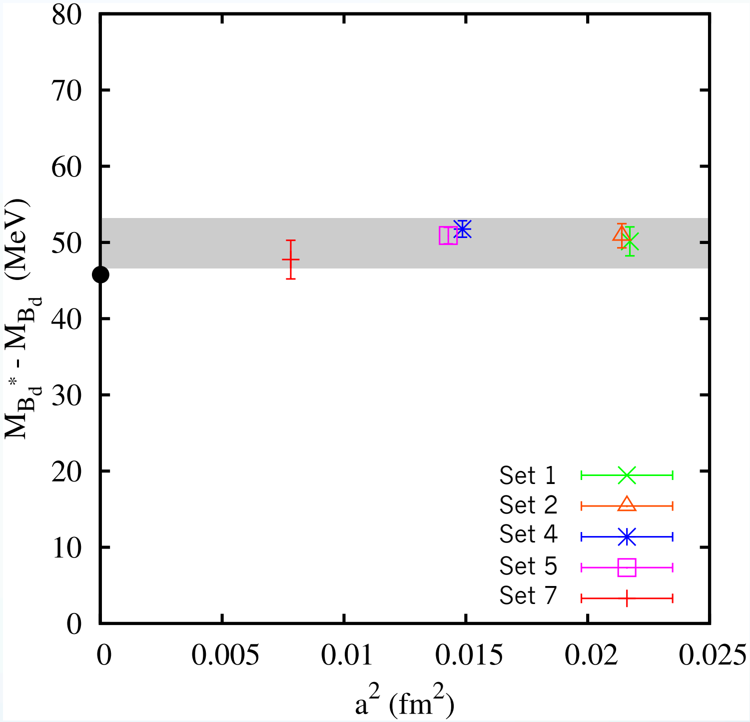
$$M_{B_s} - M_B = 84(2)\text{MeV},\ M_{B_s} = 5.366(8)\text{GeV},\ M_{B_c} = 6.278(9)\text{GeV}$$

Expt: 87.51(24)MeV, 5.36677(24)GeV, 6.2745(18)GeV

Hyperfine splittings:

Already $O(v^2)$, so improvement to spin-dependent NRQCD operators vital.





$M_\Upsilon(1S) - \eta_b(1S) = 62.8(6.7)\mathrm{MeV}$

$\Delta M(2S)/\Delta M(1S) = 0.425(25)$

$M_{B_d^*} - M_{B_d} = 50(3)\mathrm{MeV}$ $(45.8(4)\ \mathrm{MeV})$

$M_{B_s^*} - M_{B_s} = 52(3)\mathrm{MeV}$ $(46.1(1.5)\mathrm{MeV})$

$M_{B_c^*} - M_{B_c} = 54(3)\mathrm{MeV}$ $PREDICTED$

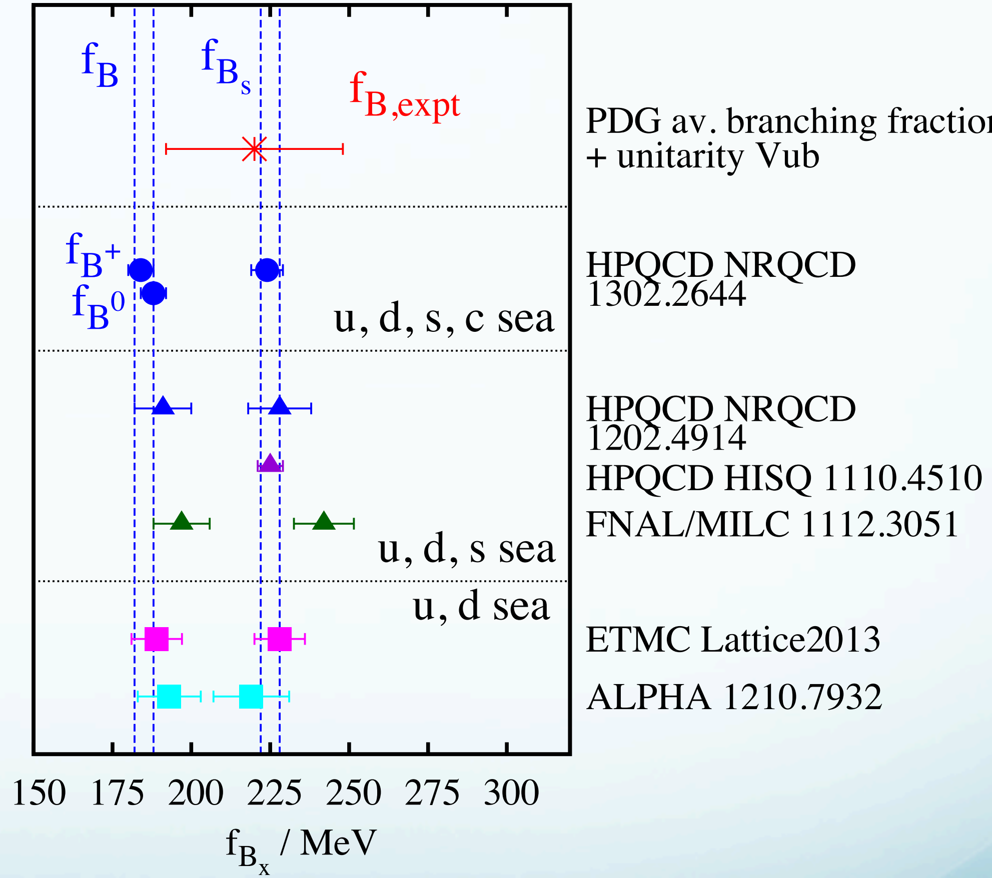
Decay Constants

B-meson decay constants, $\langle 0|A_0|B_q\rangle_{\rm QCD} = m_{B_q} f_{B_q}$:

First LQCD results for f_B, f_{B_s} with physical light quarks:

$$f_{B^+} = 184(4)\text{MeV}$$
$$f_{B_s} = 224(4)\text{MeV}$$
$$\frac{f_{B_s}}{f_{B^+}} = 1.217(8)$$

Experiment within 1-s.d. of lattice f_B. Error mainly experimental, but also some uncertainty in V_{ub} .



LQCD predicts:

$$\frac{1}{|V_{ub}|^2}\text{Br}(B^+ \to \tau\nu) = 6.05(20)$$

f_{B_s}, f_{B_d} crucial to $B_s, B_d \to \mu^+\mu^-$ decay, and hitherto major source of error:

$$\mathrm{Br}(B_q \to l^+ l^-) = A\,\tau(B_q)\,|V_{tb}^* V_{ts}|^2 f_{B_q}^2 m_b^2 m_l^2$$

Using world-average, HPQCD, results for f_{B_s}, f_{B_d} find

$$\begin{aligned} \mathrm{Br}(B_s \to \mu^+\mu^-) &= 3.17(15)(9)\,10^{-9} \\ \mathrm{Br}(B_d \to \mu^+\mu^-) &= 1.05(5)(5)\,10^{-10} \end{aligned}$$

Second error, from f_{B_q}, now competitive.

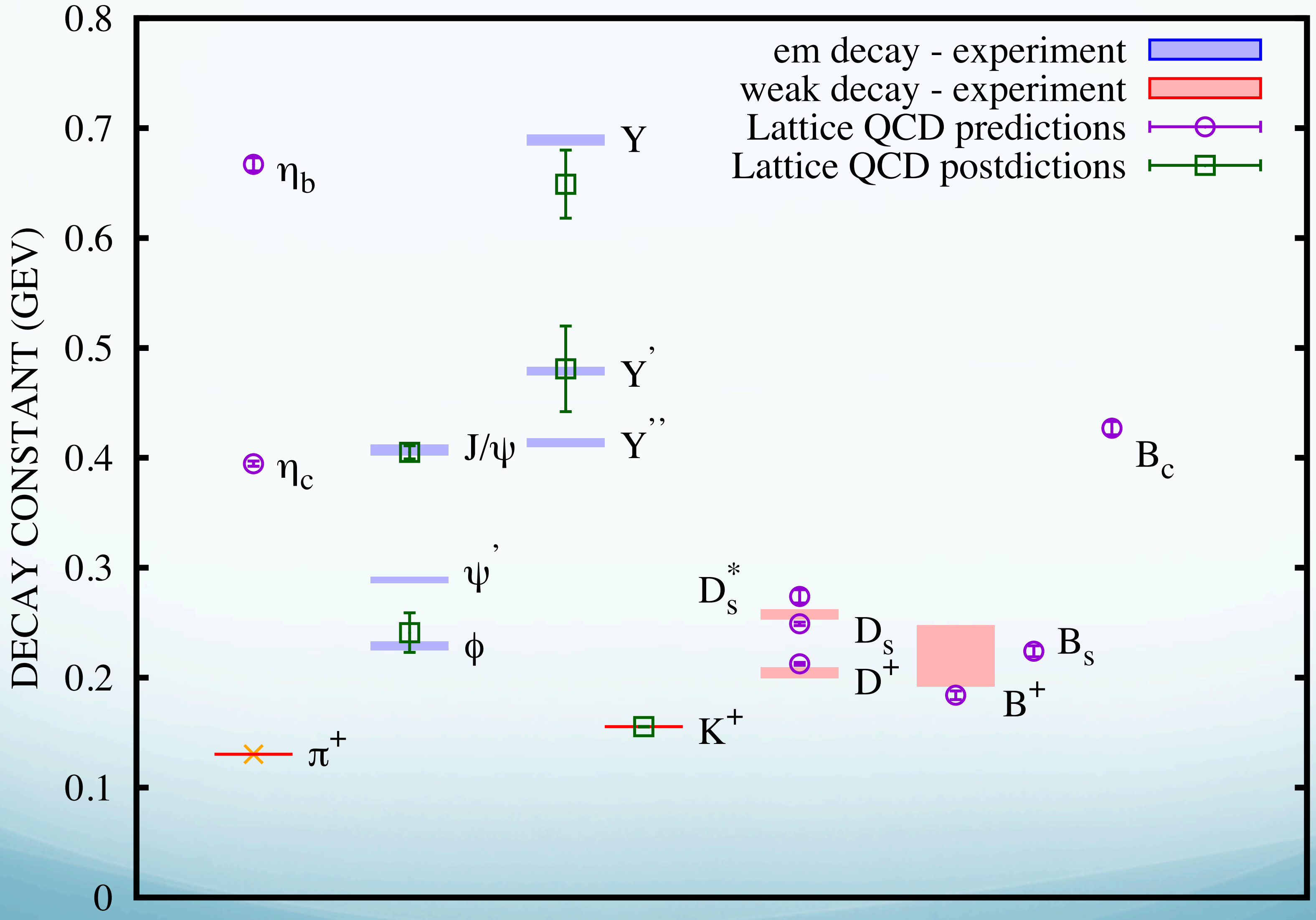
Υ electromagnetic decay constant: $\langle 0|J_i|\Upsilon_j^{(\prime)}\rangle = m_{\Upsilon^{(\prime)}} f_{\Upsilon^{(\prime)}} \delta_{ij}$

$$\boldsymbol{J} = Z_V(\boldsymbol{J}_0^{latt} + k_1 \boldsymbol{J}_1^{latt} + \ldots)$$

- $\boldsymbol{J}_n^{latt}$ is n-th term in derivative expansion of the NRQCD lattice current.
- Z_V, k_1 determined non-perturbatively using JJ correlator.

$$f_\Upsilon = 0.649(31)\mathrm{GeV} \quad f_{\Upsilon'} = 0.481(39)\mathrm{GeV}$$

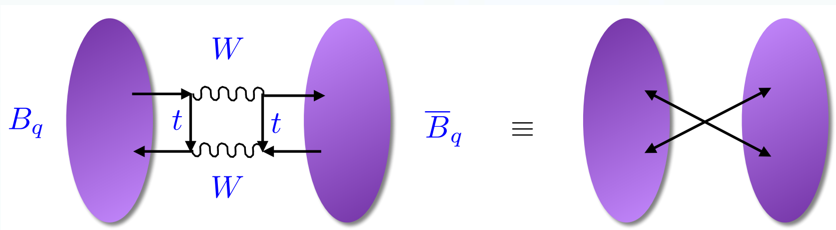
Decay constant summary plot

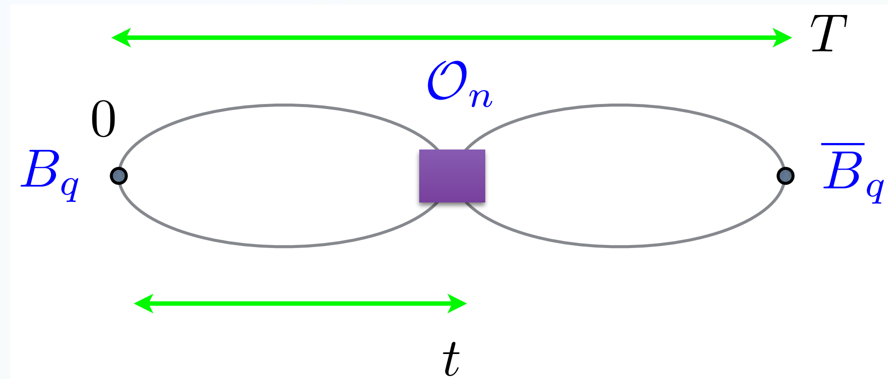


$B - \bar{B}$ mixing

First LQCD calculation of B_s, B_d mixing parameters with physical light quark masses at three lattice spacings: MILC HISQ sets 3,6,8

Compute matrix elements of effective 4-quark operators derived from box diagrams using LQCD:





$$
\begin{aligned}
O_1 &= (\bar{b}^\alpha \gamma_\mu P_L q_\alpha)(\bar{b}^\beta \gamma_\mu P_L q_\beta) \\
O_2 &= (\bar{b}^\alpha P_L q_\alpha)(\bar{b}^\beta P_L q_\beta) \\
O_3 &= (\bar{b}^\alpha P_L q_\beta)(\bar{b}^\beta P_L q_\alpha)
\end{aligned}
$$

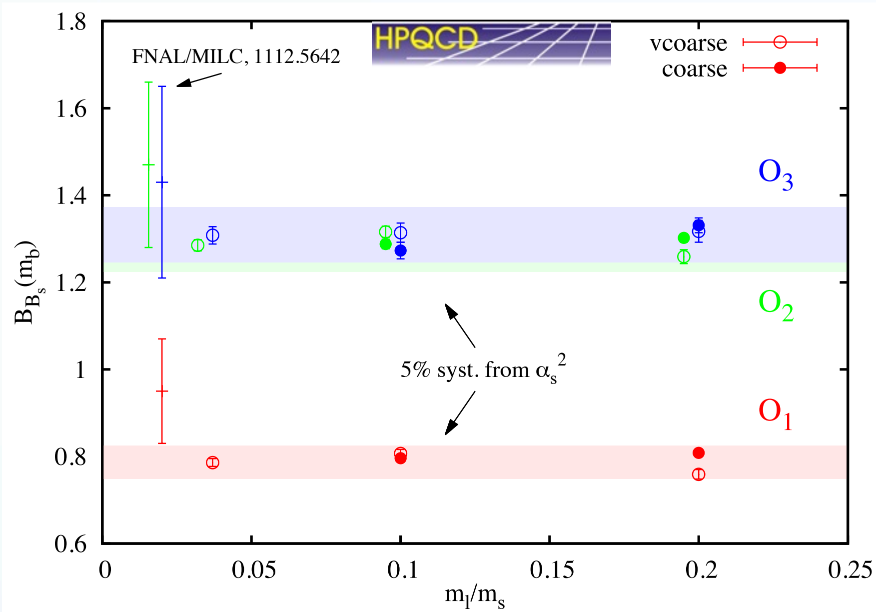
- O_1 needed for B_d, B_s oscillations
- All three appear in B width difference
- Use radiatively improved NRQCD

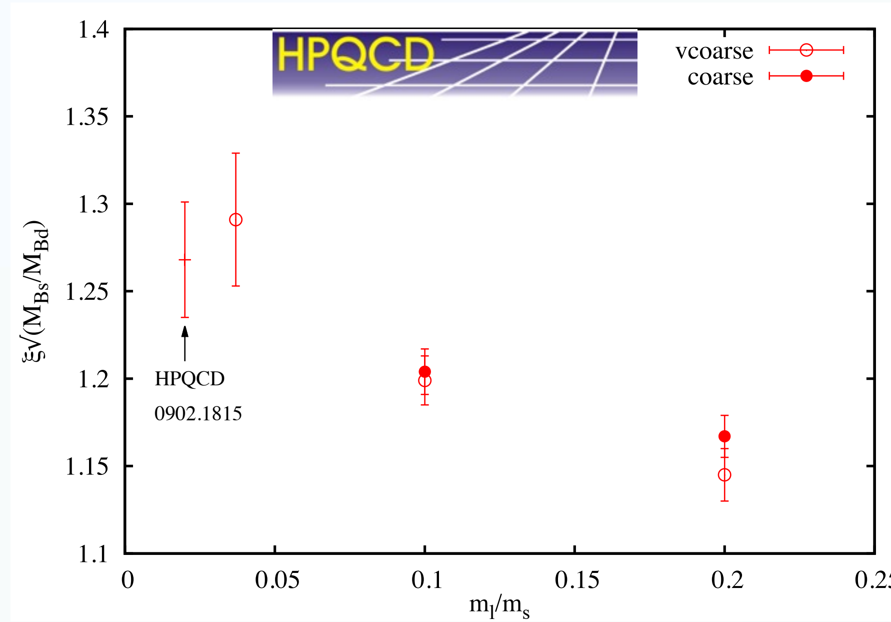
Bag parameters defined by $\langle O_1 \rangle_{\bar{M}S}(\mu) = \frac{8}{3} f_B^2 B_B(\mu) M_B^2$

Similarly for $O_2,\ O_3$ with $8/3 \longrightarrow -5/3,\ -1/3$, respectively

Operators: NRQCD b-quark and HISQ light quark matched to continuum:

$$\langle O_1\rangle_{\overline{MS}}(m_b) = [1+\alpha_s\rho_{11}]\langle O_{1,\mathrm{NRQCD}}\rangle + \alpha_s\rho_{12}\langle O_{2,\mathrm{NRQCD}}\rangle + \ldots$$





$B_{B_s}(m_b)$ versus m_l/m_s

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$ (PDG: $1.237(32)$)

Using expt., current HPQCD analysis gives: $|V_{td}|/|V_{ts}| = 0.214(1)(5)$

(PDG: $0.211(1)(6)$)

Results preliminary: more accurate results at physical point imminent.

b-quark mass

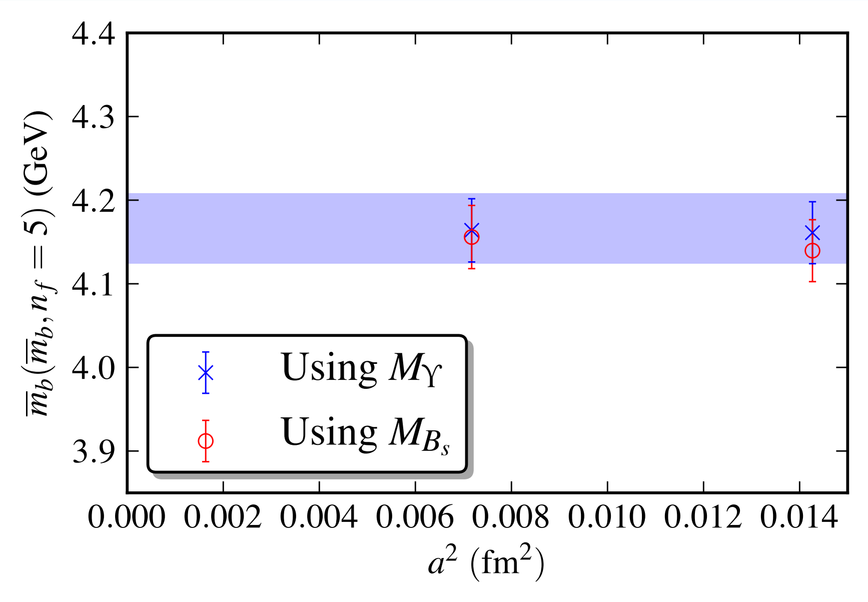
$$\overline{m}_b(\mu) = \frac{1}{2} Z_M^{-1}(\mu) \underbrace{\left[M_\Upsilon^{\text{expt}} - a^{-1}\left(aE_{\text{sim}} - 2aE_0\right)\right]}_{m_{\text{pole}}}$$

- $Z_M(\mu)$ known to 3-loop order
- E_{sim} is the energy of the Υ meson at rest using NRQCD on the lattice
- E_0 is computed fully to 2-loops in perturbation theory as follows:

1. Measure E_0 on high-β quenched gluon configurations using heavy quark propagator in Landau gauge with t'Hooft twisted boundary conditions.
2. Fit E_0 to 3rd-order series in $\alpha_{\overline{MS}}(1/a)$ and extract quenched 2-loop coefficient.
3. Compute 2-loop n_f contribution using automated perturbation theory for b-quark self-energy at p = 0.

Similar theory using B meson instead.

- Fit to E_0 consistent with known 1-loop automated pert. th. result
- Include 3-loop quenched coefficient in E_0 .
- Error dominated by $\boldsymbol{n_f}$ 3-loop contribution.



$\overline{m}_b(\overline{m}_b, n_f = 5)$ for two lattice spacings using both $\Upsilon, \ B$

$$\overline{m}_b(\overline{m}_b, n_f = 5) = 4.166(43)\mathrm{GeV}$$

- Most accurate to use $\langle J_5 J_5 \rangle$ and $\langle J_i^{(V)} J_j^{(V)} \rangle$ correlator method.
- Applicable for HISQ and NRQCD valence quarks.
- For HISQ valence need extrapolation in some cases from $m_{0h} \to m_{0b}$

$$G(t) = m_0^2 \langle J_5(t+t_0) J_5(t_0) \rangle_{latt} = \overline{m}(\mu)^2 \langle J_5(t+t_0) J_5(t_0) \rangle (\mu)_{\overline{MS}}$$

since $G(t)$ is not renormalized.

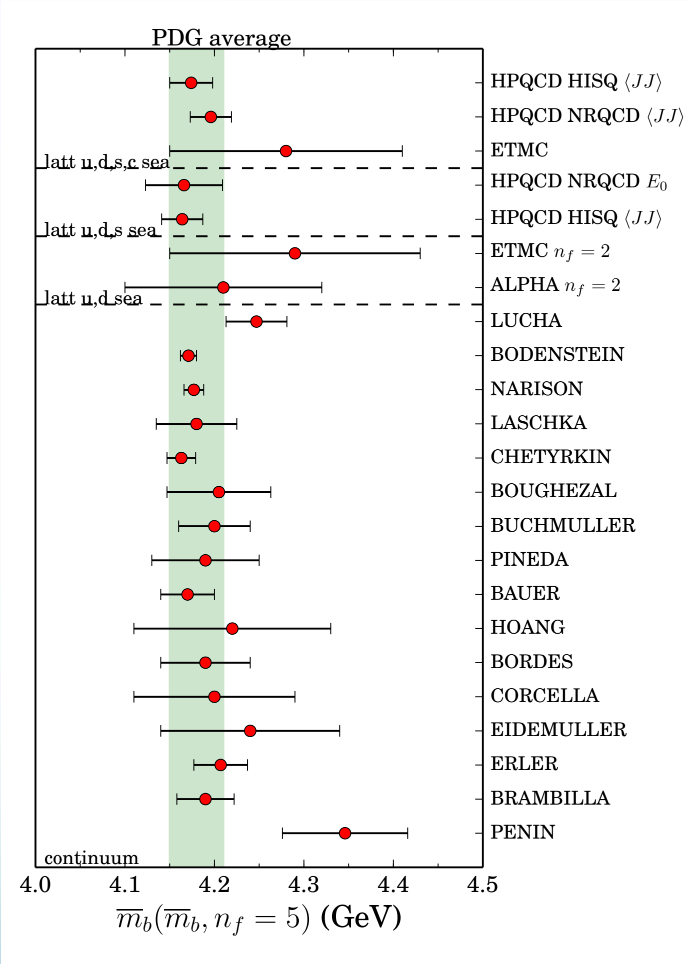
- RHS from 3-loop calculation of Chetyrkin et al.
- LHS from LQCD.
- Use moments $G_n = \sum_t t^n G(t),\ n \geq 4$.
- Fit to extract $\alpha_s(\mu),\ \overline{m}_b(\mu)$.

For $n_f = 5$ this method gives

$$\begin{aligned} \overline{m}_b(\overline{m}_b) &= 4.174(24)\text{GeV} \quad \text{HISQ} \\ &= 4.196(23)\text{GeV} \quad \text{NRQCD} \end{aligned}$$

Weighted lattice average for measurements on configs with 3,4 sea quarks and then run to $n_f = 5$:

$$\overline{m}_b(\overline{m}_b, n_f = 5) = 4.185(15)\text{GeV}$$



$B^0 \longrightarrow K^{*0}\mu^+\mu^-$ and $B_s^0 \longrightarrow \phi\,\mu^+\mu^-$

(RRH with Z. Liu, S. Meinel, M. Wingate: 1310.3887, 1310.3722)

- Highly suppressed in Standard Model: test for BSM physics
- Recently measurements by LHCb and data also from CDF, ATLAS, CMS
- Describe observables by non-perturbative form factors.
- Calculate form-factors at large q^2 with NRQCD on lattice.
- Complements results of other analyses including sum rules at low q^2

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i [C_i O_i + C_i' O_i']$$

BSM physics

Wilson coeffs

Relevant operators:

$$O_7^{(\prime)} = e\, m_b/(16\pi^2)\, \bar{s}\sigma_{\mu\nu} P_{R(L)} b\, F^{\mu\nu}$$

$$O_9^{(\prime)} = e^2/(16\pi^2)\, \bar{s}\gamma_\mu P_{L(R)} b\, \bar{l}\gamma^\mu l$$

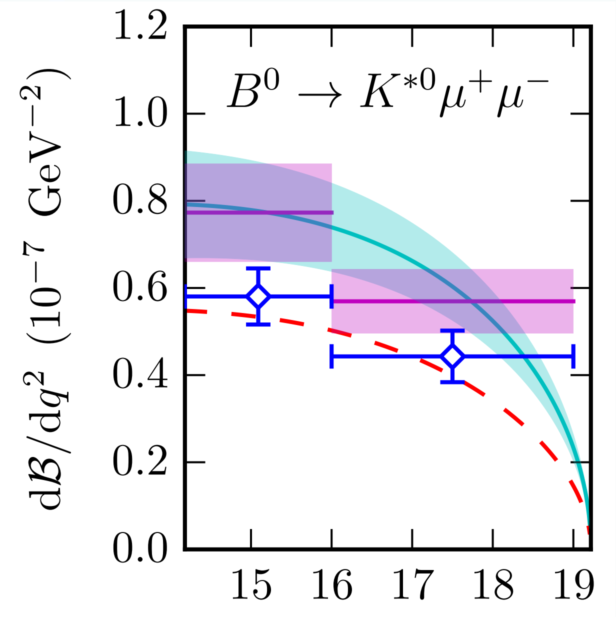
$$O_{10}^{(\prime)} = e^2/(16\pi^2)\, \bar{s}\gamma_\mu P_{L(R)} b\, \bar{l}\gamma^\mu \gamma_5 l$$

Look for departure from SM ➔ BSM physics

Example of tension between SM and data

$$\frac{d\mathcal{B}}{dq^2} = \tau_{B_s^0}\,\frac{d\Gamma}{dq^2}$$

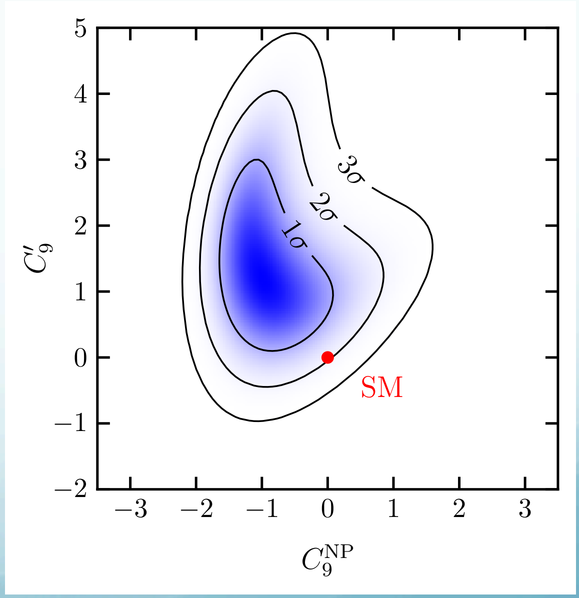
- Shaded bands show theory errors.
- Average over bins conforming to data bins.



- Best fit allowing BSM coefficients $C_9^{NP},\ C_9'$:

$$C_9^{NP} = -1.0 \pm 0.6,\ C_9' = 1.2 \pm 1.0$$

- An indication that BSM physics contributes?
- Agrees with other analyses: 1307.5683, 1308.1501, 1308.4379, 1310.2478
- Caution: signal intriguing but not yet properly significant.
- Caution: need analysis of charmonium resonance effects



The Future: NRQCD and HISQ

- NRQCD: Have radiatively improved coefficents and operators.
- HISQ: Fully relativistic and applicable for $am_0 \lesssim 1.0$.
- Next generation configs: physical sea quarks; incorporate QED effects.
- Improve QCD parameters: α_s , quark masses and hadronic matrix elements.
- See 1404.0319 for relevance of accurate $\alpha_s,\ \overline{m}_h(\mu)$, h=c,b, to higgs physics and future collider programmes.

Selected quantities:

Quantity	CKM/ expt. process	Current expt. error	Current latt. error	2018 latt. error
f_B	$\lvert V_{ub}\rvert$	12%	2%	1%
f_{B_s}	$B_s \to \mu^+\mu^-$	25%	2%	1%
$f^2_{B_s} B_{B_s}(\Delta M_s)$	$\lvert V_{ts}V_{tb}\rvert^2$	0.24%	10%	3%
$B \to \pi l \nu$	V_{ub}	4.1%	9%	2%
$B \to D/D^* l\nu$	$\lvert V_{cb}\rvert$	1.3%	2%	$< 1\%$
$B_s \to \phi\mu^+\mu^-$		20%	10%	4%

HPQCD: recent past and present

C. Davies, J. Koponen, B. Chakraborty, B. Colquhoun,
G. Donald, B. Galloway (Glasgow)
G.P. Lepage (Cornell)
G. von Hippel (Mainz)
C. Monahan (William and Mary)
A. Hart (Edinburgh)
C. McNeile (Plymouth)
RRH, R. Dowdall, T. Hammant, A. Lee (Cambridge)
J. Shigemitsu (Ohio State)
K. Hornbostel (Southern Methodist Univ.)
H. Trottier (Simon Fraser)
E. Follana (Zaragoza)
E. Gamiz (CAFPE, Granada)