On the effect of pion condensate on the spectrum of neutron stars

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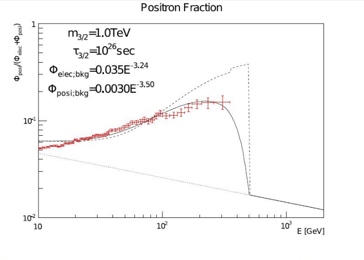
- Introduction.
- Neutron star.
- Effective model.
- Numerical calculations.
- Influence on spectrum.
- Photon decay.
- Experimental data.
- Conclusion.

Introduction

Dark Matter

Fermi-LAT, PAMELA, AMS-2:
anomalous excess of e^+e^-.
arXiv:1304.1483 [hep-ph]



1: The positron fraction expected in the decaying gravitino dark matter

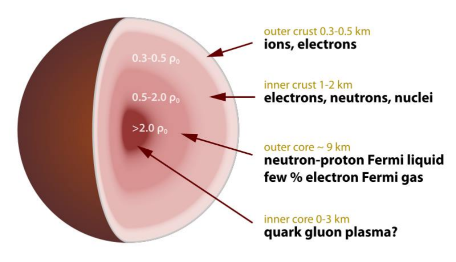
XMM-Neuton, *Chandra*: 3.55-3.57 keV.
arXiv1403.2370 [hep-ph]: dark matter decay into ALP.

Heavy ion physics

NA60, PHENIX: abnormal yield of lepton pairs (e, μ)

Neutron stars

Structure of a neutron star

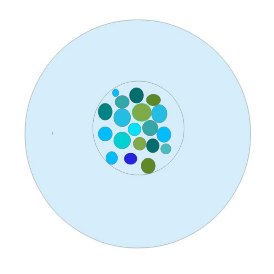


Formation standing wave of σ and π^0 fields: axial wave condensation inside neutron stars.
(Koichi Takahashi, 2002)

Density of this condensate decreases along the radius and disappears outside a core of the star.

- In such assumption there will be an area inside the star with broken parity in this region, which modify a spectrum of the star.

Structure of a neutron star



More realistic is "bubble" model, when there are a lot of small regions with pseudoscalar condensate, which brakes parity.
For every "bubble" $a_{c\ell}$ may be different.

Effective Lagrangian e.g. (Andrianov, S.K., Soldati arXiv:1109.3440)

$$\mathcal{L} = -\frac{1}{4} F^{\alpha\beta}(x) F_{\alpha\beta}(x) - \frac{1}{4} F^{\mu\nu}(x) \widetilde{F}_{\mu\nu}(x)\, a_{c\ell}(x)$$

- Inside every bubble $a_{c\ell}$ is constant
- There is a thin shell of a bubble with a big gradient of condensate density

Model

Effective Lagrangian for the boundary of the bubble

$$\mathcal{L} = -\frac{1}{4} F^{\alpha\beta}(x)F_{\alpha\beta}(x) - \frac{1}{4} F^{\mu\nu}(x)\widetilde{F}_{\mu\nu}(x)\, a_{c\ell}(x)$$

A_μ and $a_{c\ell}$: the vector and effective background pseudoscalar fields,

$$a_{c\ell}(x) = \zeta x_1\, \theta(-x_1)$$

The corresponding field equations are,

$$\Box A^\nu + m^2 A^\nu + \zeta \varepsilon^{1\nu\sigma\rho}\, \theta(-x_1)\partial_\sigma A_\rho = 0$$

But we don't know

- The typical size of the bubble
- The distribution of the bubbles
- The mean free path of the photon inside neutron star

Model

We consider concrete simplified model in order to get some quantitative result.

- In the core there is a pseudoscalar background
- There is a thin spherical symmetric shell around, where background vanishes
- The mean free path and wave length of photons are small compared to the thickness of the shell
- Hot matter inside the neutron star gives thermal spectrum

field equations

$$\Box A^{\nu} + m^2 A^{\nu} + \zeta \varepsilon^{1\nu\sigma\rho} \theta(-x_1) \partial_\sigma A_\rho = 0$$

$$S^\nu_\lambda = \delta^{\,\nu}_{\,\lambda}\,\mathtt{D} + k^\nu\, k_\lambda\, \zeta^2 + \zeta^{\,\nu}\,\zeta_\lambda\, k^2 - \zeta\cdot k\,(\zeta_\lambda\, k^\nu + \zeta^\nu\, k_\lambda); \quad \mathtt{D}\ \equiv\ (\zeta\cdot k)^2 - \zeta^2\, k^2$$

Transversal polarizations are,

$$\boldsymbol{\pi}^{\,\mu\nu}_{\pm} \equiv \frac{S^{\,\mu\nu}}{2\,\mathtt{D}}\ \pm\ \frac{i}{2}\,\varepsilon^{\mu\nu\alpha\beta}\,\zeta_\alpha\, k_\beta\, \mathtt{D}^{\,-\frac{1}{2}}; \quad \varepsilon^{\,\mu}_{\,\pm}(k) = \boldsymbol{\pi}^{\,\mu\lambda}_{\pm}\,\epsilon^{(0)}_\lambda$$

Scalar and longitudinal polarizations,

$$\varepsilon^{\,\mu}_{\,S}(k)\ \equiv\ \frac{k^{\,\mu}}{\sqrt{k^2}}, \qquad \varepsilon^{\,\mu}_{\,L}(k)\ \equiv\ \left(\mathtt{D}\,k^2\right)^{-\frac{1}{2}}\left(k^2\,\zeta^{\,\mu} - k^{\,\mu}\,\zeta\cdot k\,\right)$$

$$S_\lambda^\nu = \delta^{\,\nu}_{\,\lambda}\,\mathtt{D} + k^\nu\, k_\lambda\, \zeta^2 + \zeta^{\,\nu}\, \zeta_\lambda\, k^2 - \zeta\cdot k\, (\zeta_\lambda\, k^\nu + \zeta^\nu\, k_\lambda); \quad \mathtt{D} \;\equiv\; (\zeta\cdot k)^2 - \zeta^2\, k^2$$

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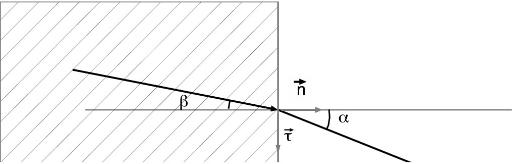
Spatial CS vector. $\zeta_\mu = (0, -\zeta_x, 0, 0)$: dispersion laws

$$\begin{cases} k_{1L} = k_{10} = \sqrt{\omega^2 - m^2 - k_\perp^2} \\ k_{1\pm} = \sqrt{\omega^2 - m^2 - k_\perp^2 \mp \zeta_x \sqrt{\omega^2 - k_\perp^2}} \end{cases}$$

Neutron star. Photon escaping (Andrianov, S.K. arXiv:1212.5723)



- We now consider the one direction of falling photons.
- $\vec{k} = k_n \vec{n} + k_\perp \vec{\tau}$
- $\frac{k_n}{k_\perp} = \cot(\alpha); \frac{k_n^{CS}}{k_\perp} = \cot(\beta)$

After the propagation through the boundary, direction is changed (angle α) in correspondence to the polarization of the falling particles.

$$k_{n\pm}^{CS} = \sqrt{\omega^2 - k_\perp^2 \mp \zeta\sqrt{\omega^2 - k_\perp^2}}; k_n = \sqrt{\omega^2 - k_\perp^2} \qquad \Rightarrow k_{n\pm}^{CS} = \sqrt{k_n^2 \mp \zeta k_n}$$

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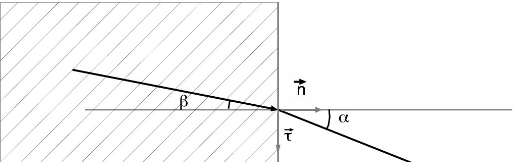

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Now it is easy to write our transmission coefficient,

$$k_{tr}^{\pm} = \frac{2k_{n\pm}^{CS}}{k_{n\pm}^{CS} + k_n} = \frac{2\cot(\alpha)}{\cot\alpha + \cot(\beta)}.$$

- In our purpose it is necessary to express $k_{tr}^{\pm}$ as a function of β, which means to express α in terms of β.
- We consider the photons on the mass shell, so in vacuum $\omega = |\vec{k}|$, and one can use $k_n = \omega \cos \alpha$.

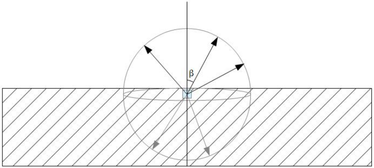
$$\cot \alpha = \cot \beta \frac{\omega \cos \alpha}{\sqrt{\omega^2 \cos^2 \alpha \mp \zeta \omega \cos \alpha}}.$$

  Where the $\mp$ stands for different polarizations.
- Solving this equation one can find the expression for $\cot \beta$ for different polarization and the value of $k_{tr}^{\pm}$,

$$k_{tr}^{\pm}(\beta, \zeta, \omega) = \frac{2 \cot \beta}{\cot \beta + \frac{\pm\zeta + \sqrt{\zeta^2 + 4\omega^2 \cot^2 \beta (1 + \cot^2 \beta)}}{\sqrt{4\omega^2 (1 + \cot^2 \beta) - 2\zeta^2 \mp 2\zeta \sqrt{\zeta^2 + 4\omega^2 \cot^2 \beta (1 + \cot^2 \beta)}}}}$$

Now we consider the all flux of outgoing photons from parity breaking medium. To do it we at first consider a small volume near the boundary.

We assume that it radiates uniformly in all directions, but for us is important a flux of energy which propagates to the surface of neutron star.

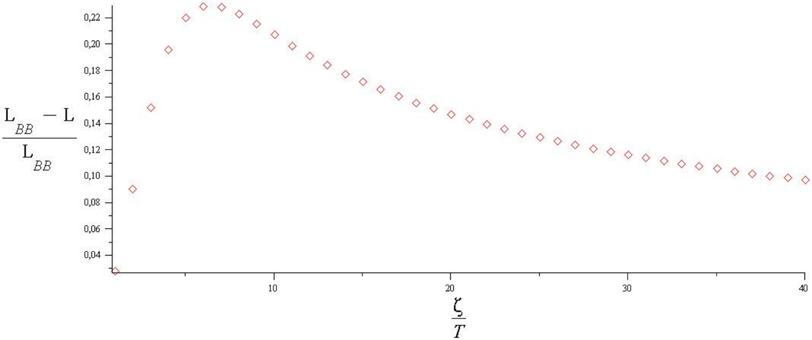


We assume that our small volume radiates at certain frequency N_Ω^ω in a unit of solid angle. In order to find the total luminosity of our layer we should take integrals over the solid angle, frequency and surface of the layer. The last integration will be the same for the case of absence of pion condensate. That is why it is not so important.

$$L \propto \int_\zeta^\infty d\omega N_\Omega^\omega \int_0^{\frac{\pi}{2}} d\beta k_{tr}^+(\beta,\omega,\zeta) + \int_0^\infty d\omega N_\Omega^\omega \int_0^{\frac{\pi}{2}} d\beta k_{tr}^-(\beta,\omega,\zeta)$$

To show the qualitative effect of pion condensate we assume, that the whole thin layer at the boundary of parity-breaking medium has a temperature T and radiates as a black body $N^{\omega}_{\Omega} \propto \frac{\omega^3}{e^{\frac{\omega}{T}}-1}$.

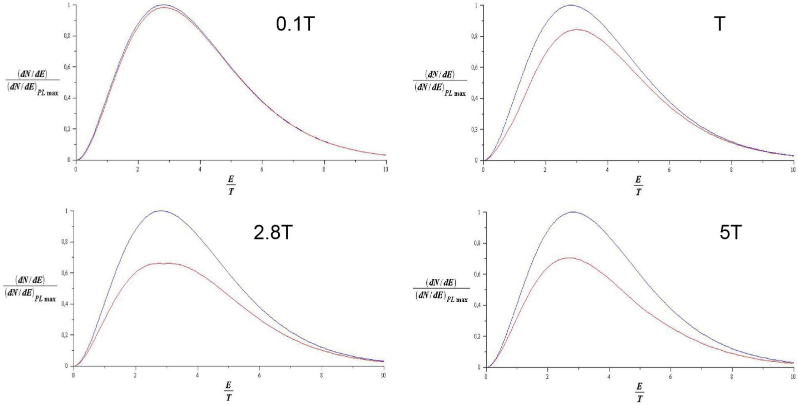
For different values of ζ - parameter the flux of radiated outside energy will differ.



One can see that for $\zeta > T$ there is a strong effect on the flux of outgoing energy which should slow down the cooling of the core.

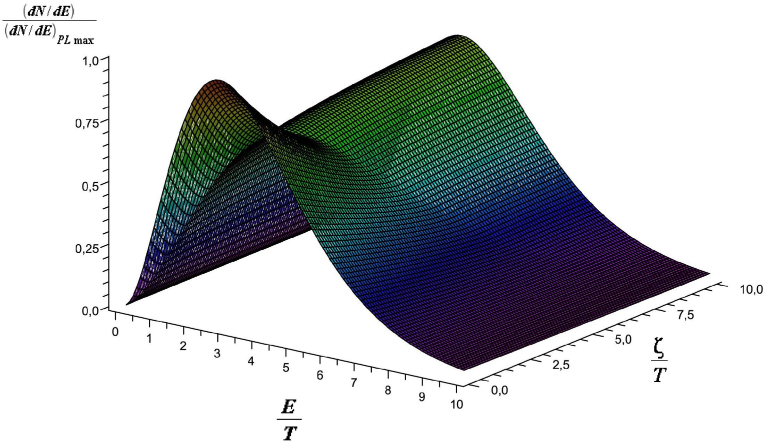
Spectrum

We present here the influence of a described pion condensate on the thermal radiation with effective temperature T and spectral radiance $B(\omega, T) \sim \frac{\omega^3}{e^{\frac{\omega}{T}}-1}$.



Spectrum

We may also plot a three-dimensional picture, where ζ will be a changing parameter. After some value ($\zeta \approx 2.8T$) with increasing ζ an effect decreases.



Photon decay Andrianov et al. arXiv:0907.3709

The photon of positive polarization may decay in presence of gradient of pseudoscalar field in $e+e-$ pair. This process will suppress the number of outgoing photons with positive chirality and possibly increase the number of outgoing electrons, positrons and antineutrino due to the process,

$$e^{+} + n^{0} \to p^{+} + \bar{\nu}_{e}.$$

The total decay width for high-energy photons with positive polarization in pseudoscalar background is,

$$\Gamma_{+} \simeq \frac{\alpha\zeta}{3}$$

We assume that the total flux of positive polarized photons outgoing from the layer is $N_0(\omega)$. After the photons with positive polarization propagate in parity breaking area, the number of photons should decrease,

$$N(t,\omega) = N_0 e^{-\frac{\Gamma_{+}\cdot t}{\gamma}},$$

where $\gamma = \frac{1}{1-v^2}$ stays for the Lorenz factor of the particle.

$$v = \frac{|\mathbf{k}_+|}{\omega} = \frac{\sqrt{\omega^2 - \zeta\omega}}{\omega}; \qquad \gamma = \frac{1}{\sqrt{1 - v^2}} = \sqrt{\frac{\zeta}{\omega}}.$$

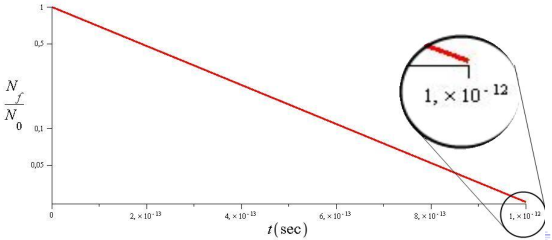
And we get,

$$N(t, \omega) = N_0 e^{-\frac{t \cdot \alpha\zeta\sqrt{\zeta}}{3\sqrt{\omega}}}$$

There is a threshold of described decay,

$$\omega \geqslant \frac{4m_e^2}{\zeta}.$$

Number of "surviving" photons on the propagation time.
$(\omega = 1 GeV; \zeta = 1 keV)$



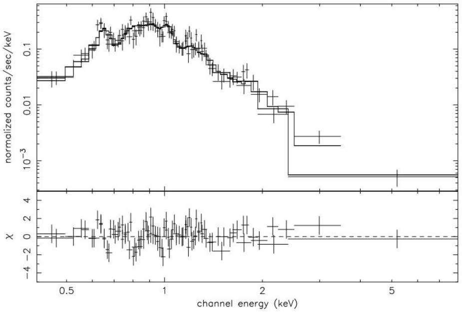


Fig. 9. Energy spectrum of the rim emission underneath RX J0822−4300 as observed in April 2001 with the EPIC-MOS1/2 detector and simultaneously fitted to an absorbed non-equilibrium ionization collisional plasma model (*upper panel*) and contribution to the χ^2 fit statistic (*lower panel*).

Bartlett et al. (arXiv:1309.2658)

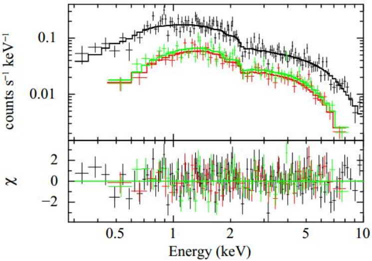


Figure 5. The 0.2–10.0 keV EPIC-pn (black), EPIC-MOS1 (red), EPIC-MOS2 (green) spectra of Swift J045106.8-694803. Top panel displays the background subtracted spectrum with best fit *phabs*vphabs(powerlaw+bbody)* model, bottom panel shows the residuals.

Hui et al. (arXiv:0508655)

Typical effective temperature of a neutron star is $T \sim keV$, so the boundary effects arises for $\zeta \sim O(keV)$.

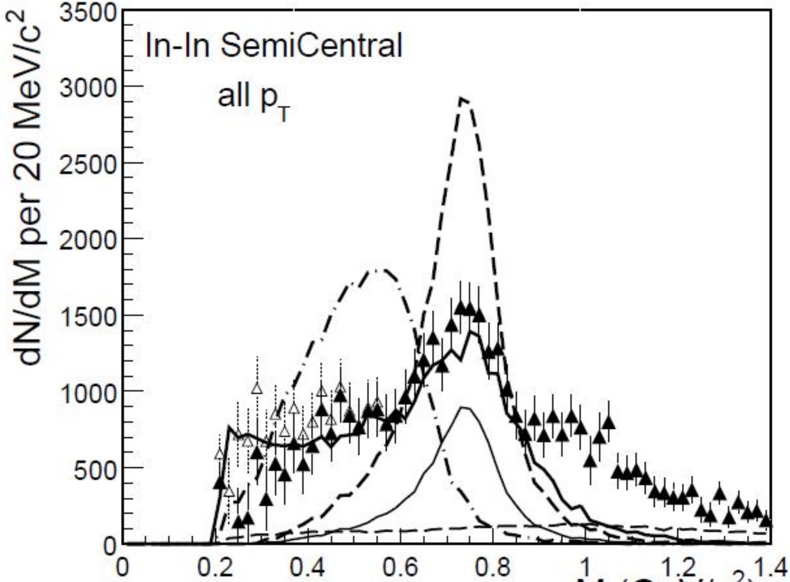
Conclusion

- Results:
  - * We have discussed the simplest model, which give an idea how may the spectrum of neutron star change;
  - * We have shown, that even in this model there is a significant (observable) effect.
- X-ray neutron star physics, including *Chandra* and *XMM-Newton*. However, an accuracy of modern experiments does not allow us to say confidently, is there any anomalies in spectrum or the strange regions are just fluctuations and instruments' errors.

Of course, discussing model is not precise, one should take into account lots of processes taking place in neutron stars to make a theoretical prediction of the spectrum of these objects.

BACKUP SLIDES

arXiv:nucl-ex/0605007v1 10 May 2006



$\hat{k} = (\omega, k_2, k_3),\ \hat{x} = (x_0, x_2, x_3) \, : \ \hat{k}\cdot\hat{x} = -\omega x_0 + k_2 x_2 + k_3 x_3 .$

Proca-Stückelberg solution

$$A^{\mu}_{\mathrm{PS}}(x) = \int \mathrm{d}\hat{k}\, \theta(\omega^2 - k_{\perp}^2 - m^2) \sum_{r=1}^{3} \left[\mathbf{a}_{\hat{k}\,,\,r}\, u^{\mu}_{\hat{k}\,,\,r}(x) + \mathbf{a}^{\dagger}_{\hat{k}\,,\,r}\, u^{\mu\,*}_{\hat{k}\,,\,r}(x) \right]$$

$$u^{\nu}_{\hat{k}\,,\,r}(x) = [\,(2\pi)^3\, 2k_{10}\,]^{-1/2}\, e^{\nu}_{r}(\hat{k})\, \exp\{\, i\, k_{10} x_1 + i\, \hat{k}\cdot\hat{x}\} \qquad (\, r = 1, 2, 3\,)$$

Chern-Simons solution

$$A^{\nu}_{\mathrm{CS}}(x) = \int \mathrm{d}\hat{k} \sum_{A=\pm, L} \theta(k_{1A}^2(\omega, k_{\perp})) \left[c_{\hat{k},A}\, v^{\nu}_{\hat{k}\,A}(x) + c^{\dagger}_{\hat{k},A}\, v^{\nu *}_{\hat{k}\,A}(x) \right]$$

$$v^{\nu}_{\hat{k}\,A}(x) = \left[\, (2\pi)^3\, 2k_{1A} \,\right]^{-\frac{1}{2}} \varepsilon^{\nu}_{A}(k)\ \exp\{\, i k_{1A}\, x_1 + i\hat{k}\cdot\hat{x}\} \quad (A = L, \pm)$$

$$\left[\, A^{\mu}_{\mathrm{PS}}(x) - A^{\mu}_{\mathrm{CS}}(x) \,\right] |_{\zeta\cdot x = 0} = 0$$

Bogolubov Transformations

$$v^{\nu}_{\hat{k},A}(\hat{x}) = \sum_{s=1}^{3} \left[\alpha_{sA}(\hat{k})\, u^{\nu}_{\hat{k},s}(\hat{x}) - \beta_{sA}(\hat{k})\, u^{\nu *}_{\hat{k},s}(\hat{x})\right]$$

relations between the creation-destruction operators are,

$$\mathbf{a}_{\hat{k},\, r} = \sum_{A=\pm,L} \left[\alpha_{rA}(\hat{k})\, c_{\hat{k},A} - \beta^{*}_{rA}(\hat{k})\, c^{\dagger}_{\hat{k},A}\right]$$

$$c_{\hat{k},A} = \sum_{r=1}^{3} \left[\alpha^{*}_{Ar}(\hat{k})\, \mathbf{a}_{\hat{k},\, r} + \beta^{*}_{Ar}(\hat{k})\, \mathbf{a}^{\dagger}_{\hat{k},\, r}\right]$$

There are two different Fock vacua,

$$\mathbf{a}_{\hat{k},\, r}|0\rangle = 0 \qquad c_{\hat{k},A}\,|\,\Omega\,\rangle = 0$$

$$\langle\, 0\,|\,\mathbf{a}_{\hat{p},s}\, c^{\dagger}_{\hat{k},A}\,|\,0\,\rangle = \delta(\hat{k} - \hat{p})\,\alpha_{As}(\hat{k})$$

The latter quantity can be interpreted as the relative probability amplitude that particle is transmitted from the left face to the right face.

Vacuum as a squeezed state

In the correct normalization, $\langle 0|0\rangle = 1$, $\langle\Omega|\Omega\rangle = 1$.
Going to the continuum limit for $\hat{k}$,

$$|0\rangle = \exp\left[\int \left(\sum_{A=\pm,L} \frac{\beta^*_{rA}(\hat{k})}{2\alpha_{rA}(\hat{k})} (c^\dagger_{\hat{k},A})^2 \theta(k^2_{1A}(\hat{k}))\right) d\hat{k}\right] |\,\Omega\,\rangle$$

$$|\,\Omega\,\rangle = \exp\left[\int \theta(\omega^2 - m^2 - k^2_\perp) \left(\sum_{r=1,2,3} \frac{-\beta^*_{Ar}(\hat{k})}{2\alpha^*_{Ar}(\hat{k})} (\mathbf{a}^\dagger_{\hat{k},\,r})^2\right) d\hat{k}\right] |0\rangle$$

$\zeta_\mu = (0, -\zeta, 0, 0)$

$$\Box A^\nu + m^2 A^\nu + \zeta \varepsilon^{1\nu\sigma\rho}\theta(-x_1)\partial_\sigma A_\rho = 0$$

A_1 may be found in the whole space,

$$A_1 = \int \frac{d\hat{k}}{(2\pi)^3} \left(\tilde{u}_{1\rightarrow}(\omega, k_2, k_3)e^{ik_{10}x_1} + \tilde{u}_{1\leftarrow}(\omega, k_2, k_3)e^{-ik_{10}x_1}\right)e^{i\hat{k}\hat{x}}$$

Solution for A_ν $(\nu = 0, 2, 3)$

$$A_\nu = \int \frac{d\hat{k}}{(2\pi)^3} \tilde{A}_\nu e^{i\hat{k}\hat{x}}$$

$$\tilde{A}_\nu = \begin{cases} \tilde{u}_{\nu\rightarrow}(\omega, k_2, k_3)e^{ik_{10}x_1} + \tilde{u}_{\nu\leftarrow}(\omega, k_2, k_3)e^{-ik_{10}x_1}, \; x_1 > 0 \\ \sum_A \left[\tilde{v}_{\nu A\rightarrow}(\omega, k_2, k_3)e^{ik_{1A}x_1} + \tilde{v}_{\nu A\leftarrow}(\omega, k_2, k_3)e^{-ik_{1A}x_1}\right], \; x_1 < 0 \end{cases}$$

Matching conditions

$$\tilde{u}^{(A)}_{\nu\rightarrow} = \frac{1}{2}(\tilde{v}_{\nu A\rightarrow}(\frac{k_{1A}+k_{10}}{k_{10}}) - \tilde{v}_{\nu A\leftarrow}(\frac{k_{1A}-k_{10}}{k_{10}}))$$

$$\tilde{u}^{(A)}_{\nu\leftarrow} = \frac{1}{2}(-\tilde{v}_{\nu A\rightarrow}(\frac{k_{1A}-k_{10}}{k_{10}}) + \tilde{v}_{\nu A\leftarrow}(\frac{k_{1A}+k_{10}}{k_{10}}))$$

Escaping from the parity-breaking medium

Using the relations obtained before, it is possible to find, which part is reflected

$$\tilde{v}_{\nu\pm\leftarrow} = \frac{k_{1\pm}-k_{10}}{k_{1\pm}+k_{10}}\tilde{v}_{\nu\pm\rightarrow}$$

and which pass through the boundary,

$$\tilde{u}^{(\pm)}_{\nu\rightarrow} = \frac{2k_{1\pm}}{k_{10}+k_{1\pm}}\tilde{v}_{\nu\pm\rightarrow}$$