Center vortices as composites of monopole fluxes

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Abstract

There are numerical evidences supporting the center vortex theory of quark confinement. Furthermore, according to Monte Carlo data, monopoles are also playing the role of agents of confinement. Therefore one can expect some kind of relations between these two topological defects. Lattice simulations by Faber and *et al.* [1] show that a center vortex configuration after transforming to maximal abelian gauge and then abelian projection will appear in the form of a monopole-antimonopole chain in *SU(2)* gauge group. Therefore monopoles and center vortices correlate to each other.

In *SU(3)* gauge group monopoles and vortices form nets instead of the chains and three vortices may meet at a single point [2,3]. Motivated by these evidences, we combine different fluxes of abelian monopoles and obtain the fluxes of center vortices for *SU(2)* and *SU(3)* gauge groups.

We study quark confinement of the fundamental representation in *SU(3)* gauge group by creating the nets corresponding to fractional fluxes of the monopoles, where combinations of them may produce center vortex fluxes on the minimal area of a Wilson loop in a "center vortex model". Comparing the potential induced by fractional fluxes of monopoles with the one induced by the center vortex model, it seems that the nets of fractional fluxes attract each other.

The magnetic charges of abelian monopoles

By a gauge transformation which diagonalizes a scalar field in the adjoint representation, monopoles appear in the theory. In *SU(2)* gauge, the gluon field can be separated into a regular part and a singular part by abelian gauge fixing:

$$\vec{A} = \vec{A}_a T_a = \vec{A}_a^R T_a - \frac{1}{e}\vec{n}_\varphi \frac{1+\cos\theta}{r\sin\theta} T_3 \qquad (1)$$

The singular part of the gauge field corresponds to the magnetic monopole with the magnetic charge [4]

$$g = -\frac{4\pi}{e} T_3 \qquad (2)$$

For *SU(3)* gauge, the topological defects of abelian gauge fixing are sources of magnetic monopoles with magnetic charges equal to:

$$\begin{aligned} g_1 &= -\tfrac{4\pi}{e} T_3 \\ g_2 &= -\tfrac{4\pi}{e}\left(-\tfrac{1}{2}T_3 + \tfrac{\sqrt{3}}{2}T_8\right) \\ g_3 &= \tfrac{4\pi}{e}\left(\tfrac{1}{2}T_3 + \tfrac{\sqrt{3}}{2}T_8\right) \end{aligned} \qquad (3)$$

Thick Center vortex model

let's briefly explain the model [5]. The effect of a thick center vortex on a fundamental Wilson loop is multiplying the loop by a group factor

Cartan generators ↑

$$\mathcal{G}_f(\alpha^{(n)})\mathbb{I}_{d_f} = \frac{1}{d_f}\mathrm{Tr}\left(\exp\left[i\vec{\alpha}^{(n)}\cdot\vec{T}\right]\right)\mathbb{I}_{d_f} \qquad (4)$$

↓ Dimension of the fundamental representation

If a thick center vortex is entirely contained within the loop, then

$$\exp\left[i\vec{\alpha}^{(n)}\cdot\vec{T}\right] = z_n \mathbb{I}_{d_f} = e^{\frac{i2\pi n}{N}}\mathbb{I}_{d_f} \qquad (5)$$

Using equation (5) for *SU(2)*, we get

$$e^{i2\pi T_3} = z_1 \mathbb{I}_{d_f} \qquad (6)$$

And for *SU(3)*

$$e^{\frac{i4\pi}{\sqrt{3}}T_8} = z_1 \mathbb{I}_{d_f} \qquad (7)$$

The induced potential between static sources is

$$V_r(R) = -\sum_x \ln\left\{1 - \sum_{n=1}^{N-1} f_n\left(1 - \mathrm{Re}\mathcal{G}_r[\vec{\alpha}^n_C(x)]\right)\right\} \qquad (8)$$

f_n is the probability that any given unit area is pierced by a center vortex of type *n*.

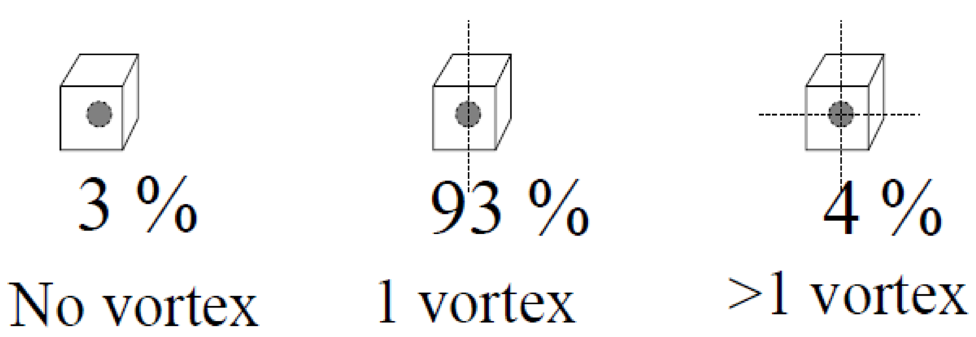
Abelian monopoles and center vortices

Now we combine fluxes of magnetic charges to obtain the center vortex flux. When the center vortex completely contains the minimal area of the loop, equations (2) and (6) for *SU(2)* gauge group gives

$$e^{i2\pi T_3} = e^{-ie\left(\frac{-4\pi T_3}{2e}\right)} = e^{-ie\frac{g}{2}} \qquad (9)$$

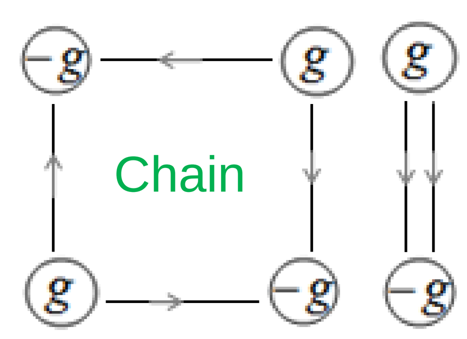
Thus, it seems that for *SU*(2) gauge group the center vortex carries a magnetic flux which is equal to the half of the total magnetic flux of a monopole.

On the one hand, according to the Monte Carlo simulations, almost all monopoles are sitting on top of the vortices by abelian projection [1].



The monopole-vortex junctions are also discussed by Cornwall [2] where they are called nexuses. In *SU(N)* gauge group, each nexus is a source of *N* vortices which meet at a point.

Configurations of *SU(2)* monopole vacuum



In *SU(3)* gauge group, magnetic charges satisfy a constraint

$$g_1 + g_2 + g_3 = 0 \qquad (10)$$

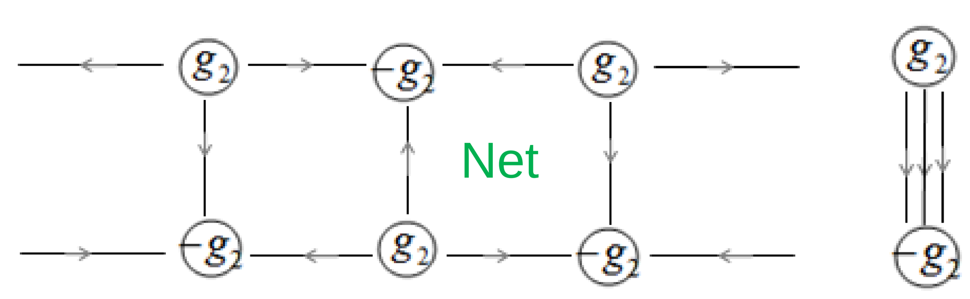
Therefore the number of independent magnetic charges reduces to 2.

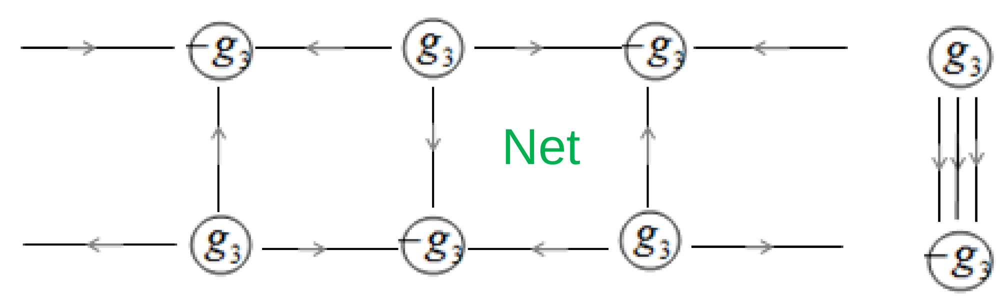
We combine fractional fluxes of magnetic charges to obtain a center vortex. When center vortex pierces the minimal area of a loop, equations (3) and (7) gives

$$e^{i\frac{4\pi}{\sqrt{3}}T_8} = e^{ie\left(\frac{g_3}{3} - \frac{g_2}{3}\right)} \qquad (11)$$

In other words, for *SU*(3) gauge group, the vortex carries one third of the total monopole flux g_3 plus one third of the total monopole flux g_2 ,pointing in opposite direction *i.e*.

Configurations of *SU(3)* monopole vacuum





Potential for fundamental representation

Creation of a configuration (net) of *SU(3)* monopole vacuum linked to a fundamental representation Wilson loop in a "vortex model" has the effect of multiplying the Wilson loop by a phase, *i.e*.

$$W_f(C) \to e^{ie\frac{g_n}{3}} W_f(C), \qquad n = 2,3 \qquad (12)$$

Where g_n are monopole charges in equation (3). The static potential induced by nets corresponding to fractional fluxes of the monopoles in this "vortex model", where combinations of them produce center vortex flux is as follows:

$$V_f(R) = -\sum_x \ln\left\{1 - \sum_{n=2}^{3} f_n[1 - Re(\mathcal{G}_n)]\right\} \qquad (13)$$

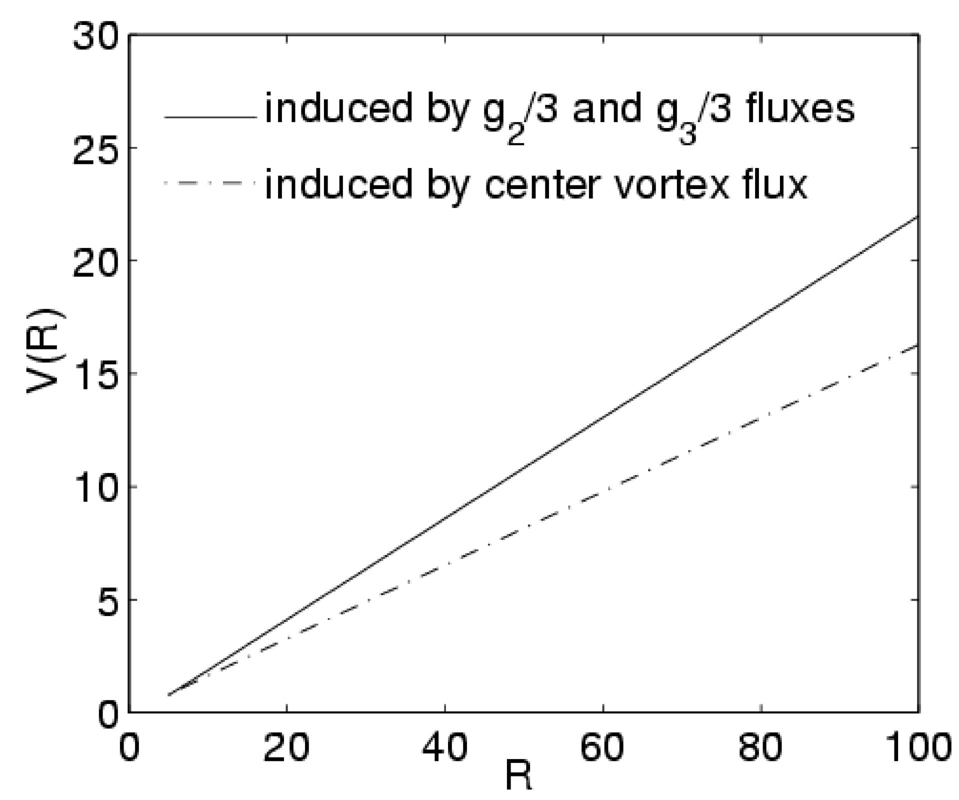
Where

$$\mathcal{G}_n = \frac{1}{d_f}\mathrm{Tr}\, e^{ie\frac{g_n}{3}} \qquad (14)$$

On the other hand, using the thick center vortex model for large distances, the static potential is

$$V_f(R) = -\sum_x \ln\{1 - f_1[1 - Re(z_1)]\} \qquad (15)$$

The results are plotted in the figure below.



The free parameters f_n are chosen to be 0.1. As shown in figure, the potential induced by fractional fluxes of monopoles is linear.

The extra negative potential energy of static potential induced by center vortex compared with the potential induced by fractional fluxes of monopoles shows that the nets corresponding to fractional fluxes of the monopoles, attract each other.

Conclusion

We study combinations of monopole fluxes and try to obtain a center vortex flux. We use center vortex model and calculate induced potentials from the center vortices and the monopoloe fractional fluxes. Comparing the potentials, we conclude that the fractional fluxes obtained from monopoles attract each other.

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