Magnetic properties of QCD matter: lattice results

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Confinement XI, IX.IX.XIV

Based on

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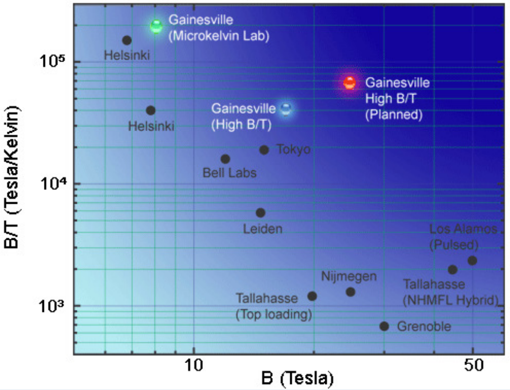
Content

- Motivation
- Simulation details
- The QCD phase diagram(s)
- The equation of state: magnetization and pressure
- Spin contribution
- Paramagnetic squeezing
- Summary

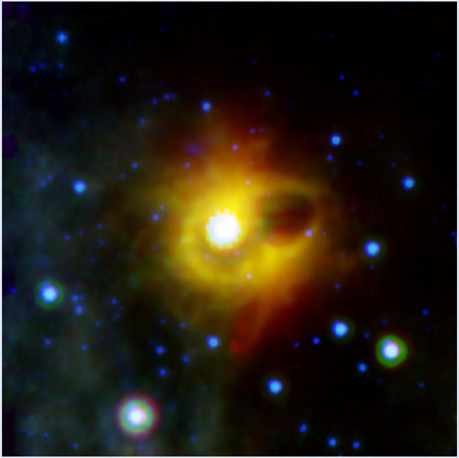
(Wo)man-made magnets

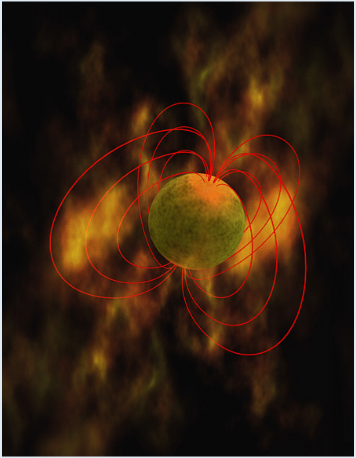
neodymium magnets,
€ 5.00 at amazon.de,
up to 10^4 G.



non-permanent at condensed matter labs,
up to $4.5 \cdot 10^5$ G.
Highest pulsed field: $2.8 \cdot 10^7$ G
non-destructive: 10^6 G.

Magnetars

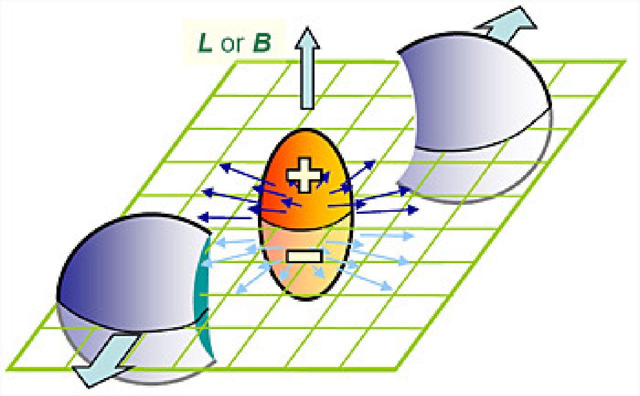




$10^{13}\,\mathrm{G} < B < 10^{15}\,\mathrm{G}$ near surface but even larger in the interior.

$(100\ \mathrm{MeV})^2 \approx 1.69 \cdot 10^{18} e\mathrm{G} = 1.69 \cdot 10^{14} e\mathrm{T}$ ($e \approx 0.3$ in natural units.)

Collider experiment: non-central heavy ion collisions



Two very big currents at very short distances produce extremely strong magnetic fields.

LHC: $eB \lesssim 0.3\,\mathrm{GeV}^2 \approx 5 \cdot 10^{19}\,e$G, RHIC: $eB \lesssim 0.04\,\mathrm{GeV}^2 \approx 6 \cdot 10^{18}\,e$G.
This is bigger than $m_\pi^2 \approx 0.02\,$GeV^2!
Fields have life time of only $\sim 0.1\,\mathrm{fm} = 10^{-16}\,\mathrm{m} \approx 5 \cdot 10^{-24}$ s.

Magnetic background field on the lattice

4-potential $(A_\nu) = (0, Bx, 0, 0) \Longrightarrow \mathbf{B} = (0, 0, B)$

Lattice: multiply links U_ν with $u_\nu = e^{iaqA_\nu} \in \mathrm{U}(1)$

$$\begin{aligned} u_y(n) &= e^{ia^2 qBn_x} & \\ u_x(n) &= 1 & n \neq N_x - 1 \\ u_x(N_x - 1, n_y, n_z, n_t) &= e^{-ia^2 qBN_x n_y} & \\ u_\nu(n) &= 1 & \nu \neq x, y \end{aligned}$$

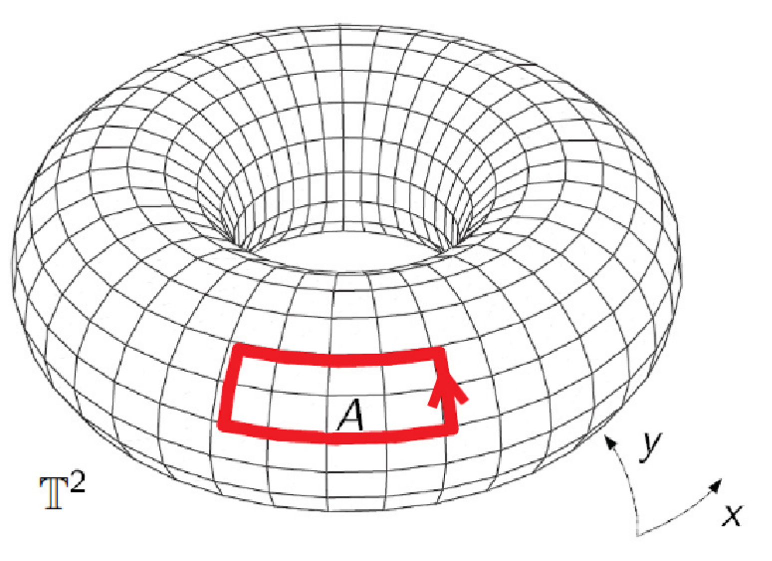
The magnetic flux through the x-y plane is constant:

$$\exp\left(iq \int_F \mathrm{d}\sigma\, \mathbf{B}\right) = \exp\left(iq \int_{\partial F} \mathrm{d}x_\nu\, A_\nu\right) = e^{ia^2 N_x N_y qB}$$

Flux quantization due to the finite volume + boundary conditions:

$$a^2 N_x N_y \cdot qB = 2\pi N_b \qquad N_b \in \mathbb{Z}$$

Flux quantization



Implementation and limitations

- B is invariant under $N_b \leftrightarrow N_b + N_x N_y$ (periodicity)
- Therefore $0 < N_b < N_x N_y / 4$
- Apply quantization for smallest charge $q = q_u/2 = -q_d = e/3$
- Typical lattice spacings:
  Maximal B $qB^{\max} = \pi/(2a^2)$: $\sqrt{eB} \approx 1\,\mathrm{GeV} \to 10^{20} e$ Gauss
- Typical aspect ratios:
  Minimal B $qB^{\min} = 2\pi T^2 (N_t/N_s)^2$: $\sqrt{eB} \approx 0.1\,\mathrm{GeV} \to 10^{18} e$ Gauss

Phenomenologically interesting range !

Renormalization

Electric charge renormalization $e \cdot B = e_r(\mu) \cdot B_r(\mu)$:

$$Z_e(\mu) = 1 + 2b_1 e^2 \log(\mu a)\,, \quad e^2 = Z_e^{-1}(\mu) e_r^2(\mu)\,, \quad B^2 = Z_e(\mu) B_r^2(\mu)$$

(Free) energy density at zero temperature:

$$f_B = f_0 + \Delta f = f_0 + \underbrace{\frac{2b_1 (eB)^2 \log(\mu a)}{2} + \frac{B_r^2(\mu)}{2}}_{B^2/2} + c(eB)^4 \cdot \text{finite}$$

Renormalization of the free energy density f_B at $T \geq 0$:

subtract the $T = 0$ $(eB)^2$-term in the limit of small eB

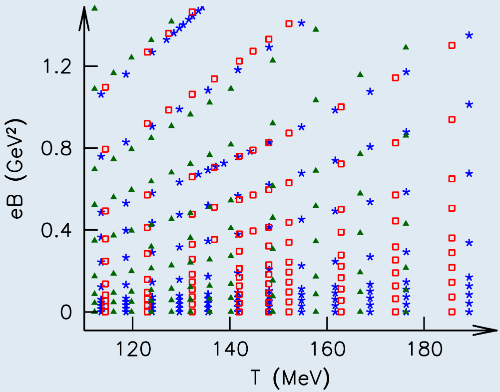
(so that $\sqrt{eB} \ll \mu$ where μ may be some hadronic scale)

The QED β-function coefficient $b_1 = b_1(a^{-1})$ depends on the lattice spacing, due to QCD-corrections.

The (trivial) background energy density $B_r^2/2$ is not included in the simulated Lagrangian.

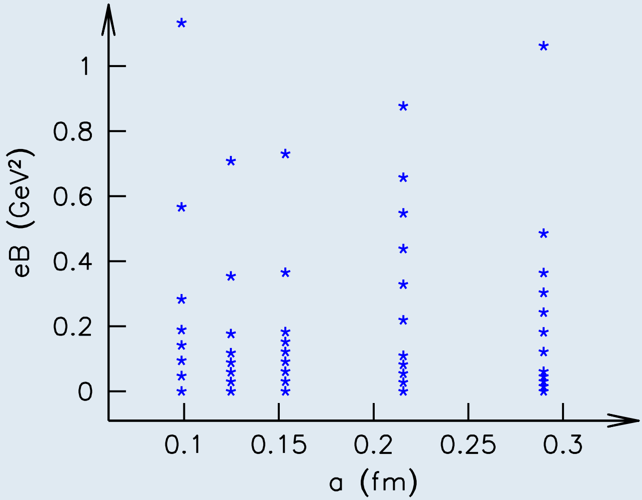
Simulation and analysis details

Symanzik improved gauge action, $N_f = 2+1$ stout smeared staggered quarks at physical masses (Budapest-Wuppertal) action.



- Simulate at various T and N_b.
- Fit all points by a 2D spline function.
- Keep physical B fixed.
- Study finite volume effects with $N_s/N_t = 3, 4, 5$
- Extrapolate to the continuum limit with $N_t = 6, 8, 10$ $= 1/(aT_c)$

$T = 0$ simulation points



Observables

- Partition function for three flavors ($\mu_f = 0$ in the simulation)

$$\mathcal{Z} = \int [dU]\, e^{-\beta S_g} \prod_{f=u,d,s} \left[\det M(q_f \cdot B, m_f, \mu_f)\right]^{1/4}$$

- Observables

$$\bar{\psi}\psi_f = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial m_f}, \quad \chi_f = \frac{T}{V}\frac{\partial^2 \log \mathcal{Z}}{\partial m_f^2}, \quad c_2^s = \frac{T}{V}\frac{\partial^2 \log \mathcal{Z}}{\partial \mu_s^2}$$

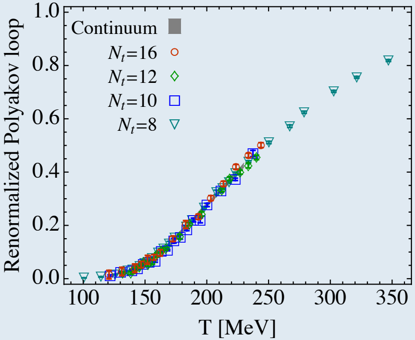
- Cancel additive divergences of $\bar{\psi}\psi$ by computing

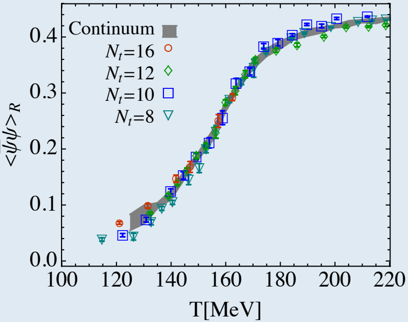
$$\Sigma_{u,d}(B,T) = \frac{2m_{ud}}{M_\pi^2 F^2}\left[\bar{\psi}\psi_{u,d}(B,T) - \bar{\psi}\psi_{u,d}(0,0)\right] + 1\,,$$

$$\Delta\Sigma_{u,d}(B,T) = \Sigma_{u,d}(B,T) - \Sigma_{u,d}(0,T)\,.$$

$B = 0$: Polyakov line and chiral condensate

S Borsányi et al JHEP 1009 (10) 073

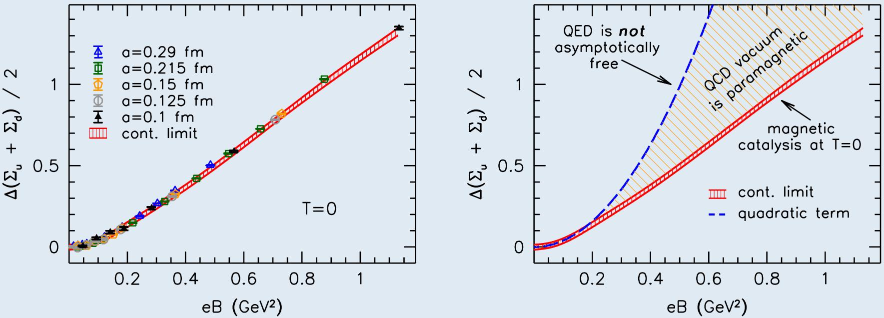




$T_c(\bar{\psi}\psi) = 155(3)(3)$ MeV$= 1.80(5) \cdot 10^{12}$ K.

(Cross-over: other quantities may have different pseudocritical temperatures.)

Magnetic catalysis: $T = 0$



$$\Delta f = \int_{m_\text{phys}}^{\infty} dm\, \Delta\bar{\psi}\psi\,, \quad \mathcal{P}[X] = (eB)^2 \lim_{eB\to 0} \frac{X}{(eB)^2}$$

$$\underbrace{\mathcal{P}[\Delta f]}_{\sim b_1} + \underbrace{(\mathbb{1}-\mathcal{P})\,[\Delta f]}_{\sim -\mathcal{M}} = \int_{m_\text{phys}}^{\infty} dm\, \left[\mathcal{P}[\Delta\bar{\psi}\psi] + (\mathbb{1}-\mathcal{P})\,[\Delta\bar{\psi}\psi]\right]$$

$\Longrightarrow$ magnetization $\mathcal{M} > 0$ at $T = 0$.

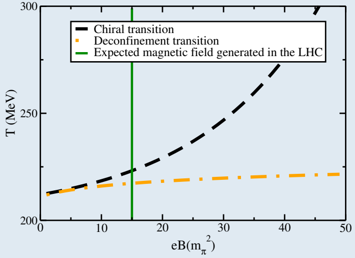
The QCD phase diagram in the B-T plane

Low energy effective models of QCD predict(ed):

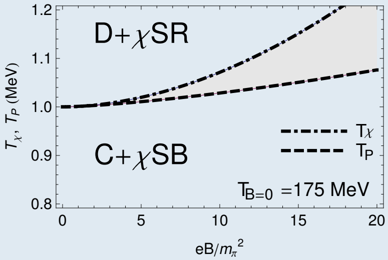
- increasing pseudocritical temperature $T_c(B)$
- increasing strength $1/W(B)$ Mizher et al 10

Supported by (P)NJL models, large-N_c arguments, low-dimensional models, Schwinger-Dyson equations

Gatto et al 11, Johnson et al 09, Alexandre et al 01, Klimenko et al 92, Kanemura et al 98

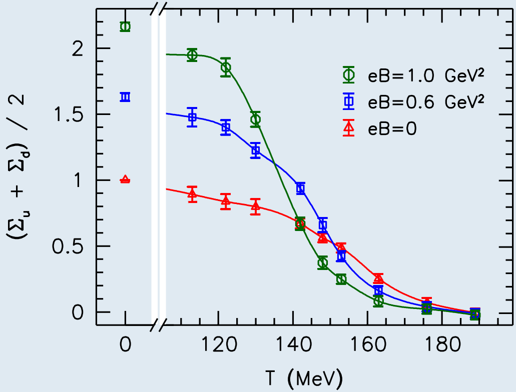


AJ Mizher et al PRD 82 (10) 105016



R Gatto, M Ruggieri PRD 83 (11)

Chiral condensate (continuum limit)

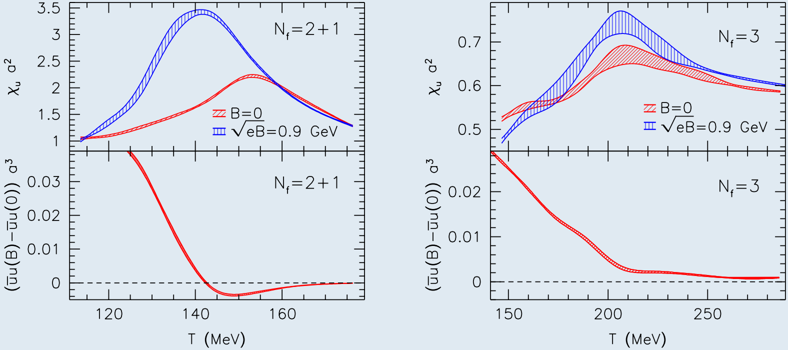


Inflection point moves to lower T with larger B !

$\Rightarrow T_c(B)$ decreases with B.

Hard to predict in models that are based on $T < T_c$ degrees of freedom.

Quark mass dependence



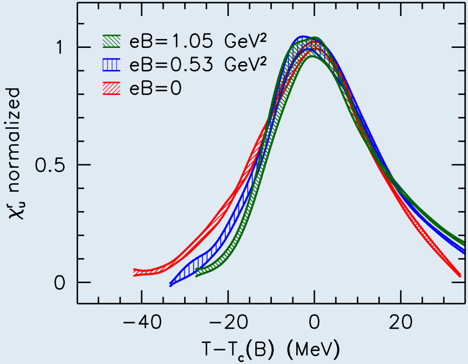
$\bar{u}u(B,T) - \bar{u}u(0,T) \propto -\Delta\Sigma_u(B,T)$: condensates are significantly different!

Shift in the χ_u peak positions!

$\Rightarrow$ dramatic dependence on the light quark mass!

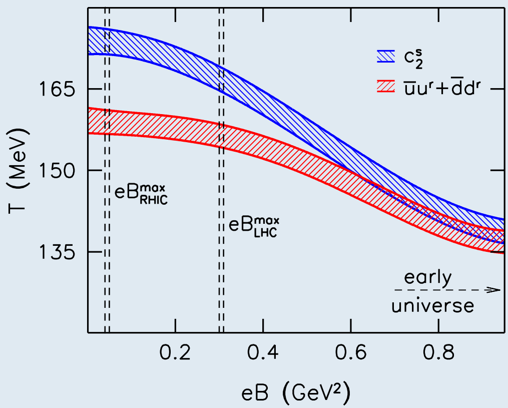
Width of the transition (continuum limit)

At $B = 0$: broad crossover. What happens at $B > 0$?
Height of the peak increases.
However when normalized to the same height...



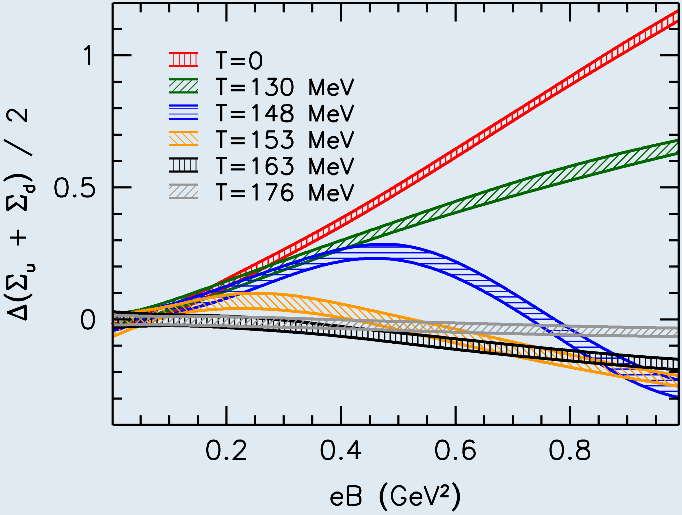
... not much changes. No indication of a critical end point!

Phase diagram (continuum limit)

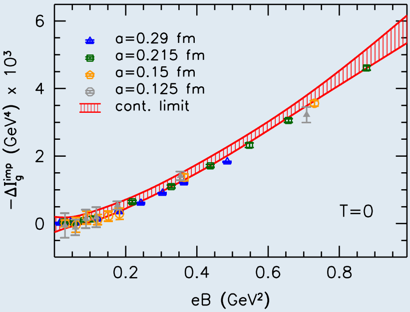


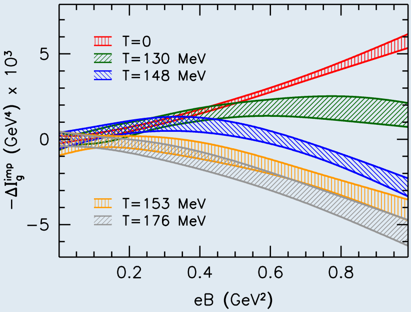
The effect is negligible for RHIC.
The temperature reduces by less than $5-10\%$ for LHC.

Inverse magnetic catalysis (continuum limit)



… and gluonic (inverse) catalysis





Interaction measure: $I = -\frac{T}{V}\frac{d\log\mathcal{Z}}{d\log a} = I_g + \sum_f I_f$, $-\Delta I_f = m_f \Delta\bar{\psi}_f\psi_f$

Gluon action density: $\beta s_g = \beta\frac{T}{V}S_g$, $\beta = \frac{6}{g^2(a)}$, $\Delta s_g = s_g(B,T) - s_g(0,T)$

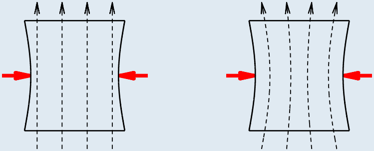
Gluon contribution to the interaction measure difference:

$$-\Delta I_g = -\frac{\partial\beta}{\partial\log a}\Delta s_g + \sum_f\left[\frac{\partial\log(am_f)}{\partial\log a} - 1\right] m_f\Delta\bar{\psi}_f\psi_f$$

Thermodynamics in an external magnetic field

Pressure	$p = \frac{T}{V} \log \mathcal{Z} = -f$
Interaction measure	$I = -\frac{T}{V} \frac{d \log \mathcal{Z}}{d \log a} = \epsilon - 3p = \langle T_{\mu\mu} \rangle$
With magnetic field	$f = \epsilon - sT = \epsilon^{\text{total}} - sT - \underbrace{\mathcal{M} \cdot eB}_{\epsilon^{\text{field}}}$

Is the pressure isotropic?



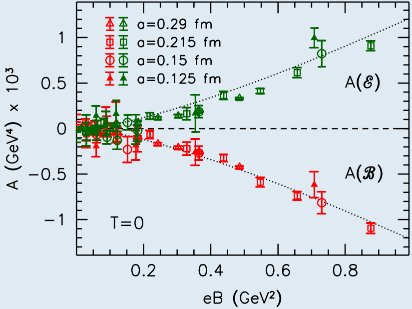
For qB fixed	$\langle T_{xx} \rangle = \langle T_{yy} \rangle = \langle T_{zz} \rangle = -p$
We work at fixed flux $\Phi = L_x L_y qB$	$p_x = p_y = p - \mathcal{M} \cdot eB$, $p_z = p$

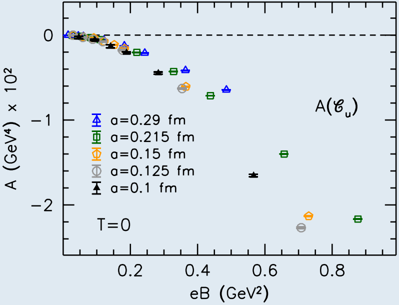
We pursued two approaches:

- Macroscopic determination of $\mathcal{M}$, using generalized integral method.
- Microscopic determination from $T_{\mu\mu}$ (non-trivial renormalization).

Microscopic (anisotropy) method

$$\mathcal{M} \cdot eB = p_x - p_z = -\frac{T}{V}\left[\frac{(\partial \beta \xi_{g0})}{\partial \xi}\Big|_{\xi=1} \langle -P_x + P_z \rangle + \frac{\partial(\beta/\xi_{g0})}{\partial \xi}\Big|_{\xi=1} \left\langle -\hat{P}_x + \hat{P}_z \right\rangle + \frac{\partial \xi_{f0}}{\partial \xi}\Big|_{\xi=1} \sum_f \langle \bar{\psi}_f D_x \gamma_x \psi_f - \bar{\psi}_f D_z \gamma_z \psi_f \rangle \right].$$





Volume: $V = L_x L_y L_z = a^3 N_s^3$, temperature: $T = 1/(aN_t)$, magnetization:

$$\mathcal{M} = -\frac{1}{V}\frac{\partial \mathcal{F}}{\partial (eB)}, \quad \chi_B = -\frac{1}{V}\left.\frac{\partial^2 \mathcal{F}}{\partial (eB)^2}\right|_{B=0}.$$

At constant $\Phi = L_x L_y eB$, eB will change as L_x or L_y are compressed:

$$p_z = -\frac{1}{V} L_z \frac{\partial \mathcal{F}}{\partial L_z} = -\frac{1}{V}\left(L_x \frac{\partial \mathcal{F}}{\partial L_x} + eB \frac{\partial \mathcal{F}}{\partial (eB)}\right) = p_x + eB \cdot \mathcal{M}$$

$$p_z = p \overset{V\to\infty}{\longrightarrow} -f = -\frac{\mathcal{F}}{V} = \frac{T}{V}\log \mathcal{Z} = \frac{1}{N_s^3 N_t a^4}\log \mathcal{Z}\,.$$

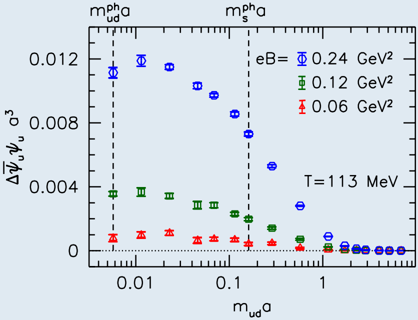
$\Delta f = -\Delta p$ known $\Rightarrow$ obtain $\mathcal{M}$ etc., as derivatives with respect to eB.

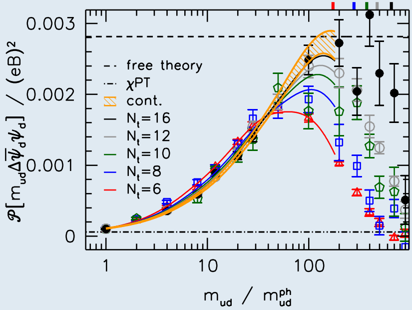
$$\frac{\partial [a^4 \Delta f(am_f, \Phi, T, \beta)]}{\partial (am_f)} = -a^3 \Delta \bar{\psi}_f \psi_f$$

Obviously, $\Delta p(am_f = \infty, \Phi, T, \beta) = -\Delta f(am_f = \infty, \Phi, T, \beta) = 0$.

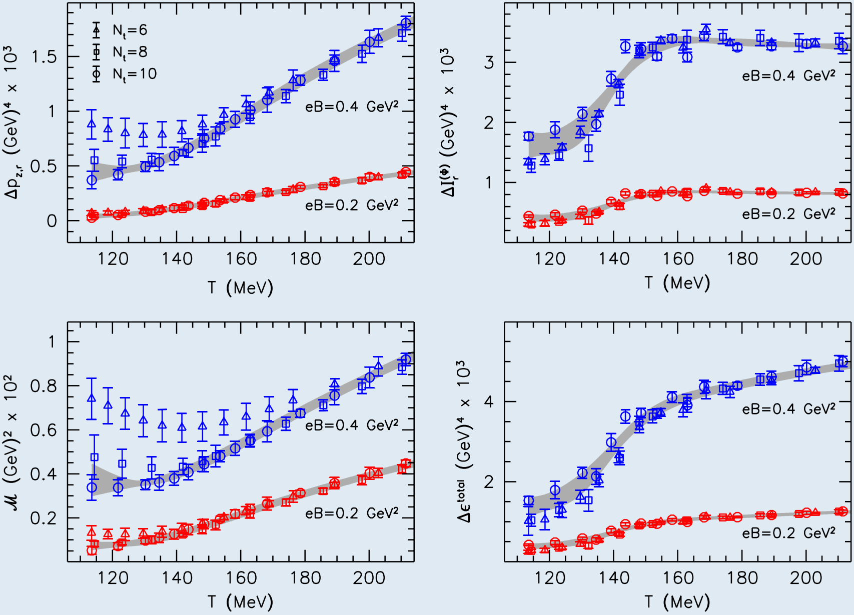
Generalized integral method

$$\frac{\Delta p_z(\Phi, T, \beta)}{T^4} = -N_t^4 \sum_f \int_{am_f^{\rm ph}}^{\infty} d(am_f)\, a^3 \Delta\bar{\psi}_f\psi_f$$





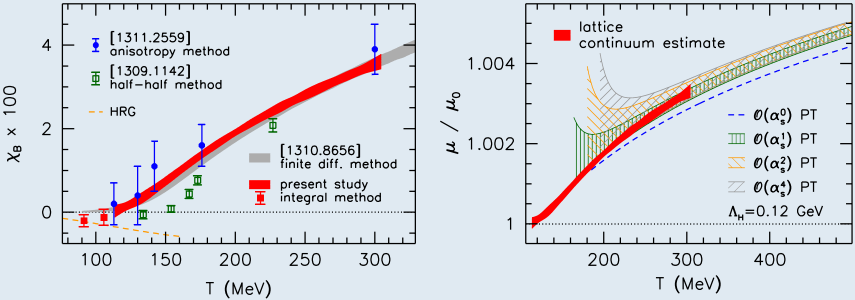
$$\frac{\mathcal{P}\left[m_f \Delta\bar{\psi}_f\psi_f\right]}{(eB)^2} \to \begin{cases} b_{1f} & (m_f \to \infty) \\ b_{1f}/(16N_c) & (\chi\mathrm{PT}) \end{cases}, \qquad \Delta p_{z,r} = (\mathbb{1} - \mathcal{P})[\Delta p_z]$$



N_t=6
N_t=8
N_t=10
eB=0.4 GeV²
eB=0.2 GeV²
$\Delta p_{z,r}$ (GeV)⁴ × 10³
T (MeV)
eB=0.4 GeV²
eB=0.2 GeV²
$\Delta I_r^{(\Phi)}$ (GeV)⁴ × 10³
T (MeV)
eB=0.4 GeV²
eB=0.2 GeV²
$\mathcal{M}$ (GeV)² × 10²
T (MeV)
eB=0.4 GeV²
eB=0.2 GeV²
$\Delta\epsilon^{total}$ (GeV)⁴ × 10³
T (MeV)

Susceptibility and permeability

half-half method: L Levkova, C DeTar, PRL 112 (14) 012002
finite diff. method: C Bonati, F Negro et al, PRD89 (14) 054506



(Linear) magnetic susceptibility and permeability ($f = \frac{1}{2}B_r^2(1 - e_r^2\chi_B)$):

$$\chi_B = \left.\frac{\partial \mathcal{M}}{\partial(eB)}\right|_{B=0}, \quad B = H + e\mathcal{M}, \quad \mu = \frac{B}{H} = \frac{1}{1 - 4\pi\alpha_{\rm em}\chi_B}.$$

Spin contribution to the magnetic susceptibility

Decomposition:

$$\chi_B = \sum_f \chi_{B,f}, \qquad \chi_{B,f} = \chi^S_{B,f} + \chi^L_{B,f},$$

$$\chi^S_{B,f} = \frac{q_f/e}{2m_f} \frac{\partial}{\partial(eB)} \left\langle \bar{\psi}_f \sigma_{xy} \psi_f \right\rangle \Big|_{eB=0} = \frac{(q_f/e)^2}{2m_f} \tau_f, \qquad \sigma_{\mu\nu} = \frac{1}{2i}[\gamma_\mu, \gamma_\nu].$$

$$\left\langle \bar{\psi}_f \sigma_{xy} \psi_f \right\rangle = q_f B \cdot \left\langle \bar{\psi}_f \psi_f \right\rangle \cdot \xi_f \equiv q_f B \cdot \tau_f$$

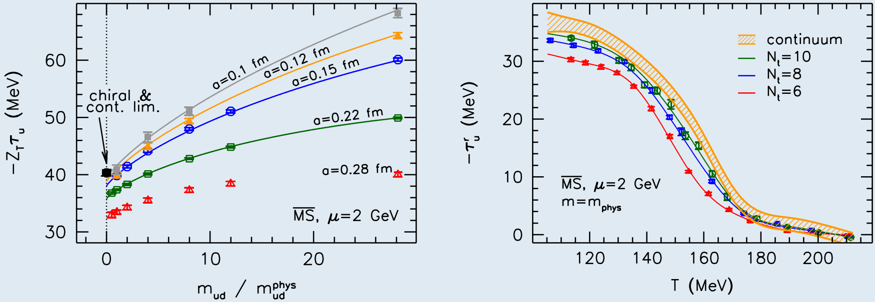
τ_f undergoes additive and multiplicative renormalization:

$$\tau^r_f \equiv \left(1 - m_f \frac{\partial}{\partial m_f}\right) \tau_f \cdot Z_T \equiv \tau_f Z_T - \tau_f^{\mathrm{div}}, \quad \tau_f^{\mathrm{div}} = m_f \left(\frac{1}{2\pi^2} + \cdots\right)$$

Note that this is the spin contribution to the $T = 0$ term $\propto (eB)^2$ that we usually subtract.

Vacuum: Landau-paramagnetism and Pauli-diamagnetism. Subtracting the $T = 0$ vacuum: Pauli-paramagnetism and Landau-diamagnetism.

The tensor coefficient



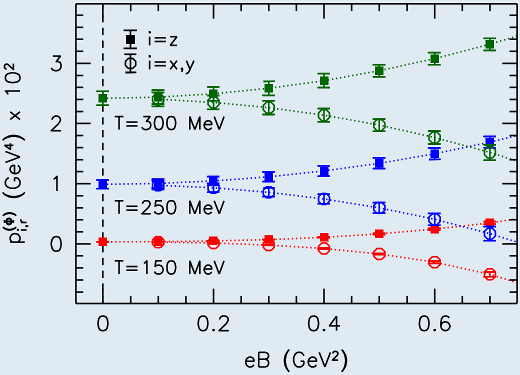
$\overline{\mathrm{MS}}$-scheme at 2 GeV for physical quark masses:

$$\tau_u^r = -40.7(1.3)\,\mathrm{MeV}\,, \quad \tau_d^r = -39.4(1.4)\,\mathrm{MeV}\,, \quad \tau_s^r = -53.0(7.2)\,\mathrm{MeV}\,.$$

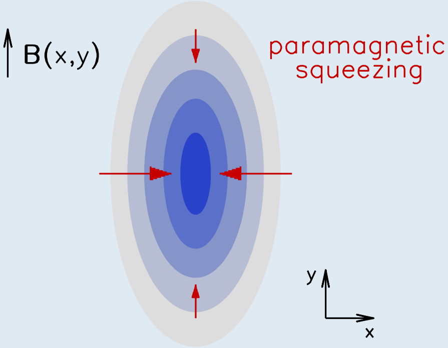
Fundamental property of the QCD vacuum.

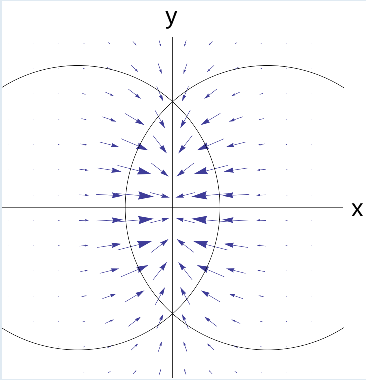
Contributes, e.g., to the hadronic light-by-light correction to muon $g-2$!

Pressure anisotropy



Magnetization $\sim eB \Rightarrow$ force density ($=$ pressure gradient) $\sim (eB)^2$.





$\longrightarrow$ effect on elliptic flow v_2?

Summary

- Phase diagram: crossover with **decreasing** $T_c(B)$
  - complex, non-monotonic dependence in $\bar{\psi}\psi(B,T)$ (inverse catalysis)
  - similar for the gluonic contribution to the interaction measure
  - no critical endpoint at $\sqrt{eB} \lesssim 1$ GeV
- Once the QED running coupling (magnetic catalysis) is accounted for, the response at $T = 0$ and $T > 100$ MeV is paramagnetic.
- At low temperatures and low fields (magnetars?) QCD is slightly diamagnetic.
- As a by-product the tensor coefficient was precisely determined.
- Can pressure anisotropy affect the elliptic flow for heavy ion collisions?
- Equation of state computed.