Partition Function of Interacting Calorons Ensemble

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Introduction

KvBLL instantons or calorons with non-trivial holonomy can be considered as the degrees of freedom in Yang-Mills vacuum in any temperature.
The non-interacting ensemble of calorons has been studied by Diakonov and Petrov [1]. In this study they computed the heavy quark potential by correlation function of two Polyakov loops:

$$e^{-\beta\Delta F_{q\bar{q}}} = \langle TrL(x)TrL^{\dagger}(x)\rangle \qquad (1)$$

To compute the correlation function, the partition function of caloron ensemble has to be calculate. Therefore, the contribution of caloron in the partition function should be derived. Diakonov and Gromov [2] computed this contribution using the metric of the caloron moduli space in terms of N different SU(N) BPS monopoles

$$Z = e^{-S_{cl}}\int J\prod_{P}\frac{dY_P}{\sqrt{2\pi g}}\int Da_\mu D\chi D\bar{\chi}\ \times$$
$$\exp\left(-\frac{1}{2g^2}\int d^4x\, a_\mu^a W_{\mu\nu}^{ab} a_\nu^b - \int d^4x\, \bar{\chi}^a (D^2)^{ab}\chi^b\right).$$

Where Y_P are caloron collective coordinates and the Jacobian J is the determinant from the moduli space metric tensor. Diakonov and *et al.* [3] computed the full partition function for KvBLL instanton in SU(2) gauge theory:

$$Z = \int d^3z_1\, d^3z_2 dt\, dOT^6 C\left(\frac{8\pi^2}{g^2}\right)^4\left(\frac{\Lambda e^{\gamma_E}}{4\pi T}\right)^{\frac{22}{3}}$$
$$\left(2\pi + \frac{\nu\bar{\nu}}{T}r_{12}\right)(\nu r_{12}+1)^{\frac{4\nu}{3\pi T}-1}(\bar{\nu}r_{12}+1)^{\frac{4\bar{\nu}}{3\pi T}-1}$$
$$\left(\frac{1}{Tr_{12}}\right)^{\frac{5}{3}} \times \exp[-VP(\nu) - 2\pi r_{12}P''(\nu)] \qquad (2)$$

Where C is a constant and

$$P(\nu) = \frac{1}{12\pi^2 T}\nu^2\bar{\nu}^2$$
$$P''(\nu) = \frac{1}{\pi^2 T}\left[\pi T\left(1-\frac{1}{\sqrt{3}}\right) - \nu\right]\left[\bar{\nu} - \pi T\left(1-\frac{1}{\sqrt{3}}\right)\right].$$

In this research we try to add the contribution of the interaction between calorons to the calculations. As a result of this change, the metric of the interacting calorons moduli space should be modified. However, one may keep the same moduli space with the same metric as the non interacting calorons, but introducing a potential on the moduli space which corresponds to the interaction between calorons. The main task of this research is to give some ideas about computing partition function of interacting calorons and finally computing the potential between quarks.
In the next two sections we use sum ansatz idea and caloron-Dirac string interaction to suggest how one may calculate the partition function.

Sum Ansatz

Consider two calorons with identical holonomy in algebraic gauge,

$$A_\mu = A_\mu^{(1)} + A_\mu^{(2)}$$

The field strength of this ansatz can be written as

$$F_{\mu\nu}\left(A^{(1)} + A^{(2)}\right) = F_{\mu\nu}\left(A^{(1)}\right) + F_{\mu\nu}\left(A^{(2)}\right)$$
$$+F_{\mu\nu}\left(A^{(1)}, A^{(2)}\right),$$

where the additional term is due to the non-linear nature of the theory. Computing the action, we have a term in addition to the action of individual calorons, that can be interpreted as the potential due to the interaction between two calorons.

$$exp(-S_{int}) \equiv exp\left(-\frac{1}{4g^2}\int d^4x\left(F_{\mu\nu}^2 - \sum_{i=1}^{2}F_{\mu\nu}^2(i)\right)\right)$$

Therefore we can write the partition function for these two calorons in periodic gauge:

$$Z_{2c} = \frac{1}{2!}\prod_{i=1}^{2} Z_i\, e^{-S_{int}},$$

where Z_i has been derived in eq. (2). Now we study the ensemble of K calorons. To simplify the calculations we restrict ourselves to a system which consists of many pairs of calorons. The calorons in each pair interact with each other but the pairs do not interact with each other. The grand canonical partition function of these K pairs can be written as:

$$Z = \sum_K f^K Z_K = \sum_K \frac{(fZ_{2c})^K}{K!}.$$

Where *f* is the fugacity and should be calculated.

Caloron-Dirac String Interaction

The next idea for computing the partition function of interacting calorons is using the caloron-Dirac string interaction which has been studied by Gerhold *et al.* [4]. The interaction between the caloron and the Dirac string had not been taken into account in non interacting caloron partition function and therefore this is a new effect in computing the partition function of interacting calorons when two calorons are located on top of each other. It means that two calorons interact with each other by their Dirac strings.
In the large caloron radius limit, the vector potential of caloron is dominantly Abelian and governed by the third component in color space

$$\left|A_\mu^{(1,2)}(\tilde{x})\right| \ll A_{1,2}^{(3)}(\tilde{x})\ ,\left|A_{3,4}^{(3)}(\tilde{x})\right| \ll A_{1,2}^{(3)}(\tilde{x})$$

This dominant component is called Dirac string.
Since the interaction due to Dirac string is not negligible, when an object (monopole or caloron) is placed on top of the Dirac string of a caloron, the ordinary sum ansatz can not be used for the superposition of caloron and this object. Gerhold et al. [4] derived the superposition by removing the Dirac string in algebraic gauge by proper gauge transformation (G), then they added this caloron with that object and finally applied the inverse gauge transformation to have the original Dirac string back. The result is:

$$A_\mu^{final}(x) = A_\mu{}^{Cal}(x) + e^{-iG(x)\tau_3}A_\mu{}^{Obj}(x)e^{iG(x)\tau_3}$$

Therefore the effect of considering Dirac string is adding an unchanged caloron to a gauge rotated object.
Using the above method, one can consider two calorons that interact with each other by their Dirac strings to compute the potential of this interaction. Again, we can restrict ourselves to a system which consists of many pairs of calorons. The calorons in each pair interact with each other but the pairs do not interact with each other. We can compute the partition function of K non-interacting pairs of calorons.

Caloron-Anticaloron Ansatz

The sum ansatz idea can be applied to a caloron and anticaloron system. Although the combination of caloron and anticaloron is not the solution of equation of motion but both of them minimize the action and may have identical integral measure and partition function. They may be saddle points of the partition function. Therefore, there is a possibility of constructing the partition function from caloron anticaloron pairs instead of calorons or anticalorons only. However, we recall that the caloron and anticaloron should have the same holonomy if one wants to gauge transform the combination appropriately from algebraic gauge to periodic gauge.
We can apply caloron-Dirac string interaction to caloron-anticaloron ansatz, because the function G which is applied to remove Dirac string of anticaloron is the same [4]. So we can consider an ensemble with equal number of calorons and anticalorons in which one caloron and one anticaloron interact (either interaction due to the ordinary sum ansatz or caloron-Dirac string interaction). Then, we assume again that these pairs do not interact and we use the same procedure as the previous section to calculate the partition function.

Conclusion

In this research we try to suggest computing the potential between interacting calorons with non-trivial holonomy. Two methods of sum ansatz and caloron-Dirac string interaction may be used to obtain the interacting potential between two calorons. With this potential, one can compute the partition function of the calorons ensemble with non interacting pairs. (Interaction exists between calorons in each pair.) This partition function is essential to compute the ensemble average of two polyakov loops which calculates the heavy quark potential in eq. (1). In principle, we will have no problem except that the complexity of caloron fields may make the calculation hard.

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