2-Color QCD at High Density

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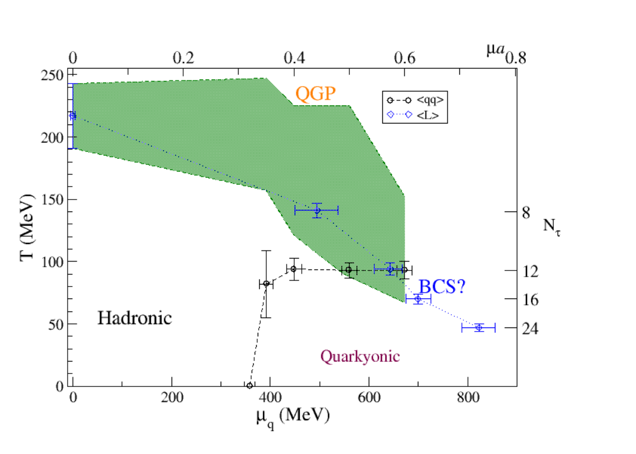
2-Color QCD on the Lattice

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \det M[U] \, e^{-S_G[U]} \mathcal{O}(U)$$

- There is *sign poblem* for real QCD for nonzero chemical potential (μ): Monte Carlo simulations can't be used
- No sign problem for two colors and even number of flavors (N_f)
- Can be viewed as a laboratory to investigate real QCD at high density
- Has confinement and chiral symmetry breaking, like real QCD

Tentative Phase Diagram for 2-Color QCD

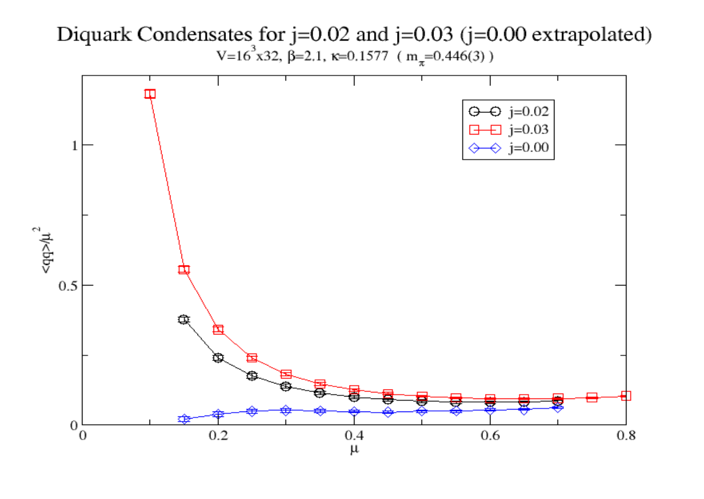


[plot by Jon-Ivar Skullerud]

Diquark Condensation in 2-Color QCD ($N_f = 2$)

- Quarks and antiquarks live in equivalent representations of the color group
- At $\mu = 0$ exact symmetry between mesons and diquarks (baryons of the theory)
- Pseudo-Goldstone multiplet ($\mu = 0$, $N_f = 2$): pion isotriplet + diquark + antidiquark
- Diquark baryons condense for $\mu \gtrsim m_\pi/2$ giving rise to a superfluid ground state
- In real QCD this corresponds to a superconducting ground state
- Order parameter for superfluid phase transition: **diquark condensate**, $\langle qq \rangle$
- To calculate $\langle qq \rangle$ on the lattice: introduce **diquark source**, j

Diquark Condensation in 2-Color QCD



Wilson Fermion Matrix, $j = 0$

- Wilson fermion matrix, M, for nonzero chemical potential, $\mu \neq 0$, and zero diquark source, $j = 0$, in position space:

$$M(\mu) = \delta_{xy} - \kappa \sum_{\nu} \Big[(\mathbf{1} - \gamma_\nu)\, e^{\mu \delta_{\nu 0}} U_\nu(x)\, \delta_{y,x+\hat{\nu}} + (\mathbf{1} + \gamma_\nu)\, e^{-\mu \delta_{\nu 0}} U_\nu^\dagger(y)\, \delta_{y,x-\hat{\nu}} \Big]$$

- In momentum space (non-interacting case):

$$M(p) = \frac{i}{a} \sum_{j=1}^{3} \gamma_j \sin(a p_j) + \frac{i}{a} \gamma_4 \sin(a\omega) + m_0 + \frac{1}{a} \sum_{j=1}^{3} [\mathbf{1} - \cos(p_j a)] + \frac{1}{a} [\mathbf{1} - \cos(\omega a)]$$

(where $\omega = p_t - i\mu$)

Fermion Action for $N_c = 2$ and $N_f = 2$

- The action for two flavors is given by:

$$S = \overline{\psi}_1 M(\mu)\psi_1 + \overline{\psi}_2 M(\mu)\psi_2 - J\overline{\psi}_1 (C\gamma_5)\tau_2 \overline{\psi}_2^{tr} + \overline{J}\psi_2^{tr}(C\gamma_5)\tau_2\psi_1.$$

- Can be written in terms of $\mathcal{M}$:

$$S = \overline{\Psi}\mathcal{M}\Psi,$$

where $\Psi \equiv \begin{pmatrix} \psi_1 \\ C^{-1}\tau_2\overline{\psi}_2^{tr} \end{pmatrix}$ and $\mathcal{M}$ is the Gorkov matrix.

Gorkov Propagator

- Wilson fermion matrix, $\mathcal{M}$, for nonzero chemical potential, $\mu \neq 0$, and nonzero diquark source, $j \neq 0$, in position space (Gorkov matrix):

$$\mathcal{M} = \begin{pmatrix} M(\mu) & -\frac{j}{2} C\gamma_5\tau_2 \\ \frac{j}{2} C\gamma_5\tau_2 & C\tau_2 M(-\mu) C\tau_2 \end{pmatrix}$$

- Inverse of this matrix is the Gorkov propagator:

$$G = \mathcal{M}^{-1} \equiv \begin{pmatrix} S & T \\ \bar{T} & \bar{S} \end{pmatrix}$$

- Diagonal block components: Normal propagation, Off-diagonal block components: Anomalous propagation (turns a quark into an antiquark)

Form Factors

- Useful tools to study *interacting* quark propagators
- The fermion matrix can be written in terms of the form factors A, B, C and D:

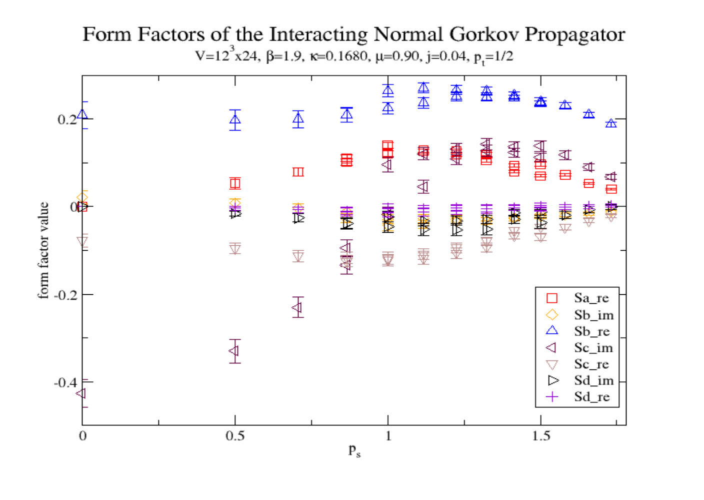
$$S^{-1}(p) = \mathrm{i}\not{\mathbf{p}}A(p) + B(p) + \mathrm{i}\omega\gamma_4 C(p) + \not{\mathbf{p}}\gamma_4 D(p)\,.$$

- The S and T block-components of the Gorkov propagator can also be written in terms of some form factors:

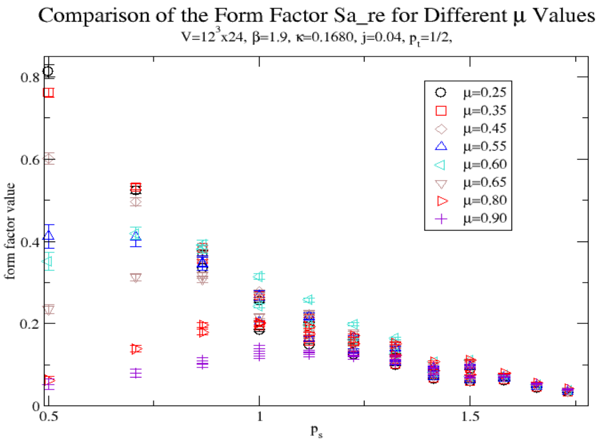
$$S(p) = \mathrm{i}\not{\mathbf{p}}S_a(p) + S_b(p) + \mathrm{i}\omega\gamma_4 S_c(p) + \not{\mathbf{p}}\gamma_4 S_d(p)\,,$$

$$T(p) = \mathrm{i}\not{\mathbf{p}}T_a(p) + T_b(p) + \mathrm{i}\omega\gamma_4 T_c(p) + \not{\mathbf{p}}\gamma_4 T_d(p)\,.$$

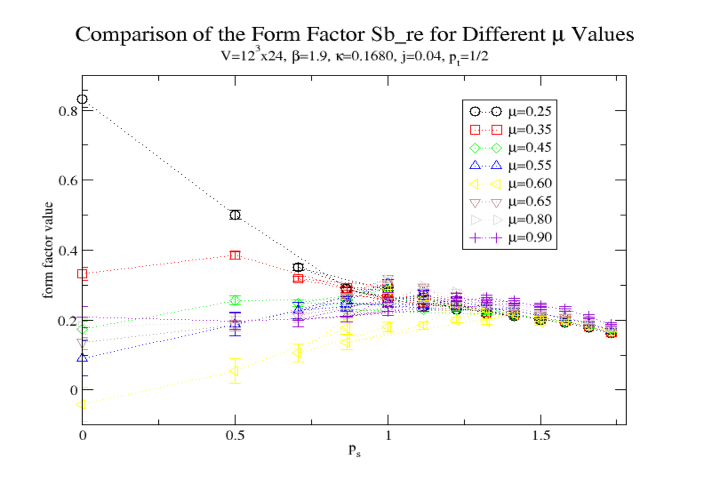
Form Factors vs Chemical Potential



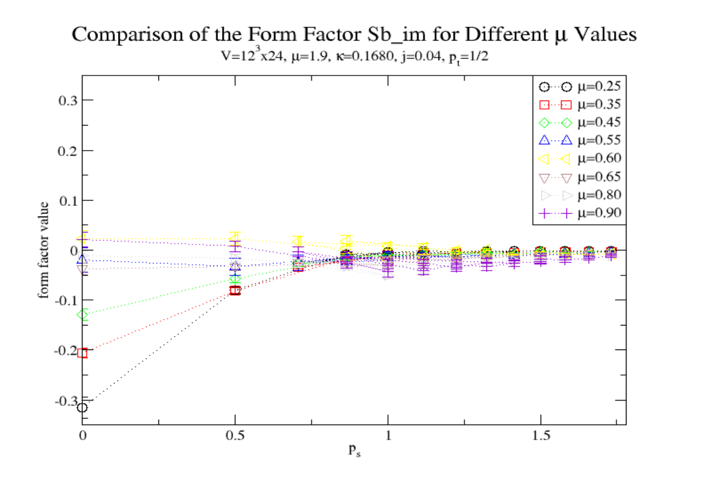
Form Factor Sa (real part)



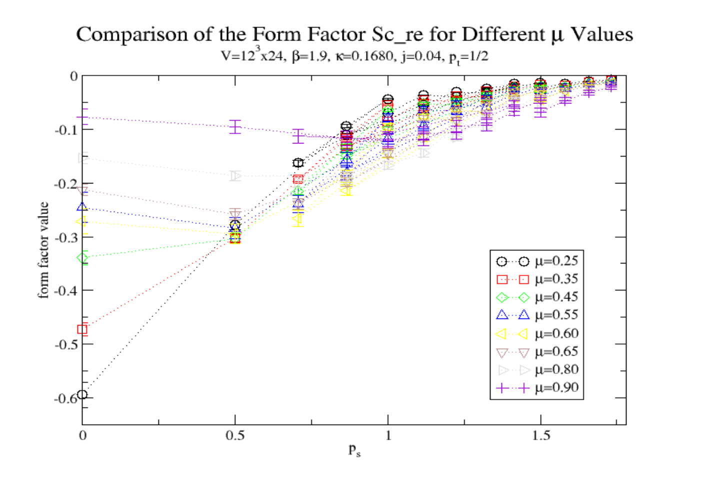
Form Factor Sb (real part)



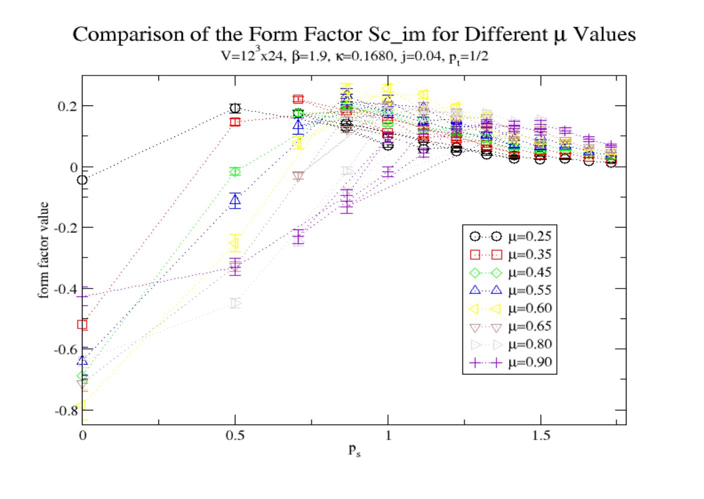
Form Factor Sb (imaginary part)



Form Factor Sc (real part)



Form Factor Sc (imaginary part)



Form Factor Sd (real part)

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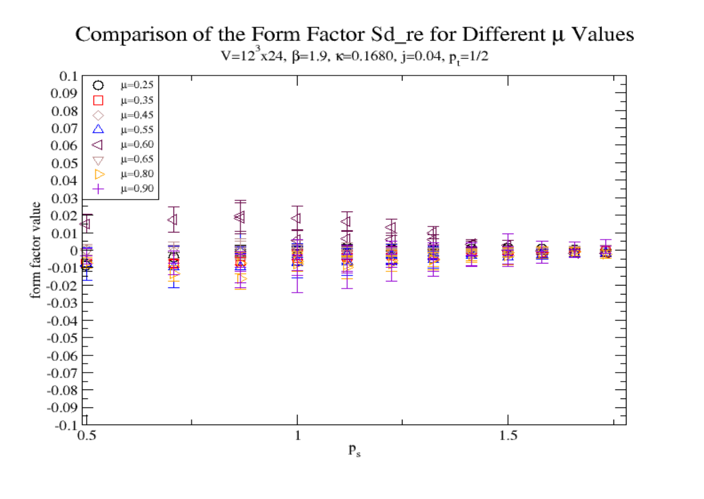
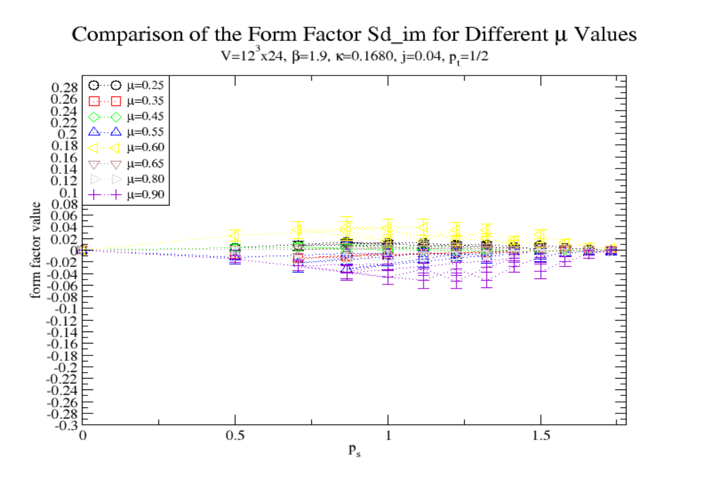


Form Factor Sd (imaginary part)



Summary

- 2-color QCD: a laboratory to investigate real QCD at high density
- diquark condensate: order parameter for superfluid phase transition
- Gorkov propagator: normal and anomalous propagation of quarks
- form factors: useful tools to study the propagator