A dispersive approach to hadronic light-by-light

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Outline

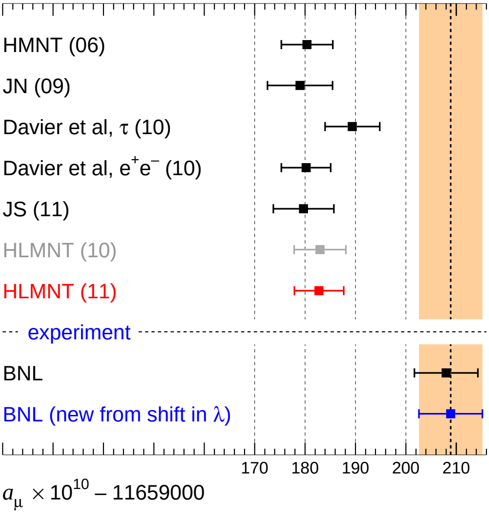
Introduction: $(g-2)_\mu$ and hadronic light-by-light

A dispersive approach to HLbL

Conclusions

arXiv:1402.7081 [to appear in JHEP]
in collaboration with M. Hoferichter, M. Procura and P. Stoffer

Status of $(g-2)_\mu$, experiment vs SM



Hagiwara et al. 2012

Status of $(g-2)_\mu$, experiment vs SM

	$a_\mu\,[10^{-11}]$	$\Delta a_\mu\,[10^{-11}]$
experiment	116 592 089.	63.
QED $\mathcal{O}(\alpha)$	116 140 973.21	0.03
QED $\mathcal{O}(\alpha^2)$	413 217.63	0.01
QED $\mathcal{O}(\alpha^3)$	30 141.90	0.00
QED $\mathcal{O}(\alpha^4)$	381.01	0.02
QED $\mathcal{O}(\alpha^5)$	5.09	0.01
QED total	116 584 718.95	0.04
electroweak, total	153.6	1.0
HVP (LO) [Hagiwara et al. 11]	6 949.	43.
HVP (NLO) [Hagiwara et al. 11]	−98.	1.
HLbL [Jegerlehner-Nyffeler 09]	116.	40.
HVP (NNLO) [Kurz, Liu, Marquard, Steinhauser 14]	12.4	0.1
HLbL (NLO) [GC, Hoferichter, Nyffeler, Passera, Stoffer 14]	3.	2.
theory	116 591 855.	59.

Hadronic light-by-light: irreducible uncertainty?

- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) can be systematically improved
  (but going much below 1% is hard – dealing with radiative corrections poses serious problems)
- Hadronic light-by-light (HLbL) is more problematic:
  - "it *cannot* be expressed in terms of measurable quantities"
  - reliability of uncertainty estimate based more on consensus than on a systematic method
  - only first-principle method in sight: lattice QCD
    (when will it become competitive?)

Different evaluations of HLbL

Jegerlehner Nyffeler 2009

Table 13

Summary of the most recent results for the various contributions to $a_\mu^{\rm LbL;had} \times 10^{11}$. The last column is our estimate based on our new evaluation for the pseudoscalars and some of the other results.

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
π^0, η, η'	85 ± 13	82.7 ± 6.4	83 ± 12	114 ± 10	–	114 ± 13	99 ± 16
π, K loops	-19 ± 13	-4.5 ± 8.1	–	–	–	-19 ± 19	-19 ± 13
π, K loops + other subleading in N_c	–	–	–	0 ± 10	–	–	–
Axial vectors	2.5 ± 1.0	1.7 ± 1.7	–	22 ± 5	–	15 ± 10	22 ± 5
Scalars	-6.8 ± 2.0	–	–	–	–	-7 ± 7	-7 ± 2
Quark loops	21 ± 3	9.7 ± 11.1	–	–	–	$2.3\pm$	21 ± 3
Total	83 ± 32	89.6 ± 15.4	80 ± 40	136 ± 25	110 ± 40	105 ± 26	116 ± 39

- large uncertainties (and differences among calculations) in individual contributions
- pseudoscalar pole contributions most important
- second most important: pion loop, *i.e.* two-pion cuts (Ks are subdominant)
- heavier single-particle poles decreasingly important (unless one models them to resum the high-energy tail)

Approaches to Hadronic light-by-light

- Model calculations
  - ENJL — Bijnens, Pallante, Prades (95-96)
  - NJL and hidden gauge — Hayakawa, Kinoshita, Sanda (95-96)
  - nonlocal χQM — Dorokhov, Broniowski (08)
  - AdS/CFT — Cappiello, Cata, D'Ambrosio (10)
  - Dyson-Schwinger — Goecke, Fischer, Williams (11)
  - constituent χQM — Greynat, de Rafael (12)
  - resonances in the narrow-width limit — Pauk, Vanderhaeghen (14)
- Impact of rigorously derived constraints
  - high-energy constraints taken into account in several models above

    addressed specifically by — Knecht, Nyffeler (01)
  - high-energy constraints related to the axial anomaly — Melnikov, Vainshtein (04) and Nyffeler (09)
  - sum rules for $\gamma^*\gamma \to X$ — Pascalutsa, Pauk, Vanderhaeghen (12)

    see also: workshop MesonNet (13)
  - low-energy constraints–pion polarizabilities — Engel, Ramsey-Musolf (13)
- Lattice — Blum et al. (05,12)

Some notation

HLbL tensor:

$$\Pi^{\mu\nu\lambda\sigma} = i^3 \int dx \int dy \int dz \, e^{-i(x\cdot q_1 + y\cdot q_2 + z\cdot q_3)} \langle 0 | T\left\{ j^\mu(x) j^\nu(y) j^\lambda(z) j^\sigma(0) \right\} | 0 \rangle$$

where $j^\mu(x) = \sum_i Q_i \bar{q}_i(x)\gamma^\mu q_i(x)$, $i = u, d, s$

$$k = q_1 + q_2 + q_3 \qquad k^2 = 0$$

Helicity amplitudes

$$\begin{aligned} H_{\lambda_1\lambda_2,\lambda_3\lambda_4}(s,t,u) &\equiv \mathcal{M}(\gamma^*(q_1,\lambda_1)\gamma^*(q_2,\lambda_2) \to \gamma^*(-q_3,\lambda_3)\gamma(k,\lambda_4)) \\ &= \epsilon_\mu(\lambda_1,q_1)\epsilon_\nu(\lambda_2,q_2)\epsilon^*_\lambda(\lambda_3,-q_3)\epsilon^*_\sigma(\lambda_4,k)\Pi^{\mu\nu\lambda\sigma} \end{aligned}$$

with Mandelstam variables

$$s = (q_1+q_2)^2 \quad t = (q_1+q_3)^2 \quad u = (q_2+q_3)^2$$

Contribution to a_μ

From gauge invariance:

$$\Pi_{\mu\nu\lambda\sigma}(q_1,q_2,k-q_1-q_2) = -k^\rho \frac{\partial}{\partial k^\sigma}\Pi_{\mu\nu\lambda\rho}(q_1,q_2,k-q_1-q_2).$$

Contribution to a_μ:

$$a_\mu = \lim_{k\to 0} \mathrm{Tr}\left\{(\not{p}+m)\Lambda^\rho(p',p)(\not{p}'+m)\Gamma_\rho(p',p)\right\}$$

$$\Gamma_\rho = e^6\int\frac{d^4q_1}{(2\pi)^4}\int\frac{d^4q_2}{(2\pi)^4}\frac{1}{q_1^2q_2^2q_3^2}\frac{\gamma^\mu(\not{p}'+\not{q}_1+m)\gamma^\lambda(\not{p}-\not{q}_2+m)\gamma^\nu}{((p'+q_1)^2-m^2)((p-q_2)^2-m^2)}k^\sigma\partial_{k^\rho}\Pi_{\mu\nu\lambda\sigma}$$

with the projector

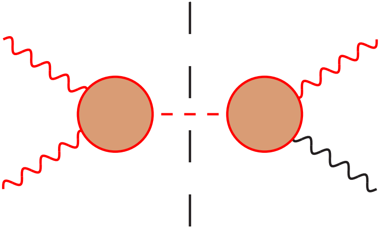
$$\Lambda^\rho(p',p) = \frac{m^2}{k^2(4m^2-k^2)}\left\{\gamma^\rho + \frac{k^2+2m^2}{m(k^2-4m^2)}(p+p')^\rho\right\}$$

m denotes the mass of the muon, p and $p' = p-k$ the momenta of the incoming and outgoing muon, respectively

Setting up the dispersive calculation

We split the HLbL tensor as follows:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\text{FsQED}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \cdots$$



Pion pole: known

Setting up the dispersive calculation

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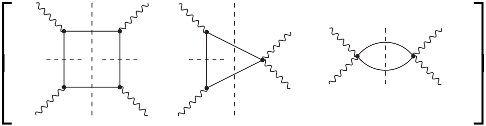
$$F^V_\pi(q_1^2) F^V_\pi(q_2^2) F^V_\pi(q_3^2) \times$$



Contribution with two simultaneous cuts

– analytic properties like the box diagram in sQED

– triangle and bulb diagram required by gauge invariance

– multiplication with F^V_π gives the correct q^2 dependence

it is not an approximation!

Setting up the dispersive calculation

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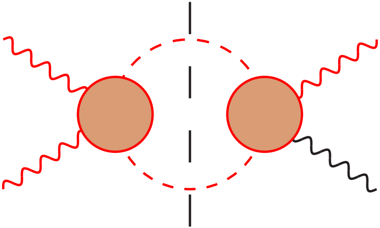

The "rest" with 2π intermediate states has cuts only in one channel and is what will be calculated dispersively

Setting up the dispersive calculation

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Contributions of cuts with anything else other than one and two pions in intermediate states will be neglected

Our dispersive representation of the HLbL tensor

$$\bar{\Pi}^{\mu\nu\lambda\sigma} = \sum_{i=1}^{15} \left(A_{i,s}^{\mu\nu\lambda\sigma} \Pi_i(s) + A_{i,t}^{\mu\nu\lambda\sigma} \Pi_i(t) + A_{i,u}^{\mu\nu\lambda\sigma} \Pi_i(u) \right)$$

- the $\Pi_i(s)$ are single-variable functions having only a right-hand cut
- for the discontinuity we keep only the lowest partial wave
- the dispersive integral that gives the $\Pi_i(s)$ in terms of its discontinuity has the required soft-photon zeros
- soft-photon zeros constrain
  the subtraction polynomial to vanish
  (unless one wanted to subtract more, which is unnecessary)

Dispersion relations for the $\Pi_i(s)$

Imposing the same form of the soft-photon zeros as in the subamplitudes $\gamma^*\gamma^* \to \pi\pi$ we obtain the following dispersion relations:

$$\Pi_1^s = \bar{h}^0_{++,++}(s) = \frac{s-q_3^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'-q_3^2}\left(\frac{1}{s'-s}-\frac{s'-q_1^2-q_2^2}{\lambda'_{12}}\right)\mathrm{Im}\bar{h}^0_{++,++}(s')$$

$$y\Pi_2^s = \bar{h}^0_{00,++}(s) = \frac{s-q_3^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'-q_3^2}\left(\frac{1}{s'-s}-\frac{s'-q_1^2-q_2^2}{\lambda'_{12}}\right)\mathrm{Im}\bar{h}^0_{00,++}(s')$$

with $y = -\frac{q_1^2 q_2^2}{\xi_1 \xi_2}$ [and similarly for the others]

Non-diagonal terms

Diagonal terms guarantee that the $\Pi_i^{s,t,u}$ are non-singular

Changing to other bases will in general produce kinematic singularities

$\Rightarrow$ must determine which basis has to be free of kinematic singularities

General procedure for finding such a basis: Bardeen, Tung (1968), Tarrach (1975)

Non-diagonal terms

Problem solved for the S-waves:

$$\Pi_1^s = \frac{s-q_3^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'-q_3^2}\left[K_1\,\mathrm{Im}\bar{h}^0_{++,++}(s') + \frac{2\xi_1\xi_2}{\lambda'_{12}}\,\mathrm{Im}\bar{h}^0_{00,++}(s')\right]$$

$$y\Pi_2^s = \frac{s-q_3^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'-q_3^2}\left[K_1\,\mathrm{Im}\bar{h}^0_{00,++}(s') + \frac{2q_1^2q_2^2}{\xi_1\xi_2\lambda'_{12}}\,\mathrm{Im}\bar{h}^0_{++,++}(s')\right]$$

$$K_1 := \frac{1}{s'-s} - \frac{s'-q_1^2-q_2^2}{\lambda'_{12}}$$

Remark: Non-diagonal terms are polynomial in s

Complete determination of non-diagonal terms is in progress (D-waves)

Master formula

$$a_\mu^{\pi\pi} = e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{I^{\pi\pi}}{q_1^2 q_2^2 s \left((p+q_1)^2 - m^2\right)\left((p-q_2)^2 - m^2\right)},$$

$$I^{\pi\pi} = \sum_{i \in \{1,2,3,6,14\}} \left(T_{i,s} I_{i,s} + 2 T_{i,u} I_{i,u} \right) + 2 T_{9,s} I_{9,s} + 2 T_{9,u} I_{9,u} + 2 T_{12,u} I_{12,u}$$

with $I_{i,(s,u)}$ dispersive integrals and $T_{i,(s,u)}$ integration kernels

$$I_{1,s} = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \left[\frac{ds'}{s'-s} K_1 \, \mathrm{Im} \bar{h}^0_{++,++}(s') + \frac{2\xi_1\xi_2}{\lambda'_{12}} \mathrm{Im} \bar{h}^0_{00,++}(s') \right],$$

$$T_{1,s} = \frac{16}{3} s \left\{ m^2 + \frac{8 P_{21}\, p \cdot q_1}{\lambda_{12}} \right\}, \qquad T_{1,u} = \frac{16}{3} \left\{ \frac{4 P_{12}^2}{\lambda_{12}} - P_{12} - Z_u \right\},$$

Master formula

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$$I_{6,s} = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{\left(s' - q_1^2 - q_2^2\right)\left(s' - s\right)^2} \mathrm{Im}\bar{h}^{2}_{+-,+-}(s') \left(\frac{75}{8}\right)$$

Helicity amplitudes contribute up to $J = 2$ (S and D waves)

Master formula

The bars on the helicity amplitudes mean that we must subtract the FsQED contribution.

The unitarity relation for the barred imaginary parts read

$$\begin{aligned}
&\mathrm{Im}_s \bar{h}_{J,ij}(s) = \\
&= h^{\mathrm{c}}_{J,i}(s;q_1^2,q_2^2)\left(h^{\mathrm{c}}_{J,j}(s;q_3^2,0)\right)^* - N_{J,i}(s;q_1^2,q_2^2)N_{J,j}(s;q_3^2,0) \\
&+ \frac{1}{2}h^{\mathrm{n}}_{J,i}(s;q_1^2,q_2^2)\left(h^{\mathrm{n}}_{J,j}(s;q_3^2,0)\right)^*
\end{aligned}$$

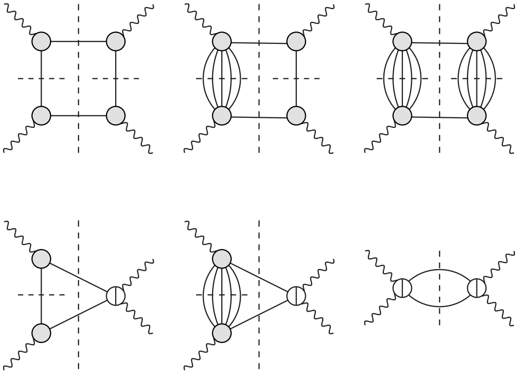
where:

$h^{\mathrm{c,n}}_{J,i}$ = helicity amplitudes for $\gamma^*\gamma^* \to \pi^+\pi^-$ and $\pi^0\pi^0$ resp.

$N_{J,i}$ = partial-wave projection of the $\gamma^*\gamma^* \to \pi^+\pi^-$ Born term

Master formula

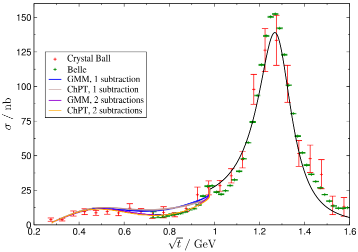
What contributions are included? How?



Dispersion relations for $\gamma^*\gamma^* \to \pi\pi$

Roy-Steiner eqs. $=$ Dispersion relations $+$ partial-wave expansion
$+$ crossing symmetry $+$ unitarity $+$ gauge invariance

- On-shell $\gamma\gamma \to \pi\pi$: prominent *D*-wave reson. $f_2(1270)$ Moussallam (10) Hoferichter, Phillips, Schat (11)
- $\gamma^*\gamma \to \pi\pi$ Moussallam (13)
- $\gamma^*\gamma^* \to \pi\pi$, new feature: anomalous thresholds Hoferichter, GC, Procura, Stoffer (13)



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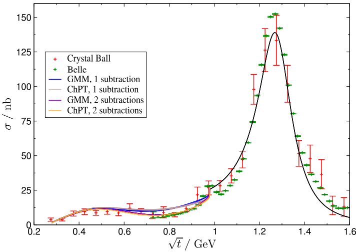
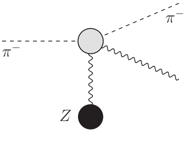
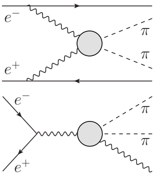


- Constraints
  - Low energy: pion polar., ChPT
  - Primakoff: $\gamma\pi \to \gamma\pi$ at COMPASS, JLAB
  - Scattering: $e^+e^- \to e^+e^-\pi\pi$, $e^+e^- \to \pi\pi\gamma$
  - Decays: $\omega, \phi \to \pi\pi\gamma$





Dispersion relations for $\gamma^*\gamma^* \to \pi\pi$

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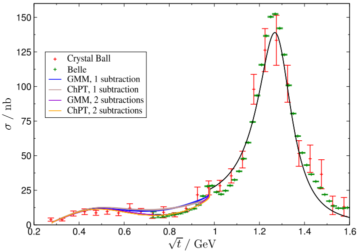
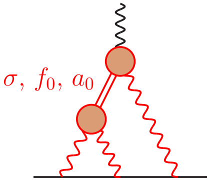
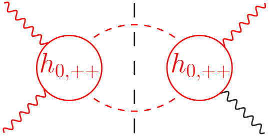


Analysis of the Roy-Steiner equations for $\gamma^*\gamma^* \to \pi\pi$ is in progress: any experimental input most welcome

Physics of $\gamma^*\gamma^* \to \pi\pi$

- $\pi\pi$ rescattering $\Leftrightarrow$ resonances, e.g. $f_2(1270)$
- S-wave provides model-independent implementation of the σ



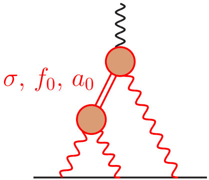


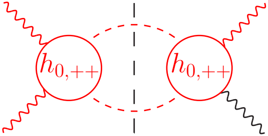
Physics of $\gamma^*\gamma^* \to \pi\pi$

- $\pi\pi$ rescattering $\Leftrightarrow$ resonances, e.g. $f_2(1270)$
- S-wave provides model-independent implementation of the σ
- Analytic continuation with dispersion theory: resonance properties
  - Precise determination of σ-pole from $\pi\pi$ scattering Caprini, GC, Leutwyler 2006

$$M_\sigma = 441^{+16}_{-8}\,\text{MeV} \qquad \Gamma_\sigma = 544^{+18}_{-25}\,\text{MeV}$$

  - Coupling $\sigma \to \gamma\gamma$ from $\gamma\gamma \to \pi\pi$ Hoferichter, Phillips, Schat 2011





$f_0(500)$ or σ
was $f_0(600)$

$I^G(J^{PC}) = 0^+(0^{++})$

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$f_0(500)$ PARTIAL WIDTHS

$\Gamma(\gamma\gamma)$ **Γ_2**

VALUE (keV)	DOCUMENT ID	TECN	COMMENT
• • • We do not use the following data for averages, fits, limits, etc. • • •			
1.7 ± 0.4	[54] HOFERICHTER 11	RVUE	Compilation
3.08 ± 0.82	[55] MENNESSIER 11	RVUE	Compilation
$2.08 \pm 0.2 {}^{+0.07}_{-0.04}$	[56] MOUSSALLAM 11	RVUE	Compilation
2.08	[57] MAO 09	RVUE	Compilation
1.2 ± 0.4	[58] BERNABEU 08	RVUE	
3.9 ± 0.6	[55] MENNESSIER 08	RVUE	$\gamma\gamma \to \pi^+\pi^-,\ \pi^0\pi^0$
1.8 ± 0.4	[59] OLLER 08	RVUE	Compilation

$f_0(500)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma \approx 2\,\text{Im}(\sqrt{s_{\text{pole}}})$.

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
(400–550)$-i$(200–350) OUR ESTIMATE			
• • • We do not use the following data for averages, fits, limits, etc. • • •			
$(445 \pm 25)-i(278^{+22}_{-18})$	[1,2] GARCIA-MAR..11	RVUE	Compilation
$(457^{+14}_{-13})-i(279^{+11}_{-7})$	[1,3] GARCIA-MAR..11	RVUE	Compilation
$(442^{+5}_{-8})-i(274^{+6}_{-5})$	[4] MOUSSALLAM 11	RVUE	Compilation
$(452 \pm 13)-i(259 \pm 16)$	[5] MENNESSIER 10	RVUE	Compilation
$(448 \pm 43)-i(266 \pm 43)$	[6] MENNESSIER 10	RVUE	Compilation
$(455 \pm 6^{+31}$[illegible]$)-i(278 \pm 6^{+34}$[illegible]$)$	[7] CAPRINI 08	RVUE	Compilation

Some preliminary numbers

S-wave contributions:

a_μ^{HLbL} in 10^{-11} units

δ_1^1 input	$I=0$ [DR1]	$I=0$ [DR2]	$I=2$ [DR1]	$I=2$ [DR2]
CCL	-7.13 ± 0.03	-6.75 ± 0.06	1.82 ± 0.01	1.68 ± 0.01
CCL $+\rho\prime, \rho\prime\prime$	-7.79 ± 0.03	-7.38 ± 0.06	2.00 ± 0.01	1.84 ± 0.01

Add FsQED:

δ_1^1 input	FsQED	sum [DR1]	sum [DR2]
CCL	-13.77 ± 0.01	-19.08 ± 0.03	-18.84 ± 0.06
CCL $+\rho\prime, \rho\prime\prime$	-14.65 ± 0.03	-20.44 ± 0.03	-20.19 ± 0.06

Compare to estimates in the literature:

Table 13

Summary of the most recent results for the various contributions to $a_\mu^{\text{LbL;had}} \times 10^{11}$. The last column is our estimate based on our new evaluation for the pseudoscalars and some of the other results.

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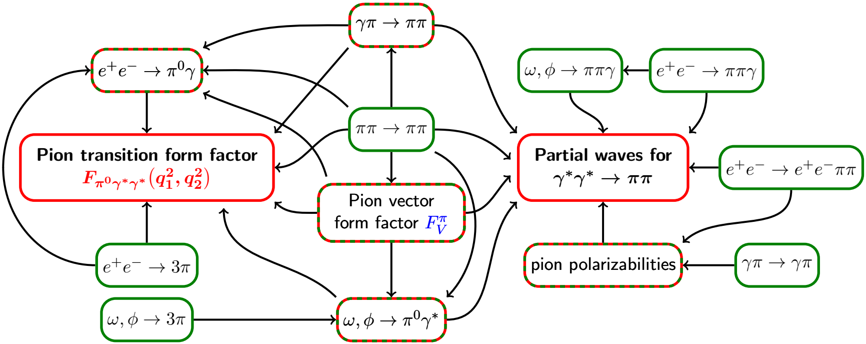
π^0, η, η'	85 ± 13	82.7 ± 6.4	83 ± 12	114 ± 10	–	114 ± 13	99 ± 16
π, K loops	-19 ± 13	-4.5 ± 8.1	–	–	–	-19 ± 19	-19 ± 13
π, K loops + other subleading in N_c	–	–	–	0 ± 10	–	–	–
Axial vectors	2.5 ± 1.0	1.7 ± 1.7	–	22 ± 5	–	15 ± 10	22 ± 5
Scalars	-6.8 ± 2.0	–	–	–	–	-7 ± 7	-7 ± 2
Quark loops	21 ± 3	9.7 ± 11.1	–	–	–	$2.3\pm$	21 ± 3
Total	83 ± 32	89.6 ± 15.4	80 ± 40	136 ± 25	110 ± 40	105 ± 26	116 ± 39

Hadronic light-by-light: a roadmap

GC, Hoferichter, Kubis, Procura, Stoffer `arXiv:1408.2517` (PLB, to appear)



Artwork by M. Hoferichter

A reliable evaluation of the HLbL requires many different contributions by and a collaboration among theorists and experimentalists

Outlook

- the complete derivation of non-diagonal (non-Cauchy) kernels is in progress
  (basis obtained via the Bardeen-Tung-Tarrach procedure)
- path to a numerical evaluation of the Master Formula:
  - take into account all experimental constraints on $\gamma^{(*)}\gamma \to \pi\pi$
  - estimate the dependence on the q^2 of the second photon (theoretically, there are no data yet on $\gamma^*\gamma^* \to \pi\pi$)
  - $\Rightarrow$ solve the dispersion relation for $\gamma^*\gamma^* \to \pi\pi$
- input the outcome into the (upgraded) master formula

Conclusions

- I have presented a dispersive framework for the calculation of the HLbL contribution to a_μ
- which takes into account only single- and double-pion intermediate states
  [and all other 1-particle intermediate states (η, $\eta', \ldots$)]
- we have derived a master formula which expresses the contribution of 2π intermediate states to a_μ in terms of (integrals over) $\gamma^*\gamma^* \to \pi\pi$ helicity amplitudes
- this is a first step towards a model-independent calculation of the HLbL contribution to a_μ

SM contributions to $(g-2)_\mu$: QED

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ \; 0.765857426\,(16)\,(\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$+ \; 24.05050988\,(28)\,(\alpha/\pi)^3$

Remiddi, Laporta, Barbieri ... ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

$+ \; 130.8796\,(63)\,(\alpha/\pi)^4$

Kinoshita & Lindquist '81, ... , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa, Kinoshita & Nio, 2007, Kinoshita et al. 2012,
Steinhauser et al. 2013 (analytic, in progress).

$+ \; 753.29\,(1.04)\,(\alpha/\pi)^5$ **COMPLETED!**

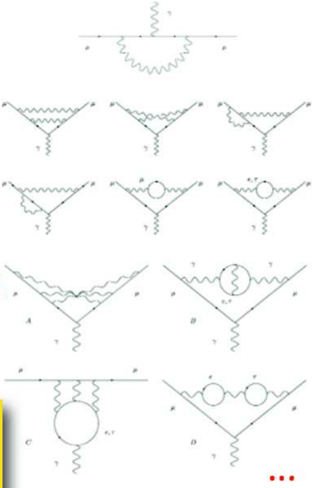
Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,
Karshenboim,..., Kataev, Kinoshita & Nio '06, Kinoshita et al. 2012

Adding up, we get:

$a_\mu^{QED} = 116584718.951\,(22)(77) \times 10^{-11}$

from coeffs, mainly from 4-loop unc — from $\delta\alpha$(Rb)

with α=1/137.035999049(90) [0.66 ppb]



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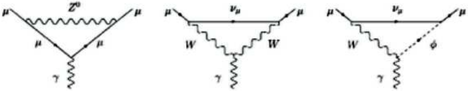
SM contributions to $(g-2)_\mu$: electroweak

- **One-loop term:**



$$a_\mu^{\mathrm{EW}}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2+O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;
Studenikin et al. '80s

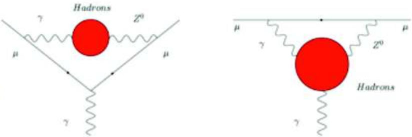
- **One-loop plus higher-order terms:**

$a_\mu^{EW} = 153.6\,(1) \times 10^{-11}$

with M_{Higgs} = 125.6 (1.5) GeV

Hadronic loop uncertainties and 3-loop nonleading logs.

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.



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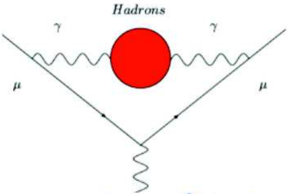
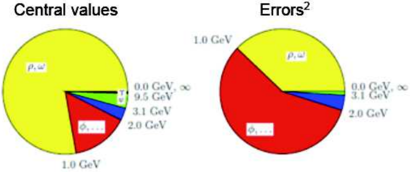
SM contributions to $(g-2)_\mu$: HVP





F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)s/m_\mu^2}$$

$$a_\mu^{\mathrm{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s)\, R(s)$$

$a_\mu^{\mathrm{HLO}} = 6903\ (53)_{\mathrm{tot}} \times 10^{-11}$ F. Jegerlehner, A. Nyffeler, Phys. Rept. 477 (2009) 1

$= 6923\ (42)_{\mathrm{tot}} \times 10^{-11}$ Davier et al, EPJ C71 (2011) 1515 (incl. BaBar & KLOE10 2π)

$= 6949\ (37)_{\mathrm{exp}}\ (21)_{\mathrm{rad}} \times 10^{-11}$ Hagiwara et al, JPG 38 (2011) 085003

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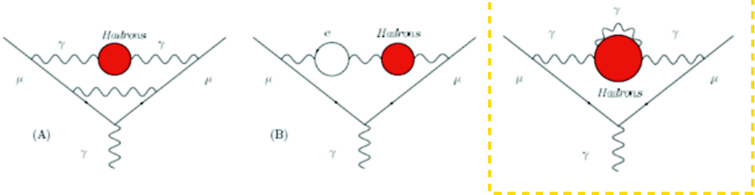
SM contributions to $(g-2)_\mu$: Higher-order HVP

- **HHO: Vacuum Polarization**

Already included in a_μ^{HLO}



O(α^3) contributions of diagrams containing hadronic vacuum polarization insertions:

$$a_\mu^{HHO}(vp) = -98\ (1) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. 2011

Only tiny shifts if τ data are used instead of the e^+e^- ones

Davier & Marciano '04.

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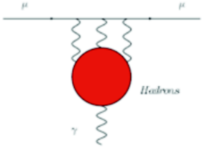
SM contributions to $(g-2)_\mu$: hadronic light-by-light

- **HHO: Light-by-light contribution**



- **Unlike the HLO term, for the hadronic l-b-l term we must rely on theoretical approaches.**
- **This term had a troubled life! Latest values:**

$a_\mu^{HHO}(lbl) = +80\ (40) \times 10^{-11}$	Knecht & Nyffeler '02
$a_\mu^{HHO}(lbl) = +136\ (25) \times 10^{-11}$	Melnikov & Vainshtein '03
$a_\mu^{HHO}(lbl) = +105\ (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein '09
$a_\mu^{HHO}(lbl) = +116\ (39) \times 10^{-11}$	Jegerlehner & Nyffeler '09

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02

- **"Bound" $a_\mu^{HHO}(lbl) <\sim 160 \times 10^{-11}$** Erler, Sanchez '06, Pivovarov '02; also Boughezal, Melnikov '11
- **Lattice? Very hard… in progress.** M. Golterman @ PhiPsi 2013; T. Blum @ Lattice 2012
- **Pion exch. in holographic QCD agrees.** D.K.Hong, D.Kim '09; Cappiello, Catà, D'Ambrosio '11
- **"By far not complete" calculation: 188×10^{-11}** Fischer et al, PRD87(2013)034013
- **Had lbl is likely to become the ultimate limitation of the SM prediction.**

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SM contributions to $(g-2)_\mu$:

$a_\mu^{EXP} = 116592089\ (63) \times 10^{-11}$

E821 – Final Report: PRD73 (2006) 072 with latest value of $\lambda = \mu_\mu/\mu_p$ from CODATA'06

$a_\mu^{SM} \times 10^{11}$	$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM}$	σ
116 591 793 (66)	$296\ (91) \times 10^{-11}$	3.2 [1]
116 591 813 (57)	$276\ (85) \times 10^{-11}$	3.2 [2]
116 591 839 (58)	$250\ (86) \times 10^{-11}$	2.9 [3]

with the "conservative" a_μ^{HHO}(lbl) = 116 (39) x 10^{-11} and the LO hadronic from:

[1] Jegerlehner & Nyffeler, Phys. Rept. 477 (2009) 1
[2] Davier et al, EPJ C71 (2011) 1515 (includes BaBar & KLOE10 2π)
[3] Hagiwara et al, JPG38 (2011) 085003 (includes BaBar & KLOE10 2π)

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