STATUS OF CHIRAL MESON PHYSICS

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Overview

1. Chiral Perturbation Theory
2. Determination of LECs in the continuum
3. Finite volume
4. Beyond QCD
5. Leading logarithms

Chiral Perturbation Theory

**Exploring the consequences of
the chiral symmetry of QCD
and its spontaneous breaking
using effective field theory techniques**

Derivation from QCD:
H. Leutwyler,
On The Foundations Of Chiral Perturbation Theory,
Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

For references to lectures see:
`http://www.thep.lu.se/~bijnens/chpt.html`

Chiral Perturbation Theory

A general Effective Field Theory:

- Relevant degrees of freedom
- A powercounting principle (predictivity)
- Has a certain range of validity

Chiral Perturbation Theory:

- Degrees of freedom: Goldstone Bosons from spontaneous breaking of chiral symmetry
- Powercounting: Dimensional counting in momenta/masses
- Breakdown scale: Resonances, so about M_ρ.

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Chiral Symmetry

Chiral Symmetry

QCD: N_f light quarks: equal mass: interchange: $SU(N_f)_V$

But $$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$$

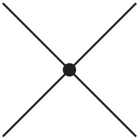
So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Spontaneous breakdown

- $\langle \bar{q} q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
- Mechanism: see talk by L. Giusti
- $SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$
- 8 generators broken $\Longrightarrow$ 8 massless degrees of freedom and interaction vanishes at zero momentum

Power counting in momenta: Meson loops, Weinberg powercounting

rules

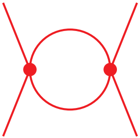


p^2

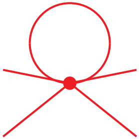
$1/p^2$

$\int d^4p$ $\quad$ p^4

one loop example



$$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$$



$$(p^2)\,(1/p^2)\,p^4 = p^4$$

Chiral Perturbation Theories

- Which chiral symmetry: $SU(N_f)_L \times SU(N_f)_R$, for $N_f = 2, 3, \ldots$ and extensions to (partially) quenched
- Or beyond QCD
- Space-time symmetry: Continuum or broken on the lattice: Wilson, staggered, mixed action
- Volume: Infinite, finite in space, finite T
- Which interactions to include beyond the strong one
- Which particles included as non Goldstone Bosons
- My general belief: if it involves soft pions (or soft K, η) some version of ChPT exists

Lagrangians: Lowest order

$U(\phi) = \exp(i\sqrt{2}\Phi/F_0)$ parametrizes Goldstone Bosons

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\,\eta_8}{\sqrt{6}} \end{pmatrix}.$$

LO Lagrangian: $\mathcal{L}_2 = \frac{F_0^2}{4}\{\langle D_\mu U^\dagger D^\mu U\rangle + \langle \chi^\dagger U + \chi U^\dagger\rangle\}$,

$D_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu$,
left and right external currents: $r(l)_\mu = v_\mu + (-)a_\mu$

Scalar and pseudoscalar external densities: $\chi = 2B_0(s + ip)$ quark masses via scalar density: $s = \mathcal{M} + \cdots$

$\langle A\rangle = Tr_F\,(A)$

Lagrangians: Lagrangian structure

	2 flavour		3 flavour		PQChPT/N_f flavour	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	52+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

- L_i LEC = Low Energy Constants = ChPT parameters
- H_i: contact terms: value depends on definition of currents/densities
- Finite volume: no new LECs
- Other effects: (many) new LECs

Chiral Logarithms

The main predictions of ChPT:

- Relates processes with different numbers of pseudoscalars
- Chiral logarithms
- includes Isospin and the eightfold way $(SU(3)_V)$
- Unitarity included perturbatively

$$m_\pi^2 = 2B\hat{m} + \left(\frac{2B\hat{m}}{F}\right)^2 \left[\frac{1}{32\pi^2}\log\frac{(2B\hat{m})}{\mu^2} + 2l_3^r(\mu)\right] + \cdots$$

$$M^2 = 2B\hat{m}$$

Overview

Let's go over to the next point: dealing with the parameters

1. Chiral Perturbation Theory
2. Determination of LECs in the continuum
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(Partial) History/References

- Original determination at p^4: Gasser, Leutwyler, Annals Phys.158 (1984) 142, Nucl. Phys. B250 (1985) 465
- p^6 2 flavour: several papers (see later)
- p^6 3 flavour: Amorós, JB, Talavera, Nucl. Phys. B602 (2001) 87 [hep-ph/0101127]
- Review article two-loops: JB, Prog. Part. Nucl. Phys. 58 (2007) 521 [hep-ph/0604043]
- Update of fits + new input: JB, Jemos, Nucl. Phys. B 854 (2012) 631 [arXiv:1103.5945]
- Recent review with more p^6 input: JB, Ecker, arXiv:1405.6488, Ann. Rev. Nucl. Part. Sc.(in press)
- Review Kaon physics: Cirigliano, Ecker, Neufeld, Pich, Portoles, Rev.Mod.Phys. 84 (2012) 399 [arXiv:1107.6001]
- Lattice: FLAG reports:, Colangelo et al., Eur.Phys.J. C71 (2011) 1695 [arXiv:1011.4408] Aoki et al., arXiv:1310.8555

Two flavour LECs

- $\bar{l}_1$ to $\bar{l}_4$: ChPT at order p^6 and the Roy equation analysis in $\pi\pi$ and F_S Colangelo, Gasser and Leutwyler, *Nucl. Phys.* B 603 (2001) 125 [hep-ph/0103088] Compatible with Rios, Nebrada, Pelaez
- $\bar{l}_5$ and $\bar{l}_6$: from F_V and $\pi \to \ell\nu\gamma$ JB,(Colangelo,)Talavera and from $\Pi_V - \Pi_A$ González-Alonso, Pich, Prades
- $$\begin{aligned} \bar{l}_1 &= -0.4 \pm 0.6\,, & \bar{l}_2 &= 4.3 \pm 0.1\,, \\ \bar{l}_3 &= 2.9 \pm 2.4\,, & \bar{l}_4 &= 4.4 \pm 0.2\,, \\ \bar{l}_5 &= 12.24 \pm 0.21\,, & \bar{l}_6 - \bar{l}_5 &= 3.0 \pm 0.3\,, \\ \bar{l}_6 &= 16.0 \pm 0.5 \pm 0.7\,. \end{aligned}$$
- $l_7 \sim 5 \cdot 10^{-3}$ from π^0-η mixing Gasser, Leutwyler 1984
- guesstimate including lattice: $\bar{l}_3 = 3.0 \pm 0.8$ $\bar{l}_4 = 4.3 \pm 0.3$

Three flavour LECs: uncertainties

- $m_K^2, m_\eta^2 \gg m_\pi^2$
- Contributions from p^6 Lagrangian are larger
- Reliance on estimates of the C_i much larger
- Typically: C_i^r: (terms with)
  kinematical dependence $\equiv$ measurable
  quark mass dependence $\equiv$ impossible (without lattice)
  100% correlated with L_i^r
- How suppressed are the $1/N_c$-suppressed terms?
- Are we really testing ChPT or just doing a phenomenological fit?

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Testing if ChPT works: relations

Yes: JB, Jemos, Eur.Phys.J. C64 (2009) 273-282 [arXiv:0906.3118]

Systematic search for relations between observables that do not depend on the C_i^r

Included:

- m_M^2 and F_M for π, K, η.
- 11 $\pi\pi$ threshold parameters
- 14 πK threshold parameters
- 6 $\eta \to 3\pi$ decay parameters,
- 10 observables in $K_{\ell 4}$
- 18 in the scalar formfactors
- 11 in the vectorformfactors
- Total: 76

We found 35 relations

Relations at NNLO: summary

- We did numerics for $\pi\pi$ (7), πK (5) and $K_{\ell 4}$ (1)
  13 relations
- $\pi\pi$: similar quality in two and three flavour ChPT
  The two involving a_3^- significantly did not work well
- πK: relation involving a_3^- not OK
  one more has very large NNLO corrections
- The relation with $K_{\ell 4}$ also did not work: related to that ChPT has trouble with curvature in $K_{\ell 4}$
- Conclusion: Three flavour ChPT "sort of" works

Fits: inputs

Amorós, JB, Talavera, Nucl. Phys. B602 (2001) 87 [hep-ph/0101127] (ABC01)

JB, Jemos, Nucl. Phys. B 854 (2012) 631 [arXiv:1103.5945] (JJ12)

JB, Ecker, arXiv:1405.6488, Ann. Rev. Nucl. Part. Sc.(in press) (BE14)

- $M_\pi, M_K, M_\eta, F_\pi, F_K/F_\pi$
- $\langle r^2\rangle^\pi_S$, c^π_S slope and curvature of F_S
- $\pi\pi$ and πK scattering lengths a^0_0, a^2_0, $a^{1/2}_0$ and $a^{3/2}_0$.
- Value and slope of F and G in $K_{\ell 4}$
- $\frac{m_s}{\hat{m}} = 27.5$ (lattice)
- $\bar{l}_1, \dots, \bar{l}_4$
- more variation with C^r_i, a penalty for a large p^6 contribution to the masses
- 17+3 inputs and 8 L^r_i+34 C^r_i to fit

Main fit

	ABC01	JJ12	L_4^r free	BE14
	old data			
$10^3 L_1^r$	0.39(12)	0.88(09)	0.64(06)	0.53(06)
$10^3 L_2^r$	0.73(12)	0.61(20)	0.59(04)	0.81(04)
$10^3 L_3^r$	$-2.34(37)$	$-3.04(43)$	$-2.80(20)$	$-3.07(20)$
$10^3 L_4^r$	$\equiv 0$	0.75(75)	0.76(18)	$\equiv 0.3$
$10^3 L_5^r$	0.97(11)	0.58(13)	0.50(07)	1.01(06)
$10^3 L_6^r$	$\equiv 0$	0.29(8)	0.49(25)	0.14(05)
$10^3 L_7^r$	$-0.30(15$	$-0.11(15)$	$-0.19(08)$	$-0.34(09)$
$10^3 L_8^r$	0.60(20)	0.18(18)	0.17(11)	0.47(10)
χ^2	0.26	1.28	0.48	1.04
dof	1	4	?	?
F_0 [MeV]	87	65	64	71

$? = (17+3) - (8+34)$

Main fit: Comments

- All values of the C_i^r we settled on are "reasonable"
- Leaving L_4^r free ends up with $L_4^r \approx 0.76$
- keeping L_4^r small: also L_6^r and $2L_1^r - L_2^r$ small (large N_c relations)
- Compatible with lattice determinations
- Not too bad with resonance saturation both for L_i^r and C_i^r
- decent convergence (but enforced for masses)
- Many prejudices went in: large N_c, resonance model, quark model estimates,...

Some results of this fit

Mass:

$$\begin{aligned}
m_\pi^2/m_{\pi phys}^2 &= 1.055(p^2) - 0.005(p^4) - 0.050(p^6)\,,\\
m_K^2/m_{K phys}^2 &= 1.112(p^2) - 0.069(p^4) - 0.043(p^6)\,,\\
m_\eta^2/m_{\eta phys}^2 &= 1.197(p^2) - 0.214(p^4) + 0.017(p^6)\,,
\end{aligned}$$

Decay constants:

$$\begin{aligned}
F_\pi/F_0 &= 1.000(p^2) + 0.208(p^4) + 0.088(p^6)\,,\\
F_K/F_\pi &= 1.000(p^2) + 0.176(p^4) + 0.023(p^6)\,.
\end{aligned}$$

Scattering:

$$\begin{aligned}
a_0^0 &= 0.160(p^2) + 0.044(p^4) + 0.012(p^6)\,,\\
a_0^{1/2} &= 0.142(p^2) + 0.031(p^4) + 0.051(p^6)\,.
\end{aligned}$$

Overview

An example of other effects:

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Finite volume

- Lattice QCD calculates at different quark masses, volumes boundary conditions,...
- A general result by Lüscher: relate finite volume effects to scattering (1986)
- Chiral Perturbation Theory is also useful for this
- Start: Gasser and Leutwyler, Phys. Lett. B184 (1987) 83, Nucl. Phys. B 307 (1988) 763
  M_π, F_π, $\langle \bar{q}q \rangle$ one-loop equal mass case
- I will stay with ChPT and the p regime ($M_\pi L >> 1$)
- $1/m_\pi = 1.4$ fm
  may need to go beyond leading $e^{-m_\pi L}$ terms
- Convergence of ChPT is given by $1/m_\rho \approx 0.25$ fm

Finite volume: selection of ChPT results

- masses and decay constants for π, K, η one-loop
  Becirevic, Villadoro, Phys. Rev. D 69 (2004) 054010
- M_π at 2-loops (2-flavour)
  Colangelo, Haefeli, Nucl.Phys. B744 (2006) 14 [hep-lat/0602017]
- $\langle \bar{q}q \rangle$ at 2 loops (3-flavour)
  JB, Ghorbani, Phys. Lett. B636 (2006) 51 [hep-lat/0602019]
- Twisted mass at one-loop
  Colangelo, Wenger, Wu, Phys.Rev. D82 (2010) 034502 [arXiv:1003.0847]
- Twisted boundary conditions
  Sachrajda, Villadoro, Phys. Lett. B 609 (2005) 73 [hep-lat/0411033]
- This talk:
  - Twisted boundary conditions and some funny effects
  - Some preliminary results on masses 3-flavours at two loop order

Twisted boundary conditions

- On a lattice at finite volume $p^i = 2\pi n^i/L$: very few momenta directly accessible
- Put a constraint on certain quark fields in some directions: $q(x^i + L) = e^{i\theta^i_q} q(x^i)$
- Then momenta are $p^i = \theta^i/L + 2\pi n^i/L$. Allows to map out momentum space on the lattice much better Bedaque,...
- But:
  - Box: Rotation invariance → cubic invariance
  - Twisting: reduces symmetry further

  Consequences:
  - $m^2(\vec{p}^2) = E^2 - \vec{p}^2$ is not constant
  - There are typically more form-factors
  - In general: quantities depend on many more components of the momenta
  - Charge conjugation involves a change in momentum

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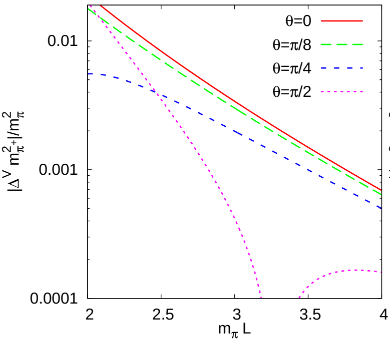
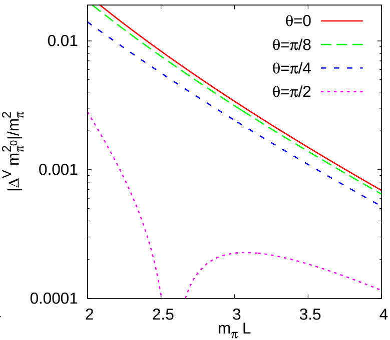
Twisted boundary conditions: volume correction masses

JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]

$m_\pi L = 2,, \vec{\theta}_u = (\theta, 0, 0), \vec{\theta}_d = \vec{\theta}_s = 0$



Charged pion mass

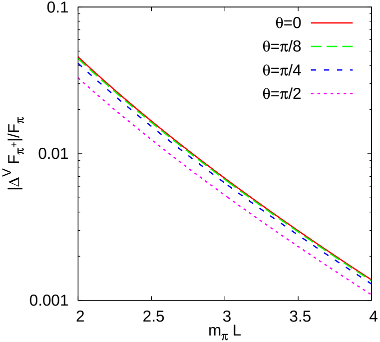


Neutral pion mass

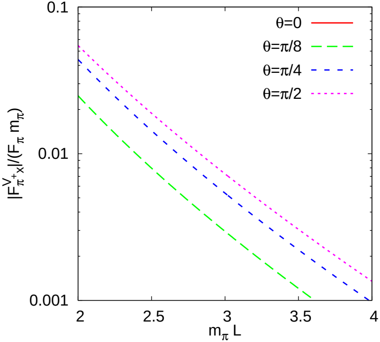
$\Delta^V X = X^V - X^\infty$ (dip is going through zero)

Volume correction decay constants: F_{π^+}

- JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]
- $\left\langle 0 \middle| A_\mu^M \middle| M(p) \right\rangle = i\sqrt{2} F_M p_\mu + i\sqrt{2} F_{M\mu}^V$
- Extra terms are needed for Ward identities



relative for F_π



Extra for $\mu = x$

Volume correction electromagnetic formfactor

- JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]
  earlier two-flavour work:
  Bunton, Jiang, Tiburzi, Phys.Rev. D74 (2006) 034514 [hep-lat/0607001]
- $\langle M'(p')|j_\mu|M(p)\rangle = f_\mu = f_+(p_\mu + p'_\mu) + f_- q_\mu + h_\mu$
- Extra terms are again needed for Ward identities
- Note that masses have finite volume corrections
  - q^2 for fixed $\vec{p}$ and $\vec{p}'$ has corrections
    small effect
  - This also affects the ward identities, e.g.
    $q^\mu f_\mu = (p^2 - p'^2)f_+ + q^2 f_- + q^\mu h_\mu = 0$
    is satisfied but all effects should be considered

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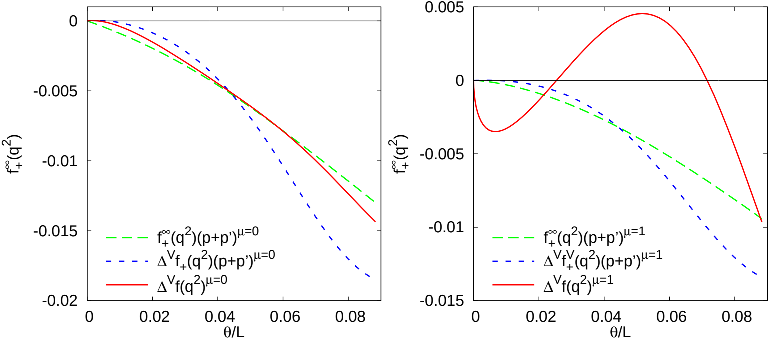
Volume correction electromagnetic formfactor

- $f_\mu = -\frac{1}{\sqrt{2}}\langle \pi^0(p')|\bar{d}\gamma_\mu u|\pi^+(p)\rangle$
  $= \left(1 + f_+^\infty + \Delta^V f_+\right)(p+p')_\mu + \Delta^V f_- q_\mu + \Delta^V h_\mu$
- Pure loop plotted: $f_+^\infty(p+p'), \Delta^V f_+(p+p')$ and $\Delta^V f_\mu$

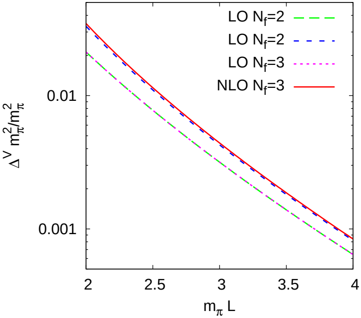


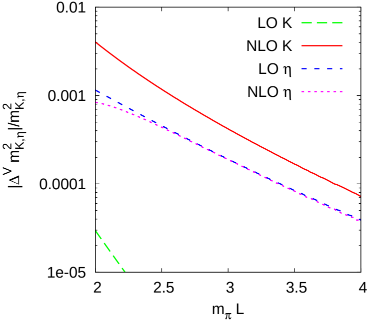
$\mu = t$ $\mu = x$

Finite volume corrections large, different for different μ

Masses at two-loop order

- Sunset integrals at finite volume done
  JB, Boström and Lähde, JHEP 01 (2014) 019 [arXiv:1311.3531]
- Loop calculations in progress JB, Rössler





- Agreement for $N_f = 2, 3$ for pion
- K has no pion loop at LO
- η large cancelation: L_i^r dependent part vs rest at NLO

Overview

ChPT for other theories:

QCDlike and/or technicolor theories

- One can also have different symmetry breaking patterns from underlying fermions
- Three generic cases
  - $SU(N) \times SU(N)/SU(N)$
  - $SU(2N)/SO(2N)$
  - $SU(2N)/Sp(2N)$
- Many one-loop results existed especially for the first case (several discovered only after we published our work)
- Equal mass case pushed to two loops JB, Lu, 2009-11

N_F fermions in a representation of the gauge group

- complex (QCD):
  - $q^T = (q_1 \ q_2 \ldots \ q_{N_F})$
  - Global $G = SU(N_F)_L \times SU(N_F)_R$
    $q_L \to g_L q_L$ and $g_R \to g_R q_R$
  - Vacuum condensate $\Sigma_{ij} = \langle \overline{q}_j q_i \rangle \propto \delta_{ij}$
  - $g_L = g_R$ then $\Sigma_{ij} \to \Sigma_{ij} \Longrightarrow$ conserved $H = SU(N_F)_V$:
- Real (e.g. adjoint): $\hat{q}^T = (q_{R1} \ \ldots \ q_{RN_F} \ \tilde{q}_{R1} \ \ldots \ \tilde{q}_{RN_F})$
  - $\tilde{q}_{Ri} \equiv C\overline{q}_{Li}^T$ goes under gauge group as q_{Ri}
  - some Goldstone bosons have baryonnumber
  - Global $G = SU(2N_F)$ and $\hat{q} \to g\hat{q}$
  - $\langle \overline{q}_j q_i \rangle$ is really $\langle (\hat{q}_j)^T C \hat{q}_i \rangle \propto J_{Sij}$ $J_S = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
  - Conserved if $g J_S g^T = J_S \Longrightarrow H = SO(2N_F)$

Lund University

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- complex (QCD):
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    $q_L \to g_L q_L$ and $g_R \to g_R q_R$
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  - $\langle \overline{q}_j q_i \rangle$ is really $\langle (\hat{q}_j)^T C \hat{q}_i \rangle \propto J_{Sij}$ $J_S = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
  - Conserved if $g J_S g^T = J_S \Longrightarrow H = SO(2N_F)$

N_F fermions in a representation of the gauge group

- complex (QCD): $q^T = (q_1\ q_2 \ldots\ q_{N_F})$
  - Global $G = SU(N_F)_L \times SU(N_F)_R \quad q_L \to g_L q_L$ and $g_R \to g_R q_R$
  - Vacuum condensate $\Sigma_{ij} = \langle \overline{q}_j q_i \rangle \propto \delta_{ij}$
  - Conserved $H = SU(N_F)_V$: $g_L = g_R$ then $\Sigma_{ij} \to \Sigma_{ij}$
- Pseudoreal (e.g. two-colours): $\hat{q}^T = (q_{R1}\ \ldots\ q_{RN_F}\ \tilde{q}_{R1}\ \ldots\ \tilde{q}_{RN_F})$
  - $\tilde{q}_{R\alpha i} \equiv \epsilon_{\alpha\beta} C \bar{q}^T_{L\beta i}$ goes under gauge group as $q_{R\alpha i}$
  - some Goldstone bosons have baryonnumber
  - Global $G = SU(2N_F)$ and $\hat{q} \to g\hat{q}$
  - $\langle \overline{q}_j q_i \rangle$ is really $\epsilon_{\alpha\beta}\langle (\hat{q}_{\alpha j})^T C \hat{q}_{\beta i}\rangle \propto J_{Aij}$ $J_A = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$
  - Conserved if $gJ_Ag^T = J_A \Longrightarrow H = Sp(2N_F)$

Lagrangians

JB, Lu, arXiv:0910.5424: 3 cases similar with $u = \exp\left(\frac{i}{\sqrt{2}F}\phi^a X^a\right)$

But the matrices X^a are:

- Complex or $SU(N) \times SU(N)/SU(N)$:
  all $SU(N)$ generators
- Real or $SU(2N)/SO(2N)$:
  $SU(2N)$ generators with $X^a J_S = J_S X^{aT}$
- Pseudoreal or $SU(2N)/Sp(2N)$:
  $SU(2N)$ generators with $X^a J_A = J_A X^{aT}$
- Note that the latter are not the usual ways of parametrizing $SO(2N)$ and $Sp(2N)$ matrices

The main useful formulae

Calculating for equal mass case goes through using:

$$
\begin{aligned}
\text{Complex}: \quad & \left\langle X^a A X^a B\right\rangle = \langle A\rangle\langle B\rangle - \frac{1}{N_F}\langle AB\rangle\,,\\
& \left\langle X^a A\right\rangle\left\langle X^a B\right\rangle = \langle AB\rangle - \frac{1}{N_F}\langle A\rangle\langle B\rangle\,.\\
\text{Real}: \quad & \left\langle X^a A X^a B\right\rangle = \frac{1}{2}\langle A\rangle\langle B\rangle + \frac{1}{2}\left\langle A J_S B^T J_S\right\rangle - \frac{1}{2N_F}\langle AB\rangle\,,\\
& \left\langle X^a A\right\rangle\left\langle X^a B\right\rangle = \frac{1}{2}\langle AB\rangle + \frac{1}{2}\left\langle A J_S B^T J_S\right\rangle - \frac{1}{2N_F}\langle A\rangle\langle B\rangle\,.\\
\text{Pseudoreal}: \quad & \left\langle X^a A X^a B\right\rangle = \frac{1}{2}\langle A\rangle\langle B\rangle + \frac{1}{2}\left\langle A J_A B^T J_A\right\rangle - \frac{1}{2N_F}\langle AB\rangle\,,\\
& \left\langle X^a A\right\rangle\left\langle X^a B\right\rangle = \frac{1}{2}\langle AB\rangle - \frac{1}{2}\left\langle A J_A B^T J_A\right\rangle - \frac{1}{2N_F}\langle A\rangle\langle B\rangle
\end{aligned}
$$

So can do the calculations for all cases

$\phi\phi \to \phi\phi$

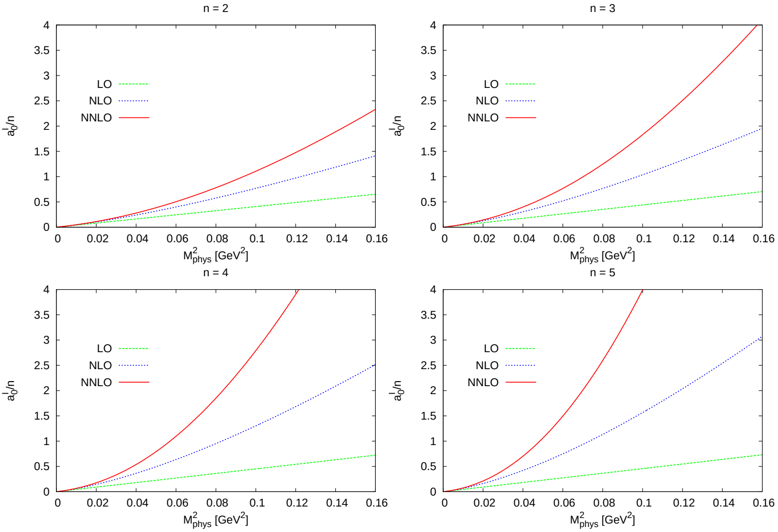
- $\pi\pi$ scattering
  - Amplitude in terms of $A(s,t,u)$
    $M_{\pi\pi}(s,t,u) = \delta^{ab}\delta^{cd}A(s,t,u) + \delta^{ac}\delta^{bd}A(t,u,s) + \delta^{ad}\delta^{bc}A(u,s,t)\,.$
  - Three intermediate states $I = 0, 1, 2$
- Our three cases
  - Two amplitudes needed $B(s,t,u)$ and $C(s,t,u)$

$$
\begin{aligned}
M(s,t,u) \quad = \quad & \left[\left\langle X^aX^bX^cX^d\right\rangle + \left\langle X^aX^dX^cX^b\right\rangle\right] B(s,t,u) \\
& + \left[\left\langle X^aX^cX^dX^b\right\rangle + \left\langle X^aX^bX^dX^c\right\rangle\right] B(t,u,s) \\
& + \left[\left\langle X^aX^dX^bX^c\right\rangle + \left\langle X^aX^cX^bX^d\right\rangle\right] B(u,s,t) \\
& + \delta^{ab}\delta^{cd}C(s,t,u) + \delta^{ac}\delta^{bd}C(t,u,s) + \delta^{ad}\delta^{bc}C(u,s,t)\,.
\end{aligned}
$$

$$B(s,t,u) = B(u,t,s) \qquad C(s,t,u) = C(s,u,t)\,.$$

  - 7, 6 and 6 possible intermediate states
- All formulas similar length to $\pi\pi$ cases but there are so many of them

$\phi\phi \to \phi\phi$: a_0^I/n



Conclusions for "Beyond QCD"

Calculations done:

- M^2_{phys}
- F_{phys}
- Meson-meson scattering
- Equal mass case: allows to get fully analytical result just as for 2-flavour ChPT
- Two-point functions relevant for S-parameter

To remember:

- Different symmetry patterns can appear for different gaugegroups and fermion representations
- Nonperturbative: lattice needs extrapolation formulae

Overview

Can we calculate something of high loop orders?

Leading Logarithms

- Take a quantity with a single scale: $F(M)$
- The dependence on the scale in field theory is typically logarithmic
- $L = \log(\mu/M)$
- $F = F_0 + F_1^1 L + F_0^1 + F_2^2 L^2 + F_1^2 L + F_0^2 + F_3^3 L^3 + \cdots$
- Leading Logarithms: The terms $F_m^m L^m$

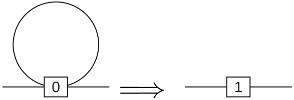
The F_m^m can be more easily calculated than the full result

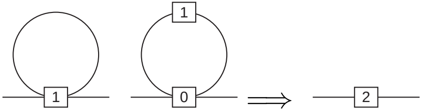
- $\mu\,(dF/d\mu) \equiv 0$
- Ultraviolet divergences in Quantum Field Theory are always local

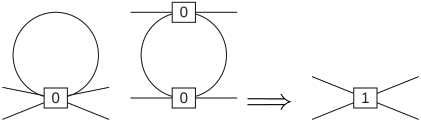
Weinberg's argument

- Weinberg, Physica A96 (1979) 327
- Two-loop leading logarithms can be calculated using only one-loop: Weinberg consistency conditions
- Proof at all orders:
  - using β-functions: Büchler, Colangelo, hep-ph/0309049
  - Proof with diagrams: JB, Carloni, arXiv:0909.5086
- Proof relies on
  - μ: dimensional regularization scale
  - $d = 4 - w$
  - at n-loop order ($\hbar^n$) must cancel:
    - $1/w^n$, $\log\mu/w^{n-1}, \ldots, \log^{n-1}\mu/w$
    - This allows for relations between diagrams
    - All needed for $\log^n\mu$ coefficient can be calculated from one-loop diagrams

Mass to $\hbar^2$

- $\hbar^1$: 

- $\hbar^2$: 

- but also needs $\hbar^1$: 

General

- Calculate the divergence
- rewrite it in terms of a local Lagrangian
    - Luckily: symmetry kept: we know result will be symmetrical, hence do not need to explicitly rewrite the Lagrangians in a nice form
    - Luckily: we do not need to go to a minimal Lagrangian
    - So everything can be computerized
- We keep all terms to have all 1PI (one particle irreducible) diagrams finite

Massive $O(N)$ sigma model

- $O(N+1)/O(N)$ nonlinear sigma model
- $\mathcal{L}_{n\sigma} = \frac{F^2}{2}\partial_\mu\Phi^T\partial^\mu\Phi + F^2\chi^T\Phi\,.$
  - Φ is a real $N+1$ vector; $\Phi \to O\Phi$; $\Phi^T\Phi = 1$.
  - Vacuum expectation value $\langle\Phi^T\rangle = (1\;0\ldots0)$
  - Explicit symmetry breaking: $\chi^T = \left(M^2\;0\ldots0\right)$
  - Both spontaneous and explicit symmetry breaking
  - N-vector ϕ
- N (pseudo-)Nambu-Goldstone Bosons
- $N = 3$ is two-flavour Chiral Perturbation Theory

Results

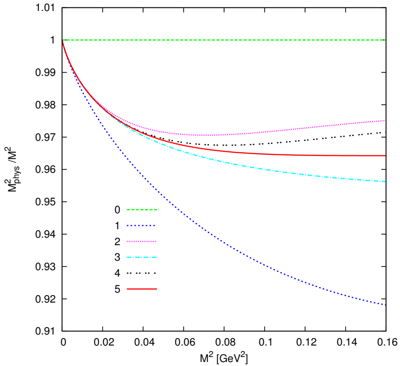
- $M^2_{\rm phys} = M^2(1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \ldots)$

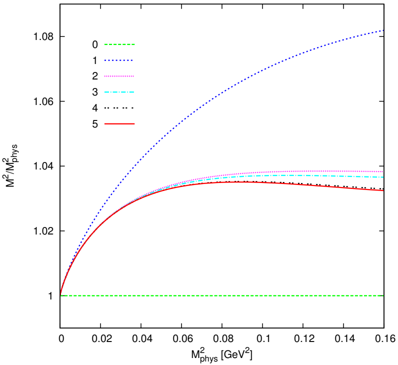
  $L_M = \frac{M^2}{16\pi^2 F^2} \log \frac{\mu^2}{M^2}$

i	a_i, $N = 3$	a_i for general N
1	$-\frac{1}{2}$	$1 - \frac{N}{2}$
2	$\frac{17}{8}$	$\frac{7}{4} - \frac{7N}{4} + \frac{5\ N^2}{8}$
3	$-\frac{103}{24}$	$\frac{37}{12} - \frac{113N}{24} + \frac{15\ N^2}{4} - N^3$
4	$\frac{24367}{1152}$	$\frac{839}{144} - \frac{1601\ N}{144} + \frac{695\ N^2}{48} - \frac{135\ N^3}{16} + \frac{231\ N^4}{128}$
5	$-\frac{8821}{144}$	$\frac{33661}{2400} - \frac{1151407\ N}{43200} + \frac{197587\ N^2}{4320} - \frac{12709\ N^3}{300} + \frac{6271\ N^4}{320} - \frac{7\ N^5}{2}$

- $F_{\rm phys}, \langle \bar{q}_i q_i \rangle$ as well done
- Anyone recognize any funny functions?
- Many more and larger tables in the papers

Numerical results (inspired from large N)





Left: $\frac{M^2_{\rm phys}}{M^2} = 1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \cdots$

Right: $\frac{M^2}{M^2_{\rm phys}} = 1 + d_1 L_{M_{\rm phys}} + d_2 L^2_{M_{\rm phys}} + d_3 L^3_{M_{\rm phys}} + \cdots$

F $= 90$ MeV, $\mu = 0.77$ GeV

Unfortunately: for other quantities less clear

Anomaly for $O(4)/O(3)$

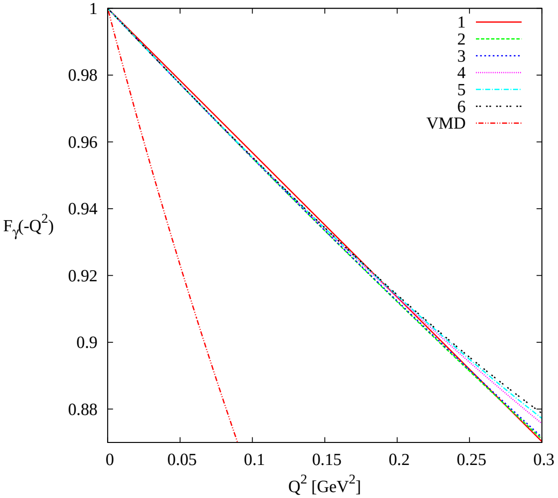
JB, Kampf, Lanz, arXiv:1201.2608

- $$\mathcal{L}_{WZW} = -\frac{N_c}{8\pi^2}\epsilon^{\mu\nu\rho\sigma}\Big\{\epsilon^{abc}\Big(\frac{1}{3}\Phi^0\partial_\mu\Phi^a\partial_\nu\Phi^b\partial_\rho\Phi^c - \partial_\mu\Phi^0\partial_\nu\Phi^a\partial_\rho\Phi^b\Phi^c\Big)v_\sigma^0 + (\partial_\mu\Phi^0\Phi^a - \Phi^0\partial_\mu\Phi^a)v_\nu^a\partial_\rho v_\sigma^0 + \frac{1}{2}\epsilon^{abc}\Phi^0\Phi^a v_\mu^b v_\nu^c\partial_\rho v_\sigma^0\Big\}.$$
- $A(\pi^0 \to \gamma(k_1)\gamma(k_2)) = \epsilon_{\mu\nu\alpha\beta}\,\varepsilon_1^{*\mu}(k_1)\varepsilon_2^{*\nu}(k_2)\,k_1^\alpha k_2^\beta\,F_{\pi\gamma\gamma}(k_1^2,k_2^2)$
- $F_{\pi\gamma\gamma}(k_1^2,k_2^2) = \dfrac{e^2}{4\pi^2 F_\pi}\hat{F}F_\gamma(k_1^2)F_\gamma(k_2^2)F_{\gamma\gamma}(k_1^2,k_2^2)$
- $\hat{F}$: on-shell photon; $F_\gamma(k^2)$: formfactor; $F_{\gamma\gamma}$ nonfactorizable part

Anomaly for $O(4)/O(3)$

- Done to six-loops
- $\hat{F} = 1 + 0 - 0.000372 + 0.000088 + 0.000036 + 0.000009 + 0.0000002 + \dots$
- Really good convergence
- $F_{\gamma\gamma}$ only starts at three-loop order (could have been two)
- $F_{\gamma\gamma}$ in the chiral limit only starts at four-loops.
- The leading logarithms thus predict this part to be fairly small.
- $F_{\gamma}(k^2)$: plot

Anomaly for $O(4)/O(3)$



Leading logs small, converge fast

Other results

- JB,Carloni, arXiv:1008.3499
  - massive case: $\pi\pi$, F_V and F_S to 4-loop order
  - large N for these cases also for massive $O(N)$.
  - done using bubble resummations or recursion eqation which can be solved analytically
- JB, Kampf, Lanz, arXiv:1201.2608
  - Mass, F_π, F_V to six loops
  - Anomaly: $\gamma^* 3\pi$ (five) and $\pi^0\gamma^*\gamma^*$ (six loops)
  - large N not relevant in this case
- JB, Kampf, Lanz, arXiv:1303.3125
  - $SU(N) \times SU(N)/SU(N)$
  - Mass, Decay constants, Form-factors
  - Meson-Meson, $\gamma\gamma \to \pi\pi$
  - No luck with guess for general N-dependence either

Other results

- Bissegger, Fuhrer, hep-ph/0612096 Dispersive methods, massless Π_S to five loops
- Kivel, Polyakov, Vladimirov, 0809.3236, 0904.3008, 1004.2197, 1012.4205
  - In the massless case tadpoles vanish
  - $\Longrightarrow$ number of external legs needed does not grow
  - All 4-meson vertices via Legendre polynomials
  - can do divergence of all one-loop diagrams analytically
  - algebraic (but quadratic) recursion relations
  - massless $\pi\pi$, F_V and F_S to arbitrarily high order
  - large N agrees with Coleman, Wess, Zumino
  - large N is not a good approximation

Conclusions Leading Logs

- Many quantities in massive $O(N)$ and $SU(N) \times SU(N)$ LL known to high loop order
- Large N in massive $O(N)$ model solved
- Had hoped: recognize the series also for other cases
- Limited essentially by CPU time and size of intermediate files
- Some studies on convergence etc.
- $\pi\pi$, F_V and F_S to four-loop order (F_V higher)
- The technique can be generalized to other models/theories
- One nucleon sector: first result mass: talk by A. Vladimirov yesterday

Conclusions

ChPT is a tool for many different areas of phenomenology.
I talked about a few of them :

1. Chiral Perturbation Theory
2. Determination of LECs in the continuum
3. Finite volume
4. Beyond QCD
5. Leading logarithms