Quark mass dependence of the nature of QCD phase transition at high temperature and density by a histogram method

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Collaboration with Norikazu Yamada (KEK)

S. Ejiri and N. Yamada, Phys. Rev. Lett. 110, 172001 (2013)

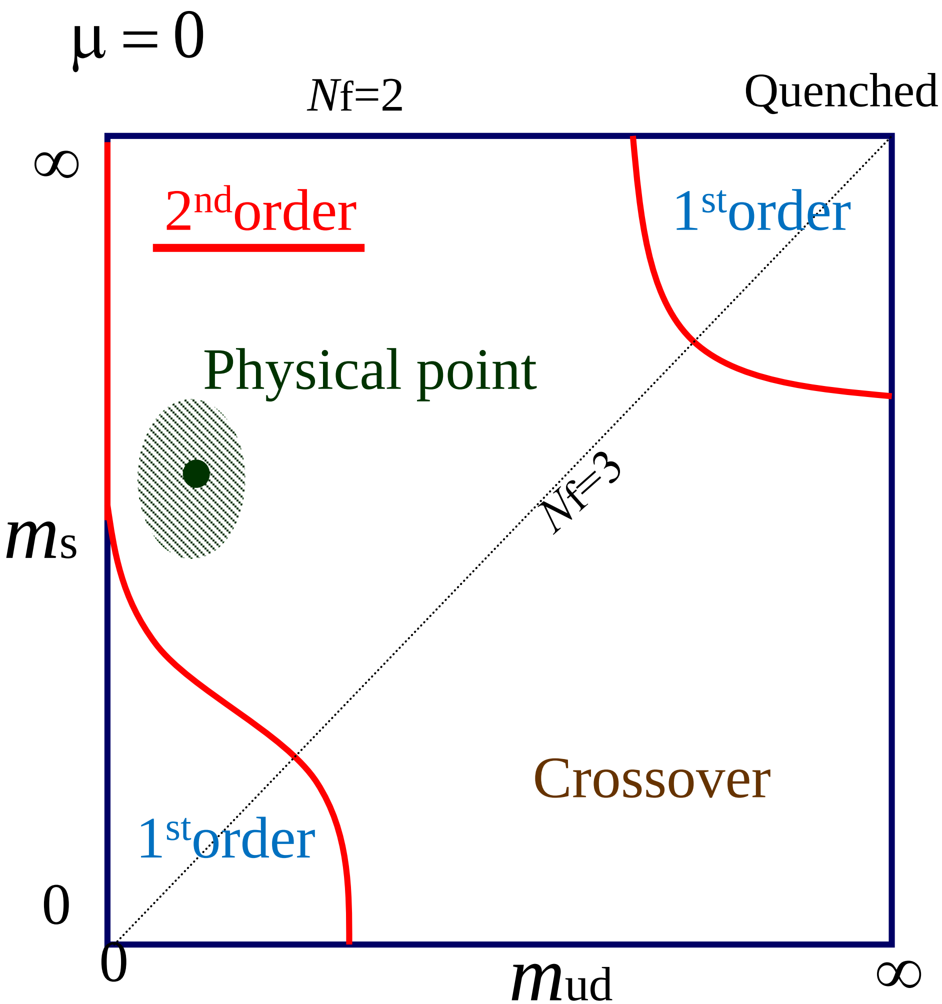
WHOT-QCD(H. Saito, S. Ejiri, S. Aoki, K. Kanaya, Y. Nakagawa, H. Ohno, K. Okuno, and T. Umeda), Phys. Rev. D89, 034507(2014)

S. Ejiri, Euro. Phys. J. A 49, 86 (2013) (mini-review)

Quark Confinement and the Hadron Spectrum XI

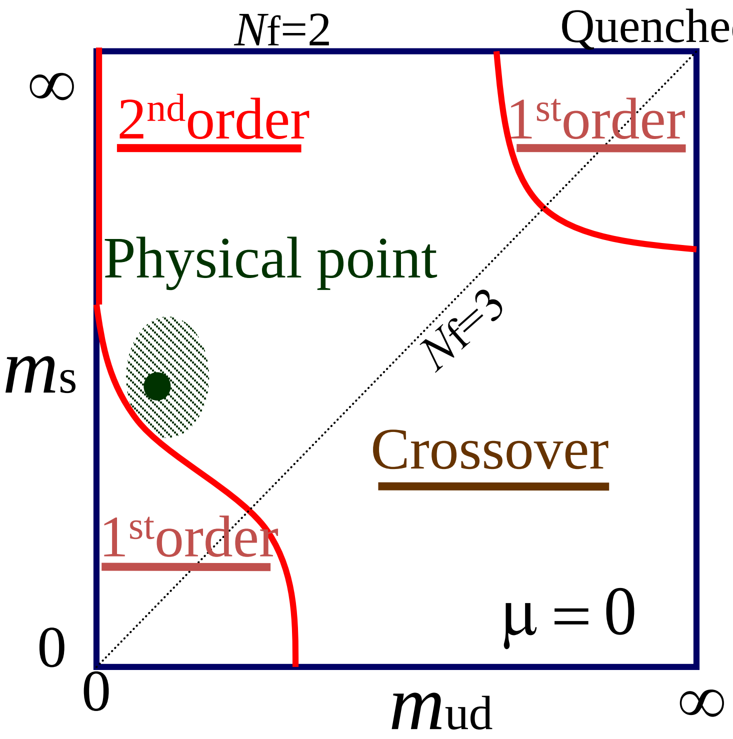
St Petersburg, Sep. 8 – 12, 2014

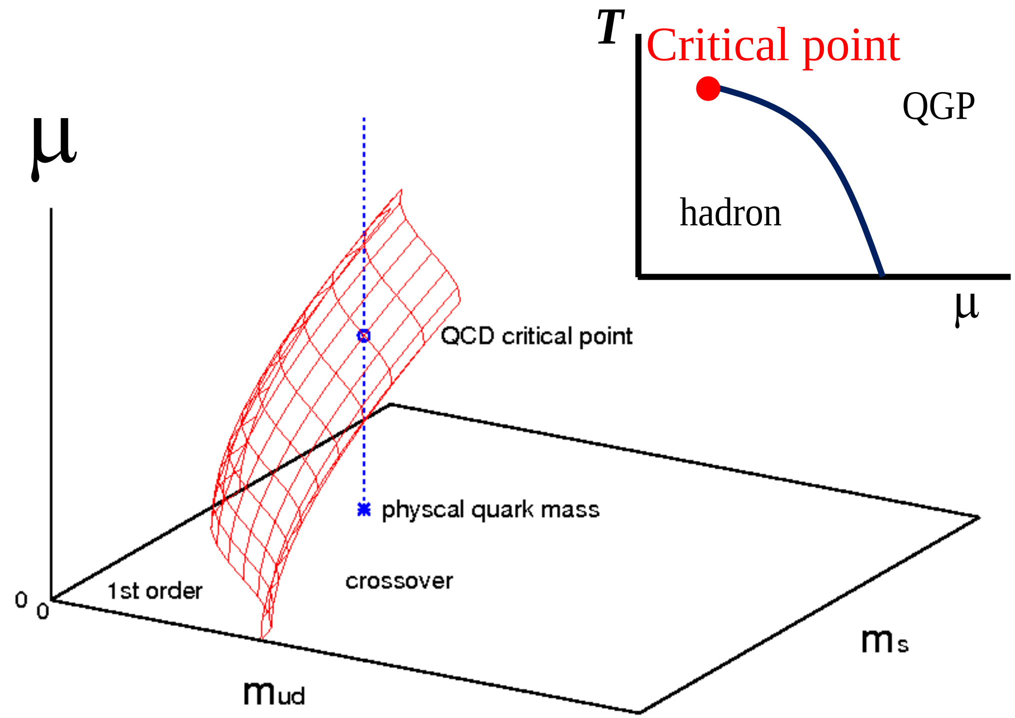
Quark Mass dependence of QCD phase trantion



- The determination of the boundary of 1st order region: important.
- 2-flavor QCD in the chiral limit
  - O(4) universality crass or First order transition
  - UA(1) symmetry restores?

Quark Mass dependence of QCD phase trantion

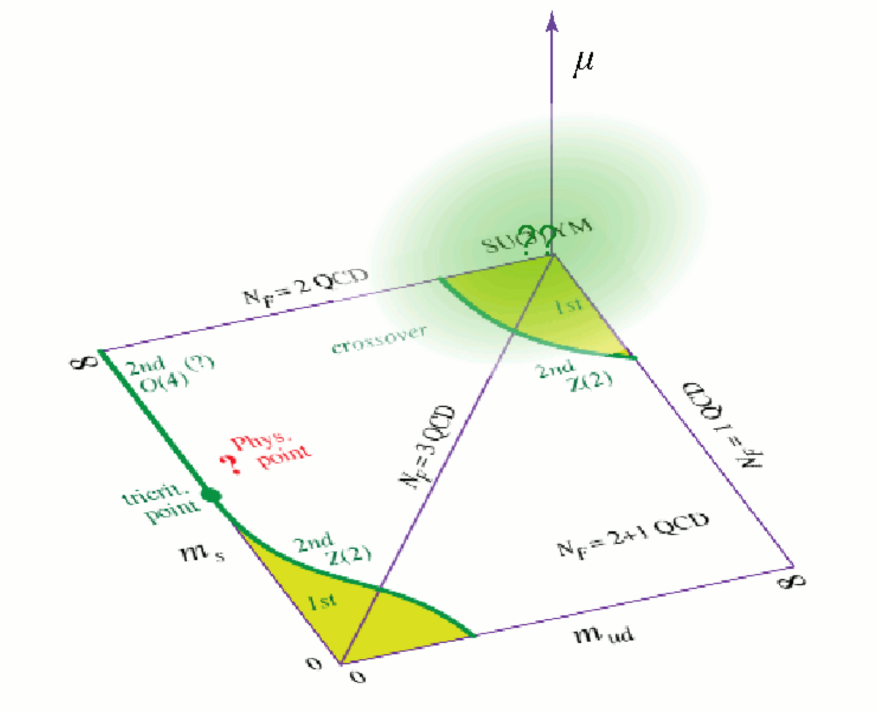




- On the line of physical mass, the crossover at low density ➡ 1st order transition at high density.
- However, the 1st order region is very small, and simulations with very small quark mass are required. ➡ Difficult to study.

Critical surface in the heavy quark region

(WHOT-QCD Collab., Phys.Rev.D84, 054502(2011); Phys.Rev.D89, 034507(2014))



- We determine the critical surface in the heavy quark region.
- Performing quenched simulations + Reweighting for dynamical ferminons.
- Standard Wilson quark action + plaquette gauge action, $S_g = -6N_{\mathrm{site}}\beta P$
- 24^3x4 lattice

Hopping parameter expansion $\boxed{\kappa \sim 1/(\text{quark mass})}$

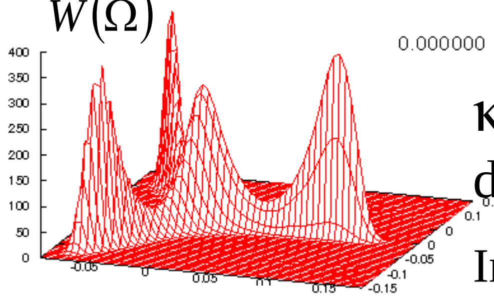
$$N_{\mathrm{f}} \ln\left(\frac{\det M(\kappa,\mu)}{\det M(0,0)}\right) = N_{\mathrm{f}}\left(288 N_{\mathrm{site}}\kappa^4 P + 12\cdot 2^{N_t} N_s^3 \kappa^{N_t}\left(\cosh(\mu/T)\Omega_R + \underline{\underline{i\sinh(\mu/T)\Omega_I}}\right) + \cdots\right)$$

phase ~ $O(\kappa^{Nt})$

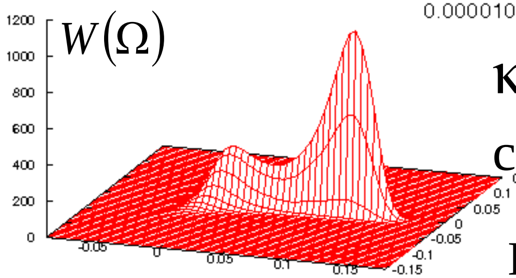
P: plaquette, Ω=Ω_R+iΩ_I : Polyakov loop $\det M(0,0) = 1$

Critical surface in the heavy quark region of (2+1)-flavor QCD

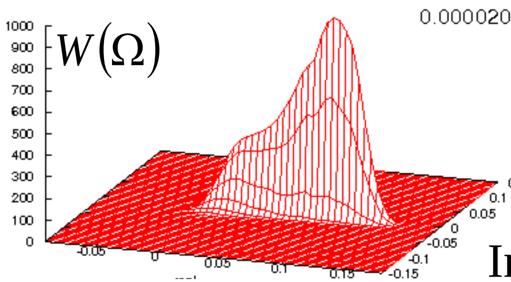
WHOT-QCD Collab., Phys.Rev.D89, 034507(2014)



$\kappa=0$: Z(3) symmetric distribution



κ small: two phases coexists (1st order tra.)



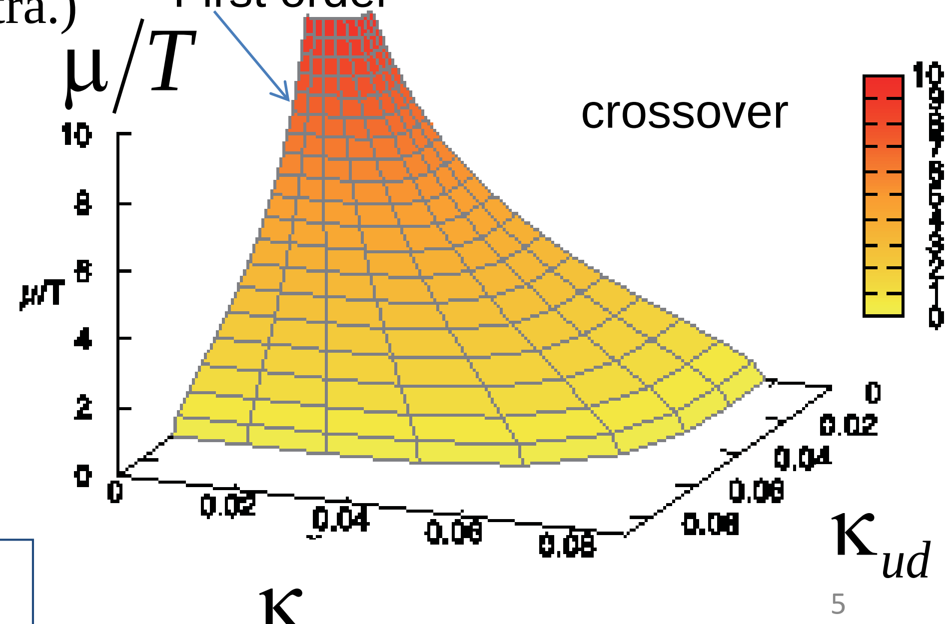
Near the critical κ: becomes one peak

$\kappa\sim 1/(\text{quark mass})$

Critical surface at finite density

$$2\kappa_{\mathrm{ud\,cp}}^{N_t}\cosh(\mu_{\mathrm{ud}}/T)+\kappa_{\mathrm{s\,cp}}^{N_t}\cosh(\mu_{\mathrm{s}}/T)\approx 4\times 10^{-5}$$

$$(N_t=4)$$



Phase transitions in many-flavor QCD

We investigate the critical surface

in QCD with 2-light flavors + N_f-massive flavors.

- Good testing ground for (2+1)-flavor QCD
- Electro-weak baryogenesis - Technicolor model

- Plan of this talk
  - Why (2+many)-flavor QCD?
  - Histogram method
  - N_f-dependence of the critical heavy quark mass.
  - μ-dependence of the critical curve.
  - Light quark mass-dependence of the critical curve
    - The chiral limit of 2-flavor QCD: 2nd order or 1st order?

Finite T and μ phase transition in (2+many)-flavor QCD

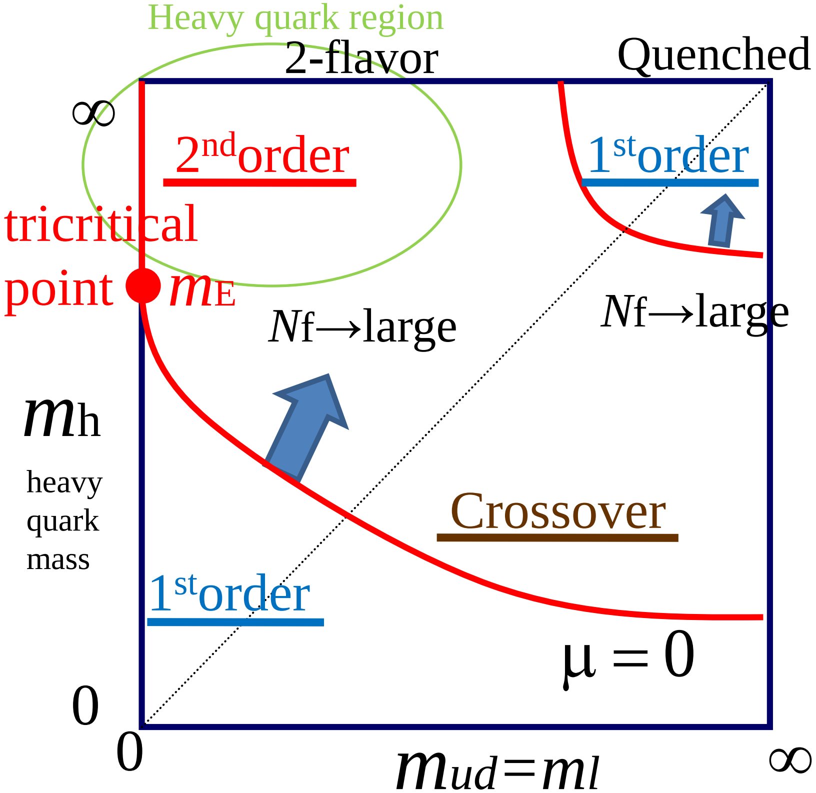
(Cf. Kikukawa, Kohda and Yasuda, Phys.Rev.D77, 015014(2008))

- Technicolor model constructed by many-flavor QCD
- Chiral phase transition of QCD

  → Electroweak phase transition at finite temperature
- Nambu-Goldstone bosons
  - 3 bosons are absorbed into gauge bosons. (3 massless bosons)
  - The other bosons have not observed yet. (The other bosons: heavy)
  - 2 techni-felmions are massless, and the others are heavy.
- Electro-weak baryogenesis
  - Strong first order transition: required. (SM: Not strong 1st order.)
  - From the analogy of 2+1-flavor QCD, 1st order at small mass; 2nd order or crossover at large mass.
- <u>It is important to determine the endpoint of the first order region in (2+many)-flavor QCD.</u>

Nature of phase transition of 2+N_f-flavor QCD

S. E. & N. Yamada, Phys. Rev. Lett. 110, 172001 (2013)

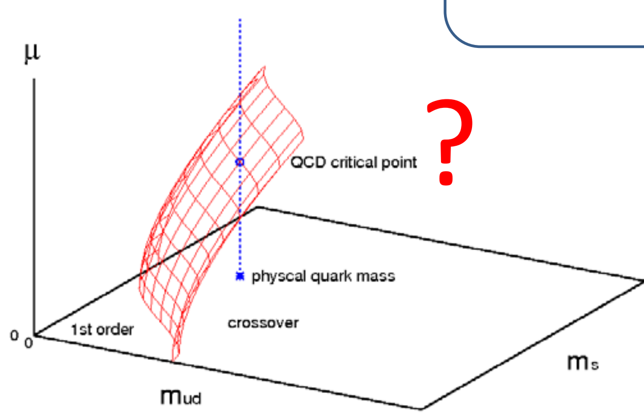


- Assumption: N_f-flavors are heavy.
  - Hopping parameter κ expansion
- Parameter: $\underline{N_f \kappa^{N_t}}$ ➡ $1/m_{h,ct} \sim \kappa_{ct} \propto 1/N_f^{1/N_t}$
- As increasing N_f, critical mass becomes larger. ➡ Easy to investigate.
- Tricritical scaling: the same as (2+1)-flavor QCD

Tricritical point m_E:	$m_{ud}^c \sim (m_E - m_h)^{5/2}$ $m_{ud}^c \sim \mu^5$

Good test ground

At finite density?



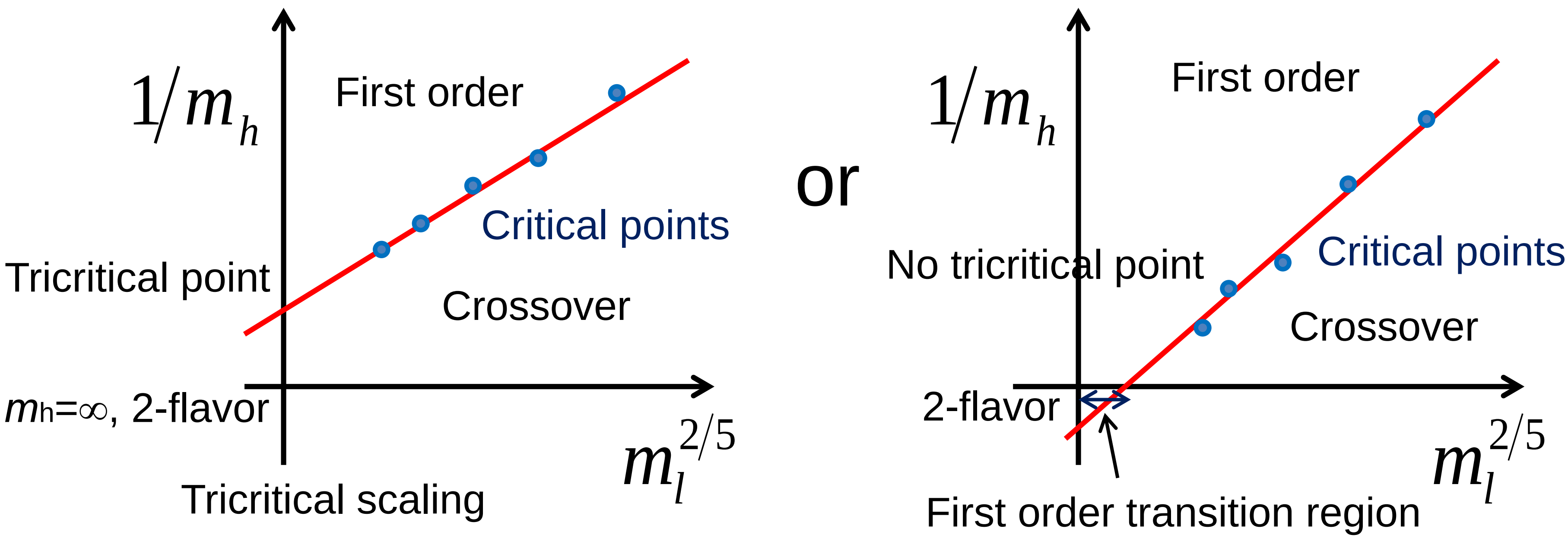
?

Nature of 2-flavor QCD in the chiral limit

2nd order or 1st order?

Light quark mass (m_l) dependence of the critical line

- Trictitical scaling behavior?
- Is there a first order transition region in 2-flavor QCD?



Similar study in QCD with an imaginary chemical potential:
Bonati, D' Elia, de Forcrand, Philipsen, Sanfilippo, arXiv:1311.0473; 1408.5086

Histogram method

Distribution function & the effective potential

(S. Ejiri, Phys. Rev. D 77 (2008) 014508)

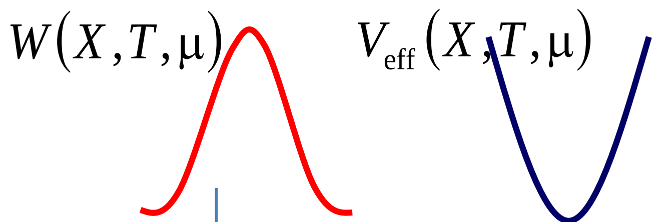
$$W(X;m,T,\mu) \equiv \int DU \delta\left(X - \hat{X}\right) \prod_{f=1}^{N_f} \det M\left(m_f, \mu_f\right) e^{-S_g} \qquad \text{(Histogram)}$$

X: order parameters, total quark number, average plaquette, etc.

$$V_{\text{eff}}(X) = -\ln W(X)$$

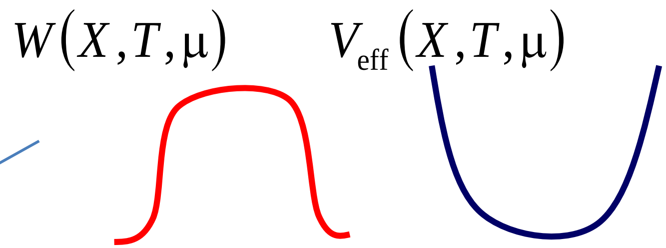
Crossover

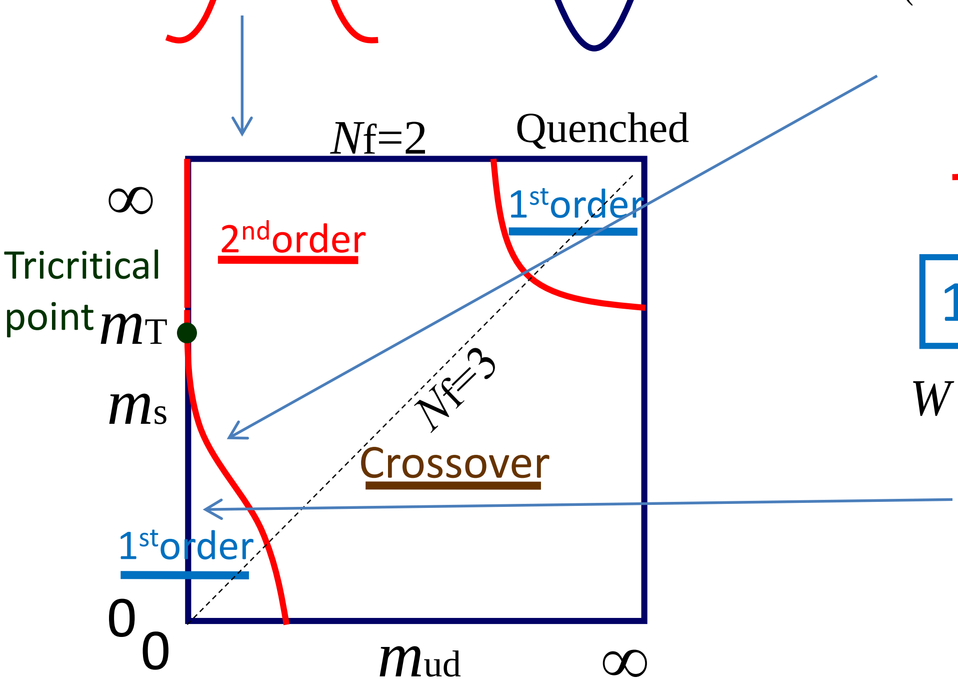
$W(X)$: Gaussian function
$V(X)$: Quadratic function



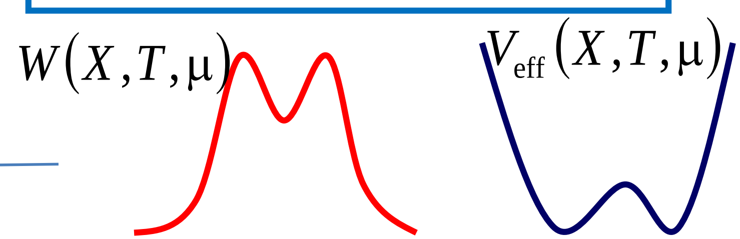
Critical point

W(X): Flat
V(X): Curvature: Zero





1st order phase transition



W(X): Two phases coexist
V(X): Double well potential

Reweighting method for plaquette distribution function

$$W(P,\beta,m,\mu)\equiv\int DU\,\delta\left(\hat{P}-P\right)\prod_{f=1}^{N_f}\det M\left(m_f,\mu_f\right)e^{6N_{\mathrm{site}}\beta\hat{P}}$$

$$S_g=-6N_{\mathrm{site}}\beta\hat{P}$$

$$\left(\beta=6/g^2\right)$$

plaquette P (1x1 Wilson loop for the standard action)

$$R\left(P,\beta,\beta_0 m,m_0,\mu\right)\equiv W\left(P,\beta,m,\mu\right)/W\left(P,\beta_0,m_0,0\right)$$

(Reweight factor)

$$R(P)=\frac{\left\langle \delta\left(\hat{P}-P\right)e^{6N_{\mathrm{site}}(\beta-\beta_0)\hat{P}}\prod_f\frac{\det M\left(m_f,\mu_f\right)}{\det M\left(m_0,0\right)}\right\rangle_{(\beta_0,\mu=0)}}{\left\langle\delta\left(\hat{P}-P\right)\right\rangle_{(\beta_0,\mu=0)}}\equiv\left\langle e^{6N_{\mathrm{site}}(\beta-\beta_0)\hat{P}}\prod_f\frac{\det M\left(m_f,\mu_f\right)}{\det M\left(m_0,0\right)}\right\rangle_{P:\mathrm{fixed}}$$

Effective potential:

$$V_{\mathrm{eff}}\left(P,\beta,m,\mu\right)=-\ln\left[W\left(P,\beta,m,\mu\right)\right]=V_{\mathrm{eff}}\left(P,\beta_0,m_0,0\right)-\ln R\left(P,\beta,\beta_0 m,m_0,\mu\right)$$

$$\ln R(P)=6N_{\mathrm{site}}\left(\beta-\beta_0\right)P+\ln\left\langle\prod_f\frac{\det M\left(m_f,\mu_f\right)}{\det M\left(m_0,0\right)}\right\rangle_{P:\mathrm{fixed}}$$

First order transition point: two phases coexist

Plaquette distribution function

- Performing simulations of 2-flavor QCD,
- Dynamical effect of N_f-flavors are included by the reweighting.
- We assume N_f-flavors are heavy.
- Hopping parameter (κ) expansion (Wilson quark)

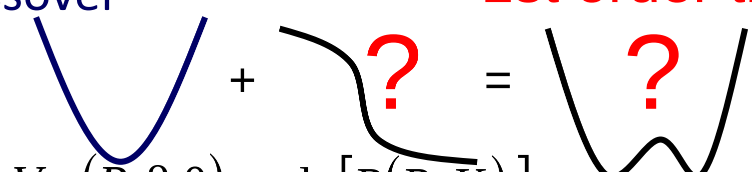
$$N_\mathrm{f} \ln\left(\frac{\det M(\kappa,\mu)}{\det M(0,0)}\right) = N_\mathrm{f}\left(288 N_\mathrm{site} \kappa^4 P + 12 \cdot 2^{N_t} N_s^3 \kappa^{N_t} \left(\cosh(\mu/T)\Omega_R + i\sinh(\mu/T)\Omega_I\right) + \cdots\right)$$

- Effective potential

2−flavor
crossover

2+Nf-flavor
1st order transition

$$V_\mathrm{eff}(P,\beta,\kappa) = -\ln\left[R(P,\kappa)W(P,\beta,0)\right] =$$



$V_\mathrm{eff}(P,\beta,0)$ $-\ln[R(P,K)]$

Negative curvature

$$\ln R(P) = \ln\left\langle e^{6N_\mathrm{site}(\beta-\beta_0)\hat{P}} \prod_f \frac{\det M(\kappa_f,\mu_f)}{\det M(\kappa_0,0)}\right\rangle_{P:\mathrm{fixed}} \approx \ln\left\langle \exp\left(6hN_s^3\hat{\Omega}_R\right)\right\rangle_{P:\mathrm{fixed}}$$ +(linear term of P)

(degenerate mass case at μ=0)

Curvature of the effective potential

$$V_{\rm eff}(P,\beta,h,\mu) = V_{\rm eff}(P,\beta_0,0,0) - \ln \bar{R}(P,h,\mu) + \text{ (linear term of } P)$$

$$\bar{R}(P) = \left\langle \exp\left(6N_s^3 h \Omega_R\right)\right\rangle_{P:\text{fixed}} \qquad \text{(for the case of } \mu=0)$$

Wilson quark

$$h = 2N_{\rm f}(2\kappa_{\rm h})^{N_t}$$

Staggered quark

$$h = N_{\rm f} \Big/ \left(4(2m_{\rm h})^{N_t}\right)$$

- Linear term of P is irrelevant to the curvature
- β-dependence is only in the linear term.
- The curvature is independent of β.

χ_P: plaquette susceptibility

$$\frac{d^2 V_{\rm eff}(0)}{dP^2} \approx \frac{6N_{\rm site}}{\chi_P}$$

$$\frac{d^2 V_{\rm eff}}{dP^2}(P,h,\mu) = \underbrace{\frac{d^2 V_{\rm eff}}{dP^2}(P,0,0)}_{\text{2-flavor}} - \frac{d^2 \ln \bar{R}}{dP^2}(P,h,\mu)$$

- If there exists the negative curvature region,

⇒ First order transition (double-well potential)

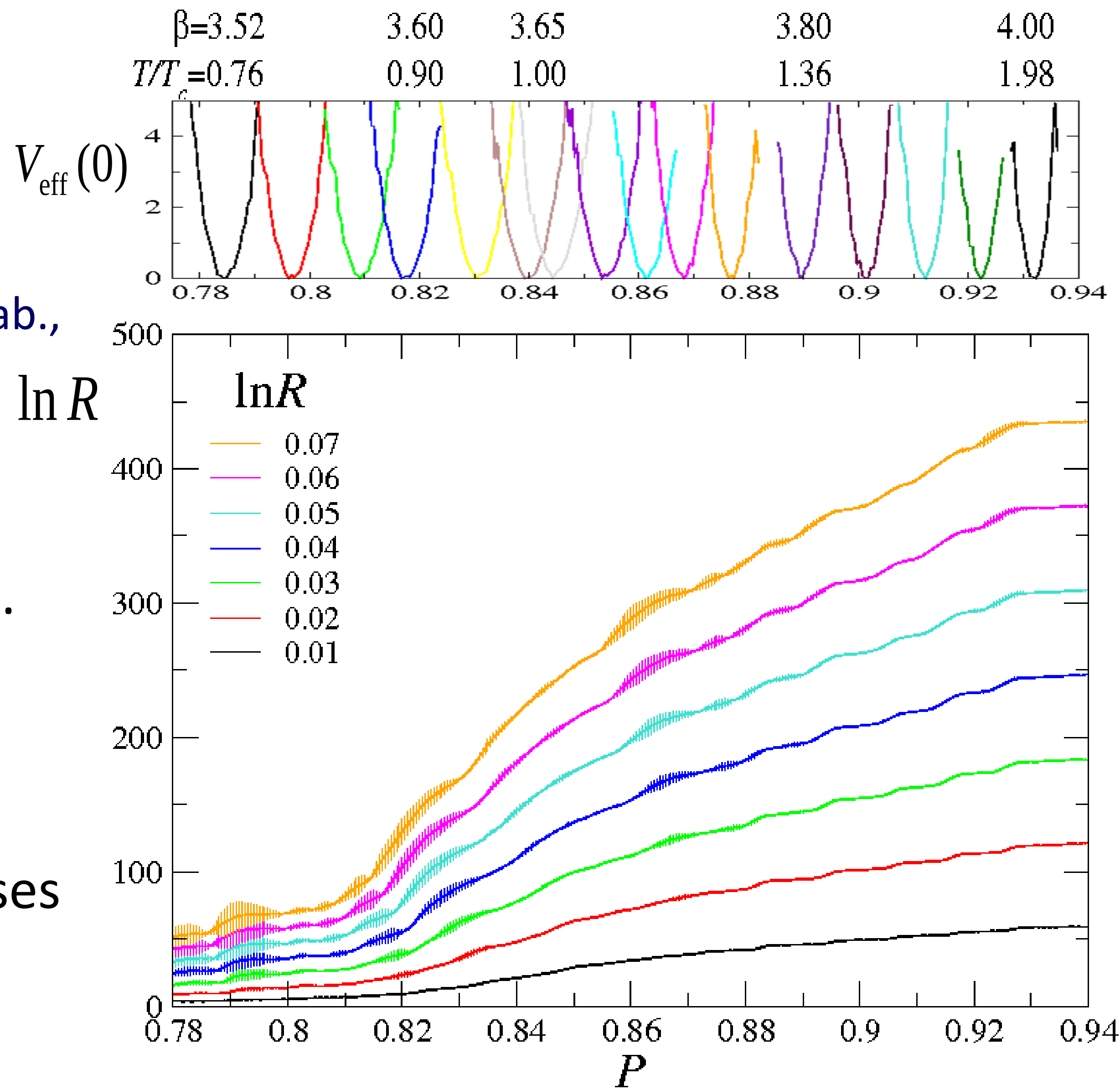
Effective potential at $h\neq 0$

$$V_{\rm eff}(P,\beta,h)=V_{\rm eff}(P,\beta,0)-\ln R(P,h)$$

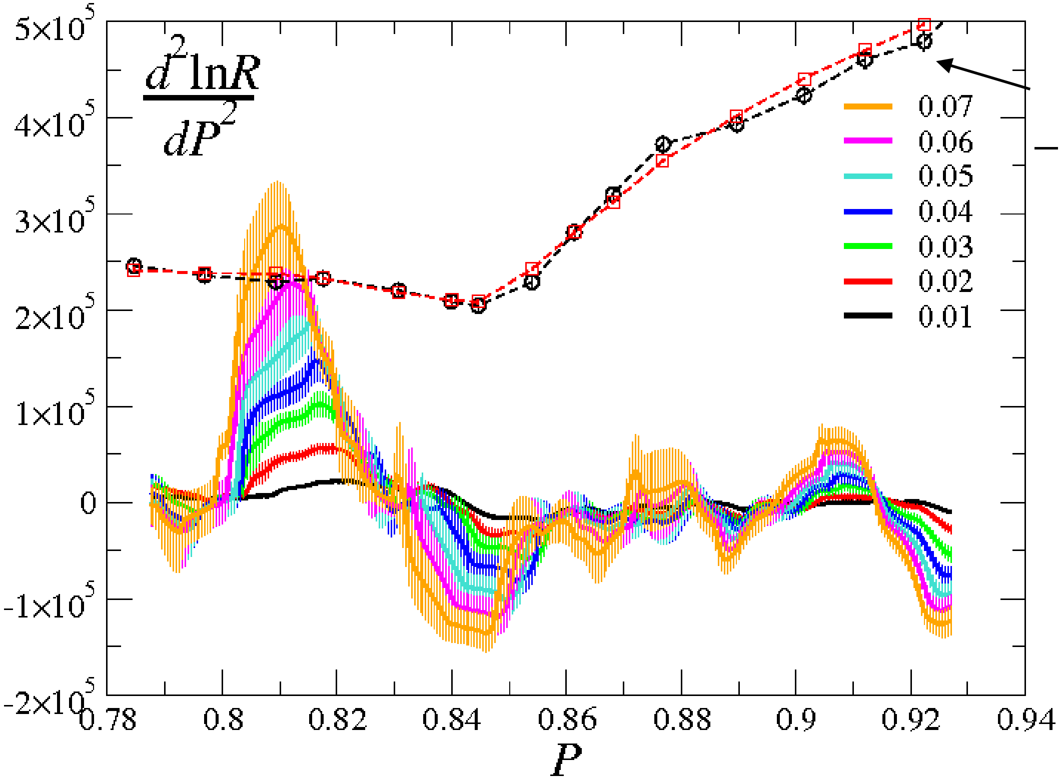
Nf=2 p4-staggared,
$m_\pi/m_\rho \approx 0.7$

data: Beilefeld-Swansea Collab., PRD71,054508(2005)

- det$M$: hopping parameter expansion.
- lnR increases as increasing h.
- The curvature increases with h.



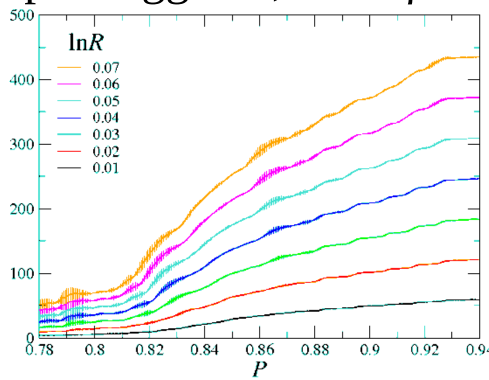
Curvature of the effective potential

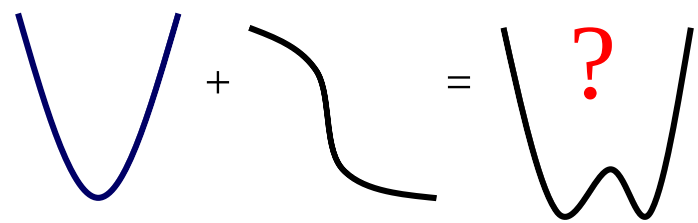


$$-\frac{d^2 \ln W}{dP^2}$$

at h=0

Nf=2 p4-staggered, $m\pi/m\rho\approx0.7$





First order transition: $$\frac{d^2V_{\text{eff}}(P,\beta,h)}{dP^2} = \frac{d^2V_{\text{eff}}(P,\beta,0)}{dP^2} - \frac{d^2 \ln \overline{R}(P,h)}{dP^2} < 0$$

$$h = 2N_{\text{f}}(2\kappa_{\text{h}})^{N_t}$$

(Wilson quarks)

- First order transition for h > 0.6

Critical value: $h_c = 0.0614(69)$

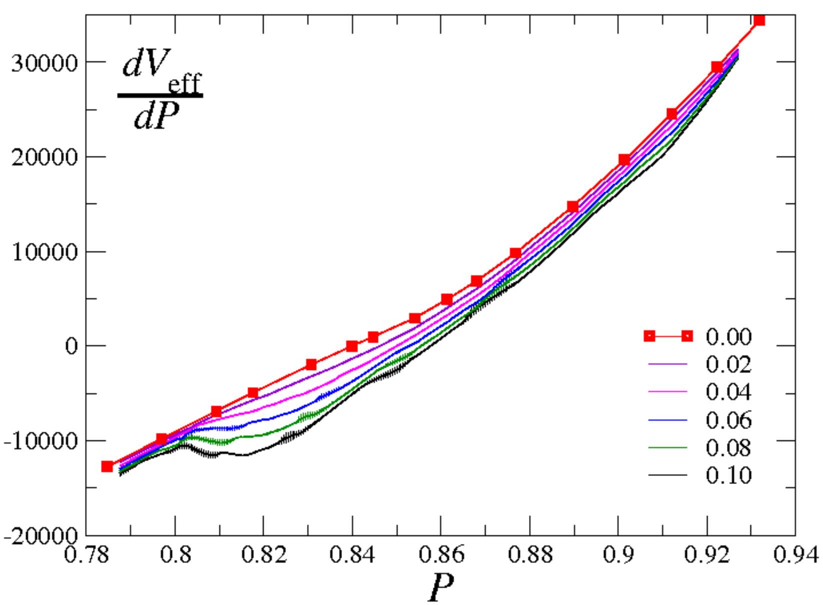
Slope of the effective potential

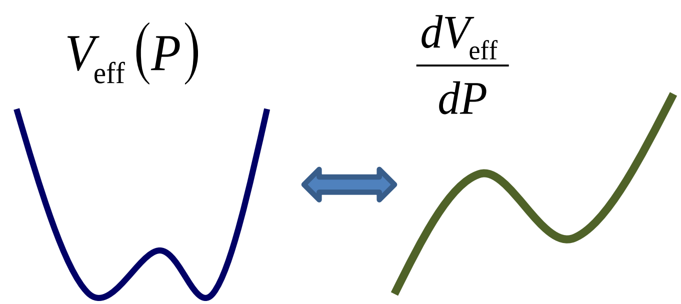
$$V_{\text{eff}}(P,\beta,h,\mu)=V_{\text{eff}}(P,\beta_0,0,0)-\ln\overline{R}(P,h,\mu)+\text{ (linear term of } P)$$

$$\Rightarrow \frac{dV_{\text{eff}}}{dP}(P,h,\mu)=\frac{dV_{\text{eff}}}{dP}(P,0,0)-\frac{d\ln\overline{R}}{dP}(P,h,\mu)+\text{ (constant term)}$$

- The shape of dV_{eff}/dP is independent of β.
- If dV_{eff}/dP is an S-shaped function,

$\Rightarrow$ First order phase transition (double-well potential).





S-shaped function at large h

$$h=2N_f(2\kappa_h)^{N_t} \quad \text{for Wilson quark}$$

N_f–dependence of the critical mass

$$h_c = 0.0614(69)$$ (p4-staggared, $m_\pi/m_\rho \approx 0.7$)

- Critical mass increases as N_f increases.

$$h = 2N_f(2\kappa_h)^{N_t} \Rightarrow \kappa_h^c = \frac{1}{2}\left(\frac{h_c}{2N_f}\right)^{1/N_t}$$

  – When N_f is large, κ is small. Then, the hopping parameter (κ) expansion is good.

  – On the hand, when N_f is small, the κ-expansion is bad.

- In a quenched simulation with N_t=4, the first and second terms becomes comparable around κ=0.18.

- For N_f=10, N_t=4, $h_c = 0.0614(69) \Rightarrow \kappa_h^c \approx 0.118$

  – It may be applicable for N_f~10.

The effective potential at finite μ

Reweighting factor

$$\ln R(P) = \ln \left\langle \underbrace{\left(\frac{\det M(m,\mu)}{\det M(m,0)} \right)^2}_{\text{light quarks}} \underbrace{\left(\frac{\det M(h,\mu_h)}{\det M(0,0)} \right)^{N_f}}_{\text{heavy quarks}} \right\rangle_{P:\text{fixed}}$$

Evaluation of $\ln R$ at finite μ

- Quark determinant: Taylor expansion up to $O(\mu^6)$

data: Beilefeld-Swansea Collab.,PRD71,054508(2005)

$$N_f \ln\det M(\mu) = N_f \sum_{n=0}^{N} \left[\frac{1}{n!} \left(\frac{\mu}{T} \right)^n \frac{d^n \ln\det M}{d(\mu/T)^n} \right]$$

θ: complex phase

$$\theta \equiv \mathrm{Im} \ln\det M$$

- Cumulant expansion method (SE,PRD77,014508(2008), WHOT-QCD,PRD82,014508(2010))

$$\left\langle e^{i\theta} \right\rangle = \exp\left[\cancel{i\langle\theta\rangle_C}_{\to 0} - \frac{1}{2}\langle\theta^2\rangle_C - \cancel{\frac{i}{3!}\langle\theta^3\rangle_C}_{\to 0} + \frac{1}{4!}\langle\theta^4\rangle_C + \cancel{\cdots} \right]$$

cumulants

$$\langle\theta\rangle_C = \langle\theta\rangle, \quad \langle\theta^2\rangle_C = \langle\theta^2\rangle - \langle\theta\rangle^2, \quad \langle\theta^3\rangle_C = \langle\theta^3\rangle - 3\langle\theta^2\rangle\langle\theta\rangle + 2\langle\theta\rangle^3, \quad \langle\theta^4\rangle_C = \cdots$$

– Odd terms vanish from a symmetry under $\mu \leftrightarrow -\mu$ $(\theta \leftrightarrow -\theta)$

Source of the complex phase

– If the distribution of θ is Gaussian, $\langle\theta^2\rangle_C$ term dominates.

– Assuming the Gaussian distribution, we approximate

$$\left\langle e^{i\theta} \right\rangle \approx \exp\left[-\frac{1}{2}\langle\theta^2\rangle_C \right]$$

Curvature of the effective potential at finite μ

$$\frac{d^2 V_{\rm eff}}{dP^2}(P,h,\mu)=\frac{d^2 V_{\rm eff}}{dP^2}(P,0,0)-\frac{d^2 \ln R}{dP^2}(P,h,\mu)$$

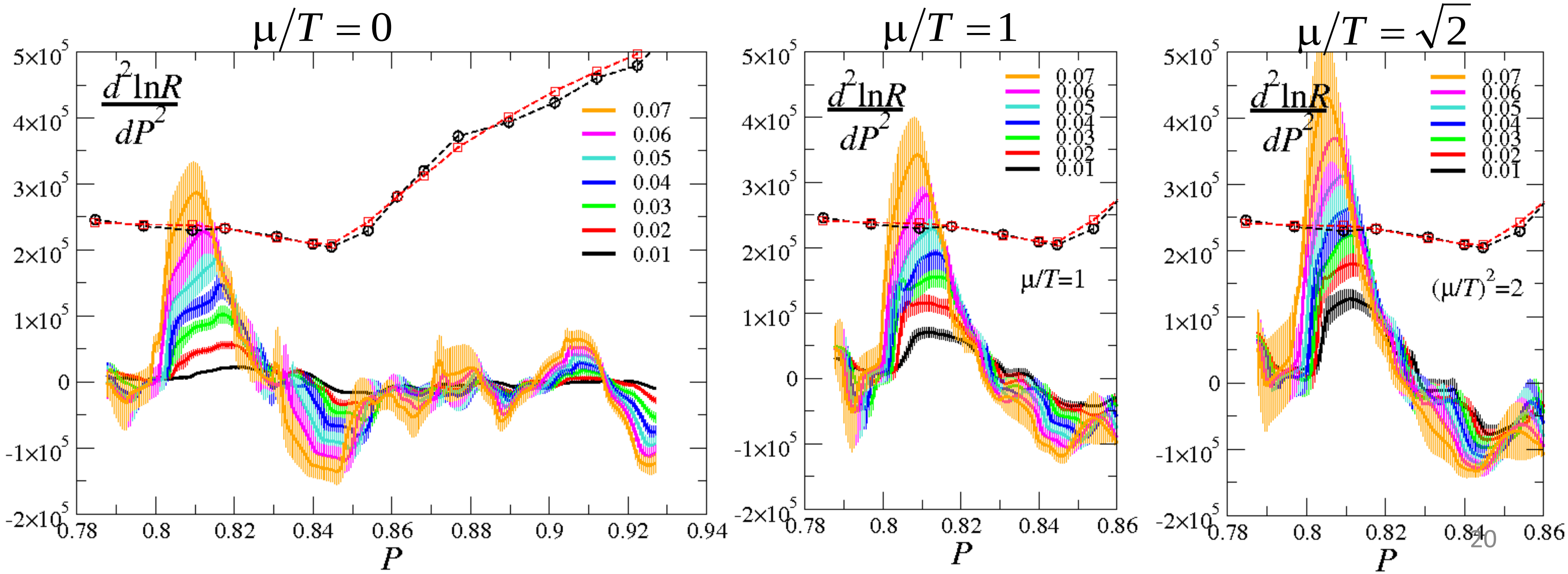
$$h = 2N_{\rm f}\,(2\kappa_{\rm h})^{N_t}$$

for Wilson quarks

- Reweighting factor

$$\ln R(P)=\ln\left\langle \underbrace{\left(\frac{\det M(m,\mu)}{\det M(m,0)}\right)^2}\underbrace{\left(\frac{\det M(h,\mu_h)}{\det M(0,0)}\right)^{N_{\rm f}}} \right\rangle_{P:{\rm fixed}}$$

$$\mu_h/T=0$$



Critical line at finite density

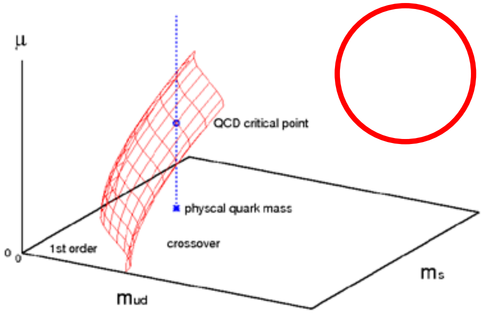
$$h = 2N_f \left(2\kappa_h\right)^{N_t}$$

for Wilson quarks

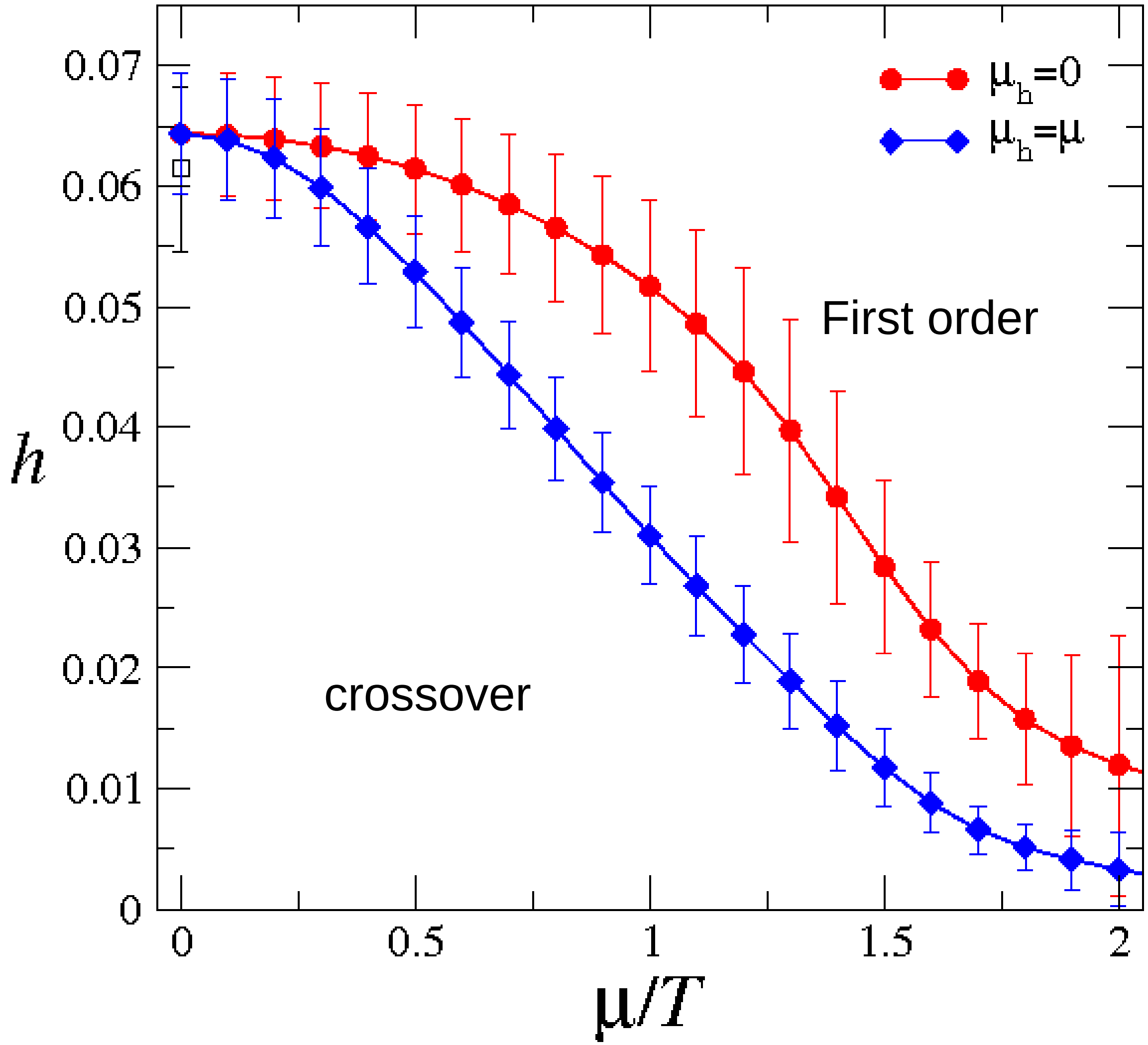
$$h = N_f \Big/ \left(4\left(2m_h\right)^{N_t}\right)$$

for staggered quarks

- Calculations of detM: Taylor expansion up to O(μ^6)
- Distribution function of the complex phase of detM: approximated by a Gaussian function



S. E. & N. Yamada, Phys. Rev. Lett. 110, 172001 (2013)



Phase structure of (2+many)-flavor QCD using Wilson quark action

- Light quark mass dependence
- (Chemical potential dependence: in progress)

Simulations

Iwasaki gauge action + N_f=2 clover -Wilson fermion action,

κ=0.145, 0.475, 0.150, 0.1505,

m_π/m_ρ = 0.6647, 0.5761, 0.4677, 0.4575,

16^3x4 lattice.

Dynamical heavy quark effect is added by the reweighting method.

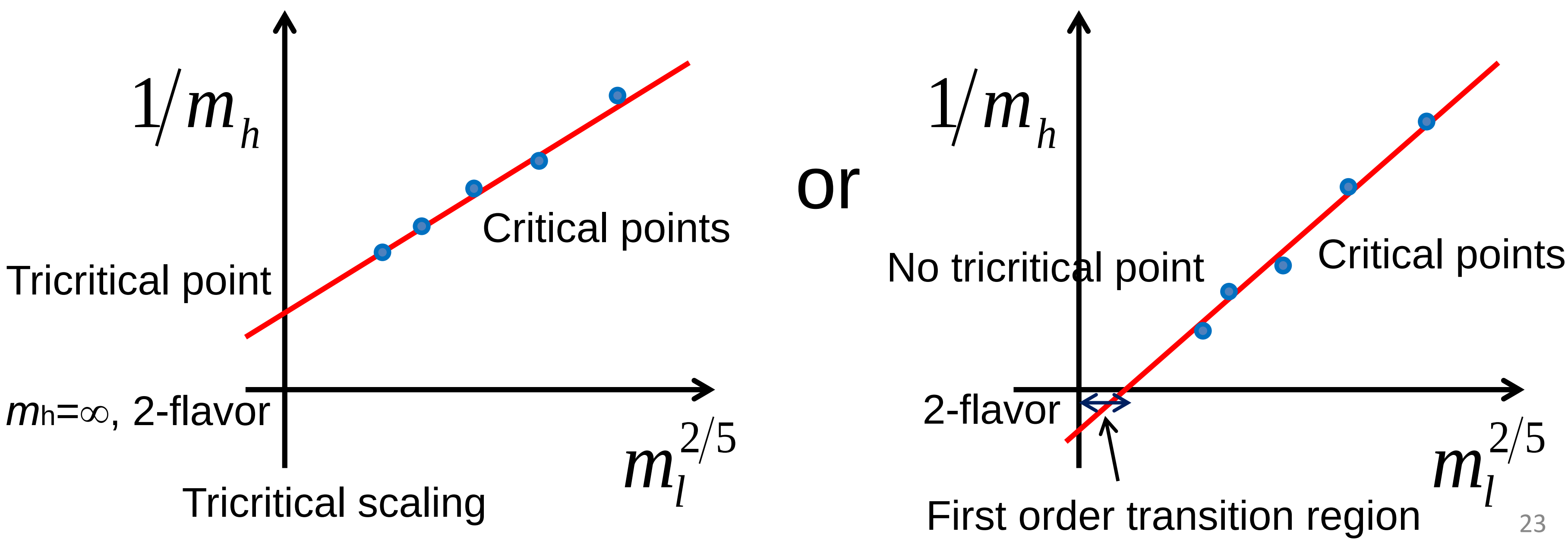
det$M$: Hopping parameter expansion

Phase structure of (2+many)-flavor QCD using Wilson quark action

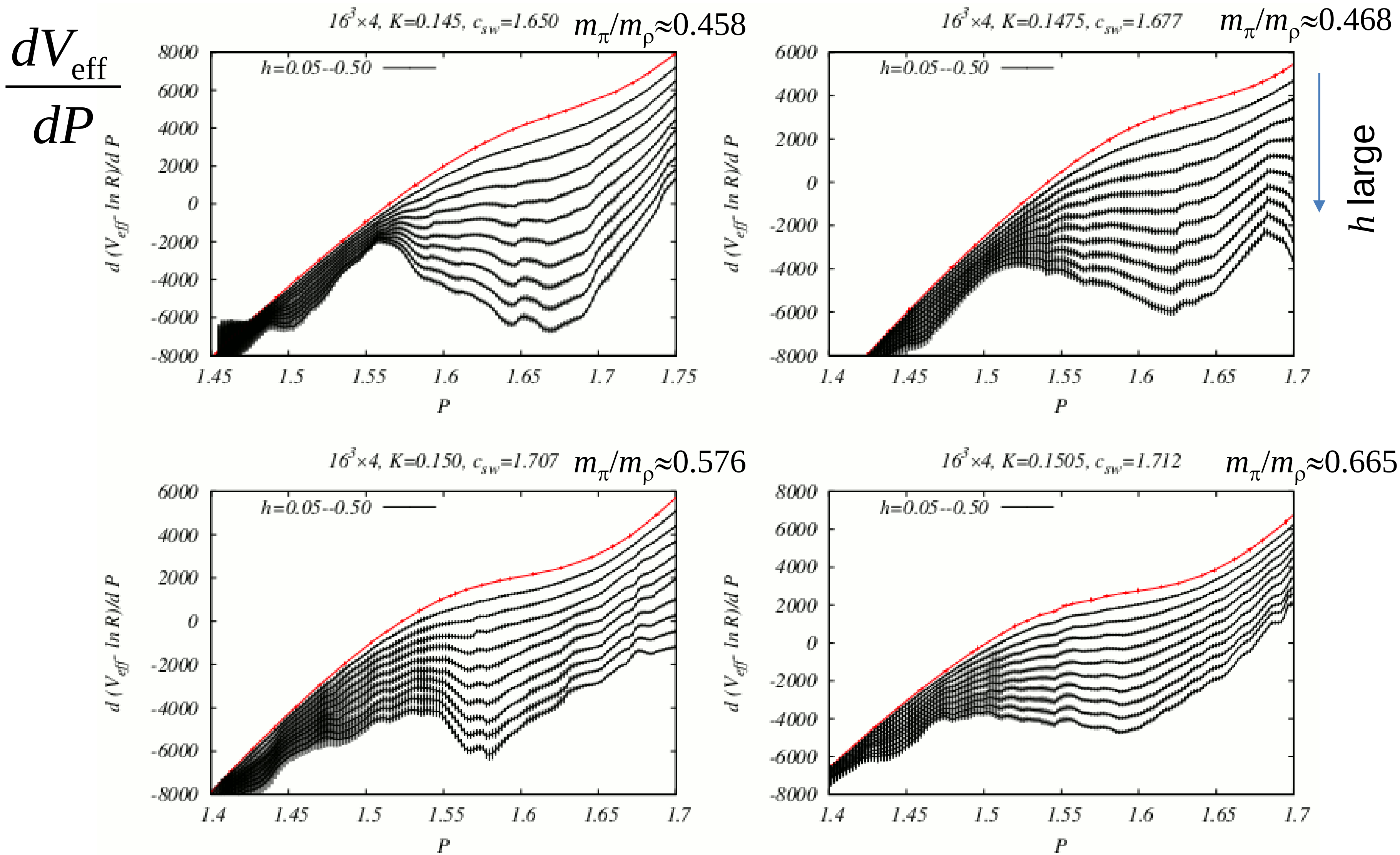
2-flavor QCD simulations + reweighting

Light quark mass dependence of the critical line

- Trictitical scaling behavior?
- Is there a first order transition region in 2-flavor QCD?



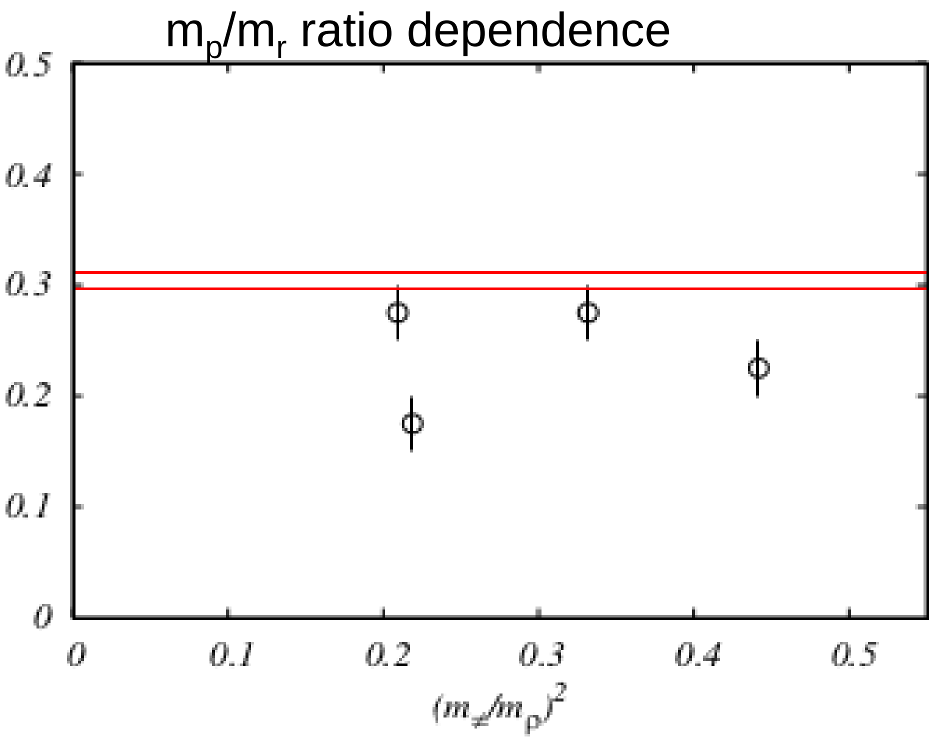
Light quark mass dependence (preliminary)

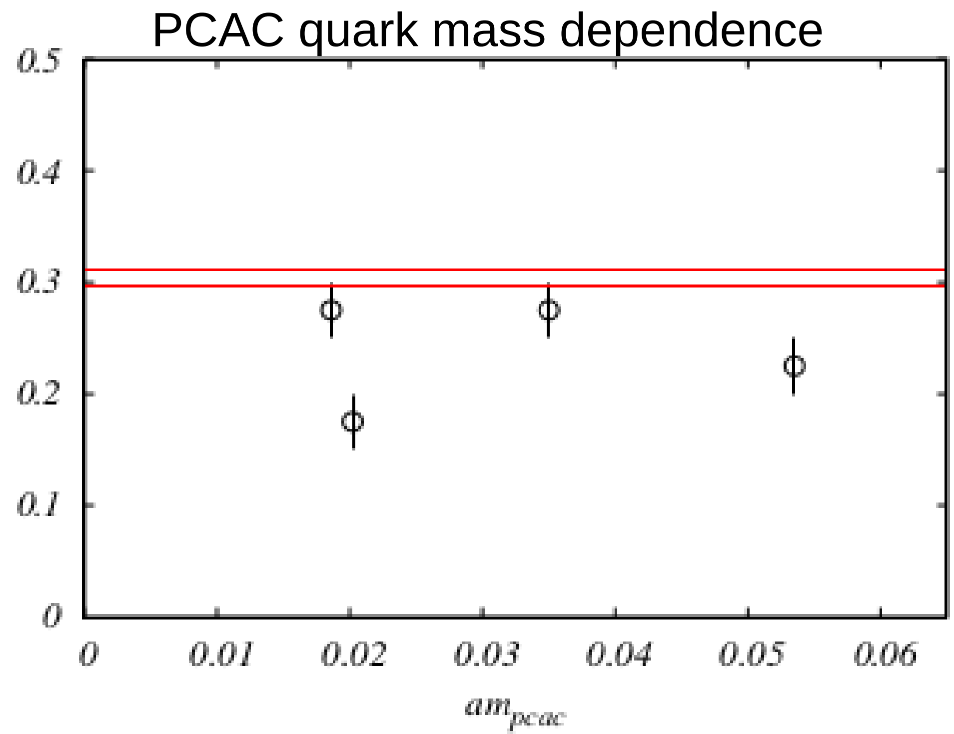


- The derivative of V_{eff} becomes an S-shaped function at large h.
- Critical point: light quark mass dependence is small in this region.

Light quark mass dependence (preliminary)

$$h = 2N_f \left(2\kappa_h\right)^{N_t} \text{ for Wilson quarks}$$





- Critical point: light quark mass dependence is small in the region we investigated.
- The red line is the critical point in the $m_l = \infty$ limit.
- The first order transition in the massless 2-flavor QCD is not suggested.

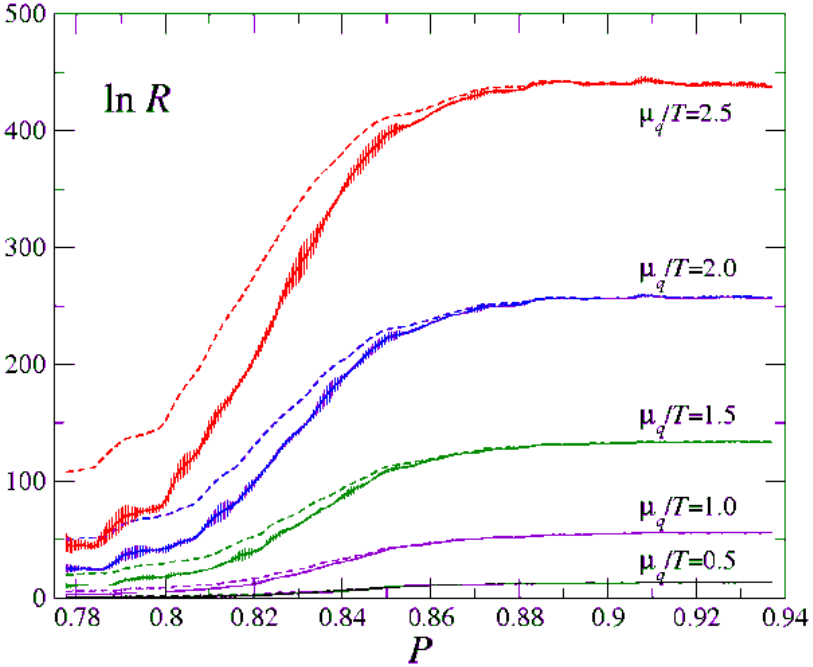
Summary

- We investigated the phase structure of (2+Nf)-flavor QCD.
  - This model is interesting for the feasibility study of the electroweak baryogenesis in the technicolor scenario.
  - An appearance of a first order phase transition at finite temperature is required for the baryogenesis.
- Applying the reweighting method, we determine the critical mass of heavy flavors terminating the first order region.
  - The critical mass becomes larger with N_f.
  - The first order region becomes wider as increasing μ. (p4-staggered)
  - The light quark mass dependence the critical heavy quark mass is small in the region we investigated. (clover-Wilson)
  - The first order transition in 2-flavor QCD is not suggested.
- This may be a good approach for the determination of boundary of the first order region in (2+1)-flavor QCD at finite density.

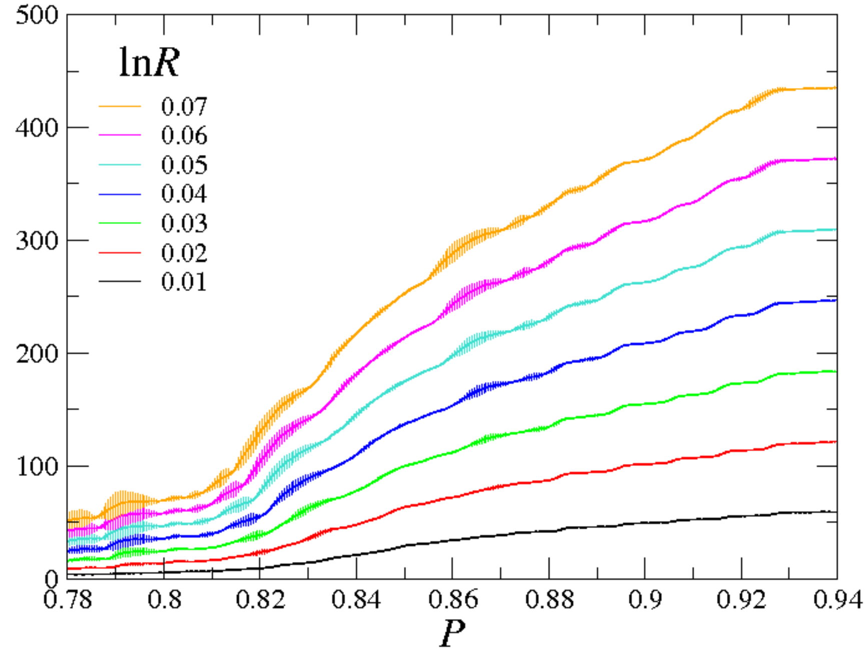
Reweighting factors at $h\neq 0$ $\mu\neq 0$

$$\ln R(P;h,\mu)=\ln\left\langle\left(\frac{\det M(m,\mu)}{\det M(m,0)}\right)^2\left(\frac{\det M(h,0)}{\det M(0,0)}\right)^{N_f}\right\rangle_{P:\text{fixed}} \approx \ln R(P;0,\mu)+\ln R(P;h,0)$$

$\ln R(P;0,\mu)$ $\qquad$ $\ln R(P;h,0)$



(S. Ejiri, Phys. Rev. D 77 (2008) 014508)



N_f=2 p4-staggared, $m_\pi/m_\rho\approx 0.7$ [data in PRD71,054508(2005)]

- The curvatures of lnR(P;μ,0) and lnR(P;0,h) are large at the same P.

→ The curvature of lnR(P;μ,h) is enhanced.