A model of light front QCD zero mode and description of quark-antiquark bound states

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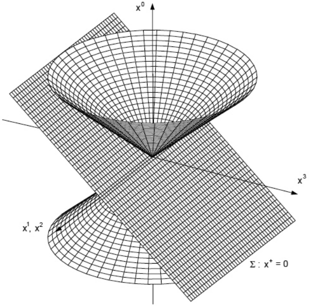
A few words on definition of the Light Front

Instead of usual Lorentz coordinates x^0, x^1, x^2 and x^3 the so called LF coordinates are introduced:

$$x^{\pm} = \frac{x^0 \pm x^3}{\sqrt{2}} \quad x^{\perp} = (x^1, x^2) = x^k$$

where the x^+ plays the role of time and the LF is defined by the equation $x^+ = 0$.



Traditional Light Cone quantization and its difficulties

Usually the surface of quantization is $x^+ = 0$. But this has the difficulty that $p_- = 0$ is a singular field mode.

There are several aproaches to deal with it:

- $|p_-| \geq \delta > 0$, i.e., discarding zero modes.
  This approach has difficulties describing vacuum effects.
- $|x^-| \leq L$ with (anti)periodic boundary conditions, i.e., DLCQ.
  Here zero modes are not dynamical and can be obtained from Hamiltonian constraints. But these constraints appears to be too difficult to deal with.

Our approach to quantization: η-frame

To quantize the theory we use a surface that is near to the light front. The corresponding coordinate system is:

$$y^0 = x^+ + \frac{\eta^2}{2} x^-, \qquad y^3 = x^-, \qquad y^\perp = x^\perp.$$

and the surface of quantization is now $y^0 = 0$, it coincides with the LF in the limit $\eta \to 0$.

The momenta become now $p_+ \to p_0, \quad p_- \to p_3$, and p_0 now plays the role of the Hamiltonian. The mass squared is

$$M^2 = 2P_0 P_3 + \eta^2 P_0 - P_\perp^2.$$

All the p_3 modes are now independent dynamical variables.

Our approach to quantization: Lagrangian

So now we are going from traditional Lagrangian density in the Lorentz frame

$$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \overline{\psi}(i\not{D} - m_q)\psi$$

to ...

Our approach to quantization: Lagrangian

...the Lagrangian density in the η - coordinate system:

$$L(y) = Tr\left\{F_{03}^2(y) + 2F_{0k}(y)F_{3k}(y) + \eta^2 F_{0k}^2(y) - F_{12}^2(y)\right\}$$
$$+ i\sqrt{2}\psi_+^\dagger(y)D_0\psi_+(y) + \frac{i\eta^2}{\sqrt{2}}\psi_-^\dagger(y)D_0\psi_-(y) + i\sqrt{2}\psi_-^\dagger(y)D_3\psi_-(y)$$
$$+ i\psi_-^\dagger(y)(D_\perp - m)\psi_+(y) + i\psi_+^\dagger(y)(D_\perp + m)\psi_-(y)$$

where

$$F_{\mu\nu}(y) = \partial_\mu A_\nu(y) - \partial_\nu A_\mu(y) - ig\left[A_\mu(y), A_\nu(y)\right],$$
$$D_\mu = \partial_\mu - igA_\mu(y), \quad D_\perp = \sum_{k=1,2} \sigma_k D_k, \quad \sigma_k \text{ are Pauli matrices},$$

$A_\mu(y)$ − vector gluon fields, related to η-coordinates y^μ,

$\psi = \begin{pmatrix}\psi_+ \\ \psi_-\end{pmatrix}$ − bispinor fermion (quark) field with mass m

and g − is a coupling constant.

Limit $\eta \to 0$ and our model

Limit $\eta \to 0$ can be investigated in QED_{1+1} in the formulation where $p_3 = \frac{\pi n}{L}, |y^3| \leq L$. The result is that we cannot obtain a correct description of vacuum effects (i.e. mass spectrum dependence on condensate parameters) if $\eta \to 0$ at fixed L. But the limit $L \to \infty$ at first and then $\eta \to 0$ gives correct result.

Even if $L \to \infty$, $\eta \to 0$ (and $L\eta \to$ const) we get non-zero vacuum condensate with some choise of this const.

To apply such limit to QCD we introduce the following simplification: for zero modes we consider the parameter $\eta = \eta_0$ "frozen" when $\eta \to 0$ at fixed L, and then consider the $\eta_0 \to 0$, $L \to \infty$ limit at $L\eta \to$ const. In QED_{1+1} this procedure gives correct description of vacuum effects.

At the first step we can obtain some effective Hamiltonian $H = H_0 + \widetilde{H}$, where by H_0 we denote the pure zero mode contribution, representing zero mode as dynamical variable still "living" in η_0-frame.

Discretization and ultraviolet regularization

We do the following discretization steps:

1. Use a lattice with respect to transverse coordinates $x^{\perp}$ with a lattice parameter $a_{\perp} \equiv a$
2. $A_{\mu}(y) \to \Big(A_0, A_3, M_{\perp} = (I + iga_{\perp}\widetilde{A}_{\perp})U_{\perp}\Big)$,
   $U_{\perp}$ – are link variables, $A_0, A_3, A_{\perp}(y)$ and $\psi(y)$ – are site variables,
   $D_3 U_{\perp}(y) = \partial_3 U_{\perp}(y) - igA_3(y)U_{\perp}(y) + igU_{\perp}(y)A_3(y - a_{\perp}e_{\perp})$
   $D_3 U_{\perp} = 0, \quad |D_3 M_{\perp}| \leq \Lambda, \quad U_{\perp}^{\dagger} = U_{\perp}^{-1}, \quad \widetilde{A}_{\perp}^{\dagger} = \widetilde{A}_{\perp}$
3. Gauge $A_3 = 0$. In this gauge $U_{\perp}$ is the zero mode (wrt y^3)
4. $|y^3| \leq L$ plus periodic boundary conditions, and hence $p_{3n} = \frac{\pi n}{L}$
5. Antiperiodic boundary conditions for fermions to avoid zero modes.
6. We will consider only states with $p_{\perp} = 0$, so the formula for the mass will be:

$$M^2 = 2P_0P_3 + \eta^2 P_0.$$

In such a formulation $U_{\perp}$ are the “frozen” zero modes.

Our Hamiltonian

$$H = \sum_{x^\perp} \Bigg\{ \int_{-L}^{L} dx^- \Bigg[\frac{g^2}{8L^2\eta_0^2} \Big(\pi_k^a - \frac{i}{2} \sum_{n>0} f^{abc} a_{nk}^{\dagger b} a_{nk}^c \Big)^2 + \frac{a^2}{2} \Big(F_{+-}^a \Big)^2 + a^2 \mathrm{tr} \Big(G_{12}{}^\dagger G_{12} \Big)$$

$$- \frac{i}{8} \Big(\chi^\dagger (x - ae_{k'}) \sigma_{k'} U_{k'}^{-1} \Big(I - igaA_{k'} \Big) - \chi^\dagger (x + ae_{k'}) \Big(I + igaA_{k'}(x + ae_{k'}) \Big) U_{k'}(x + ae_{k'}) \sigma_{k'} + 2ma\chi^\dagger \Big) \cdot$$

$$\cdot\, \partial_-^{-1} \Big(\Big(I + igaA_k \Big) U_k \sigma_k \chi(x - ae_k) - \sigma_k U_k^{-1}(x + ae_k) \Big(I - igaA_k(x + ae_k) \Big) \chi(x + ae_k) + 2ma\chi \Big)$$

$$+ \ \ldots \ \text{(terms from normal ordering)} \Bigg] \Bigg\} = H_0 + \widetilde{H}$$

where . . .

Our Hamiltonian

$$\chi = 2^{1/4}\psi_+, \quad \xi = 2^{-1/4}\eta\psi_-$$

$$F^a_{+-} = \mathrm{tr}\Big(\lambda^a F_{+-}\Big)$$

$$F_{+-} = \frac{1}{a}\left(A_k - \left(U_k^{-1}A_k U_k\right)(x + ae_k)\right)$$

$$- \frac{ig}{2}\partial_-^{-1}\left(\Big[\partial_- A_k, A_k\Big] + \Big[\partial_-\left(U_k^{-1}A_k U_k\right), U_k^{-1}A_k U_k\Big](x+ae_k) - \chi^\dagger\lambda^a\chi\frac{\lambda^a}{2}\right)$$

$$G_{12}(x) = -\frac{1}{ga^2}\left(\left(I + igaA_1\right)U_1\left(I + igaA_2(-a_1)\right)U_2(-a_1) - \Big(1 \leftrightarrow 2\Big)\right)$$

$$A_k(x) = \frac{1}{a\sqrt{2L}}\sum_{n>0}\left(\frac{a^a_{nk}(x^\perp)e^{-ip_n x^-}}{\sqrt{2p_n}} + h.c.\right)$$

$$\chi^i_r(x) = \frac{1}{a\sqrt{2L}}\sum_{n>0}\left(b^i_{nr}(x^\perp)e^{-ip_n x^-} + d^{i+}_{nr}(x^\perp)e^{ip_n x^-}\right)$$

π_k – canonically conjugated variables to U_k, $\quad \Big[\pi^a_\perp, U_\perp\Big] = -\dfrac{\lambda^a}{2}U_\perp$

Our Hamiltonian: purely zero modes

$$H_0 = \frac{(\pi_\perp^a)^2}{4L\eta_0^2} + \frac{2L}{g^2 a^2}\mathrm{tr}\, U_{12}^\dagger U_{12},$$
$$U_{12} = U_1 U_2(x - ae_1) - U_2 U_1(x - ae_2)$$

Definition of mass

Now we have a problem due to the broken Lorentz symmetry: How to define invariant mass?

We suggest the following procedure:

- First we define $P_0 = H_0 - E_{\text{vac}}$
- For zero modes we use the expression for M^2 in the η system of coordinates. So $M_0^2 = \eta_0^2 P_0^2$
- For non-zero modes we use the standard light cone mass expression: $\widetilde{M}^2 = 2\widetilde{P_0} P_3$

We define the "invariant" mass in our model in the following way:

$$M^2 \equiv M_0^2 + \widetilde{M}^2$$

This expression appears to be boost-invariant.

Quark-Antiquark model: Definition

The features of the model:

- $2+1$-dimensional
- q and $\bar{q}$ interact only with zero modes of the gluon field
- An arbitrary state is described as

$$\left|f_m^l\right\rangle = \sum_{m,l}\sum_{x^\perp} b_m^\dagger(x^\perp)U^\dagger(x^\perp + a_\perp e_\perp)\ldots U^\dagger(x^\perp + la_\perp e_\perp)d_{\overline{m}}^\dagger(x^\perp + la_\perp e_\perp)\varphi_{\mathrm{vac}}(U)\left|0\right\rangle$$

where φ_{vac} is the eigenfunction of H_0 corresponding to E_{vac}. In our $(2+1)$ dimensional case $E_{\mathrm{vac}} = 0$ and $\varphi_{\mathrm{vac}} = \mathrm{I}$.

Quark-Antiquark model: Hamiltonian

The simplified Hamiltonian:

$$H = \sum_{x^\perp} \Bigg\{ \int_{-L}^{L} dx^- \Bigg[\frac{g^2}{8L^2\eta_0^2}\pi_k^2 + \frac{g^2 a}{2}\, \partial_-^{-1} \left(\chi^\dagger \frac{\lambda^a}{2}\chi\right)\, \partial_-^{-1} \left(\chi^\dagger \frac{\lambda^a}{2}\chi\right)$$

$$+ -\frac{i}{8a}\left[\chi^\dagger(x^\perp - 1)\sigma_1 U^\dagger(x^\perp) - \chi^\dagger(x^\perp + a_\perp)U(x^\perp + a_\perp)\sigma_1 + 2ma\chi^\dagger(x^\perp)\right] \cdot$$

$$\cdot\, \partial_-^{-1}\left[U(x^\perp)\sigma_1\chi(x^\perp - a_\perp) - \sigma_1 U^\dagger(x^\perp + a_\perp)\chi(x^\perp + a_\perp) + 2ma\chi(x^\perp)\right]$$

$$+ \ldots \text{ (terms from normal ordering)} \Bigg] \Bigg\} \equiv H_0 + \widetilde{H}$$

All these operators are understood to be in the normal-ordered form.

Spectrum computation

Now we can find matrix element of every term in the mass formula. Solving this matrix equation we obtain the approximate spectrum of $Q - \overline{Q}$ system.

The basis set is limited by the length of state and by the total momentum $p_3 = \frac{\pi n}{L}$.

The limit $L \to \infty$ is achieved by increasing n such that p_3 is fixed. When we arrive at stable spectrum we assume that the Lorentz symmetry is restored.

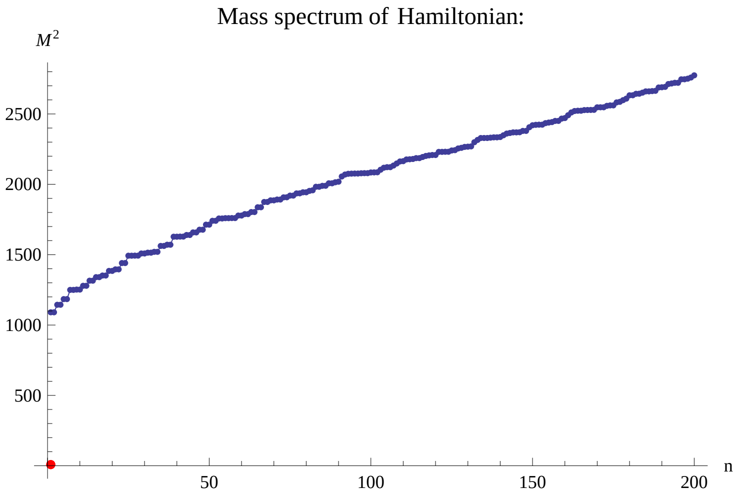
Spectrum features

- There are quite a few subtle moments in spectrum computation. For example a question about renormalization of coupling constants. Unfortunately we cannot discuss them yet.
- There are several phenomenological parameters in the model.
- The main feature we want to emphasize is the form of $Q - \overline{Q}$ potential we get: it is quadratic with respect to the transverse direction and it has a "'t Hooft" form in the longitudinal direction.
- The spectrum "stabilizes" when the longitudinal resolution increases.
- Due to the special form of potential the spectrum shows clear "Regge" trajectory.

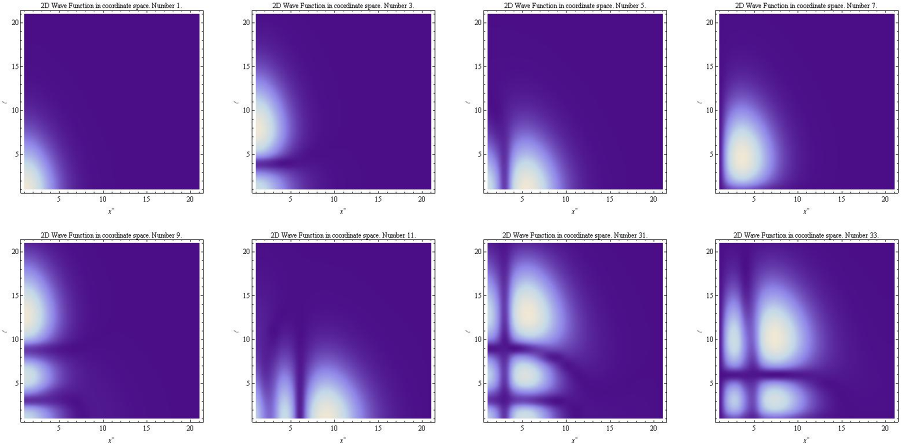
Conclusions

- We propose a new semi-phenomenological approach to the QCD spectrum computation on the Light Front.
- The spectrum obtained show (qualitatively) desired features.

Typical spectrum and wave functions



Typical spectrum and wave functions



Several first 2D wavefunctions squared of Hamiltonian $\left(|\varphi(x^{\perp}, x^{-})|^{2}\right)$.