Jet broadening at NNLL in perturbation theory

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Outline

1. Introduction

2. The Jet Quenching Parameter

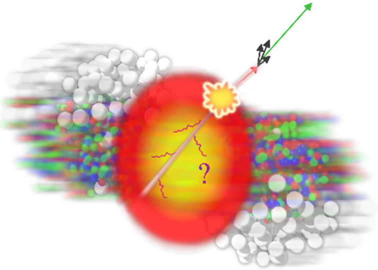
3. Effective Field Theories

4. Perturbative Calculations

5. Conclusions

Jets in the Quark-Gluon Plasma

- Jets have a clear experimental signature
- They are produced by hard interactions before the formation of the plasma
  $\rightarrow$ Production calculable at $T = 0$



lbl.gov

- Subsequently propagate through the plasma
  $\rightarrow$ By comparison with jets in p-p collisions the properties of the QGP can be analyzed

Plasma Effects on the Jet

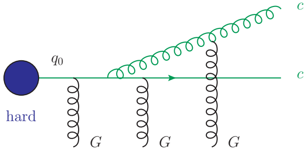
Two types of interaction

- **Radiative energy loss** through medium induced gluon radiation (radiated gluons are again subject to in medium interactions)
- **Jet broadening** without energy loss, i.e. change of momentum perpendicular to initial jet direction through interaction with medium

  $\rightarrow$ Both interfere and are relevant to the so-called **jet quenching**

- There are several approaches to calculate the effect of the medium on jets due to

  Baier, Dokshitzer, Peigne, Schiff, Zakharov, Armesto, Salgado, Wiedemann, Gyulassy, Levai, Vitev, Guo, Wang, Arnold, Moore, Yaffe, ...



- How to characterize the medium?

The Jet Quenching Parameter

- One way to parameterize effect of the medium is to introduce a **jet quenching parameter**
- It corresponds to the change of the momentum perpendicular to the original direction of the jet parton per distance traveled

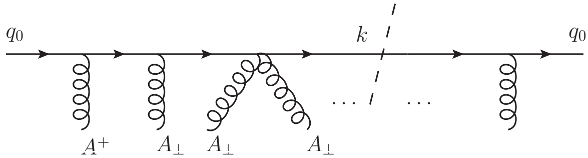
  When describing the broadening of the $k_\perp$-distribution while travelling a distance through the medium by a diffusion equation, $\hat{q}$ is related to the diffusion constant
- Introduce $P(k_\perp)$, the probability to acquire a perpendicular momentum $k_\perp$ after travelling through a medium with length L

$$P(k_\perp) \sim \frac{1}{\hat{q}L} e^{-\frac{k_\perp^2}{\hat{q}L}}$$

Calculation of $P(k_\perp)$

- Determine the probability $P(k_\perp)$ by calculating the amplitude for the interaction of the collinear quark with gluons from the medium
- Calculation may be performed in Soft-Collinear Effective Theory

  Idilbi, Majumder '08; D'Eramo, Liu, Rajagopal '10; Mi.B., Brambilla, Escobedo, Vairo '12
- Use optical theorem to determine scattering amplitude



- Initially on-shell quark scattering on an arbitrary number of medium particles

Results

- We find

$$P(k_\perp) = \frac{1}{N_c}\int d^2x_\perp\, e^{ik_\perp\cdot x_\perp} \left\langle \mathrm{Tr}\left[T^\dagger(0,-\infty,x_\perp)\, W_F^\dagger[0,x_\perp]\, T(0,\infty,x_\perp)\, T^\dagger(0,\infty,0)\, W_F[0,0]\, T(0,-\infty,0)\right]\right\rangle$$

- where we have defined the Wilson lines

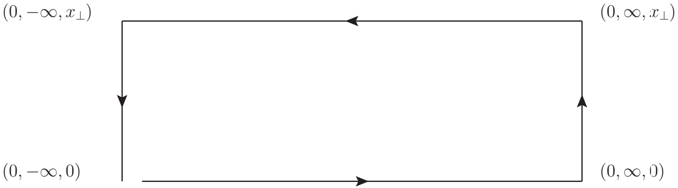
$$W_F[y^+,y_\perp] = \mathcal{P}\left\{\exp\left[ig\int_{-\infty}^{\infty} dy^- A^+(y^+,y^-,y_\perp)\right]\right\}$$

$$T(x_+,\pm\infty,x_\perp) = \mathcal{P}\, e^{-ig\int_{-\infty}^{0} ds\; l_\perp\cdot A_\perp(x_+,\pm\infty,x_\perp+l_\perp s)}$$

- Agrees with known results Casalderrey-Solana, Salgado '07 but pay attention to operator ordering

Results

- Transverse Wilson lines combine to



- Fields on the lower line are time ordered, the ones on the upper line anti-time ordered

  $\rightarrow$ Use **Schwinger-Keldysh contour** in path integral formalism

Next step

- Actually calculate $\hat{q}$ in the weak coupling regime.

The relevant scales

- Several scales appear in the process, most notably

  The scale of the medium (temperature) T

  Thermal scales, such as the Debye mass $m_D \sim gT$

  the chromomagnetic mass $g_E \sim g^2 T$ (magnetostatic screening)
- In the weak coupling limit (g small) these scales are ordered by their size

Approach

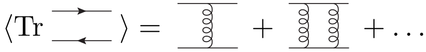
Introduce a series **effective field theories** to

- transparently derive factorization
- obtain a systematic expansion in terms of ratios of scales
- resum possible large logarithms of ratios of scales

Thermal Field Theory

- Probability $P(x_\perp)$ is related to **thermal expectation value** of a light-cone Wilson loop
- Assume a thermalized medium and a weak coupling g and calculate $\hat{q}$ in thermal field theory
- Since the Wilson loop extends along the light-cone, in principle one needs analytic continuation in the **imaginary time formalism** or a doubling of degrees of freedom in the **real time formalism**

  $\rightarrow$ Calculate $P(x_\perp)$ in perturbation theory



- Use covariant gauge

Effective Thermal Field Theory

- Due to the appearance of additional thermal scales $m_D \sim gT$ and $g_E \sim g^2 T$ (electro- and magnetostatic screening) large logarithms may appear, invalidating the perturbative expansion
  $\rightarrow$ Introduce a set of effective field theories to separate the scales
  Braaten '95
- In the imaginary time formalism an equal-time propagator can be written

$$G(t=0,\vec{x}) = T\sum_n \int \frac{d^3p}{(2\pi)^3}\, e^{i\vec{p}\cdot\vec{x}} G_E(\omega_n,\vec{p})$$

$$\omega_n = 2\pi n T \qquad \text{Matsubara frequency}$$

$$G_E(\omega_n,\vec{p}) \sim \frac{1}{\omega_n^2+\vec{p}^2} \qquad \text{Euclidean propagator}$$

- Integrate out the Matsubara modes $n > 0$ ($\sim \pi T$) and redefine $A^\mu \rightarrow \sqrt{T}A^\mu \rightarrow$ **electrostatic QCD** (EQCD)

Electrostatic QCD

- EQCD is 3D Euclidean Yang-Mills coupled to massive scalar A^0 with mass $m_E \sim gT$ and coupling $g_E \sim g^2 T$
- EQCD Lagrangian

$$\mathcal{L}_{\mathrm{EQCD}} = \frac{1}{4} G^a_{ij} G^a_{ij} + \frac{1}{2} (D_i A_0)^2 + \frac{1}{2} m_E^2 A_0^2$$

- Propagators in EQCD
  $G^{00} = -1/(\vec{q}^2 + m_E^2)$ and $G^{ij} = \delta^{ij}/\vec{q}^2$
- In principle applicable for calculating $P(k_\perp)$ if $k_\perp \sim gT$
- In order to apply EQCD to light-like correlators, **deform contour** to be slightly space-like ($v = 1 + \epsilon$), then boost to equal time and use imaginary time formalism Caron-Huot '08
- Soft modes are not sensitive to the precise velocity of the jet parton

Perturbative Contributions to $\hat{q}$

- Known perturbative contributions to $\hat{q}$
- Interference of loop and power expansion

  LO $\sim g^4 T^3$ Arnold, Xiao '08 (from $k_\perp \sim T$ and $k_\perp \sim gT$)

  NLO $\sim g^5 T^3$ Caron-Huot '08 (from $k_\perp \sim gT$ with loops)
- The region for $k_\perp \sim gT$ can be derived from the Wilson loop in EQCD (extending along the z direction and containing the fields $A_E^+(x_\perp, z) = (A_E^0 + A_E^3)/\sqrt{2}$)

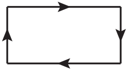
$$\begin{aligned}\hat{q}(q_{\max}) &= \frac{2g^4T^3}{3\pi}\left[\frac{3}{2}\log\left(\frac{T}{m_D}\right) + \frac{7\zeta(3)}{4\zeta(2)}\log\left(\frac{q_{\max}}{T}\right) - 0.105283\right] \\ &+ \frac{g^4T^3}{8\pi^2}\frac{m_D}{T}(3\pi^2 + 10 - 4\log(2))\end{aligned}$$

- At order $g^6 T^3$ non-perturbative contributions from the region $k_\perp \sim g^2 T$ start to appear

Magnetostatic QCD

- So far we have argued that $P(k_\perp)$ for $k_\perp \sim gT$ is related to the expectation value of a Wilson loop 3D-Yang-Mills theory (plus the A^0 field)
- The probability $P(k_\perp)$ can therefore be related to the **static energy in 3D Yang-Mills theory**

$$P(k_\perp)_{\mathrm{F.T.}} \sim \frac{1}{N_c}\langle \mathrm{Tr}$$

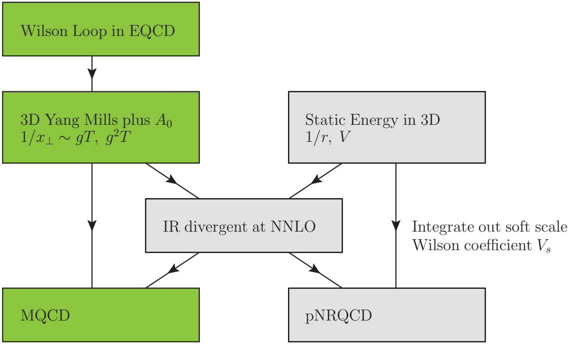


$$\rangle = e^{-h_s(x_\perp)L} \qquad \text{for } L \to \infty$$

- When also integrating out the scale $m_E \sim gT$ (i.e. the field A^0) one arrives at magnetostatic QCD (MQCD) Braaten '95

Soft Logarithms

- The relevance of the g^2T contributions at NNLO manifests in IR divergencies appearing in the EQCD calculation
- The situation is exactly analogous to the static potential in 3D QCD Schroeder '99; Pineda, Stahlhofen '10
  There it was demonstrated, that a matching onto a low energy effective theory (pNRQCD) regulates this divergence



Soft Logarithms

- The static energy can be written

$$h_s(x_\perp) = V_s(x_\perp, \mu) + \delta h_s(x_\perp, \mu)$$

- We will use the analogy to the jet quenching case to determine the **logarithmic contributions** at NNLO to $P(k_\perp)$

$$\mu \frac{d}{d\mu} V_s = B(V)$$

- The role of pNRQCD will be played by MQCD
- $B(V)$ can be derived from the poles of the ultrasoft contribution
  $\rightarrow$ work in progress

Conclusions

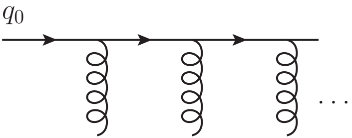
- The jet quenching parameter is related to the thermal expectation value of a light-cone Wilson loop
- In the weak coupling limit perturbative contributions may be calculated using EQCD
- There is an analogy to the static energy in 3D Yang-Mills
- Perturbative results are available at LO and NLO
- At NNLO the non-perturbative contribution can be determined from the lattice, the logarithms between the two regions derived from the static Wilson loop

Thank you for your attention!

Bonus Slides

The Jet Quenching Parameter

- The Fourier transform $P(x_\perp)$ exponentiates
$$P(x_\perp) \sim e^{C(x_\perp)L}$$
where $C(x_\perp)$ is the collision kernel



- The jet quenching parameter may then be defined as
$$\hat{q} = \int \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 \, C(k_\perp)$$
where the range of integration is restricted by a process-dependent cut-off

Non-perturbative Contributions at NNLO

- In MQCD the jet quenching parameter may be written as

$$\hat{q}|_{g^2T} = -\int \frac{d^2k_\perp}{(2\pi)^2}\, k_\perp^2\, V(k_\perp)$$

Laine '12

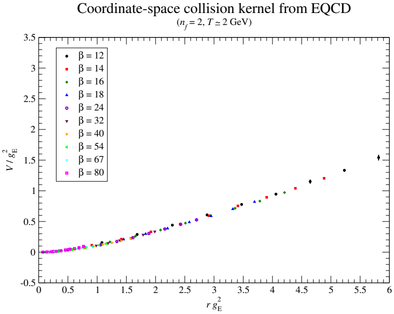
- The static potential V has been calculated on the lattice Luescher, Weisz '02
- Possible to derive the contribution from the non-perturbative region $k_\perp \sim g^2 T$ to $\hat{q}$

  Laine '12; Mi.B., Brambilla, Escobedo, Vairo '12
- It is found that the non-perturbative contribution is than the perturbative contribution at NLO

EQCD on the lattice

- Recently, the full EQCD Wilson loop has been calculated on the lattice

  Panero, Rummukainen, Schaefer '13



- Numerical result: $\hat{q}|_{gT} \approx 0.45(5) g_E^6$ for $T \approx 2\,\text{GeV}$