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Potential description of the charmonium from lattice QCD

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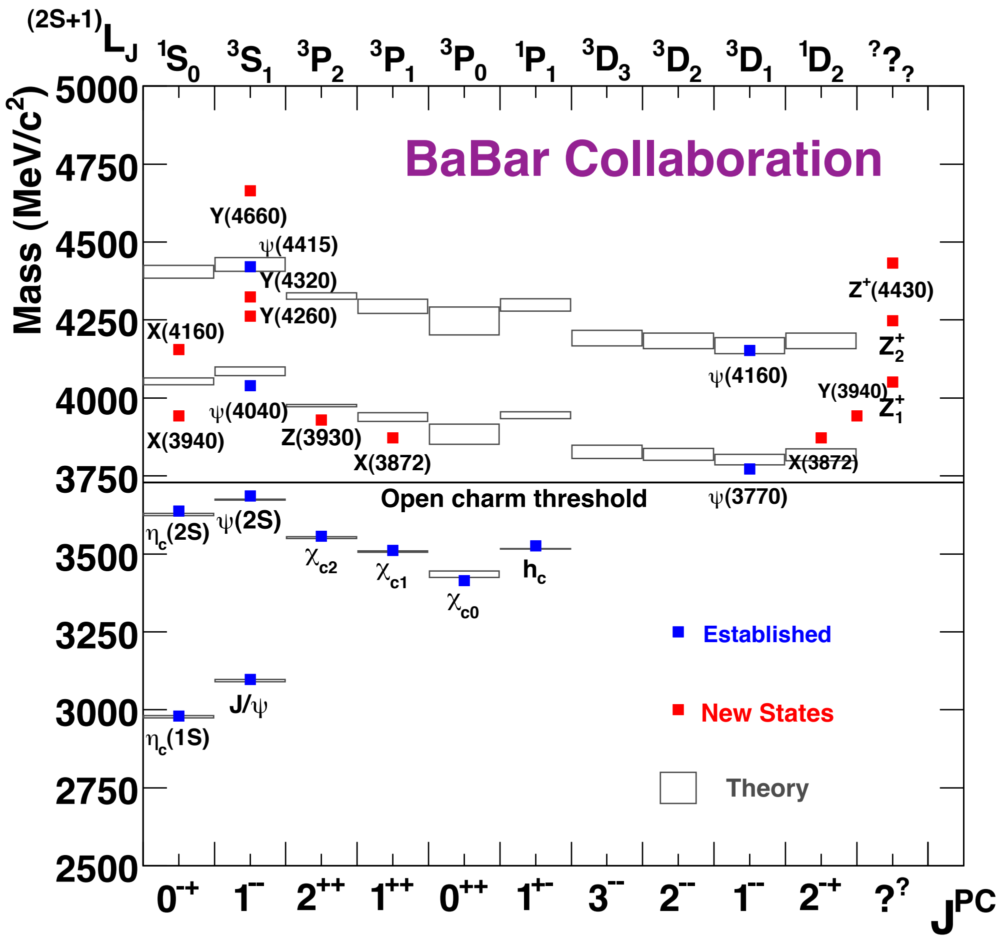
in collaboration with Shoichi Sasaki (Tohoku University)

T. Kawanai and S. Sasaki, PRL, 107, 091601 (2011)
T. Kawanai and S. Sasaki, PRD85, 091503(R) (2012)
T. Kawanai and S. Sasaki, PRD89 054507 (2014)

Motivation

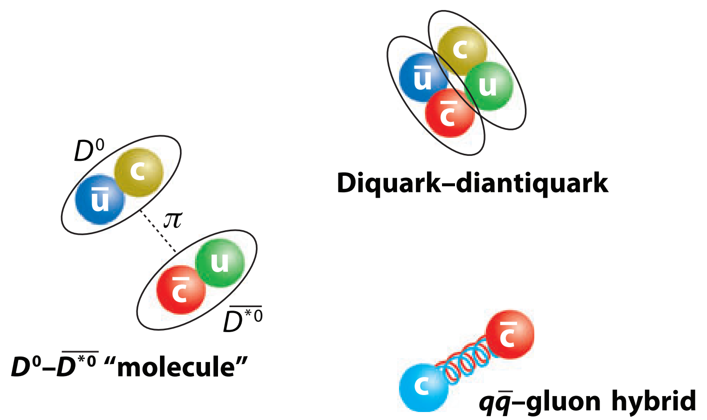
✦ Exotic XYZ charmonium-like mesons

"Standard" states can be defined in potential models



The XYZ mesons are expected to be good candidates for non-standard quarkonium mesons

S. Godfrey and S. L. Olsen, Ann. Rev. Nucl. Part. Sci. 58, 51 (2008)



"Exotic" = "Non-standard"?

Motivation

✦ $q\bar{q}$ interquark potential in quark models

S. Godfrey and N. Isgur, PRD 32, 189 (1985).
T. Barnes, S. Godfrey and E. S. Swanson, PRD 72, 054026 (2005)

$$V_{c\bar{c}} = \underbrace{-\frac{4}{3}\frac{\alpha_s}{r} + \sigma r}_{\text{Cornell potential}} + \underbrace{\frac{32\pi\alpha_s}{9m_q^2}\delta(r)\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} + \frac{1}{m_q^2}\left[\left(\frac{2\alpha_s}{r^3} - \frac{b}{2r}\right)\mathbf{L}\cdot\mathbf{S} + \frac{4\alpha_s}{r^3}T\right]}_{\text{spin-dependent potential}}$$

- Spin-spin, tensor and spin-orbit terms appear as corrections in the $1/m_q$ expansion.
- Functional forms of the spin-dependent terms are determined by one-gluon exchange.

→ Properties of higher charmonium states predicated in potential models may contain uncertainties due to perturbative method.

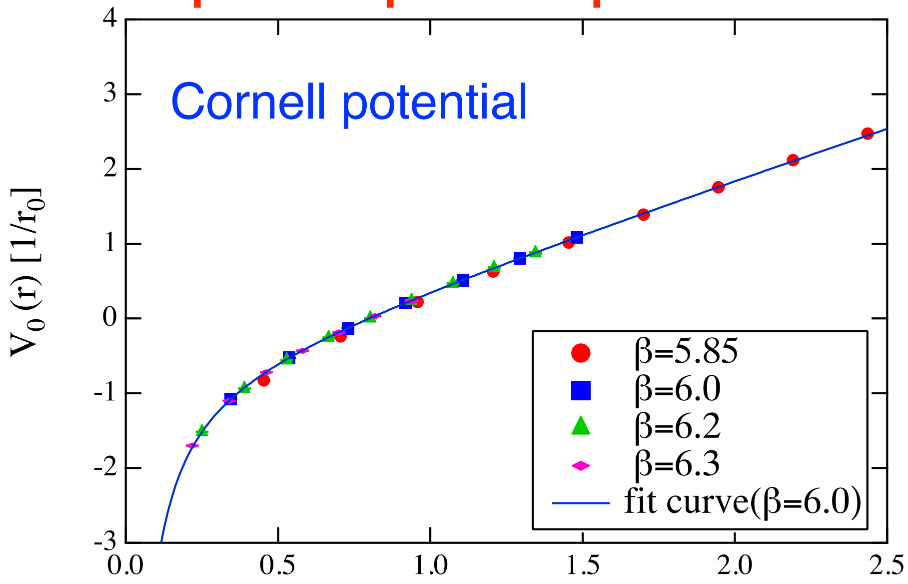
A reliable charmonium potential directly derived from first principles QCD is important.

Wilson loop approch

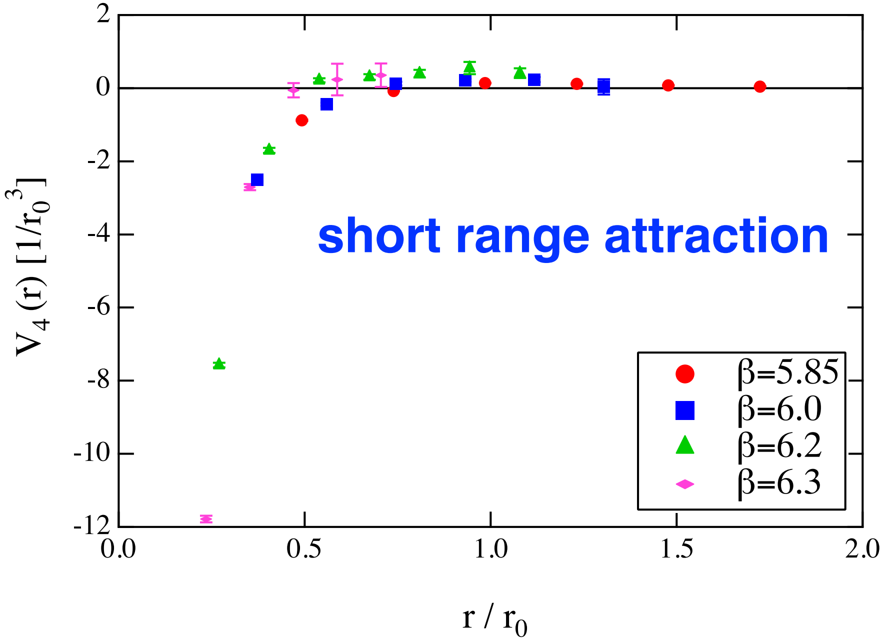
✦ Static interquark potential from Wilson loop

Koma et al., NPB769 (2007) 79
Koma et al., PRL97 (2006) 122003

Spin-independent potential



spin-spin potential



- The static potential have been precisely calculated by Wilson loop from lattice QCD.
- Relativistic corrections are classified in powers of $1/m_q$ within framework of pNRQCD.

N. Brambilla et al., Rev. Mod. Phys. 77, 1423 (2005).

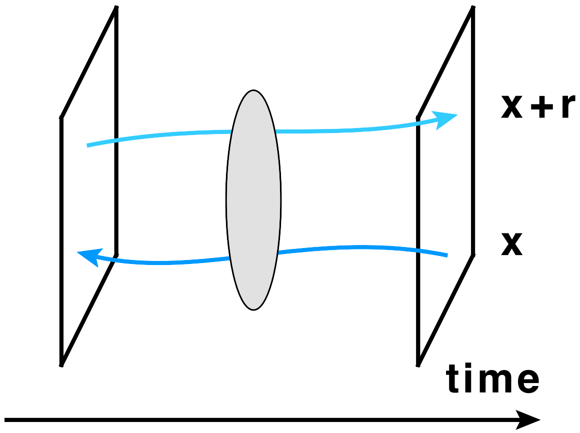
→ spin-spin potential induced by $1/m_q^2$ correction exhibits short range attraction.
cf. short range repulsion is required in phenomenology.

Potential definition

S. Aoki, T. Hatsuda and N. Ishii, Prog. Theor. Phys. 123 (2010) 89.
Y. Ikeda and H. Iida, arXiv:1102.2097 [hep-lat].

1. Equal-time BS wavefunction

$$\phi_\Gamma(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|q\bar{q}; J^{PC}\rangle$$



$$\begin{aligned}
&\sum_{\mathbf{x},\mathbf{x}',\mathbf{y}'} \langle 0|\bar{q}(\mathbf{x},t)\Gamma q(\mathbf{x}+\mathbf{r},t)\left(\bar{q}(\mathbf{x}',t_{\rm src})\Gamma q(\mathbf{y}',t_{\rm src})\right)^\dagger|0\rangle \\
&= \sum_n A_n \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|n\rangle e^{-M_n^\Gamma(t-t_{\rm src})} \\
&\xrightarrow{t\gg t_0} A_0\phi_\Gamma(\mathbf{r})e^{-M_0^\Gamma(t-t_{\rm src})}
\end{aligned}$$

2. Schrödinger equation with non-local potential

$$-\frac{\nabla^2}{2\mu}\phi_\Gamma(\mathbf{r}) + \int dr' U(\mathbf{r},\mathbf{r}')\phi_\Gamma(\mathbf{r}') = E_\Gamma\phi_\Gamma(\mathbf{r})$$

3. Velocity expansion

$$U(\mathbf{r}',\mathbf{r}) = \{V(r) + V_{\rm S}(r)\mathbf{S}_Q\cdot\mathbf{S}_{\overline{Q}} + V_{\rm T}(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + \mathcal{O}(\nabla^2)\}\delta(\mathbf{r}'-\mathbf{r})$$

Potential definition

S. Aoki, T. Hatsuda and N. Ishii, Prog. Theor. Phys. 123 (2010) 89.
Y. Ikeda and H. Iida, arXiv:1102.2097 [hep-lat].

5. Projection to "S-wave" $\phi_\Gamma(\mathbf{r}) \rightarrow \phi_\Gamma(r; A_1^+)$

$$\left\{-\frac{\nabla^2}{m_q} + V(r) + \mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} V_{\mathrm{S}}(r)\right\} \phi_\Gamma(r) = E_\Gamma \phi_\Gamma(r)$$

6. Linear combination

$$V(r) = E_{\mathrm{ave}} + \frac{1}{m_q}\left\{\frac{1}{4}\frac{\nabla^2\phi_{\mathrm{PS}}(r)}{\phi_{\mathrm{PS}}(r)} + \frac{3}{4}\frac{\nabla^2\phi_{\mathrm{V}}(r)}{\phi_{\mathrm{V}}(r)}\right\}$$

$$V_{\mathrm{S}}(r) = E_{\mathrm{hyp}} + \frac{1}{m_q}\left\{-\frac{\nabla^2\phi_{\mathrm{PS}}(r)}{\phi_{\mathrm{PS}}(r)} + \frac{\nabla^2\phi_{\mathrm{V}}(r)}{\phi_{\mathrm{V}}(r)}\right\}$$

The quark kinetic mass m_q is essentially involved in the definition of the potentials.
Under a simple, but reasonable assumption of $\lim_{r\to\infty} V_S(r) = 0$

T. Kawanai and S. Sasaki, PRL. 107, 091601 (2011).

$$m_q = \lim_{r\to\infty} \frac{-1}{\Delta E_{\mathrm{hyp}}}\left(\frac{\nabla^2\phi_{\mathrm{V}}(r)}{\phi_{\mathrm{V}}(r)} - \frac{\nabla^2\phi_{\mathrm{PS}}(r)}{\phi_{\mathrm{PS}}(r)}\right)$$

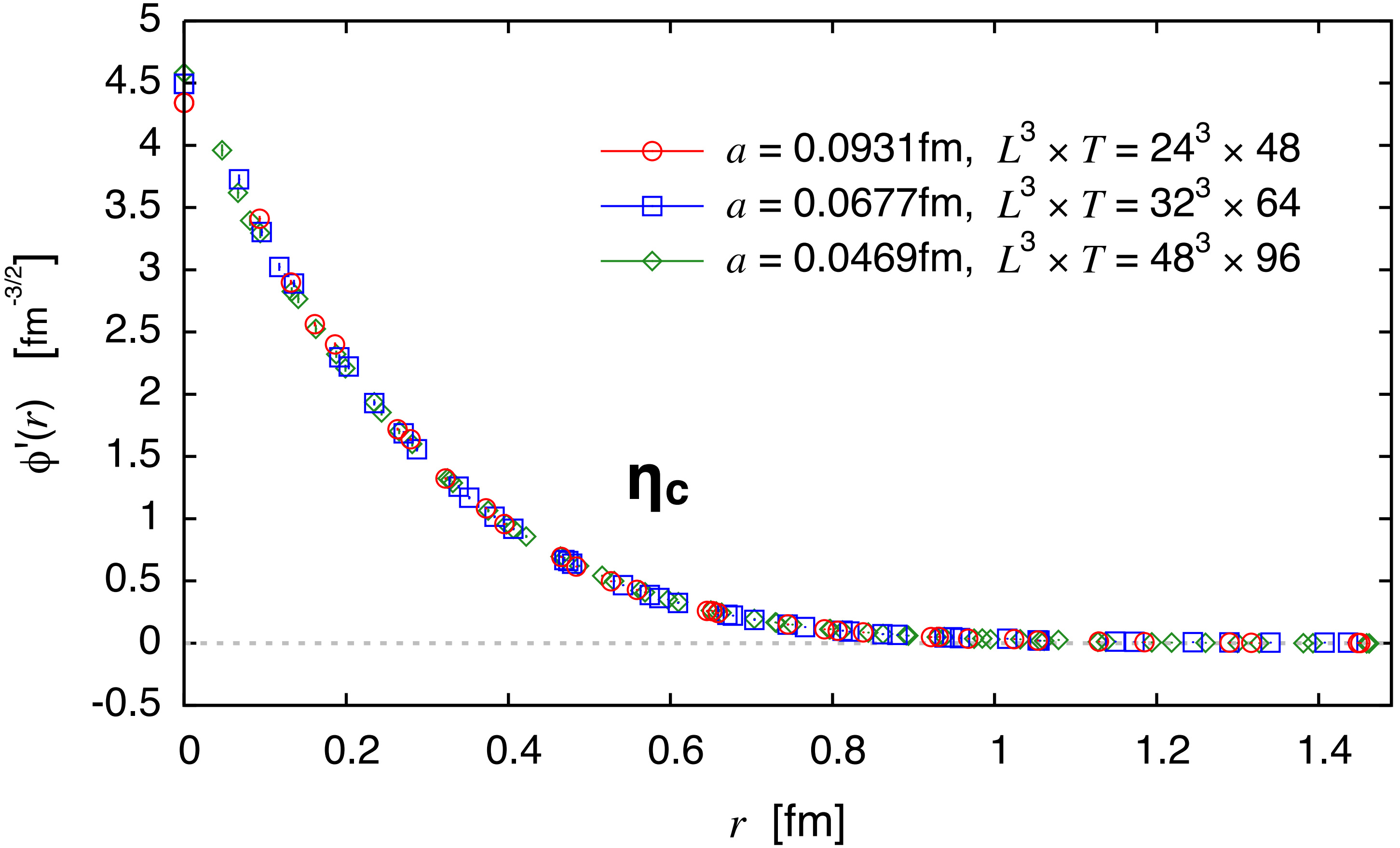
$$\Delta E_{\mathrm{hyp}} = M_{\mathrm{V}} - M_{\mathrm{PS}}$$

1. Quenched lattice QCD simulation

2. N_f =2+1 dynamical QCD simulation

qq^{bar} wave function

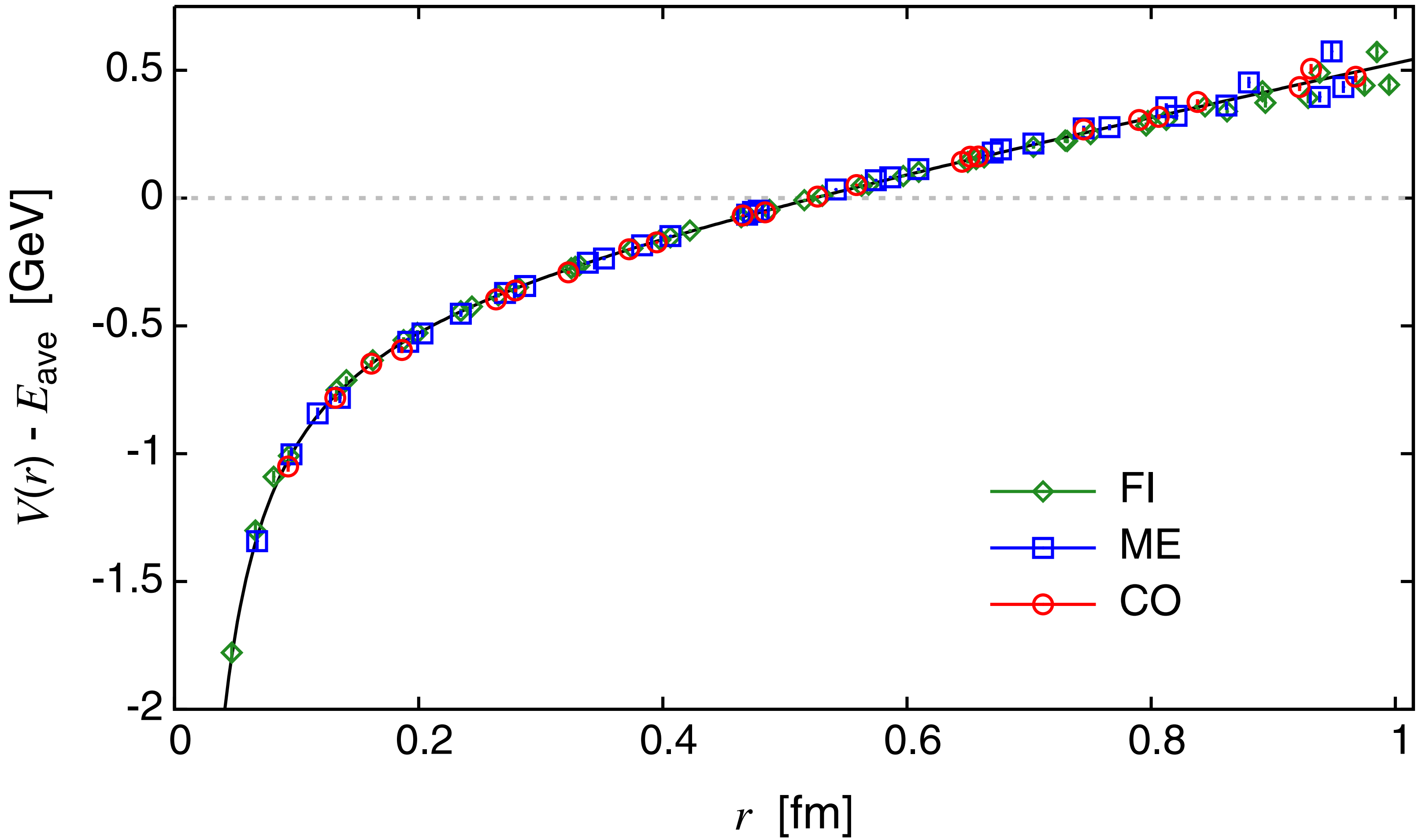
$$\phi_\Gamma(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|q\bar{q}; J^{PC}\rangle$$



- Normalization $\int dr^3\, \phi^2(r) = 1$
- BS wave functions vanish at r ~ 1fm

Spin-independent central potential

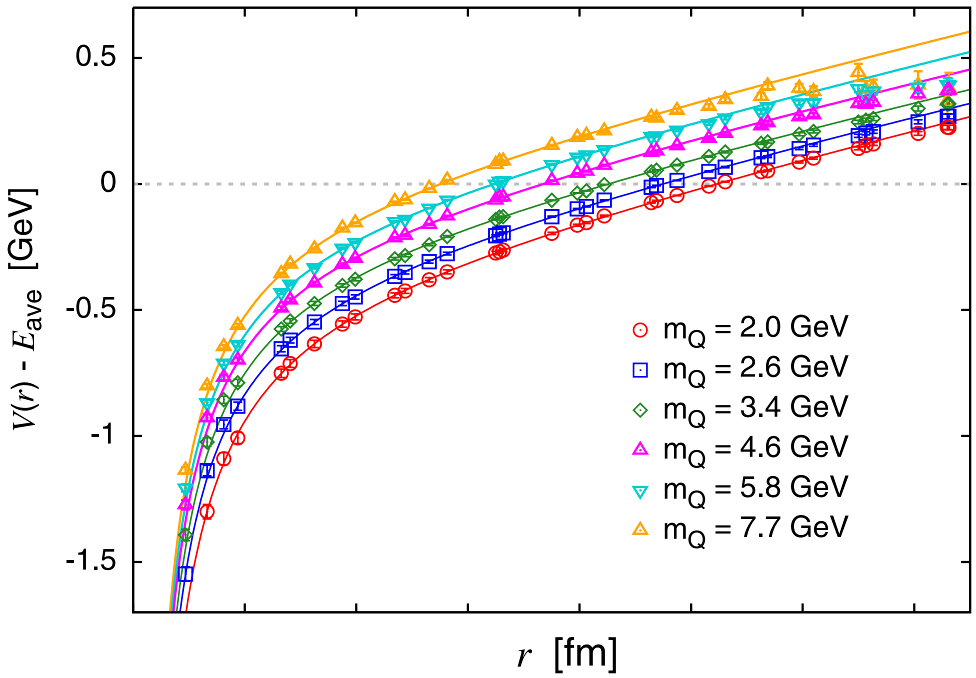
T. Kawanai and S. Sasaki, PRL. 107, 091601 (2011)
T. Kawanai and S. Sasaki, PRD89 054507 (2014)

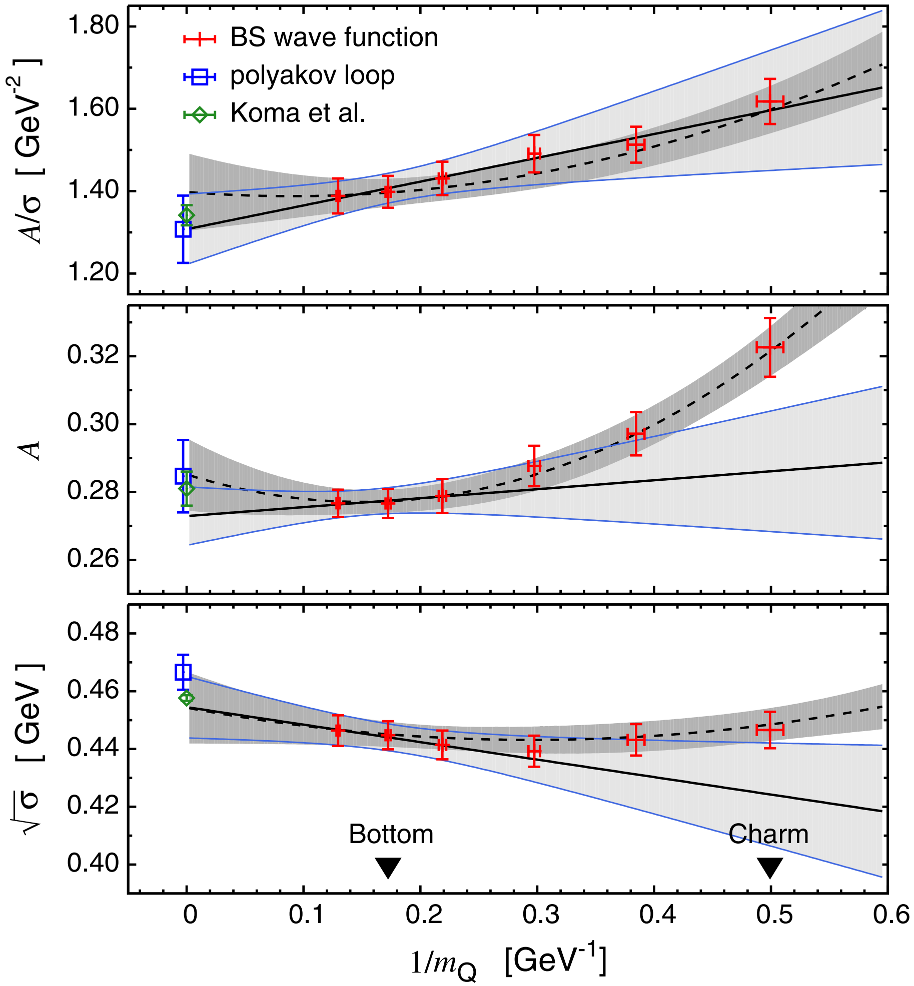


- Coulomb + linear potential
- Good scaling behavior is observed.

Quark mass dependence

T. Kawanai and S. Sasaki, PRL. 107, 091601 (2011)
T. Kawanai and S. Sasaki, PRD89 054507 (2014)





Cornell parameterization

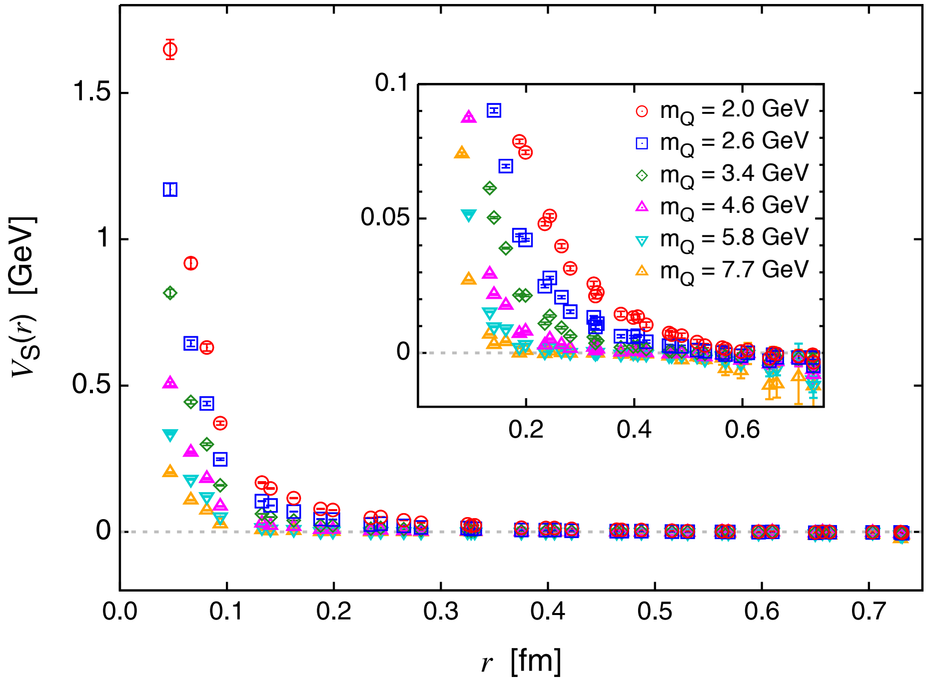
$$V(r) = -\frac{A}{r} + \sigma r + V_0$$

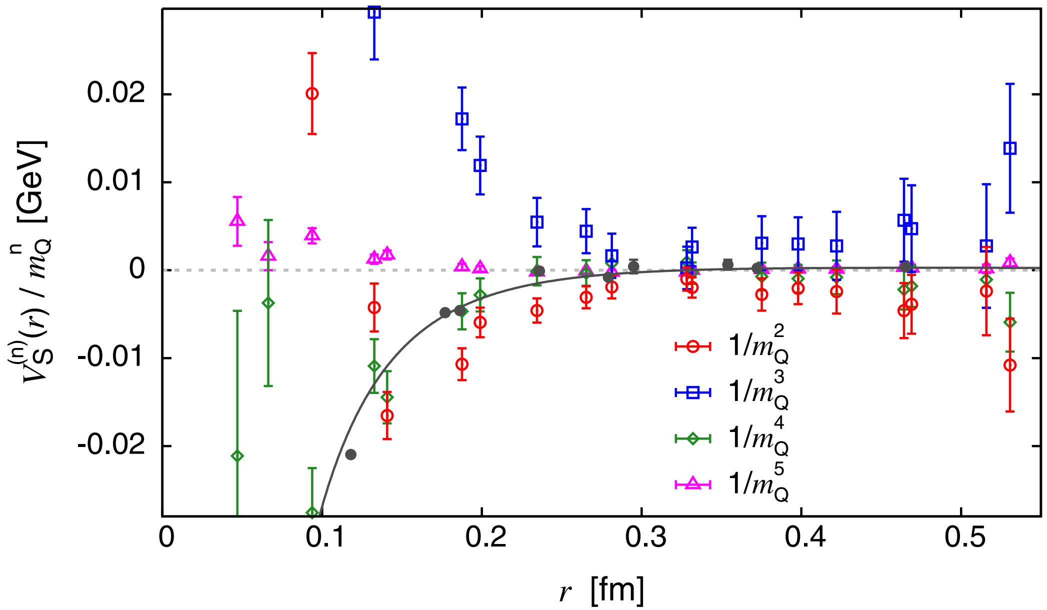
Consistent with the Wilson loops in the $m_q \to \infty$ limit

Quark mass dependence

T. Kawanai and S. Sasaki, PRD89 054507 (2014)

spin-spin potential





- $1/m_q^2$ correction (red) gives the attractive contribution.
- Finite quark mass effect changes the spin-spin potential from attractive to repulsive.

1. Quenched lattice QCD simulation

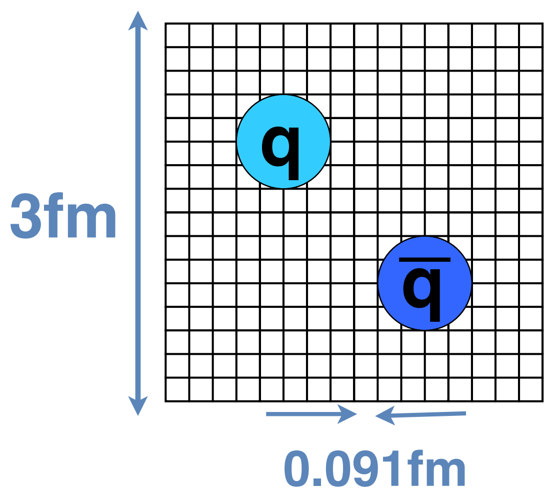
2. $N_f = 2+1$ dynamical QCD simulation

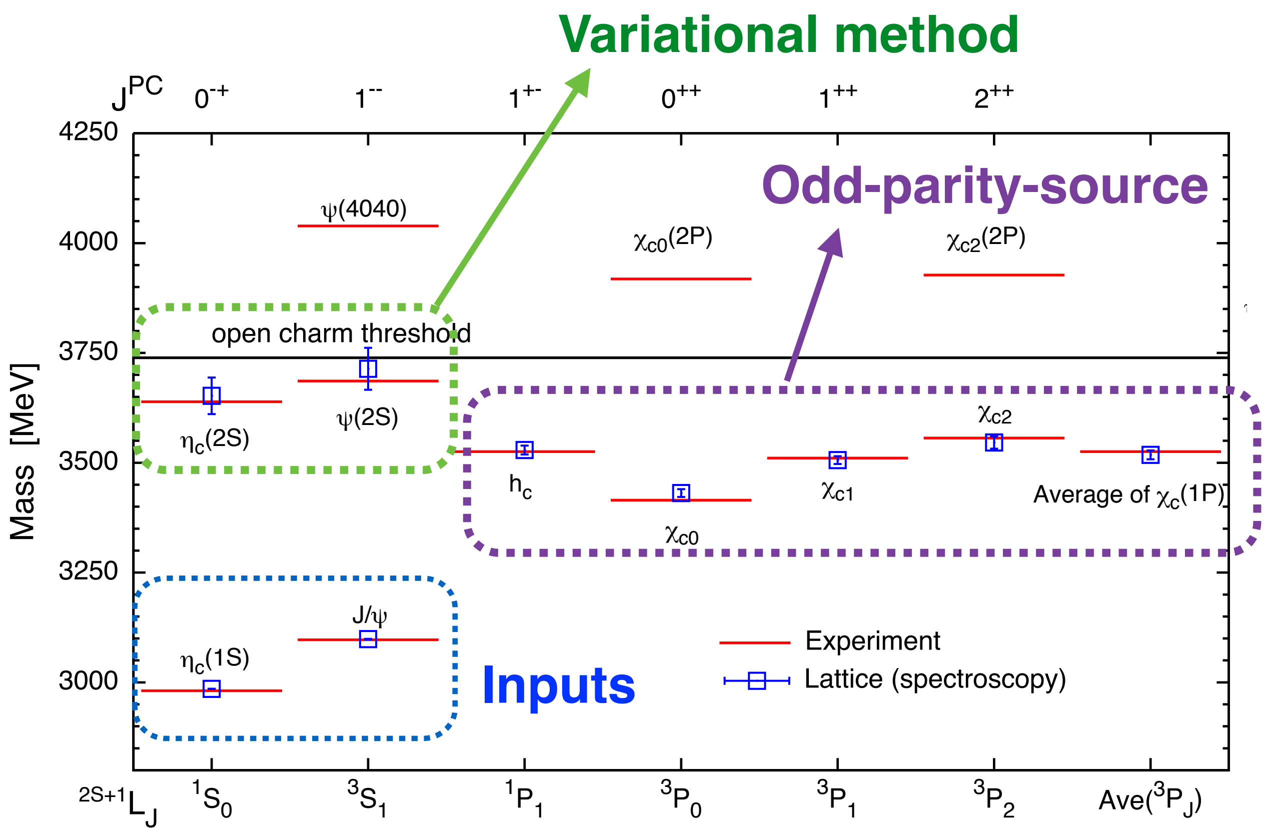
Lattice Set up and Spectrum

- 2+1 flavor dynamical gauge configurations generated by PACS-CS collaboration.
  $\rightarrow m_\pi = 156(7)$ MeV, $m_K = 553(2)$MeV

- Iwasaki gauge action $\beta=1.9$ ($a\approx0.091$ fm, $a^{-1}\approx2.3$GeV)

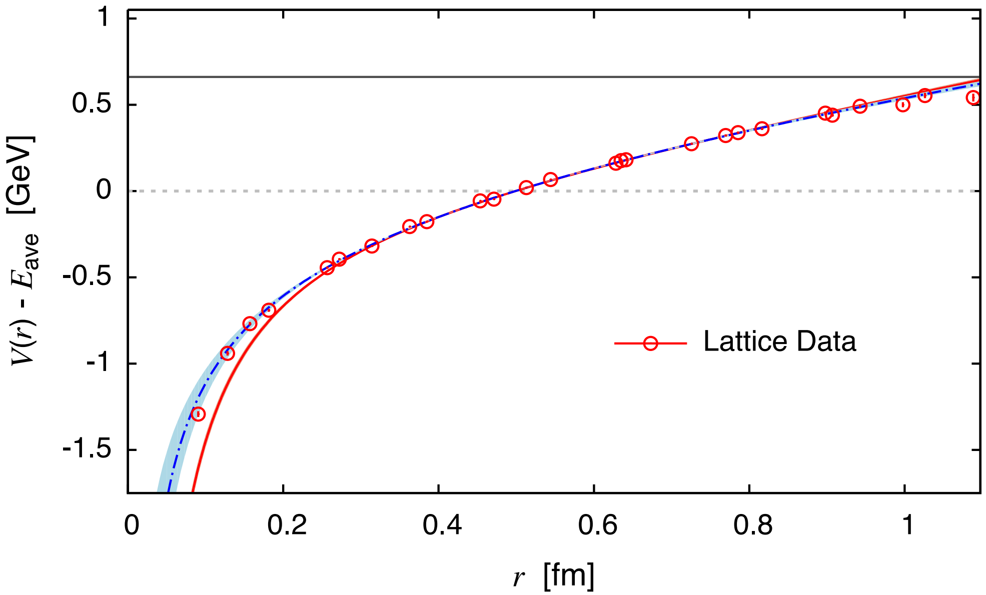
- RHQ action with RHQ parameters non-perturbativey tuned by 1S charmonium states.
  $\rightarrow m_{ave}(1S) = 3.069(2)$ GeV, $m_{hyp}(1S)=111(2)$ MeV





Charmonium potential

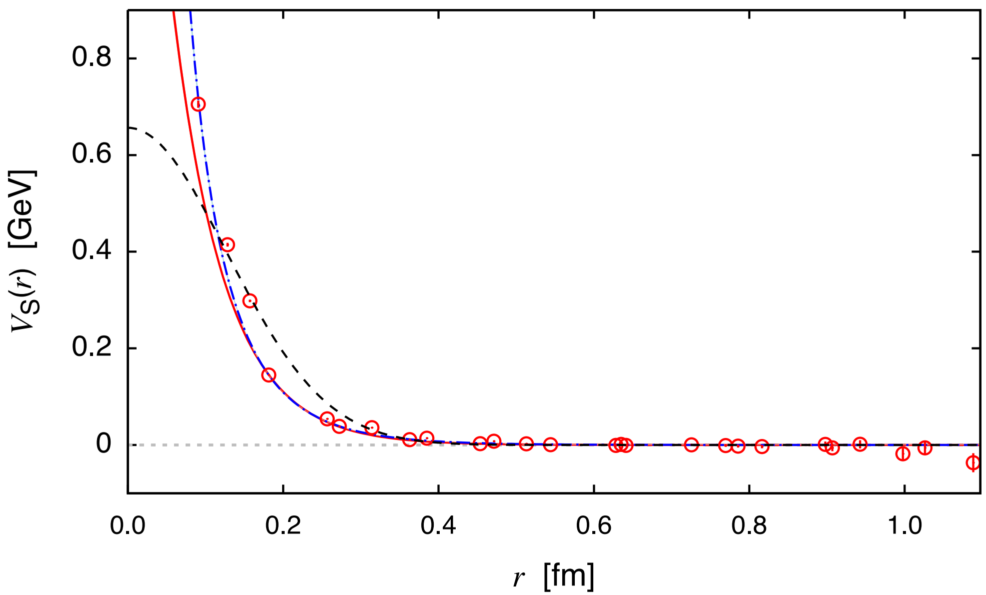
T. Kawanai and S. Sasaki, arXiv:1110.0888



- The charmonium potential obtained from the BS wave function resembles the NRp model.

	This work	NRp model	Static
A	0.713(83)	0.7281	0.403(24)
√σ [GeV]	0.402(15)	0.3775	0.462(4)

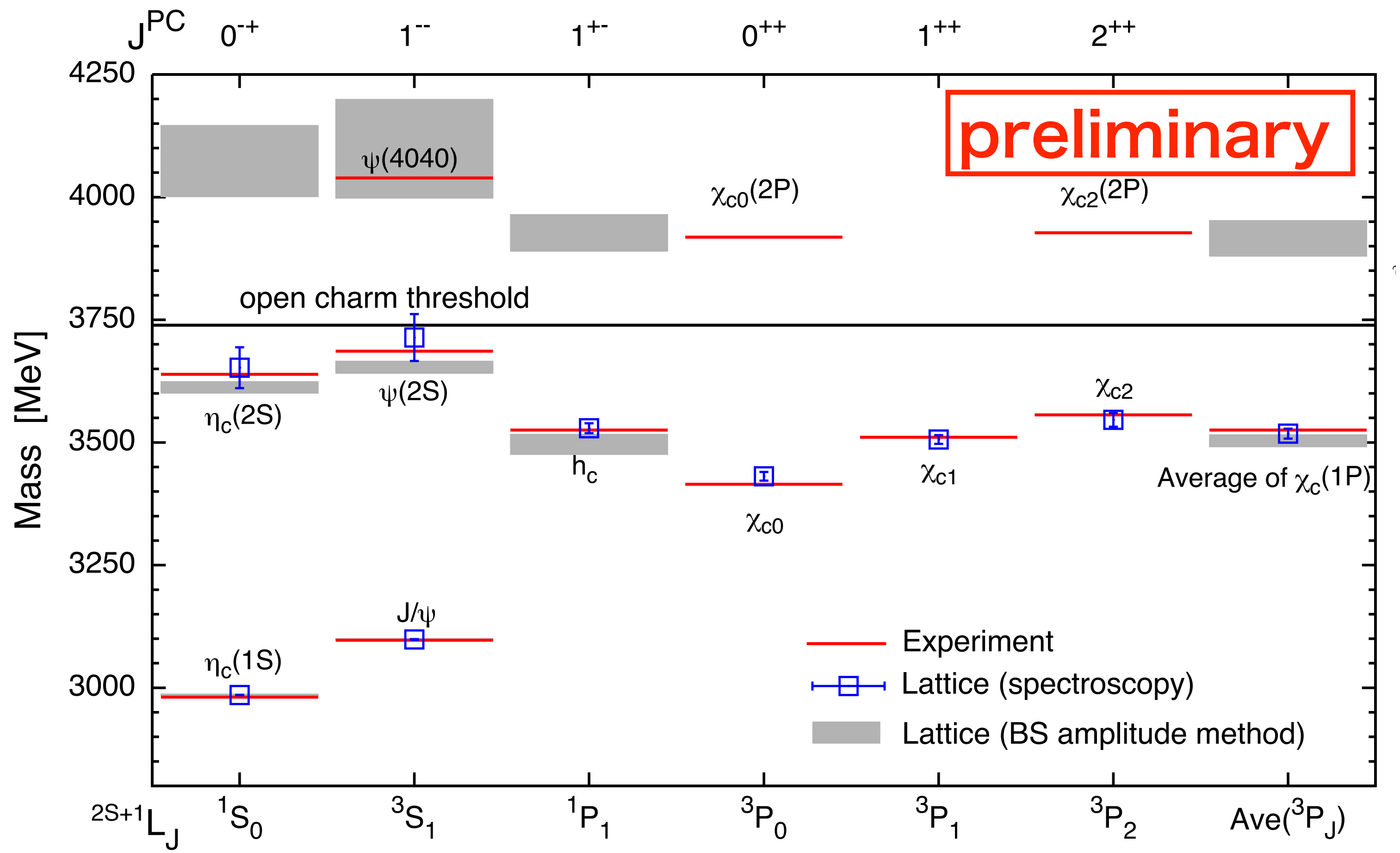
Non-relativistic potential (NRp) model
T.Barnes, S. Godfrey, E.S. Swanson, PRD72 (2005) 054026



- Short range, but non-point like, repulsive interaction
- A difference from phenomenological potential appears in the spin-dependent interaction.

Charmonium mass spectrum

Once the charmonium potential obtained, we can simply solve Schrödinger equation.



The charmonium potential from BS wavefucntion describe well the charmonium spectrum below open charm threshold.

Summary

✦We have derived $q\bar{q}$ interquark potential from the BS wave function in Quenched QCD simulation and 2+1 flavor dynamical lattice QCD simulation with almost physical quark masses.

✓spin-independent $q\bar{q}$ potential from BS wave function smoothly approaches the static $q\bar{q}$ potential from Wilson loop.

✓Attractive interaction in spin-spin potential is indued by the finite quark mass effects.

✓The spin-independent charmonium potential obtained from the BS wave function resembles the one used in the NRp model.

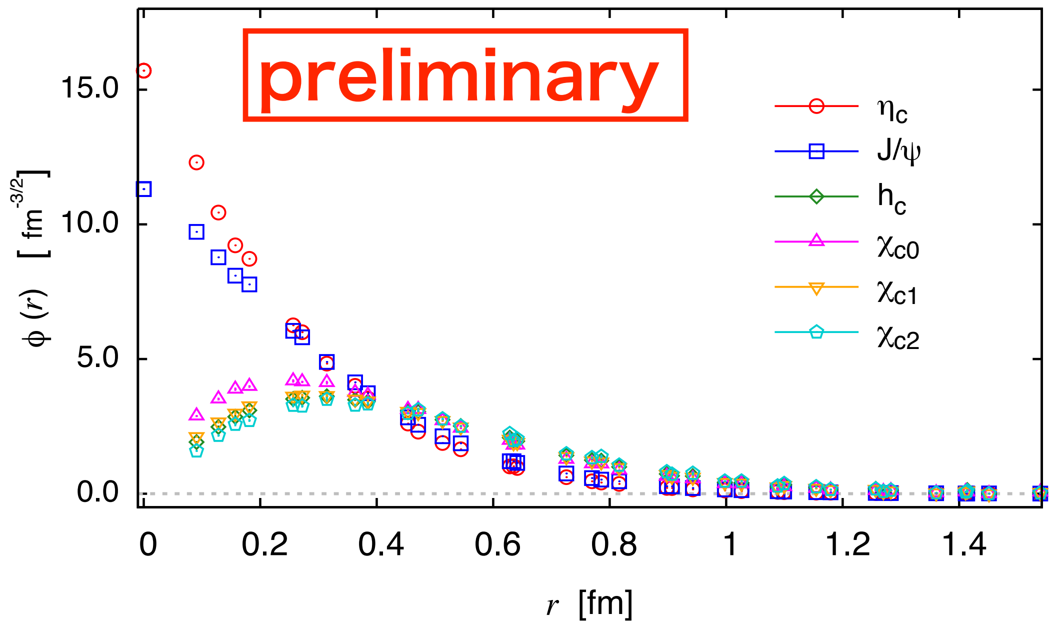
✓Our charmonium potential well describes charmonium spectrum.

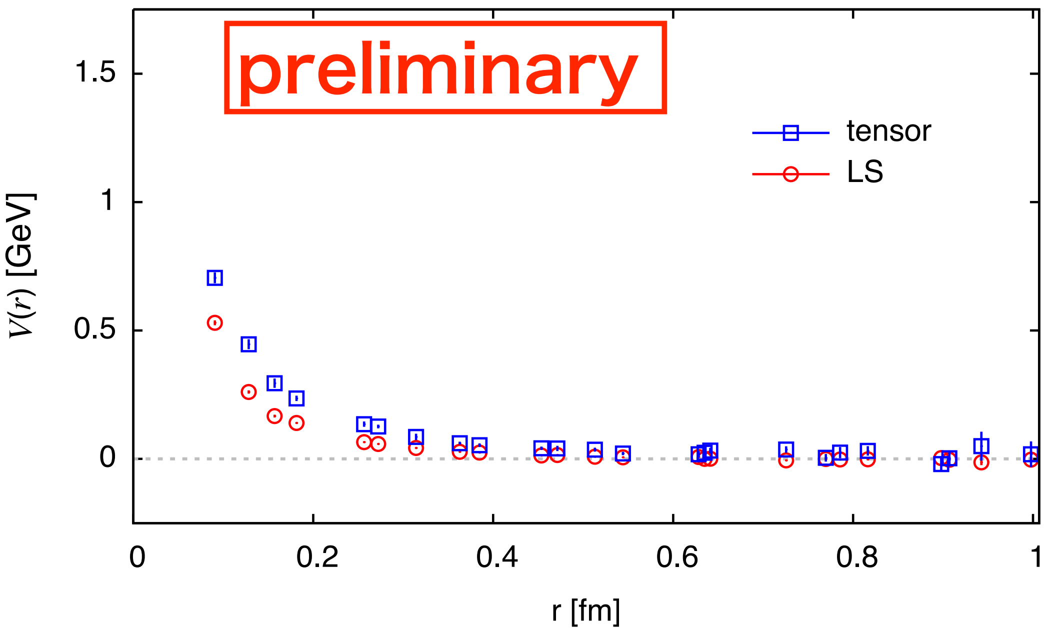
✦Future perspective

✓Other spin-dependent potential: tensor and LS force.

Thank you for your attention.

Other spin-dependent potential





- P-wave states from sine-source.

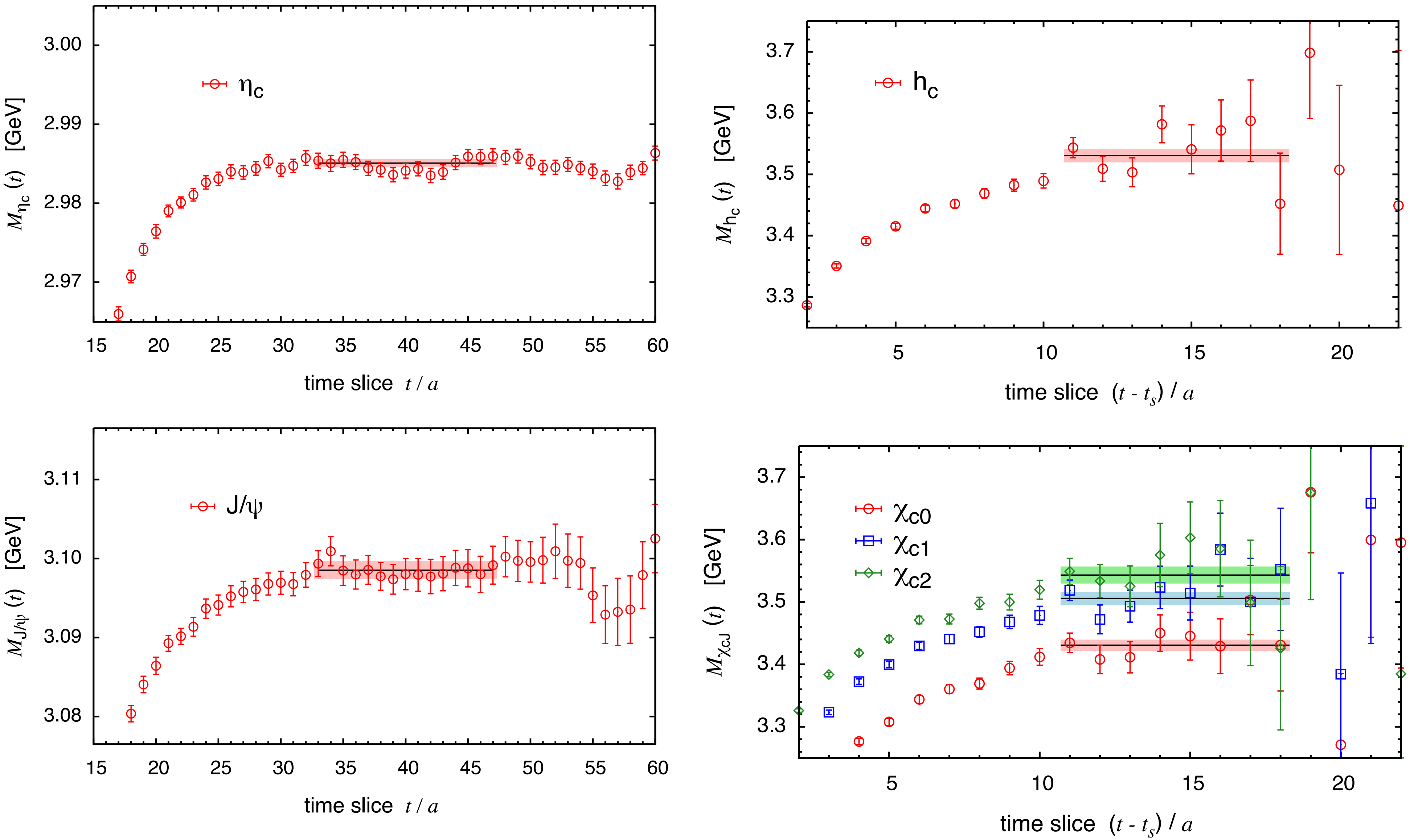
$$f_i^{T_1}(\mathbf{r}) = \frac{1}{2}\left(e^{-ip_i r_i} - e^{ip_i r_i}\right)$$

- irreducible representations can be obtained by appropriate projection.

- **short-range repulsive** interaction.

- qualitatively consistent with Wilson approach and phenomenology.

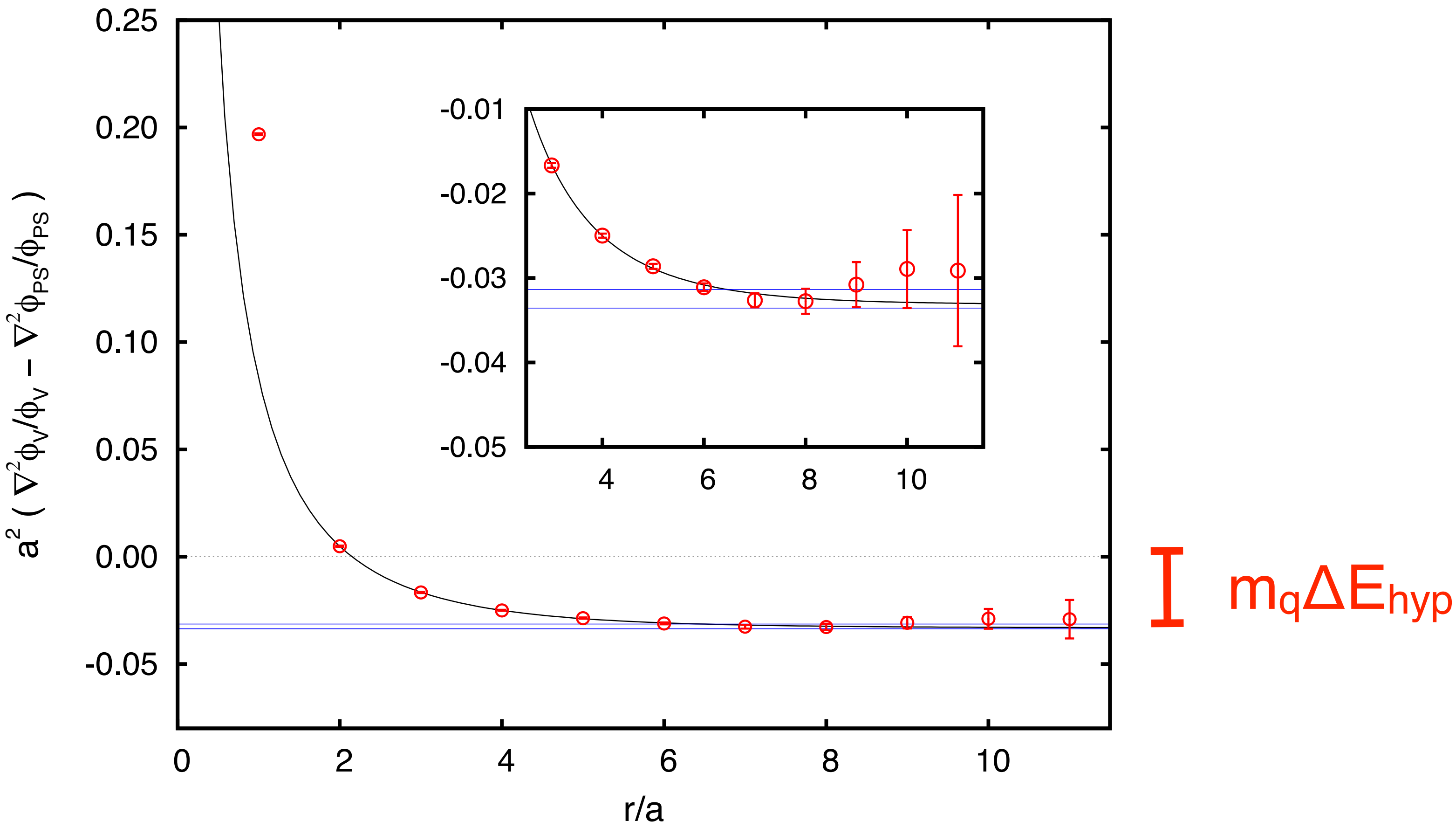
Two point correlation function



Determination of kinetic quark mass

T. Kawanai and S. Sasaki, PRL. 107, 091601 (2011)

$$m_q = \lim_{r \to \infty} \frac{-1}{\Delta E_{\mathrm{hyp}}} \left(\frac{\nabla^2 \phi_{\mathrm{V}}(r)}{\phi_{\mathrm{V}}(r)} - \frac{\nabla^2 \phi_{\mathrm{PS}}(r)}{\phi_{\mathrm{PS}}(r)} \right)$$



RHQ action

✦Heavy quark mass introduces discretization errors of **$O((ma)^n)$**

✓ At charm quark mass, it becomes severe: **m_c ~ 1.5 GeV** and **1/a ~ 2 GeV**, then **$m_c a$ ~O(1)**.

✦The Fermilab group proposed relativistic heavy quark action (RHQ) approach where all $O((ma)^n)$ errors are removed by the appropriate choice of **m_0, ξ, r_s, C_B, C_E** .

A. X. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, (1997)

$$S_{\rm lat} = \sum_{n,n'} \bar{\psi}_{n'} (\gamma^0 D^0 + \zeta \vec{\gamma} \cdot \vec{D} + m_0 a - \frac{r_t}{2} a (D^0)^2 - \frac{r_s}{2} a (\vec{D})^2 + \sum_{i,j} \frac{i}{4} c_B a \sigma_{ij} F_{ij} + \sum_i \frac{i}{2} c_E a \sigma_{0i} F_{0i})_{n',n} \psi_n$$

We take the Tsukuba procedure in our study.

S. Aoki, Y. Kuramashi, and S.-i. Tominaga, Prog. Theor. Phys. 109, 383 (2003)

Y. Kayaba et al. [CP-PACS Collaboration], JHEP 0702, 019 (2007).

Tuning RHQ parameters

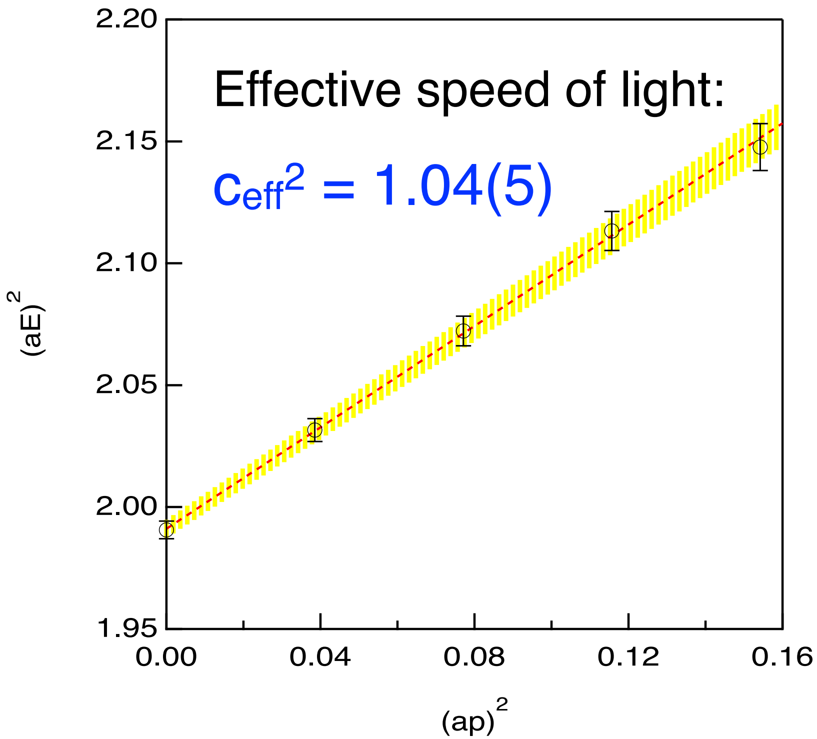
Y. Namekawa et al. [CP-PACS Collaboration], arXiv:1104.4600

RHQ action (Tsukuba-type) has 5 parameters $\varkappa_c$, ν, r_s, c_B, c_E

- The parameters r_S, c_B and c_E are determined by one-loop perturbation.
- For ν, we use a non-perturbatively determined value.

Dispersion relation: $$E^2(\mathbf{p}^2) = M^2 + c_{\text{eff}}^2|\mathbf{p}|^2$$

- $\varkappa_c$ is chosen to reproduce the experimental spin-averaged mass of 1S charmonium states M_{exp} = 3.0678(3) GeV.

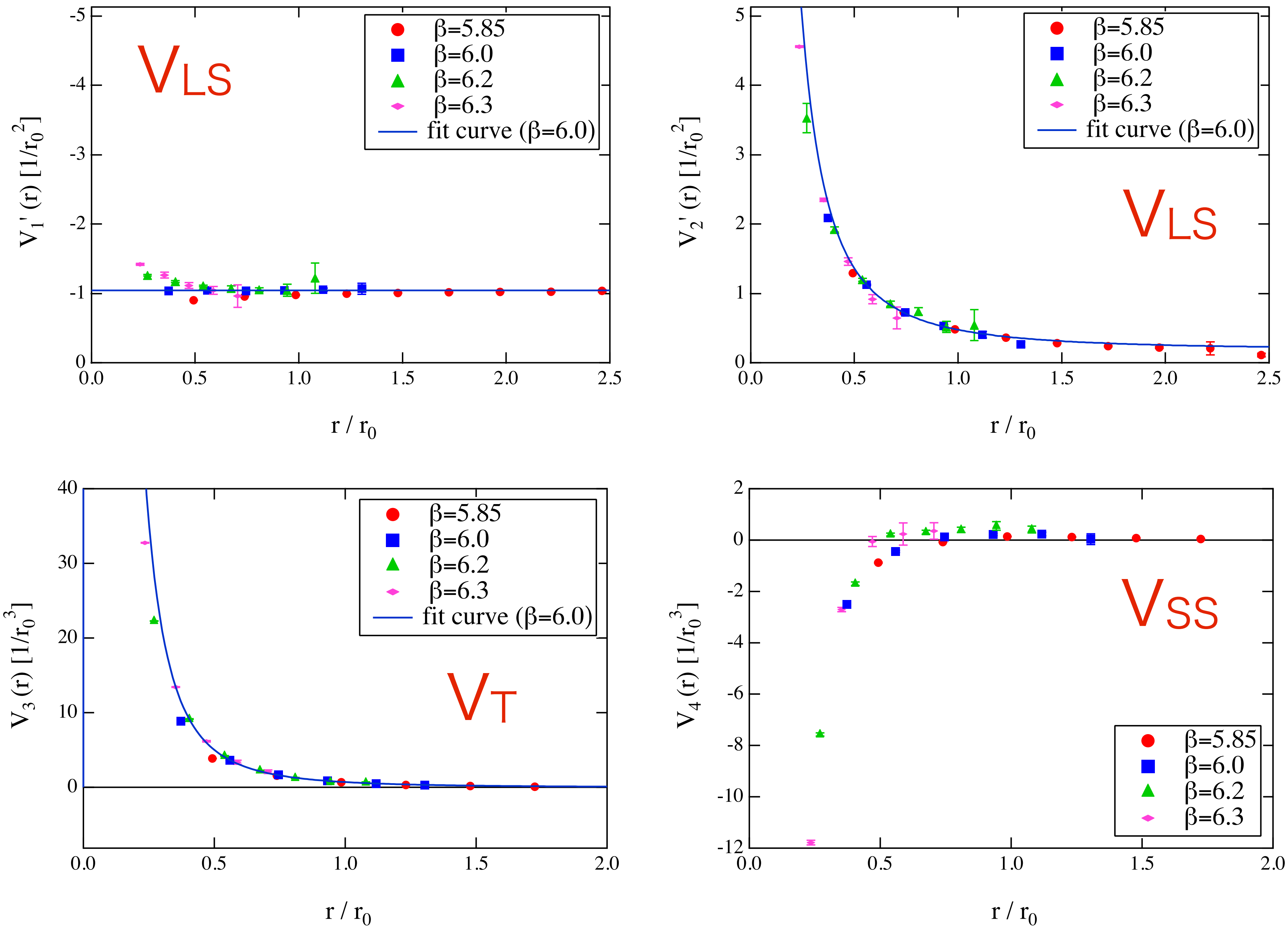


$\varkappa$	ν	r	c	c
0.10819	1.2153	1.2131	2.0268	1.7911

m_{ave} = 3.069(2) GeV,
m_{hyp} = 0.1110(17) GeV

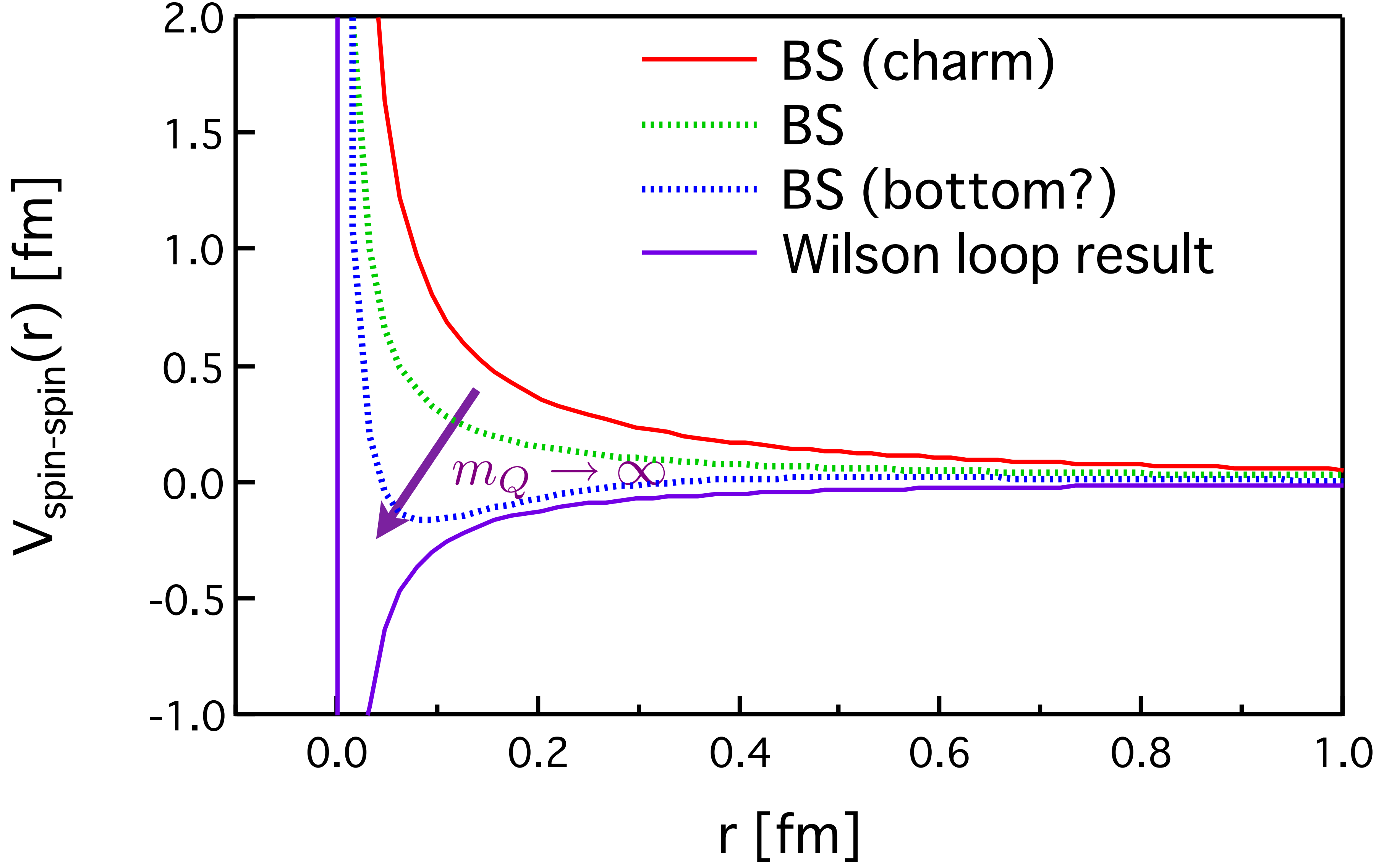
cf. m_{hyp}(exp) = 0.1165(12) GeV

Spin-dependent potentials



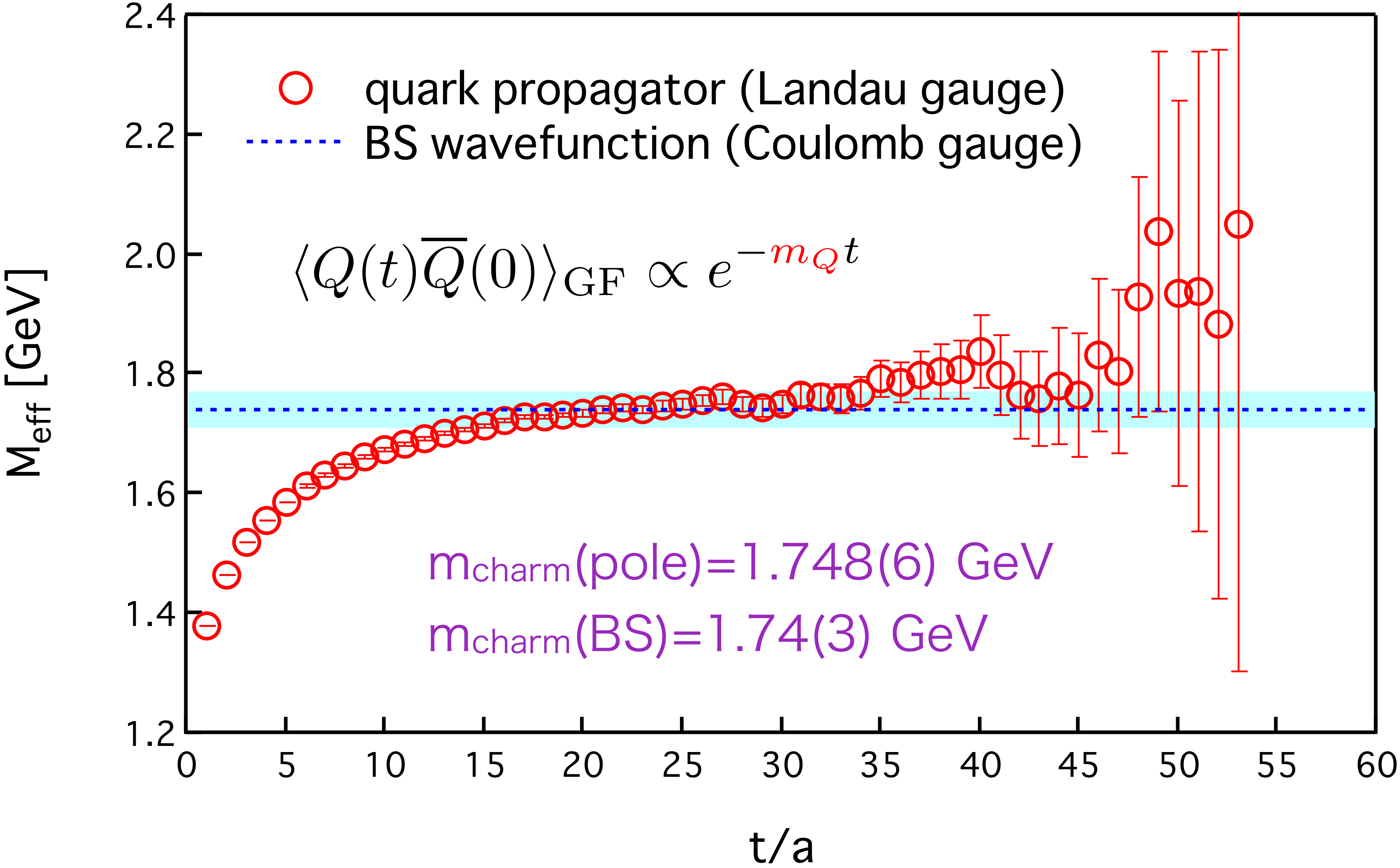
Koma Koma Wittig 05, Koma Koma 06

Conjecture



2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
$V_{\text{spin-spin}}(r)$ [fm]
BS (charm)
BS
BS (bottom?)
Wilson loop result
$m_Q \to \infty$
0.0
0.2
0.4
0.6
0.8
1.0
r [fm]

What does "quark mass" correspond to ?



Spatial information = Temporal information