Renormalized Light Front Hamiltonian in the Pauli-Villars Regularization*

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In the present report we have constructed the renormalized light front (LF) Hamiltonian for the $\lambda\varphi^4$ model in (2+1)-dimensional space-time. We have found the explicit expression for the counterterm, necessary for the renormalization, using the Pauli-Villars (PV) regularization. To do this we compare the diagrams of the covariant perturbation theory in Lorentz coordinates with the analogous diagrams of the perturbation theory generated by the LF Hamiltonian which has also the cutoff in the momentum p_- ($|p_-| \geqslant \delta > 0$). We show that both perturbation theories can be described by the same set of diagrams, with the values of the compared diagrams coinciding in the limit $\delta \to 0$. Then we renormalize the LF Hamiltonian by the counterterm found in the calculation of the divergent part of the corresponding diagram in the covariant perturbation theory in Lorentz coordinates.

Furthermore we have taken into account the possibility

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of the spontaneous symmetry breaking in this model and obtained the LF Hamiltonians corresponding to two different vacua. We arrive at these LF Hamiltonians by considering the limit transition from the theories quantized on the spacelike planes approaching the LF. It is possible to describe the vacuum on these planes using the Gaussian approximation. The Hamiltonians obtained with this approximation still require ultraviolet (UV) renormalization. And the above-mentioned comparison of perturbative theories, generated by these LF Hamiltonians, and the covariant perturbation theory in Lorentz coordinates allows to renormalize both of these Hamiltonians in the PV regularization.

Hamiltonian formulation on the light front [1] leads in quantum field theory to simple description of the vacuum state, that simplifies the nonperturbative Hamiltonian approach to the bound state and mass spectrum problem [2, 3]. The LF can be defined by the equation $x^+ = 0$ where $x^+ = \frac{x^0+x^1}{\sqrt{2}}$ plays the role of time ($x^0, x^1, x^\perp$ are Lorentz coordinates with $x^\perp$ denoting the remaining spatial coordinates). The role of usual space coordinates is played by the LF coordinates $x^- = \frac{x^0-x^1}{\sqrt{2}}$, $x^\perp$.

The generator P_- of translations in x^- is kinematical [1] (i.e. it is independent of the interaction and quadratic in fields, as a momentum in a free theory). It is nonnegative ($P_- \geqslant 0$) for quantum states with nonnegative mass squared. So the state with the minimal eigenvalue $p_- = 0$ of the momentum operator P_- can describe (in the case of the absence of the massless particles) the vacuum state, and it is also the state minimizing the P_+ in Lorentz invariant theory. Furthermore it is possible to introduce the Fock space on this vacuum and formulate in this space the eigenvalue problem for the operator P_+ (which is the LF Hamiltonian) and find the spectrum of mass m in subspaces with fixed values of the momenta $p_-, p_\perp$ [2,4]:

$$P_+|p_-, p_\perp\rangle = \frac{m^2 + p_\perp^2}{2p_-}|p_-, p_\perp\rangle. \tag{1}$$

The theory on the LF has the singularity at $p_- = 0$, and the simplest regularization is the cutoff $p_- \geqslant \delta > 0$. Other convenient translationally invariant regularization, that can treat also zero ($p_- = 0$) modes of fields, is the cutoff $|x^-| \leqslant L$ plus periodic boundary conditions for fields. This regularization discretizes the momentum p_- ($p_- = \frac{\pi n}{L}$, $n = 0, 1, 2, ...$) and clearly separates zero and nonzero modes. It is the so-called "Discretized Light Cone Quantization" (DLCQ).

All used regularizations of the singularity at $p_- = 0$ are not Lorentz invariant. This can lead to nonequivalence of the results obtained with the LF and the conventional formulation in Lorentz coordinates. It was shown in papers [4, 18, 19] that some diagrams of the perturbation theory, generated by the LF Hamiltonian, and corresponding diagrams of the conventional perturbation theory in Lorentz coordinates can differ. In papers [4,19] it was found how to restore the equivalence of the LF and conventional perturbation theories in all orders in the coupling constant by addition of new (in particular, nonlocal) terms to the canonical LF Hamiltonian. These terms must remove the above-mentioned differences of diagrams.

The method of the restoration of the equivalence between the LF and conventional perturbation theories, found in [4, 19], was applied to constructing of correct renormalized LF Hamiltonian for (3+1)-dimensional Quantum Chromodynamics [4, 20]. In the papers [4, 21] this method was applied to massive Schwinger model ((1+1)-dimensional Quantum Electrodynamics) and correct LF Hamiltonian was constructed. This Hamiltonian was used for numerical calculations of the mass spectrum [23], and the obtained results well agree with lattice calculations in Lorentz coordinates [24] for all values of the coupling (including very large ones).

The number of the above-mentioned new terms, which must be added to canonical LF Hamiltonian, and counterterms, necessary for the ultraviolet (UV) renormalization, depends essentially on the regularization scheme. For the case of QCD(3+1) [4, 20] in the light-cone gauge one gets the finite number of these terms only in the regularization of the Pauli-Villars (PV) type [25]. This regularization violates gauge invariance. However it was shown in [4, 20] that gauge invariance can be restored in renormalized LF theory with proper choice of coefficients before these new terms and counterterms. On the other side, the PV regularization involves the introduction of auxiliary ghost fields (with the large mass playing the role of the regularization parameter). These ghost fields generate the states with the indefinite metric, and one has to deal with such states in the nonperturbative (e.g. variational) calculations using the LF Hamiltonian. Attempts to do these calculations were made in papers [26–29] for nongauge theories. It is important to generalize this for gauge theories like QCD (e.g. for the formulation [4, 20], where the PV regularization introduces ghost gauge fields).

The question of using the PV regularization in the LF Hamiltonian approach isn't studied sufficiently. So we address this question in the present report. For the investigation of the problem we start with the construction of the renormalized LF Hamiltonian in the PV regularization for the scalar field theory in the (2+1)-dimensional space-time.

We compare the perturbation theory generated by the LF Hamiltonian and covariant perturbation theory in Lorentz coordinates by the method of papers [4, 19]. This allows to find the counterterm necessary for the renormalization of the LF Hamiltonian by the calculation of the divergent part of the corresponding diagram in the covariant perturbation theory in Lorentz coordinates. Let us note that there is the possibility to carry out the renormalization directly in x^+-ordered perturbation theory [9]. However the renormalized theory on the LF at that approach can, in principle, turn out to be nonequivalent to the original Lorentz covariant theory due to possible differences between finite diagrams generated by the LF Hamiltonian and corresponding to them covariant diagrams.

To take into account the different vacua appearing in considered model due to the spontaneous symmetry breaking we consider the transition to the LF Hamiltonian from the theories quantized on the spacelike planes approaching to the LF. In these theories it is possible to determine the true vacuum using the Gaussian approximation [39]. Accordingly we get two different expressions for the LF Hamiltonian for the cases without and with the spontaneous symmetry breaking. To clarify the way of the construction of the LF Hamiltonian we start from the Lagrangian formulation in the coordinates y^μ approaching the LF coordinates $x^\mu = (x^+, x^-, x^\perp)$:

$$y^0 = x^+ + \frac{\eta^2}{2} x^-, \ y^1 = x^-, \ y^\perp = x^\perp, \tag{2}$$

where $\eta > 0$ is a small parameter. The Lagrangian density of the conventional scalar field theory can be written in these coordinates as follows [4, 22]:

$$L(y) = \partial_0\varphi(y)\partial_1\varphi(y) + \frac{\eta^2}{2}(\partial_0\varphi(y))^2 - \frac{1}{2}(\partial_\perp\varphi(y))^2 - \\ - \frac{m_B^2}{2}(\varphi(y))^2 - \lambda(\varphi(y))^4, \tag{3}$$

where m_B is a mass parameter (the bare mass). The equation $y^0 = 0$ defines the space-like plane, so the canonical quantization on this plane is equivalent to the ordinary quantization on the $x^0 = 0$ plane in Lorentz coordinates. From the Lagrangian (3) we obtain the following Hamiltonian density:

$$\mathcal{H} = \frac{(\Pi - \partial_1\varphi)^2}{2\eta^2} + \frac{1}{2}(\partial_\perp\varphi)^2 + \frac{m_B^2}{2}\varphi^2 + \lambda\varphi^4, \tag{4}$$

where $\Pi(y)$ is the momentum canonically conjugated to the field $\varphi(y)$, the $\Pi(y) = \eta^2\partial_0\varphi(y) + \partial_1\varphi(y)$.

Further we consider the transition from the theories with the Hamiltonians (4) taken at different values of the parameter η, to the LF Hamiltonian in the limit $\eta \to 0$. This gives a possibility to take into account (before reaching the LF) two different vacua existing in this model. Indeed, at $\eta > 0$ we still can use known methods [44] for the description of the quantum vacuum. In particular we can apply the variational method [39] to find the minimum of the vacuum average of the Hamiltonian density. This method uses different Fock vacua and Bogolyubov transformations from one Fock vacuum to another (this method corresponds to the "Gaussian" variational approximation to the vacuum wave function). Let us apply this method to the Hamiltonian density (4). We introduce the following expressions for φ and Π (at $y^0 = 0$):

$$\varphi(y) = \frac{1}{2\pi}\int \frac{dk_1 dk_\perp}{\sqrt{2\omega(k)}}\Big(a(k) + a^+(-k)\Big)e^{-ik\cdot y} + \varphi_0, \quad (5)$$

$$\Pi(y) = \frac{-i}{2\pi}\int dk_1 dk_\perp \sqrt{\frac{\omega(k)}{2}}\Big(a(k) - a^+(-k)\Big)e^{-ik\cdot y}, \quad (6)$$

where $k = (k_1, k_\perp)$ and $k\cdot y = k_1 y^1 + k_\perp y^\perp$. Here we define the creation and annihilation operators corresponding to the varying Fock vacua $|0\rangle$:

$$a(k)|0\rangle = 0,\ \ [a(k), a^+(k')] = \delta^{(2)}(k-k'),\ \ [a(k), a(k')] = 0. (7)$$

The parameters $\omega(k)$ and φ_0 in (5), (6) play the role of variational parameters (the φ_0 doesn't depend on k). The variation of the parameters $\omega(k)$ and φ_0 is equivalent to linear transformations of operators a, a^+ that is equivalent to the variation of the vacuum state vector $|0\rangle$ in the assumed approximation. We implicitly suppose that the integration domain in the k_1 is limited by the cutoff $|k_1| \geqslant \delta$. It is related to the necessity to get in the limit $\eta \to 0$ the theory on the LF which is regularized by the cutoff $|k_-| \geqslant \delta$. Further we substitute the expressions (5) and (6) into the Hamiltonian (4) and use the equalities (7). We obtain the following result:

$$\langle 0|\mathcal{H}|0\rangle =$$

$$= \frac{1}{16\pi^2\eta^2} \int dk_1 dk_\perp \left(\omega(k) + \frac{k_1^2 + \eta^2(m_B^2 + k_\perp^2 + 12\lambda\varphi_0^2)}{\omega(k)} \right) +$$

$$+ \frac{m_B^2}{2}\varphi_0^2 + \lambda\varphi_0^4 + 3\lambda \left(\frac{1}{8\pi^2} \int \frac{dk_1 dk_\perp}{\omega(k)} \right)^2 . \quad (8)$$

This expression contains divergent integrals. So we introduce the regularization of these integrals by a cutoff in the momenta. Varying the quantity (8) w.r.t. $\omega(k)$ and equating the result to zero we get

$$\frac{1}{16\pi^2\eta^2} \left(1 - \frac{k_1^2 + \eta^2 \left(m_B^2 + k_\perp^2 + 12\lambda\varphi_0^2 \right)}{\omega^2(k)} - \right.$$

$$\left. - \frac{3\lambda\eta^2}{2\pi^2\omega^2(k)} \int \frac{dq_1 dq_\perp}{\omega(q)} \right) = 0. \quad (9)$$

Using the definition

$$m^2 \equiv m_B^2 + 12\lambda\varphi_0^2 + \frac{3\lambda}{2\pi^2} \int \frac{dq_1 dq_\perp}{\omega(q)}, \quad (10)$$

we obtain

$$\omega^2(k) = k_1^2 + \eta^2(m^2 + k_\perp^2). \quad (11)$$

Below we show that m^2 can be chosen to be finite in the regularization removing limit.

The variation of Eq. (8) with respect to φ_0 gives the equation

$$\varphi_0 \left(m_B^2 + 4\lambda\varphi_0^2 + \frac{3\lambda}{2\pi^2} \int \frac{dk_1 dk_\perp}{\omega(k)} \right) = 0 \quad (12)$$

which can be rewritten in the following form (here we use the definition (10)):

$$\varphi_0(m^2 - 8\lambda\varphi_0^2) = 0. \quad (13)$$

The solutions of this equation are $\varphi_0 = 0$ and $\varphi_0^2 = \frac{m^2}{8\lambda}$. One can check that these solutions correspond to the minimum of the $\langle 0|\mathcal{H}|0\rangle$ at $m^2 > 0$. Let us choose the bare mass m_B so that the parameter m be finite:

$$m_B^2 = -\frac{3\lambda}{2\pi^2}\int \frac{dk_1 dk_\perp}{\sqrt{k_1^2 + \eta^2 k_\perp^2}} + r, \tag{14}$$

where the r is finite in the regularization removing limit. Then the Eq. (10) takes the following form:

$$m^2 = 12\lambda\varphi_0^2 + \frac{3\lambda}{2\pi^2}\int dk_1 dk_\perp \left(\frac{1}{\sqrt{k_1^2 + \eta^2(k_\perp^2 + m^2)}} - \frac{1}{\sqrt{k_1^2 + \eta^2 k_\perp^2}} \right) + r. \tag{15}$$

The integral in the Eq. (15) is convergent and needs no regularization. Then by the change of the variable $k_1 \rightarrow \eta k_1$ one can reduce this integral to a simpler form for which the result (in the $\delta \rightarrow 0$ limit) is already known and equals to $-2\pi m$. Let us define $\mu = \frac{m}{\lambda}$ and $\rho = \frac{r}{\lambda^2}$. Then the Eq. (15) can be rewritten in the following form

$$\mu^2 + \frac{3\mu}{\pi} - \frac{12\varphi_0^2}{\lambda} - \rho = 0. \tag{16}$$

We denote the solution of this equation for the case $\varphi_0 = 0$ by $\mu_1(\rho)$, and for the case $\varphi_0^2 = \frac{m^2}{8\lambda}$ by $\mu_2(\rho)$. These solutions are shown in Fig. 1. The curves 1 and 2 show the solutions $\mu_1(\rho)$ and $\mu_2(\rho)$ correspondingly. We consider these solutions at $\mu > 0$. For any ρ in the domain $0 < \rho \leqslant \frac{9}{2\pi^2}$ (the largest value corresponds to the rightmost point of the curve 2) there are several distinct values

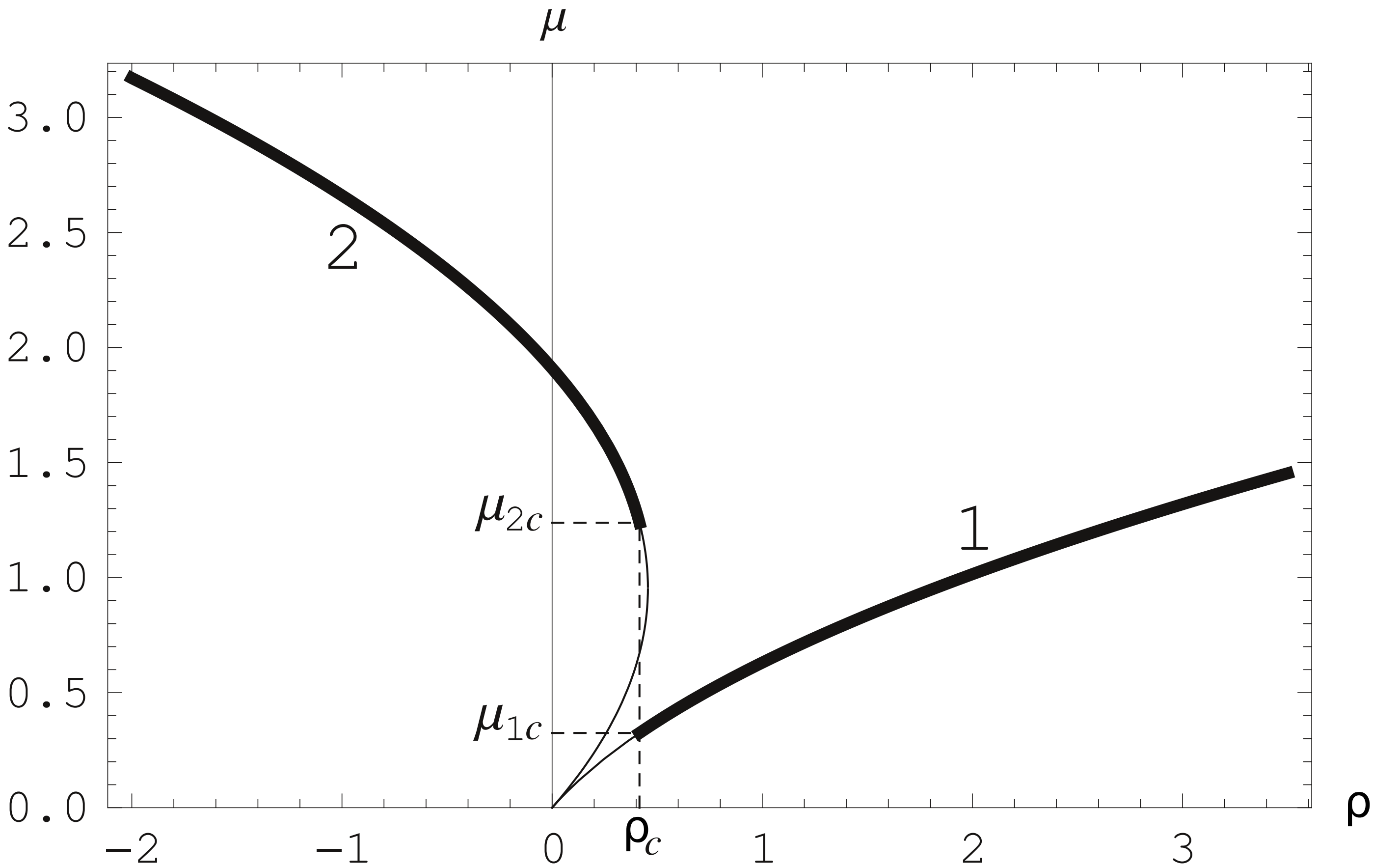


Fig. 1 *The dependence of* $\mu = \frac{m}{\lambda}$ *on* $\rho = \frac{r}{\lambda^2}$*. Quantities* r *and* m *are defined by Eqs. (14) and (15). The curves 1 and 2 represent the solutions* $\mu_1(\rho)$ *and* $\mu_2(\rho)$ *of Eq. (16). The bold curves show where these solutions correspond to the minimum of vacuum energy density (8). The* ρ_c *is the point where this minimum is common for both solutions. The* μ_{1c}*,* μ_{2c} *are limit values of* $\mu_1(\rho)$*,* $\mu_2(\rho)$ *as we approach to* ρ_c *along bold parts of curves. We find from expressions (17), (18)* $\rho_c \simeq 0.4157$*,* $\mu_{1c} \simeq 0.3248$ *and* $\mu_{2c} \simeq 1.2385$*.*

of μ on the branches of curves with $\mu > 0$. The direct evaluation of the quantity (8) shows that its minimum corresponds to points on the bold curves in Fig. 1. Indeed, we consider the r.h.s. of Eq. (8) for the curve 1 and upper part of the curve 2 at common value of ρ and take the difference of these expressions. Using Eq. (10) and Eq. (14) we find the following finite result for this difference in the regularization removing limit[1]:

$$\frac{\lambda^3}{2}\left(\frac{1}{16}\left(2\mu_1^4+\mu_2^4\right)+\frac{1}{3\pi}\left(\mu_1^3-\mu_2^3\right)+\frac{\rho}{12}\left(\mu_2^2-\mu_1^2\right)\right). \quad (17)$$

We estimate this expression numerically at different values of ρ taking into account the explicit dependence of μ_1, μ_2 on ρ according to Eq. (16). We find that this expression is positive at $\rho < \rho_c$ where ρ_c is the value of ρ for which the expression (17) is equal zero. For $\rho_c < \rho \leqslant \frac{9}{2\pi^2}$ this expression is negative. The μ_{1c} and μ_{2c} are the limit values of $\mu_1(\rho)$ and $\mu_2(\rho)$ in the limit transition $\rho \to \rho_c$ along bold parts of curves 1 and 2 respectively. Numerically we find $\rho_c \simeq 0.4157$, $\mu_{1c} \simeq 0.3248$ and $\mu_{2c} \simeq 1.2385$.

Analogous comparison for corresponding lower and upper points on the curve 2 gives the following expression:

$$\frac{\lambda^3}{2}\left(\frac{1}{16}\left(\mu_2^4-\bar{\mu}_2^4\right)+\frac{1}{3\pi}\left(\bar{\mu}_2^3-\mu_2^3\right)+\frac{\rho}{12}\left(\mu_2^2-\bar{\mu}_2^2\right)\right), \quad (18)$$

where $\bar{\mu}_2$ denotes lower point on the curve 2. Analogously we find numerically that this expression is positive at $\rho < \frac{9}{2\pi^2}$.

Thus we prove that the minimum of vacuum energy corresponds to the points on the bold curves. Therefore we have the following inequalities limiting the parameters λ, $m_1 \equiv \lambda\mu_1$, $m_2 \equiv \lambda\mu_2$ which one should use in calculations with our Hamiltonian:

$$\frac{\lambda}{m_1} < \frac{1}{\mu_{1c}}, \quad \text{i.e.} \quad \mu_1 > \mu_{1c} \quad \text{for} \quad \varphi_0 = 0;$$

$$\frac{\lambda}{m_2} < \frac{1}{\mu_{2c}}, \quad \text{i.e.} \quad \mu_2 > \mu_{2c} \quad \text{for} \quad \varphi_0^2 = \frac{m_2^2}{8\lambda}. \quad (19)$$

[1]At the first step we represent the expression for the integral in Eq. (10) through m_B^2, m^2, φ_0^2, λ and then use this representation in Eq. (8).

Let us apply these results to the Hamiltonian (4). We define the $\tilde{\varphi} = \varphi - \varphi_0$ and rewrite the Hamiltonian in the normal ordered form w.r.t. those operators $a(k)$ and $a^+(k)$ which correspond to the found vacuum. Owing to Eq. (10) the resulting expression becomes dependent on the mass parameters m_1, m_2 only. These parameters correspond to solutions shown in Fig. 1. In the case $\varphi_0 = 0$ we get the following Hamiltonian (throwing out the constant term $\langle 0|\mathcal{H}|0\rangle$):

$$H = \\ =: \int dy^1 dy^\perp \left(\frac{(\Pi - \partial_1\tilde{\varphi})^2}{2\eta^2} + \frac{1}{2}(\partial_\perp\tilde{\varphi})^2 + \frac{m_1^2}{2}\tilde{\varphi}^2 + \lambda\tilde{\varphi}^4 \right) :, \quad (20)$$

where the symbol ": :" denotes the normal ordering. Analogously, in the case $\varphi_0^2 = \frac{m_2^2}{8\lambda}$ we obtain

$$H =: \int dy^1 dy^\perp \left(\frac{(\Pi - \partial_1\tilde{\varphi})^2}{2\eta^2} + \frac{1}{2}(\partial_\perp\tilde{\varphi})^2 + \frac{m_2^2}{2}\tilde{\varphi}^2 + \\ + 4\lambda\varphi_0\tilde{\varphi}^3 + \lambda\tilde{\varphi}^4 \right) : . \quad (21)$$

Here the terms linear in the fields $\tilde{\varphi}, \Pi$ are discarded because they don't contribute to the integral (21) due to the condition $|k_1| \geqslant \delta > 0$ proposed earlier for the integration in formulae (5) and (6) (see the text before Eq. (8)).

To find the form of the LF Hamiltonian let us consider the eigenvalue problem:

$$H|f\rangle = E|f\rangle, \quad (22)$$

where the H is the Hamiltonian (20) or (21). One can expand these Hamiltonians in powers of the parameter η.

We separate the η^{-2} term of these Hamiltonians and write them in the form

$$H = \frac{H_0}{\eta^2} + H_2, \tag{23}$$

where

$$H_0 = 2\int_{-\infty}^{0} dk_1 \int dk_\perp |k_1|\, a^+(k)\, a(k). \tag{24}$$

In the derivation of this expression we use the equality $\omega(k) = |k_1| + \frac{\eta^2(m^2+k_\perp^2)}{|2k_1|} + O(\eta^4)$ following from the Eq. (11). Let us write the following asymptotic expansions:

$$E(\eta) = \frac{E_0}{\eta^2} + E_2 + \cdots , \qquad |f(\eta)\rangle = |f_0\rangle + \eta^2 |f_2\rangle + \cdots \tag{25}$$

In the lowest order approximation w.r.t. η we obtain the equations:

$$H_0|f_0\rangle = E_0|f_0\rangle\,, \quad (H_0 - E_0)|f_2\rangle + (H_2 - E_2)|f_0\rangle = 0. \tag{26}$$

In the limit $\eta \to 0$ we have $x^1 \to x^-$, $|f\rangle \to |f_0\rangle$, i.e. the states $|f_0\rangle$ form the state space on the LF. To get finite eigenvalues for the LF Hamiltonian we demand $E_0 = 0$. Then from the Eq. (24) and the first of Eqs. (26) we obtain

$$a(k)|f_0\rangle = 0 \quad \text{at} \quad k_1 < 0. \tag{27}$$

Therefore in the limit $\eta \to 0$ the LF state space is the subspace of our Fock space in which only the quanta with $k_- > 0$ are present. Now let us take the projection of the second of the Eqs. (26) on the subspace of states $|f_0\rangle$ and denote by $\mathcal{P}$ the projector on this subspace. Then we get the equation which can be interpreted as the eigenvalue equation for the LF Hamiltonian. So now we have

$$H_{LF} = \mathcal{P} H_2 \mathcal{P}. \tag{28}$$

Using the expressions (20) and (21) and the equality (27) we obtain the following results:

$$H_{LF} =: \int dx^- dx^\perp \left(\frac{1}{2} (\partial_\perp \Phi)^2 + \frac{m_1^2}{2} \Phi^2 + \lambda \Phi^4 \right) : \quad (29)$$

for the case $\varphi_0 = 0$ and

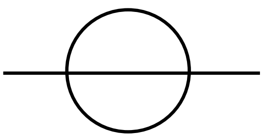
$$H_{LF} =$$
$$=: \int dx^- dx^\perp \left(\frac{1}{2} (\partial_\perp \Phi)^2 + \frac{m_2^2}{2} \Phi^2 + 4\lambda \varphi_0 \Phi^3 + \lambda \Phi^4 \right) : (30)$$

for the case $\varphi_0^2 = \frac{m_2^2}{8\lambda}$. Here we denote by $\Phi(x)$ the field on the LF,

$$\Phi(x) = \frac{1}{2\pi} \int_\delta^\infty \frac{dk_-}{\sqrt{2k_-}} \int dk_\perp \big(a(k_-, k_\perp)\, e^{-ik\cdot x} +$$
$$+ a^+(k_-, k_\perp)\, e^{ik\cdot x} \big), \quad (31)$$

where $k \cdot x = k_- x^- + k_\perp x^\perp$. The operators $a^+(k_-, k_\perp)$ and $a(k_-, k_\perp)$ play the role of creation and annihilation operators in the LF Fock space. They satisfy canonical commutation relations on the LF. Note that the integration range in Eq. (31) is limited from below by a small parameter δ which we implicitly use in the Eqs. (5), (6) (see the text before Eq. (8)).

For the renormalization of our model in the conventional Feynman covariant perturbation theory it is necessary to consider only one logarithmically divergent diagram (32). This diagram in the PV regularization has the following form:



$$= I(\vec{p}) = \frac{96 i \lambda^2}{(2\pi)^6} \int d^3\vec{k}_1\, d^3\vec{k}_2\, d^3\vec{k}_3\, \delta \left(\sum_{j=1}^{3} \vec{k}_j - \vec{p} \right) \times$$

$$\times \prod_{j=1}^{3} \left(\frac{1}{\vec{k}_j^2 + m^2} - \frac{1}{\vec{k}_j^2 + M^2} \right), \quad (32)$$

where the integration is over the Euclidean momenta, the parameter m is the mass parameter, the M is PV regularization parameter, the factor $96 = (4!)^2/6$ includes the symmetry coefficient $1/6$ of this diagram (the factor $(4!)^2$ is related to the definition of the coupling λ in the Lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{m^2}{2}\varphi^2 - \gamma\varphi - g\varphi^3 - \lambda\varphi^4. \quad (33)$$

To find the counterterm we need to calculate only the divergent (at $M \to \infty$) part of the $I(\vec{p})$. This divergent part can be evaluated as the divergent part of the $I(\vec{p})|_{\vec{p}=0}$. We obtain for $I(0)$ the following result:

$$I(0) = \frac{6i\lambda^2}{\pi^2} \ln \frac{M}{m} + O(1). \quad (34)$$

Using this result one can find the corresponding counterterm in the standard way [44]. The corresponding Lagrangian and Hamiltonian:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\varphi\partial^\mu\varphi - \tilde{m}^2\varphi^2\right) - \frac{1}{2}\left(\partial_\mu\varphi_g\partial^\mu\varphi_g - M^2\varphi_g^2\right) - $$
$$-\frac{3\lambda^2}{\pi^2}\left(\ln\frac{M}{\tilde{m}}\right)(\varphi + \varphi_g)^2 - \tilde{g}\,(\varphi + \varphi_g)^3 - \lambda\,(\varphi + \varphi_g)^4, \quad (35)$$

$$H_{LF} =: \int dx^- dx^\perp \Bigg(\frac{1}{2}(\partial_\perp\Phi)^2 - \frac{1}{2}(\partial_\perp\Phi_g)^2 + \frac{\tilde{m}^2}{2}\Phi^2 - \frac{M^2}{2}\Phi_g^2 + $$
$$+\frac{3\lambda^2}{\pi^2}\left(\ln\frac{M}{\tilde{m}}\right)(\Phi + \Phi_g)^2 + \tilde{g}\,(\Phi + \Phi_g)^3 + \lambda\,(\Phi + \Phi_g)^4 \Bigg) : . (36)$$

The theory with this LF Hamiltonian turns out to be equivalent in all orders of perturbation theory to the conventional renormalized covariant perturbation theory in Lorentz coordinates in the limit $\delta \to 0$ (and then $M \to \infty$).

Therefore the Eq. (36) gives the perturbatively renormalized LF Hamiltonian. We notice that the LF Hamiltonian (36) can be considered, at definite choice of its parameters, as one of the LF Hamiltonians (29), (30), correspondingly regularized and renormalized. The coupling constant $\tilde{g}$ can be identified with $4\lambda\varphi_0$ ($\varphi_0 = 0$ or $\varphi_0^2 = \frac{m_2^2}{8\lambda}$), while $\tilde{m}$ can be identified with m_1 or m_2 for the LF Hamiltonians (29) and (30) respectively. Thus we obtain the renormalized LF Hamiltonians for the cases without and with the spontaneous symmetry breaking. The parameters m_1, m_2 and λ satisfy the inequalities (19) in the Gaussian approximation. Nevertheless one can assume that these inequalities are approximately valid for the renormalized LF Hamiltonians in the PV regularization.

Having such LF Hamiltonians one can start nonperturbative calculation of the mass spectrum solving the eigenvalue problem:

$$(2P_- H_{LF} - P_\perp^2)|p_-, p_\perp\rangle = m^2 |p_-, p_\perp\rangle. \qquad (37)$$

This calculation could help to study the peculiarities of the PV regularization related to the presence of ghost fields and states with the indefinite metric. That is important for the application of this method to QCD.

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11.38.189.2014.

References

[1] Dirac, P.A.M.: Forms of relativistic dynamics. Rev. Mod. Phys. **21**(3), 392-398 (1949)

[2] Brodsky, S.J., Pauli, H.-C., Pinsky, S.S.: Quantum Chromodynamics and other field theories on the light cone. Phys. Rep. **301**(4-6), 299-486 (1998), arXiv:hep-ph/9705477 and references therein

[3] Bakker, B.L.G., Bassetto, A., Brodsky, S.J., Broniowski, W., Dalley, S., Frederico, T., Glazek, S.D., Hiller, J.R., Ji, C.-R., Karmanov, V., Kulshreshtha, D., Mathiot, J.-F. at al. Light-Front Quantum Chromodynamics: A framework for the analysis of hadron physics, arXiv:1309.6333 [hep-ph]

[4] Franke, V.A., Novozhilov, Yu.V., Paston, S.A., Prokhvatilov, E.V.: Quantization of Field Theory on the Light Front. In: Kovras, O. (ed.) Focus on quantum field theory, pp. 23-81. Nova science publishers, New York (2005), arXiv:hep-th/0404031

[5] Annenkova A.M., Prokhvatilov, E.V., Franke, V.A.: The solution of Schrödinger equation on the light front for Sine-Gordon model. Vestn. Leningrad. Univ. (Ser. 4. Fiz. Khim.) **18**, 80-83 (1985)

[6] Brodsky, S.J., Pauli, H.-C. Discretized light-cone quantization: Solution to a field theory in one space and one time dimension. Phys. Rev. D **32**(8), 2001-2013 (1985)

[7] Eller, T., Pauli, H.-C., Brodsky, S.J. Discretized light-cone quantization: The massless and the massive Schwinger model. Phys. Rev. D **35**(4), 1493-1507 (1987)

[8] Hornbostel, K., Brodsky, S.J., Pauli, H.-C.: Light-cone-quantized QCD in 1+1 dimensions. Phys. Rev. D **41**(12), 3814-3821 (1990)

[9] Brodsky, S.J., Roskies, R, Suaya, R: Quantum Electrodynamics and Renormalization Theory in the Infinite-Momentum Frame. Phys. Rev. D **8**(12), 4574-4594 (1973)

[10] Srivastava, P.P., Brodsky, S.J. Light-front-quantized QCD in the light-cone gauge: The doubly transverse gauge propagator. Phys. Rev. D **64**(4), 045006 (2001), arXiv:hep-ph/0011372v2

[11] Glazek, S.D., Wilson, K.G.: Renormalization of Hamiltonians. Phys. Rev. D **48**(8), 4214-4218 (1993)

[12] Glazek, S.D., Wilson, K.G.: Perturbative renormalization group for Hamiltonians. Phys. Rev. D **49**(12), 5863-5872 (1994)

[13] Wilson, K.G., Walhout, T.S., Harindranath, A., Zhang, W.-M., Perry, R.J., Glazek, S.D.: Nonperturbative QCD: A Weak-Coupling Treatment on the Light Front. Phys. Rev. D **49**(12), 6720-6766 (1994), arXiv:hep-th/9401153

[14] Glazek, S.D.: Renormalization of Hamiltonians in the Light-Front Fock Space, arXiv:hep-th/9706212

[15] Glazek, S.D.: Renormalization group and bound states. Acta Phys.Polon. B **39**, 3395-3421 (2008), arXiv:0810.5258 [hep-th]

[16] Glazek, S.D.: Dynamics of Effective Gluons. Phys. Rev. D **63**(11), 116006 (2001), arXiv:hep-th/0012012

[17] Glazek, S.D., Wieckowski, M.: Large-momentum convergence of Hamiltonian bound-state dynamics of effective fermions in quantum field theory. Phys. Rev. D **66**(1), 016001 (2002), arXiv:hep-th/0204171

[18] Burkardt, M., Langnau, A.: Hamiltonian formulation of (2+1)-dimensional QED on the light cone. Phys. Rev. D **44**(4), 1187-1197 (1991). Rotational invariance in light-cone quantization. Phys. Rev. D. **44**(12), 3857-3867 (1991)

[19] Franke, V.A., Paston, S.A.: Comparison of quantum field perturbation theory for the light front with the theory in Lorentz coordinates. Theor. Math. Phys. **112**(3), 1117-1130 (1997), arXiv:hep-th/9901110

[20] Paston, S.A., Prokhvatilov, E.V., Franke, V.A.: Constructing the light-front QCD Hamiltonian. Theor. Math. Phys. **120**(3), 1164-1181 (1999), arXiv:hep-th/0002062

[21] Paston, S.A., Prokhvatilov, E.V., Franke, V.A.: The Light-Front Hamiltonian Formalism for Two-Dimensional Quantum Electrodynamics Equivalent to the Lorentz-Covariant Approach. Theor. Math. Phys. **131**(1), 516-526 (2002), arXiv:hep-th/0302016

[22] Ilgenfritz, E.M., Franke, V.A., Paston, S.A., Pirner, H.J., Prokhvatilov, E.V.: Quantum fields on the light front, formulation in coordinates close to the light front, lattice approximation. Theor. Math. Phys. **148**(1), 948-959 (2006), arXiv:hep-th/0610020

[23] Paston, S.A., Prokhvatilov, E.V., Franke, V.A.: Calculation of the Mass Spectrum of QED-2 in Light-Front Coordinates. Phys. Atom. Nucl. **68**, 267-278 (2005), arXiv:hep-th/0501186

[24] Sriganesh, P., Hamer, C.J., Bursill, R.J.: New finite-lattice study of the massive Schwinger model. Phys. Rev. D **62**(3), 034508 (2000), arXiv:hep-lat/9911021

[25] Pauli, W., Villars, F.: On the Invariant Regularization in Relativistic Quantum Theory. Rev. Mod. Phys. **21**(3), 434-444 (1949)

[26] Brodsky, S.J., Hiller, J.R., McCartor, G.: Application of Pauli-Villars regularization and discretized light-cone quantization to a single-fermion truncation of Yukawa theory. Phys. Rev. D **64**(11), 114023 (2001), arXiv:hep-ph/0107038

[27] Brodsky, S.J., Hiller, J.R., McCartor, G.: Exact Solutions to Pauli-Villars-Regulated Field Theories. Ann. Phys. **296**(2), 406-424 (2002), arXiv:hep-th/0107246

[28] Brodsky, S.J., Hiller, J.R., McCartor, G.: Application of Pauli-Villars regularization and Discretized Light-Cone Quantization to a (3+1)-Dimensional Model. Phys. Rev. D **60**(5), 054506 (1999), arXiv:hep-ph/9903388

[29] Brodsky, S.J., Hiller, J.R., McCartor, G.: Pauli-Villars as a Nonperturbative Ultraviolet Regulator in Discretized Light-Cone Quantization. Phys. Rev. D **58**(2), 025005 (1998), arXiv:hep-th/9802120

[30] Brodsky, S.J., Franke, V.A., Hiller, J.R., McCartor, G., Paston, S.A., Prokhvatilov, E.V.: A nonperturbative calculation of the electron's magnetic moment. Nucl. Phys. B **703**, 333-362 (2004), arXiv:hep-ph/0406325

[31] Chabysheva, S.S., Hiller, J.R.: A nonperturbative calculation of the electron's magnetic moment with truncation extended to two photons. Phys. Rev. D **81**(7), 074030 (2010), arXiv:0911.4455v2 [hep-ph]

[32] Chabysheva, S.S., Hiller, J.R.: Nonperturbative Pauli-Villars regularization of vacuum polarization in light-front QED. Phys. Rev. D **82**(3), 034004 (2010), arXiv:1006.1077v2 [hep-ph]

[33] Chabysheva, S.S., Hiller, J.R.: First nonperturbative calculation in light-front QED for an arbitrary covariant gauge. Phys. Rev. D **84**(3), 034001 (2011), arXiv:1102.5107v2 [hep-ph]

[34] Chabysheva, S.S., Hiller, J.R.: A Light-Front Coupled-Cluster Method for the Nonperturbative Solution of Quantum Field Theories. Phys.Lett. B **711** 417-422 (2012), arXiv:1103.0037v2 [hep-ph]

[35] Chabysheva, S.S., Hiller, J.R.: An application of the light-front coupled-cluster method to the nonperturbative solution of QED, arXiv:1203.0250 [hep-ph]

[36] Karmanov, V.A., Mathiot, J.-F., Smirnov, A.V.: Nonperturbative calculation of the anomalous magnetic moment in the Yukawa model within truncated Fock space. Phys. Rev. D **82**(5), 056010 (2010), arXiv:1006.5640 [hep-th]

[37] Carbonell, J., Desplanques, B., Karmanov, V.A., Mathiot, J.-F.: Explicitly covariant light-front dynamics and relativistic few-body systems. Phys. Rep. **300**, 215-347 (1998), arXiv:nucl-th/9804029

[38] Mathiot, J.-F., Smirnov, A.V., Tsirova, N.A., Karmanov, V.A.: Nonperturbative renormalization in light-front dynamics and applications. Few Body Syst. **49**, 183-203 (2011), arXiv:1009.5269 [hep-th]

[39] Stevenson, P.M.: Gaussian effective potential: Quantum mechanics. Phys. Rev. D, **30**(8), 1712-1726 (1984); Gaussian effective potential. II. $\lambda\varphi^4$ field theory. Phys. Rev. D **32**(6), 1389-1408 (1985). See also the development of this method in the paper by Siringo, F.: Higher order extensions of the Gaussian effective potential. Phys. Rev. D **88**(5), 056020 (2013), arXiv:1308.1836 [hep-ph]

[40] Hornbostel, K.: Nontrivial vacua from equal time to the light cone. Phys.Rev. D **45**(10), 3781-3801 (1992)

[41] Srivastava, P.P., Brodsky, S.J.: Light-front formulation of the standard model. Phys. Rev. D **66**(4), 045019 (2002), arXiv:hep-ph/0202141v1

[42] Brodsky, S.J., de Teramond, G.F.: Hadronic Spectra and Light-Front Wave Functions in Holographic QCD. Phys. Rev. Lett. **96**, 201601 (2006), arXiv:hep-ph/0602252v2

[43] Brodsky, S.J., de Teramond, G.F. Light-Front Holography: A First Approximation to QCD. Phys. Rev. Lett. **102**, 081601 (2009), arXiv:0809.4899v3 [hep-ph]

[44] Weinberg, S.: The Quantum Theory of Fields. V.1. Foundations. V.2. Modern Applications. Cambridge University Press, Cambridge (2000)

[45] Ligterink, N.E., Bakker, B.L.G.: Equivalence of Light-Front and Covariant Field Theory, Phys. Rev. D **52**(10), 5954-5979 (1995), arXiv:hep-ph/9412315

[46] Franke, V.A., Novozhilov, Yu.V., Prokhvatilov, E.V.: On the Light Cone Formulation of Classical Nonabelian Gauge Theory. Lett. Math. Phys. **5**(3), 239-245 (1981). On the Light Cone Quantization of Nonabelian Gauge Theory. Lett. Math. Phys. **5**(5) 437-444 (1981).

[47] De Teramond, G.F., Dosch, H.G., Brodsky, S.J.: Kinematical and dynamical aspects of higher-spin bound-state equations in holographic QCD. Phys. Rev. D **87**(7), 075005 (2013), arXiv:1301.1651v1 [hep-ph]

[48] Brodsky, S.J., de Teramond, G.F., Dosch, H.G.: Threefold Complementary Approach to Holographic QCD, arXiv:1302.4105v6 [hep-th]

[49] Vary, J.P., Honkanen, H., Li, J., Maris, P., Brodsky, S.J., Harindranath, A., de Teramond, G.F., Sternberg, P., Ng, E.G., Yang, C.: Hamiltonian light-front field theory in a basis function approach. Phys.Rev. C **81**, 035205 (2010), arXiv:0905.1411v2 [nucl-th]

[50] Li, Y., Wiecki, P.W., Zhao, X., Maris, P, Vary, J.P.: Introduction to Basis Light-Front Quantization Approach to QCD Bound State Problems, arXiv:1311.2980v1 [nucl-th]

[51] Vary, J.P., Zhao, X., Ilderton, A., Honkanen, H., Maris, P., Brodsky, S.J. Basis Light-front Quantization: a New Approach to Non-perturbative Scattering and Time-dependent Production Processes. Acta. Phys. Pol. A. Proc. Suppl. **6**(1) 257 (2013)