The dynamical QCD string

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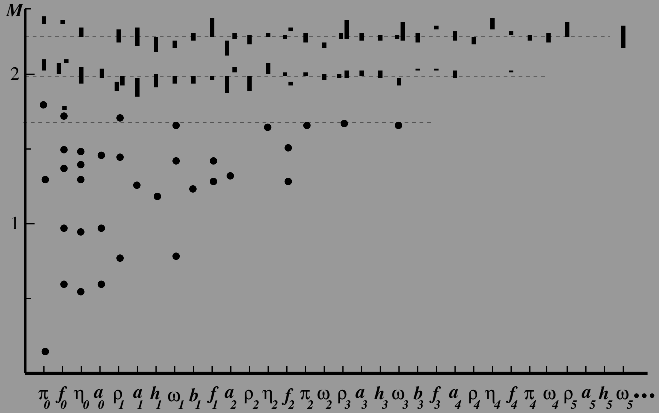
Low and high lying meson spectra.



The high-lying mesons are from $\bar{p}p$ annihilation at LEAR (Anisovich, Bugg, Sarantsev,...).

The quark condensate and the Dirac operator

Banks-Casher: A density of the lowest quasi-zero eigenmodes of the Dirac operator represents the quark condensate of the vacuum:

$$< 0|\bar{q}q|0 >= -\pi\rho(0).$$

Sequence of limits:

$$V \to \infty; m_q \to 0$$

The lattice volume is finite and the spectrum is descrete. We remove an increasing number of the lowest Dirac modes from the valence quark propagators and study the effects of the remaining chiral symmetry breaking on the masses of hadrons.

$$S(k) = S - \sum_{i \leq k} \frac{|\lambda_i >< \lambda_i|}{\lambda_i},$$

S - standard quark propagator in a given gauge configuration;
λ_i - eigenvalues of the manifestly chirally symmetric overlap Dirac operator;
$|\lambda_i >$ - eigenvectors;
k - number of the removed lowest eigenmodes.

Extraction of the physical states on the lattice

E.g., we want to study the $\rho(I=1,1^{--})$ spectrum.
Then a basis of interpolators:

$$\mathcal{O}_V = \bar{q}(x)\tau\gamma^i q(x);$$

$$\mathcal{O}_T = \bar{q}(x)\tau\sigma^{0i} q(x);$$

with a few different exponential smearings of the quark fields in spatial directions in the source and sink.

Some lattice details:
• 100 gauge configurations with 2 dynamical flavors with the overlap Dirac operator from JLQCD.
• $L = 1.9$ fm; $a = 0.12$ fm
• $m_\pi = 289$ MeV

We subtract the low-lying chiral modes from the valence quarks.

Extraction of the physical states on the lattice

Assume we have hadrons (states) with energies $n = 1, 2, 3, ...$ with fixed quantum numbers.

$$C(t)_{ij} = \langle \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)\rangle = \sum_n a_i^{(n)} a_j^{(n)*} \mathrm{e}^{-E^{(n)}t} \qquad (1)$$

where

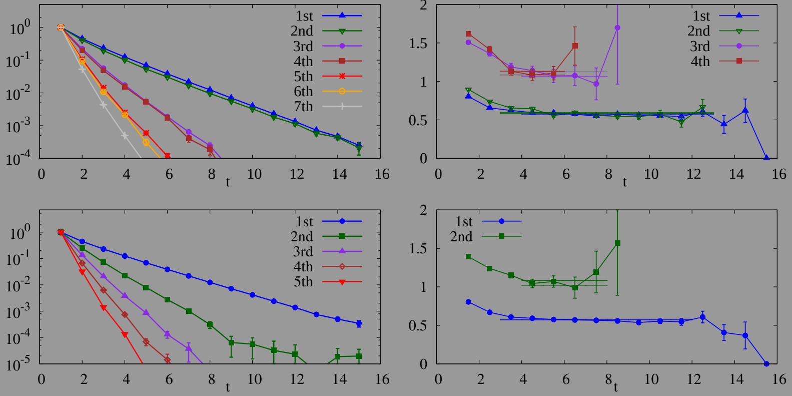
$$a_i^{(n)} = \langle 0|\mathcal{O}_i|n\rangle \ .$$

The generalized eigenvalue problem:

$$\widehat{C}(t)_{ij} u_j^{(n)} = \lambda^{(n)}(t, t_0)\widehat{C}(t_0)_{ij} u_j^{(n)}. \qquad (2)$$

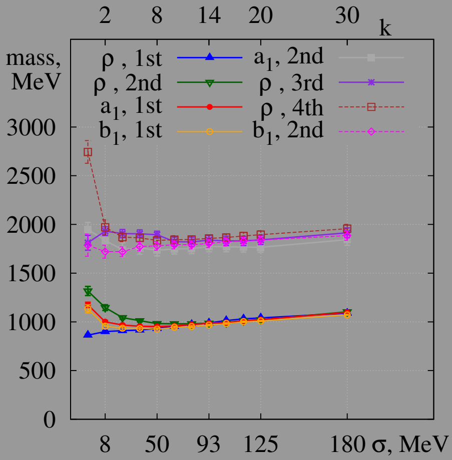
Each eigenvalue and eigenvector corresponds to a given state. If a basis $\mathcal{O}_i$ "wave functions" of all states.

$\rho(I=1,1^{--})$ and $a_1(I=1,1^{++})$ with 10 subtracted eigenmodes



The correlators $\lambda_n(t) \sim \exp(-E_n t)$ for all eigenstates (left) and the effective mass plots $E_n(t) = \log(\lambda_n(t)/\lambda_n(t+1))$ the lowest states (right).

Observations



What do meson degeneracies tell us?

The $SU(2)_L \times SU(2)_R$ multiplets:

$(0,0)$	:	$\omega(0,1^{--})$	$f_1(0,1^{++})$
$(1/2,1/2)_a$	:	$h_1(0,1^{+-})$	$\rho'(1,1^{--})$
$(1/2,1/2)_b$	:	$\omega(0,1^{--})$	$b_1(1,1^{+-})$
$(0,1)+(1,0)$	:	$a_1(1,1^{++})$	$\rho(1,1^{--})$

The $SU(2)_L \times SU(2)_R \times U(1)_A$ multiplets:

$$h_1(0,1^{+-});\ \rho'(1,1^{--});\ \omega(0,1^{--});\ b_1(1,1^{+-}),$$

$$\omega(0,1^{--});\ f_1(0,1^{++}),$$

$$a_1(1,1^{++});\ \rho(1,1^{--}).$$

A degeneracy of all eight mesons indicates a higher symmetry that includes $SU(2)_L \times SU(2)_R \times U(1)_A$ as a subgroup. What is this symmetry?

A new symmetry

A dim=16 rep of the $SU(4) \times C_i \supset SU(2)_L \times SU(2)_R \times U(1)_A \times C_i$ group

$(0,0)$	:	$\omega(0,1^{--})$	$f_1(0,1^{++})$
$(1/2,1/2)_a$	:	$h_1(0,1^{+-})$	$\rho'(1,1^{--})$
$(1/2,1/2)_b$	:	$\omega(0,1^{--})$	$b_1(1,1^{+-})$
$(0,1)+(1,0)$	:	$a_1(1,1^{++})$	$\rho(1,1^{--})$

The $SU(4) \times C_i \supset SU(2)_L \times SU(2)_R \times U(1)_A \times C_i$ transformations rotate the (u_r, u_l, d_r, d_l) fundamental vector.

Do not mix it with the nonrelativistic heavy quark (WIGNER) SPIN-ISOSPIN $SU(4)$ symmetry $(m_u, m_d \to \infty)$!
(that rotates $(u_\uparrow, u_\downarrow, d_\uparrow, d_\downarrow)$ and combines within the nonrelativistic quark model into one dim=16 rep $\pi(1,0^{-+})$, $\eta(0,0^{-+})$, $\rho(1,1^{--})$, $\omega(0,1^{--})$).

The dynamical QCD string

Energy is independent on orientations of the quark spins, which means that there are no magnetic interactions in the system, i.e., quarks are at rest with respect to the color-electric field.

These are the energy levels of an ultrarelativistic dynamical QCD string.

$$E_{n_r} = (n_r + 1)\hbar\omega$$

$$\hbar\omega = 900 \pm 70 MeV$$

Is the dynamical QCD string of the Nambu-Goto type?

Nambu-Goto open bosonic string: $M^2 \sim L$
A unitary transformation from a chiral basis R in $\bar{q}q$ to the $\{I; {}^{2S+1}L_J\}$ basis (L.Ya.G., A.V. Nefediev, PRD 76 (2007) 096004; 80 (2009) 057901):

$$|R; IJ^{PC}\rangle = \sum_L \sum_{\lambda_q \lambda_{\bar{q}}} \chi^{RPI}_{\lambda_q \lambda_{\bar{q}}} \times \sqrt{\frac{2L+1}{2J+1}} C^{S\Lambda}_{\frac{1}{2}\lambda_q \frac{1}{2} - \lambda_{\bar{q}}} C^{J\Lambda}_{L0S\Lambda} |I; {}^{2S+1}L_J\rangle.$$

Examples of fixed L :
a_1 : $|(0,1)+(1,0); 1\ 1^{++}\rangle = |1; {}^3P_1\rangle$
h_1 : $|(1/2,1/2)_b; 0\ 1^{+-}\rangle = |0; {}^1P_1\rangle$.
However, there are two kinds of ρ -mesons:

$$|(0,1)+(1,0); 1\ 1^{--}\rangle = \sqrt{\frac{2}{3}}|1; {}^3S_1\rangle + \sqrt{\frac{1}{3}}|1; {}^3D_1\rangle,$$

$$|(1/2,1/2)_b; 1\ 1^{--}\rangle = \sqrt{\frac{1}{3}}|1; {}^3S_1\rangle - \sqrt{\frac{2}{3}}|1; {}^3D_1\rangle.$$

For the dynamical QCD string with chiral quarks at the ends fixed L is impossible!

Conclusions

- Chiral symmetry is restored but confinement is still there

- Hadrons get their large chirally symmetric mass

- Both $SU(2)_L \times SU(2)_R$ and $U(1)_A$ get simultaneously restored (consistent with the instanton mechanism of both breakings)

- A symmetry of the dynamical chirally symmetric QCD string includes $SU(4) \supset SU(2)_L \times SU(2)_R \times U(1)_A$.

- The radial spectrum of the string $E_{n_r} = (n_r + 1)\hbar\omega, \quad \hbar\omega = 900 \pm 70 MeV$

- The dynamical QCD string is not of the Nambu-Goto type