Large-N: mesons and strong decays

- Sea quark effects $\propto N_f/N \Rightarrow$ The $N=\infty$ limit is "quenched".
- OZI rule OK at $N=\infty$. Strong decays gone.
- Mixing glueballs-$\bar{q}q$-tetraquarks-polyquarks-etc. large-N suppressed.

Is $N=\infty$ close to $N=3$ QCD? Obviously $N_f/3$ is a large number!

AdS/QFT starts from $N=\infty$. Also many simplifications in chiral EFT.

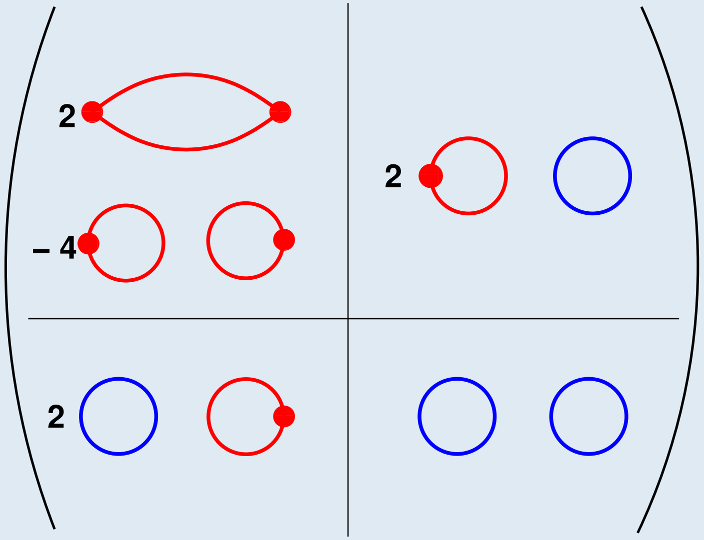
- Counting – Each closed colour loop: N
- Mesons $\hat{\pi}^\dagger|0\rangle$: $1/\sqrt{N}$, so that $\langle\pi|\pi\rangle = \langle 0|\hat{\pi}\hat{\pi}^\dagger|0\rangle = \mathcal{O}(1)$.
- Glueballs $|G\rangle = \hat{G}^\dagger|0\rangle$: $1/N$

Pion decay constant: $\langle 0|\bar{d}\gamma_\mu\gamma_5 u|\pi^+\rangle = \sqrt{2}F_\pi p_\mu = \mathcal{O}\left(\sqrt{N}\right)$.

Scale setting: define

$$\hat{F}_\infty = \lim_{N\to\infty}\sqrt{\frac{3}{N}}F_\pi(N, m_\pi = 0) = 85.9\,\mathrm{MeV}.$$

Wick contractions: mixing of singlet $\bar{q}q$ with glueball ($N_f = 2$):



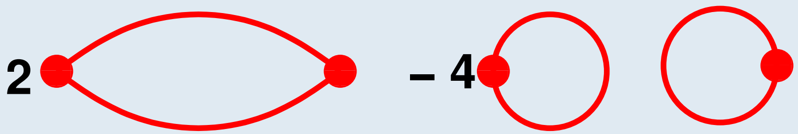
$$= \begin{pmatrix} \# + \# N & \# \sqrt{N} \\ \# \sqrt{N} & \# \end{pmatrix}$$

At large-N flavour non-singlet mesons and glueballs decouple.

Mixing between flavour-singlets and glueballs is governed by

$$\frac{C_{12}}{\sqrt{C_{11} C_{22}}} = \frac{\# \sqrt{N}}{\sqrt{\# + \# N}} \xrightarrow{N \to \infty} \mathcal{O}(1)$$

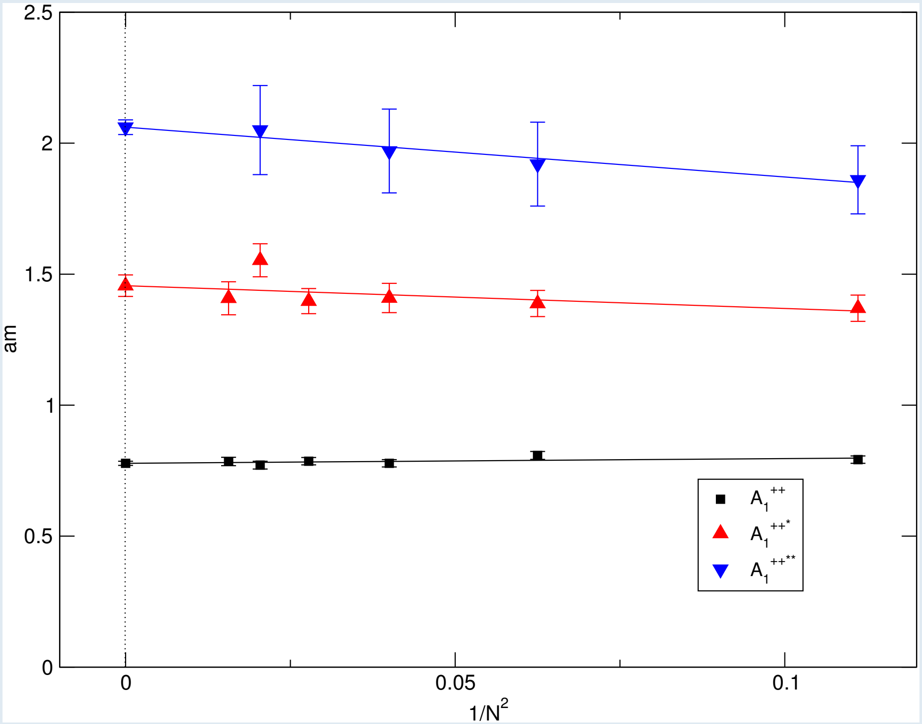
Glueballs and singlets become the same! This is not surprising:

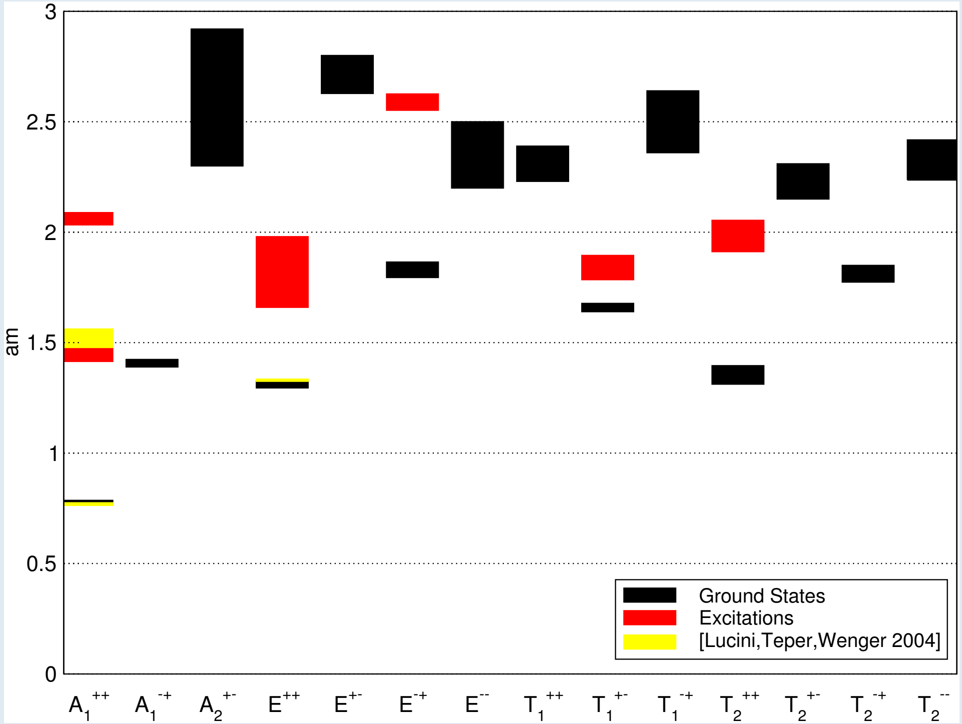


$$\propto \left[\cancel{e^{-m_\pi t}} + \left(N e^{-m_{\eta_1} t} - \cancel{e^{-m_\pi t}} \right) \right]$$

But what component dominates the $N = 3$ meson?

Glueballs at large-N



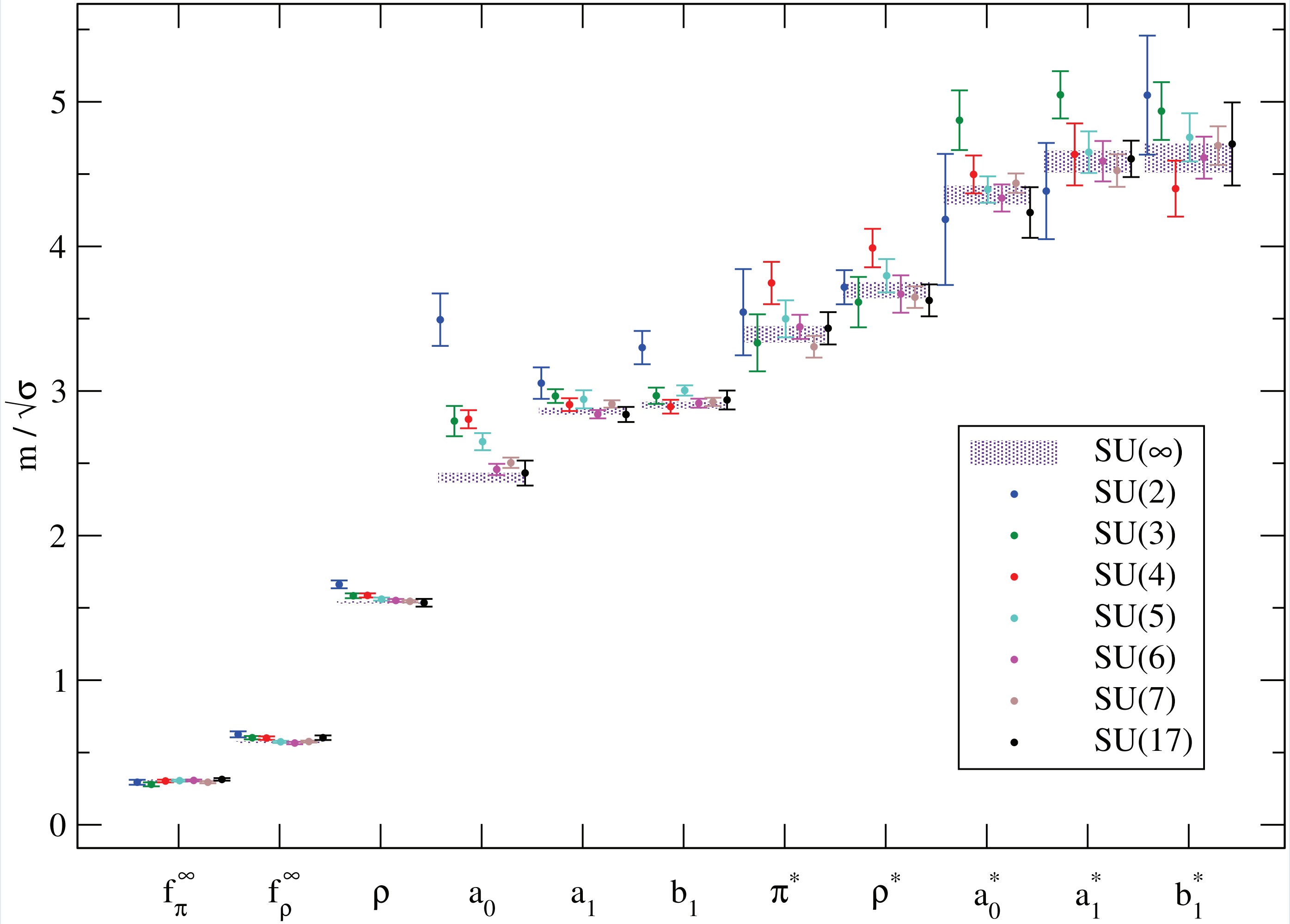


Continuum limit ($a \to 0$) extrapolation $\quad a^{-1} \approx 1.5$ GeV

from B Lucini, A Rago, E Rinaldi 10

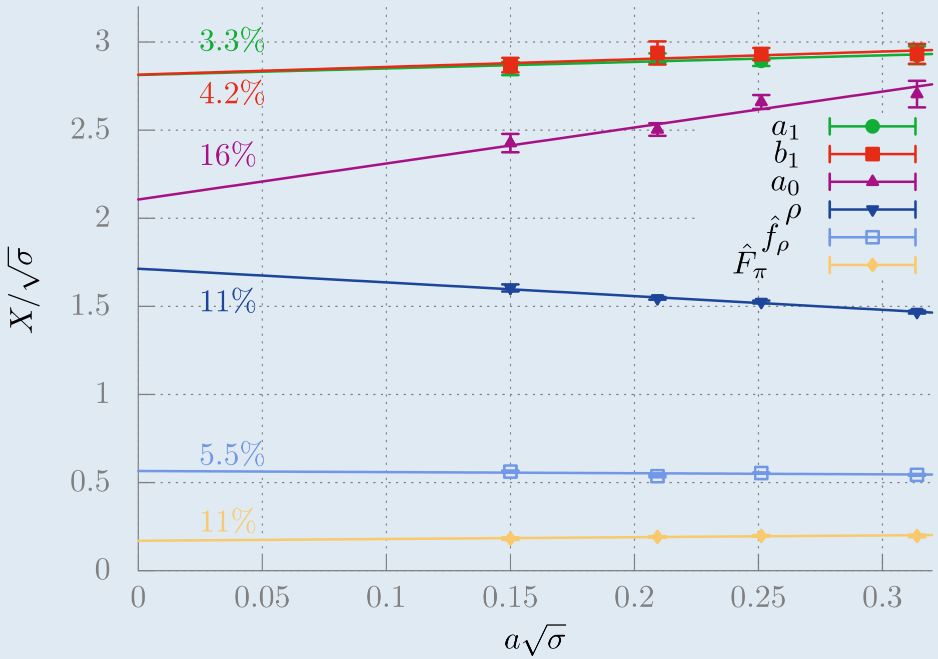
What about mesons?

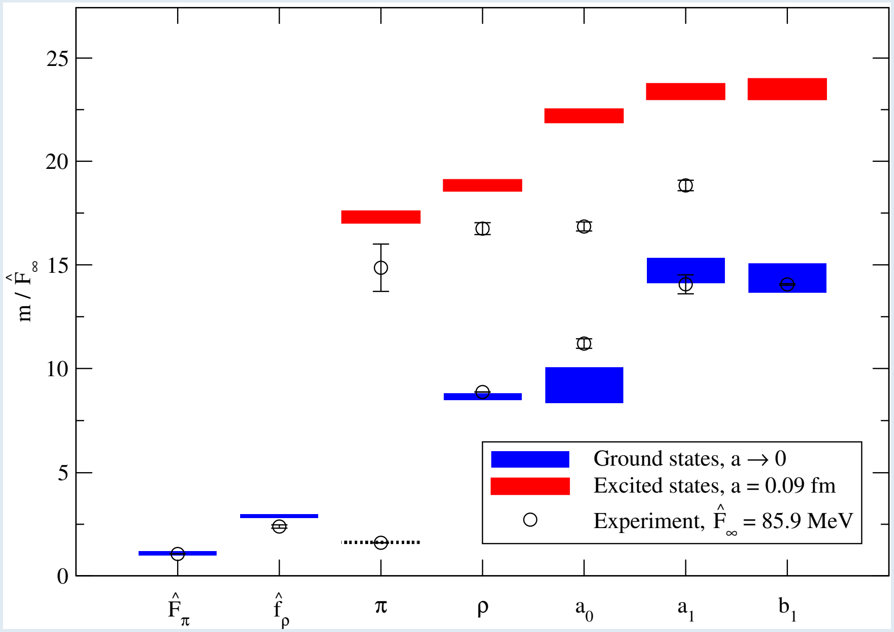
Meson spectrum: different N at $a^{-1} \approx 2.1$ GeV



from GB, F Bursa, S Collins, L Castagnini, L Del Debbio, B Lucini, M Panero 13

Meson spectrum: continuum limit



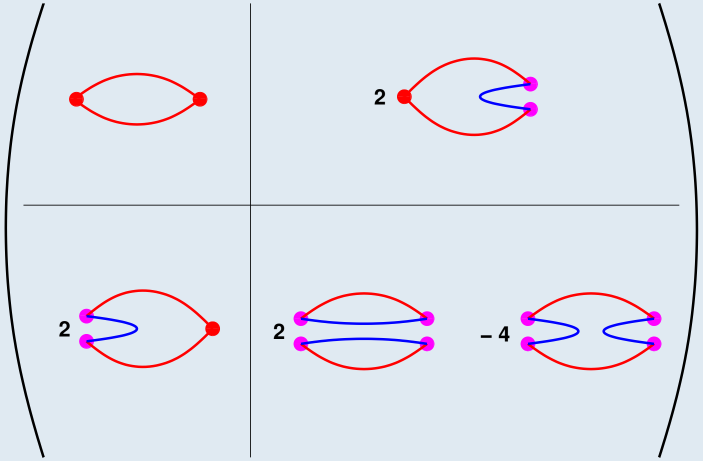


Continuum extrapolation SU(7)

Continuum limit ($N = \infty$)

GB, L Castagnini, B Lucini, M Panero 13: Lattice 13, in preparation

Mixing of $\bar{u}d$ with $\bar{u}q\bar{q}d$ ($N_f = 2$):



$$= \begin{pmatrix} \# & \#/\sqrt{N} \\ \#/\sqrt{N} & \# + \#/N \end{pmatrix}$$

Now

$$\frac{C_{12}}{\sqrt{C_{11}C_{22}}} = \mathcal{O}(1/\sqrt{N}) \quad \text{or} \quad \mathcal{O}(1),$$

depending on whether quark line disconnected diagram (with mass $m_{\pi\pi} = 2m_\pi + \mathcal{O}(1/N)$) or connected diagram dominates (Weinberg).
Assume state with $m_T < 2m_\pi + \mathcal{O}(1/N)$ exists. Then connected diagram

$$e^{-m_T t} \quad \left[-Ne^{-m_{\pi\pi} t}\right]$$

will dominate. Is this the case for $a_0 \leftrightarrow K\overline{K}, D_{s0} \leftrightarrow DK$ etc?
$\Rightarrow$ Calculation of $N = \infty$ tetraquark states should be interesting.
NB: for singlet states $\bar{q}q$/tetraquark/glueball all mix at $\mathcal{O}(1)$ and the leading diagram topologically resembles the glueball propagator.

A Planar Diagram Theory for Strong Interactions
Gerard 't Hooft (CERN). Dec 1973. 20 pp.
Published in **Nucl.Phys. B72 (1974) 461**
CERN-TH-1786
DOI: 10.1016/0550-3213(74)90154-0
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Science Direct
Detailed record - Cited by 3657 records 1000+

N_c @ 40

1) 1/Nc is a fundamental expansion of QCD: understanding it means understanding QCD itself

a) Theoretical understanding: this involves hard-core QCD theoretical progress

b) Phenomenological understanding: role of the 1/Nc expansion in hadronic observables, both experimental and from LQCD

2) 1/Nc and/or quark model?: why does the 1/Nc expansion seem to often work better than expected?

e.g. quark loops are suppressed by Nf/Nc, so they should not be so suppressed in real life, but they very often are! why?

3) Hard to understand certain notorious failures, e.g. : $\Delta I = 1/2$ rule

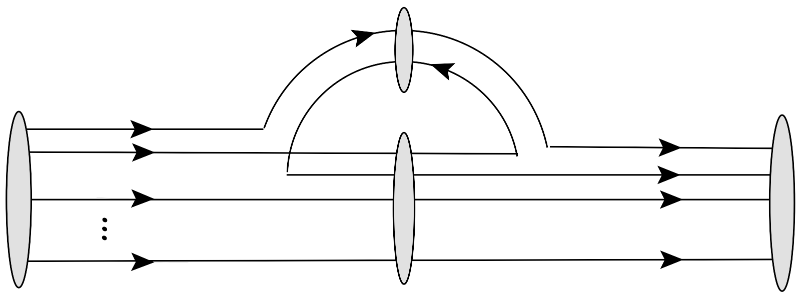
5) Opportunities: LQCD is the best new tool for the phenomenology of large Nc: Nc=5, 7, etc can be explored in mesons and baryons.
Do quenched QCD which is easier and still has a 1/Nc expansion.
Also, useful to always compare quenched with the full QCD calculation.

5) Mesons/glueballs vs baryons: fundamentally different at large Nc

Mesons: $$U_B(1) \times SU_L(N_f) \times SU_R(N_f) \to U_L(N_f) \times U_R(N_f)$$

Baryons: $$U_B(1) \times SU_L(N_f) \times SU_R(N_f) \to U_B(1) \times SU_{\mathrm{contrated}}(2N_f)$$

6) Baryons have a pion cloud contributing at LO to the baryon mass. No pion cloud for mesons. A chiral expansion for baryons in strict large Nc does not exist.



6) BChPT: fundamental role of spin-flavor symmetry to assure cancellations of Nc counting violating contributions; need to include spin 3/2 baryons as degrees of freedom in BChPT

7) 1/Nc and nuclear potential: some studies have been carried out on the potential. Problem: NN interaction is order Nc: why is the deuteron so weakly bound?

8) Defining 1/Nc expansion: 'tHooft limit, other limits: usually correspond to different choices of quark content as function of Nc. Also SUSY theories.

9) More pheno: 1/Nc expansion and excited baryons: as we include constraints from SU(3) approximate symmetry, also constraints from 1/Nc should be employed (e.g., in coupled channel models).
Easier said than done, but worth considering nonetheless.

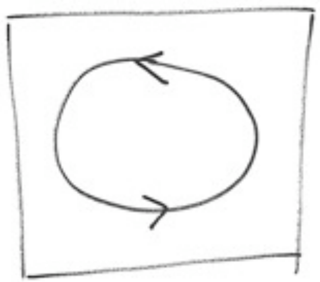
10) There should always be a kinematic domain where 1/Nc expansion fails: when ratios of energy scales to LambdaQCD/Nc go from large to small: 1/Nc is not necessarily an analytic expansion for all observables: e.g., it does not commute with chiral expansion. Still a lot to discover in this context.

Let us do more before

Nc @ 50

a bit of history:

Eguchi and Kawai (1982) showed that the infinite set of loop (Schwinger-Dyson) equations for Wilson loops in pure Yang-Mills theory is the same in small-V and infinite-V theory, to leading order in 1/N; e.g.:



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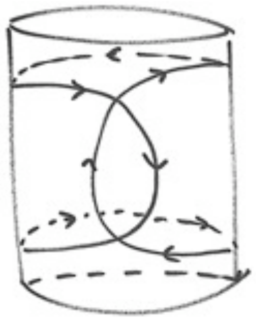

+ O(1/N)

expectation value of any Wilson loop at infinite-L

expectation value of (folded) Wilson loop at small-L

provided
all topologically nontrivial (w/ arbitrary winding) Wilson loops have vanishing expectation value
= **unbroken center**

"EK reduction" or "large-N reduction" or "large-N volume-independence"

Note: this is an exact result in QFT (one of the few!).

... potentially exciting, since:

1) **simulations may be cheaper** (use single-site lattice ?)

2) **raises theorist's hopes** (that small-L easier to solve ?)

From a modern point of view **EK reduction is a large-N orbifold with respect to the group of translations -**

a development from past 10 years, which is what I'm supposed to discuss

an orbifold = throw out of Z fields/components not invariant under some discrete symmetry (origin in stringy terminology)

Volume-independence viewed as an orbifold helps establish that VEVs and correlators of operators that are center-neutral and carry momenta quantized in units of 1/L (in compact direction) are the same on, say,  , and in infinite-L theory, to leading order in 1/N.

Thus, a working example of EK would be good for

- calculating vevs (symmetry breaking)
  - even if all dimensions small
- calculating spectra (for generic theories/reps)
  - need at least one large dimension

progress I - theory:

- large-N equivalences hold in field theory as well; nonperturbative proof
- neccessary and sufficient conditions: discrete symmetry used does not break

progress II - theory/expt.(lattice):

known that in pure YM center breaks so EK does not hold, but
in QCD(adj) argued to hold ... evidence (that center unbroken) from small size weak coupling and lattice studies, even with massive adjoints for a range of masses

future progress desired...

- is this equivalence useful/practical for actual simulations of QCD(adj)?

for pure field theory interest...

QCD(adj): Nf =1 is N=1 SUSY YM

Nf =4
- happens to be N=4 SYM without the scalars
- "minimal walking technicolor"??

Nf =5.5 asymptotic freedom lost

or for large-N QCD interest...

another orbifold (orientifold, really) equivalence relates QCD(adj) to large N QCD with antisymmetric Dirac using SYM results - predictions for one (+) flavor...

- is this equivalence useful for better theoretical understanding?

yes, for $N_c \Lambda L \ll 1$
weak-coupling calculable regime
abelian large-N limit only,
interesting but not all!

not (so far?) for $N_c \Lambda L \gg 1$
volume independence regime
ambitious, as this = solving the theory at large N
't Hooftian large-N!

progress III:

perhaps Adi and others can comment: use at finite density?
(large-N equivalence between isospin and baryon chemical potential?)