The lightest scalar-isoscalar meson, its history, new parameters and role in QCD

Robert Kamiński

Institute of Nuclear Physics PAN, Kraków, Poland

XI$_{th}$ Quark Confinement and the Hadron Spectrum,
St. Petersburg 11.09.2014

sigma meson: $f_0(500)$ (former σ)

- General information on the $f_0(500)$
- Theoretical works and experimental data '70-2014
- Shortly on what it can be

$f_0(500)$ *or* σ*:* **What** *is it?*

- lightest scalar-isoscalar meson i.e. $J^{PC}I^G$: $0^{++}0^+$,
- with mass and width $\approx$ 500 MeV, dominates $\pi\pi$ amplitude below $\approx$ 900 MeV
- hadronic decay channel: 100% $\pi\pi$,
- dramatic history:
  - until 1976 called ϵ or σ,
  - disappeared from Particle Data Tables between 1978 and 1992,
  - since 1994: $f_0(400-1200)$,
  - in years 2002-2010: $f_0(600)$,
  - now (since 2012): $f_0(500)$
- Renaissance of the sigma meson:

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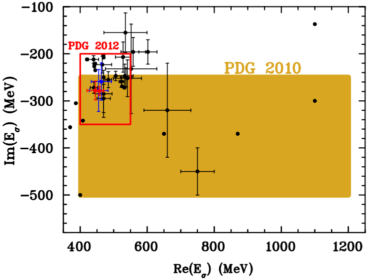
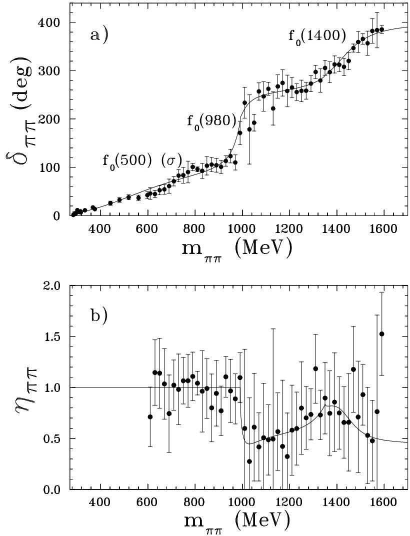
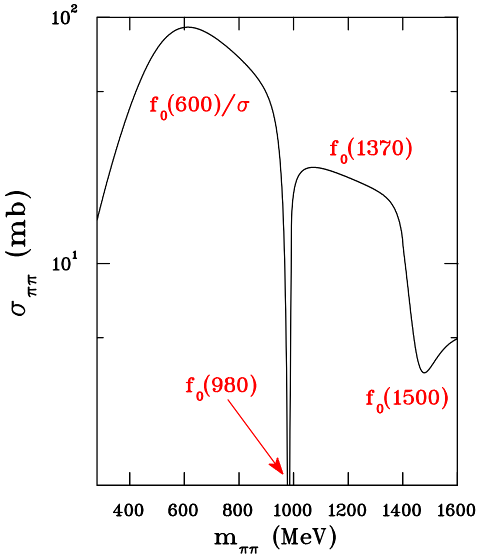


$\pi\pi$ S0-*wave phase shifts and inelasticities*

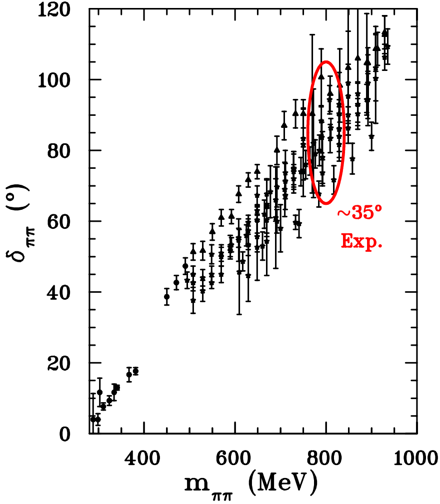


Puzzling S0 wave ππ cross section



GKPY dispersion equations with imposed

’70



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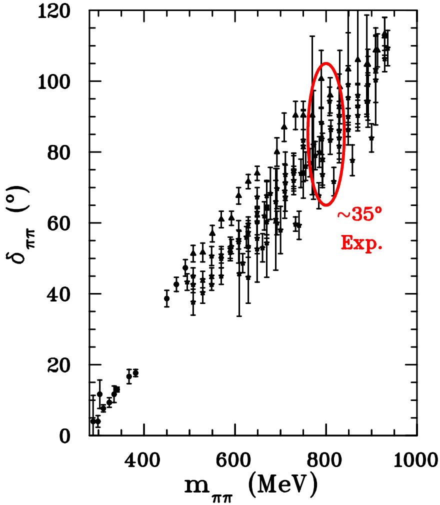
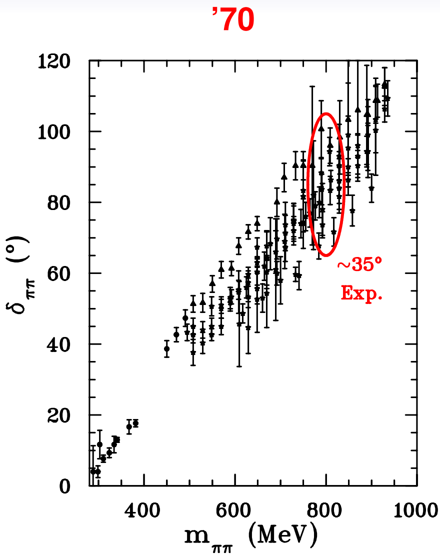


GKPY dispersion equations with imposed crossing symmetry condition

Madrid-Kraków group 2005-2011



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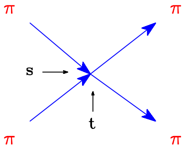

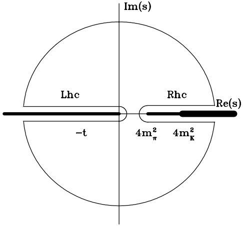

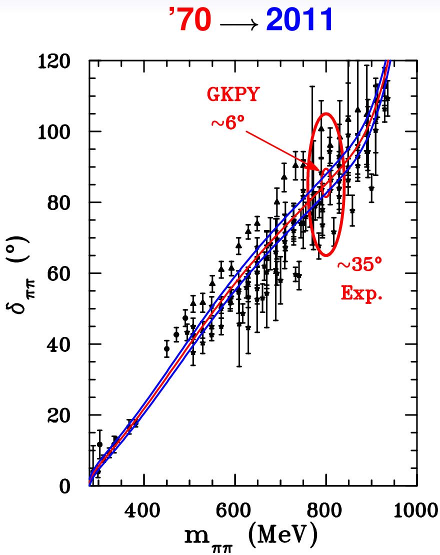

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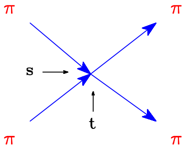
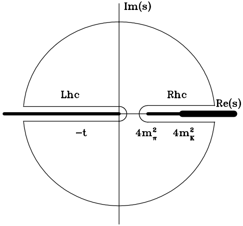




GKPY equations and poles of the $\pi\pi$ amplitudes

partial waves: *JI*

experiment

F1 D2

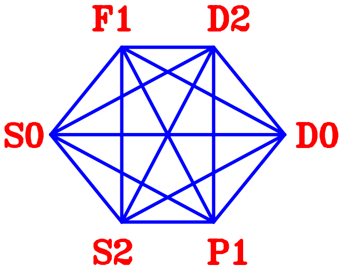
S0 D0

S2 P1

GKPY equations and poles of the $\pi\pi$ amplitudes

partial waves: *JI*

experiment + theory (GKPY)



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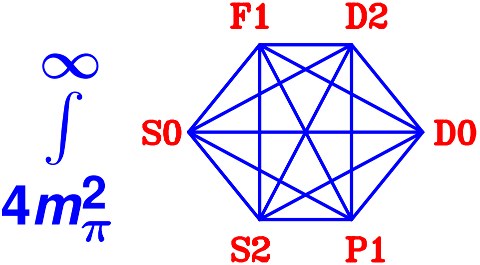

GKPY equations:

$$\text{Re } t_\ell^{I(OUT)}(s) = \sum_{I'=0}^{2} C^{II'} t_0^{I'(IN)}(4m_\pi^2) + \sum_{I'=0}^{2} \sum_{\ell'=0}^{4} \int_{4m_\pi^2}^{\infty} ds' K_{\ell\ell'}^{II'}(s,s') \text{ Im } t_{\ell'}^{I'(IN)}(s')$$

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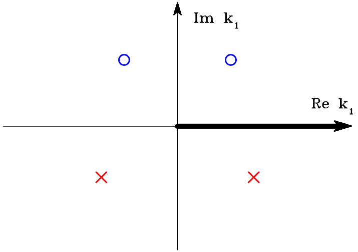
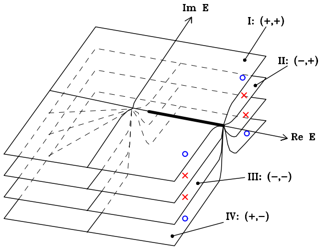
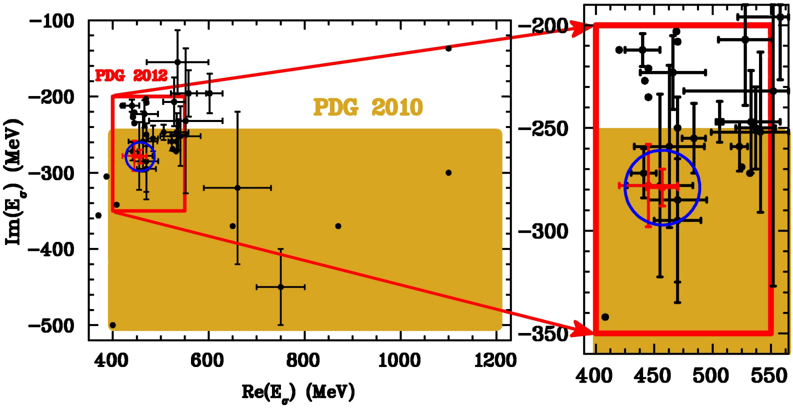




Renaissance of the σ meson



Before 2012

Citation: C. Amsler *et al.* (Particle Data Group), PL **B667**, 1 (2008) and 2009 partial update for the 2010 edition (URL: http://pdg.lbl.gov)

$f_0(600)$ or σ

$I^G(J^{PC}) = 0^+($

A REVIEW GOES HERE – Check our WWW

$f_0(600)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma \approx 2\ \text{Im}(\sqrt{s_{\text{pole}}})$.

VALUE (MeV)	*DOCUMENT ID*		*TECN*
(400–1200)−*i*(250–500) OUR ESTIMATE			
• • • We do not use the following data for averages, fits, limits, et			
$(455 \pm 6^{+31}_{-13}) - i(278 \pm 6^{+34}_{-43})$	[1] CAPRINI	08	RVUE
$(463 \pm 6^{+31}_{-17}) - i(259 \pm 6^{+33}_{-34})$	[2] CAPRINI	08	RVUE
$(552^{+84}_{-106}) - i(232^{+81}_{-72})$	[3] ABLIKIM	07A	BES2
$(466 \pm 18) - i(223 \pm 28)$	[4] BONVICINI	07	CLEO
$(484 \pm 17) - i(255 \pm 10)$	GARCIA-MAR..	07	RVUE
$(441^{+16}_{-8}) - i(272^{+9}_{-12.5})$	[5] CAPRINI	06	RVUE
$(470 \pm 50) - i(285 \pm 25)$	[6] ZHOU	05	RVUE
$(541 \pm 39) - i(252 \pm 42)$	[7] ABLIKIM	04A	BES2
$(528 \pm 32) - i(207 \pm 23)$	[8] GALLEGOS	04	RVUE
$(440 \pm 8) - i(212 \pm 15)$	[9] PELAEZ	04A	RVUE
$(533 \pm 25) - i(247 \pm 25)$	[10] BUGG	03	RVUE
$532 - i272$	BLACK	01	RVUE
$(470 \pm 30) - i(295 \pm 20)$	[5] COLANGELO	01	RVUE

Since year 2012

Citation: J. Beringer *et al.* (Particle Data Group), PR **D86**, 010001 (2012) and 2013 p

$f_0(500)$ or σ

was $f_0(600)$

$I^G(J^{PC}$

A REVIEW GOES HERE – Check our

$f_0(500)$ T-MATRIX POL

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VALUE (MeV)	*DOCUMENT ID*	
(400–550)−*i*(200–350) OUR ESTIMATE		
• • • We do not use the following data for averages, fits		
$(440 \pm 10) - i(238 \pm 10)$	[1] ALBALADEJO	12
$(445 \pm 25) - i(278^{+22}_{-18})$	[2,3] GARCIA-MAR..	11
$(457^{+14}_{-13}) - i(279^{+11}_{-7})$	[2,4] GARCIA-MAR..	11
$(442^{+5}_{-8}) - i(274^{+6}_{-5})$	[5] MOUSSALLAM	11
$(452 \pm 13) - i(259 \pm 16)$	[6] MENNESSIER	10
$(448 \pm 43) - i(266 \pm 43)$	[7] MENNESSIER	10
$(455 \pm 6^{+31}_{-13}) - i(278 \pm 6^{+34}_{-43})$	[8] CAPRINI	08
$(463 \pm 6^{+31}_{-17}) - i(259 \pm 6^{+33}_{-34})$	[9] CAPRINI	08
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$(466 \pm 18) - i(223 \pm 28)$	[11] BONVICINI	07
$(472 \pm 30) - i(271 \pm 30)$	[12] BUGG	07
$(484 \pm 17) - i(255 \pm 10)$	GARCIA-MAR..	07

Roy's equations and up-down ambiguity in the $\pi\pi$ S0 wave

$$\text{Re}\, t_\ell^{I(OUT)}(s) = a_0^0 + (2a_0^0 - 5a_0^2)(s-4) + \sum_{I'=0}^{2} \sum_{\ell'=0}^{4} \fint_{4m_\pi^2}^{\infty} ds' \bar{K}_{\ell\ell'}^{II'}(s,s')\, \text{Im}\, t_{\ell'}^{I'(IN)}(s')$$

precision of the Roy and GKPY equations

Roy' 1971	GKPY' 2011
two subtractions	one subtraction
$K_{\ell\ell'}^{II'}(s,s') \sim s'^{-3}$-fast convergence	$K_{\ell\ell'}^{II'}(s,s') \sim s'^{-2}$
$ST_0^0 = a_0^0 + (2a_0^0 - 5a_0^2)(s-4)$	$ST_0^0 = a_0^0 + 5a_0^2$ - no error propagation!

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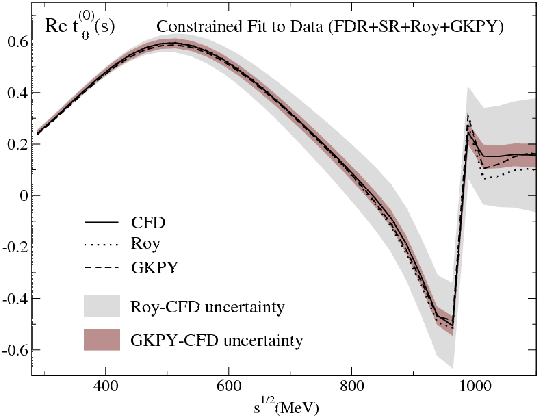

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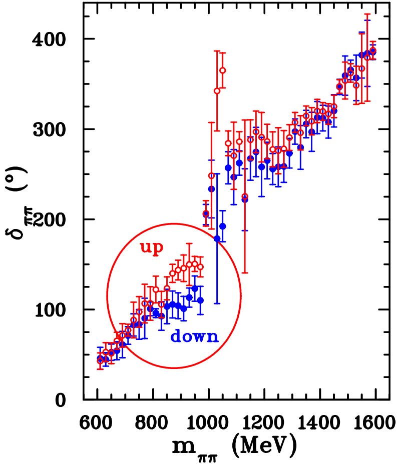


Another group - "Bern" group:

H. Leytwyller, J. Gasser, G. Colangelo, I. Caprini: 2001-2011

FAQ: ... for sure your solution is not unique

The Role of the input in Roy's equations for pi pi scattering" G. Wanders, Eur. Phys. J. C17 (2000) 323-336

In the abstract:

An updated survey of known results on the dimension of the manifold of solutions is presented. The solution is unique for a low energy interval with upper end at 800 MeV. We determine its response to small variations of the input: S-wave scattering lengths and absorptive parts above 800 MeV.

I.e.:
Fixed two boundary conditions for the $\pi\pi$ amplitude:

- at the threshold ($S0$ wave scattering length) and
- at 800 MeV

precise determination of the pole, its couplings to the $\pi\pi$ channel and amplitude

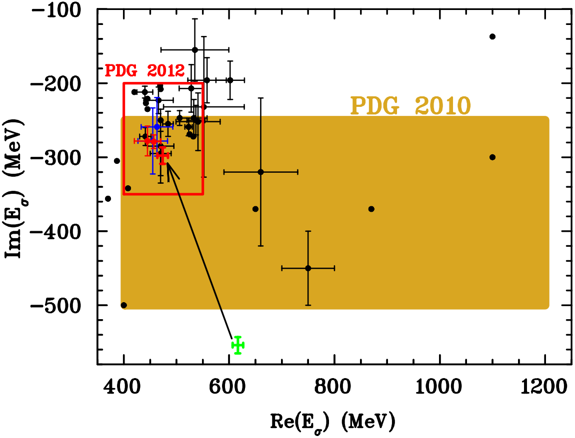
$$g^2 = -16\pi \lim_{s \to s_{pole}} (s - s_{pole})\, t_\ell(s)\, (2\ell + 1)/(2p)^{2\ell}$$

	$\sqrt{s_{\rm pole}}$ (MeV)	$\lvert g \rvert$
$f_0(500)^{\rm GKPY}$	$(457^{+14}_{-13}) - i(279^{+11}_{-7})$	$3.59^{+0.11}_{-0.13}$ GeV
$f_0(500)^{\rm Roy}$	$(445 \pm 25) - i(278^{+22}_{-18})$	3.4 ± 0.5 GeV
$f_0(980)^{\rm GKPY}$	$(996 \pm 7) - i(25^{+10}_{-6})$	2.3 ± 0.2 GeV
$f_0(980)^{\rm Roy}$	$(1003^{+5}_{-27}) - i(21^{+10}_{-8})$	$2.5^{+0.2}_{-0.6}$ GeV
$\rho(770)^{\rm GKPY}$	$(763.7^{+1.7}_{-1.5}) - i(73.2^{+1.0}_{-1.1})$	$6.01^{+0.04}_{-0.07}$
$\rho(770)^{\rm Roy}$	$(761^{+4}_{-3}) - i(71.7^{+1.9}_{-2.3})$	$5.95^{+0.12}_{-0.08}$

S0 scattering length

- ChPT + Roy eqs (Bern group): $0.220 \pm 0.005\, m_\pi^{-1}$
- GKPY: $0.220 \pm 0.008\, m_\pi^{-1}$

shift of the σ pole caused by crossing symmetry condition



what forces GKPY eqs to pull up-left the sigma pole?

$$\text{Re}\; t_\ell^{I(OUT)}(s) = \sum_{I'=0}^{2} C^{II'}\, t_0^{I'(IN)}(4m_\pi^2) + \sum_{I'=0}^{2}\sum_{\ell'=0}^{4} \fint_{4m_\pi^2}^{\infty} ds'\, K_{\ell\ell'}^{II'}(s,s')\, \text{Im}\; t_{\ell'}^{I'(IN)}(s')$$

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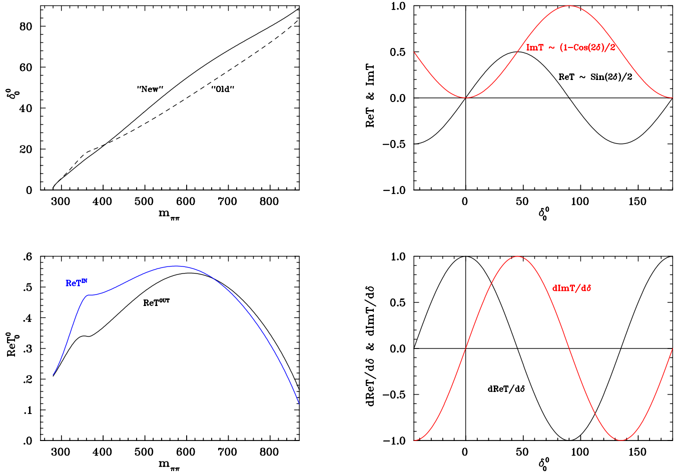
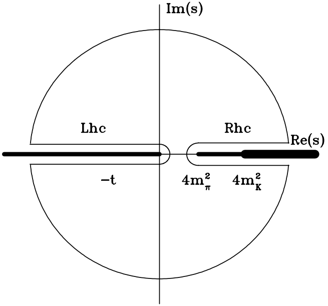
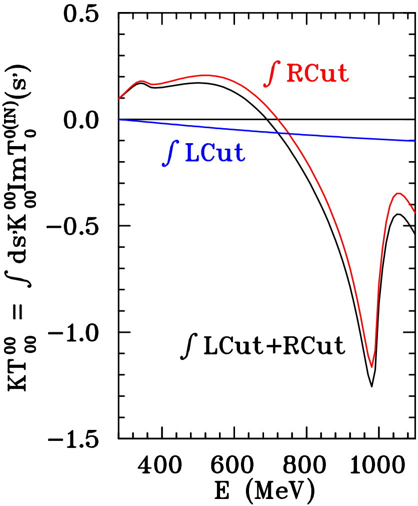


shape of the $KT^{00}_{00} = \int_{4m_\pi^2}^{\infty} ds' K^{ll'}_{\ell\ell'}(s,s')\, Imt^{l'(IN)}_{\ell'}(s')$

The shape is given by coefficients in the crossing symmetry matrix C_{st} and integrated amplitudes. It is produced by the integration along the right cut





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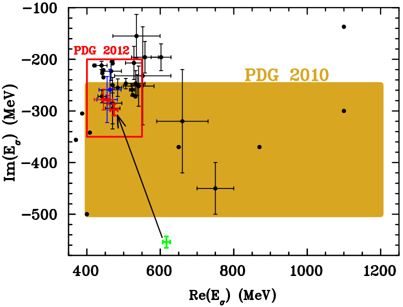

Two things: trigonometry and crossing symmetry algebra lead to narrower and lighter σ.

Nothing more and nothing instead of it is needed.

What it really can be?

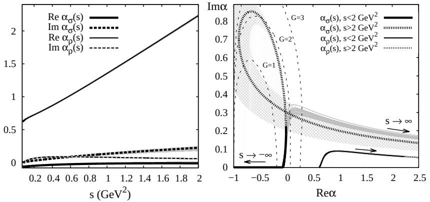

Fig. 1. (Left) $\alpha_\rho(s)$ and $\alpha_\sigma(s)$ Regge trajectories, from our constrained Regge-pole amplitudes. (Right) $\alpha_\sigma(s)$ and $\alpha_\rho(s)$ in the complex plane. At low and intermediate energies (thick continuous lines), the trajectory of the σ is similar to those of Yukawa potentials $V(r) = -Ga\exp(-r/a)/r$ [8] (thin dashed lines). Beyond 2 Gev^2 we plot our results as thick discontinuous lines because they should be considered just as extrapolations.

Furthermore, in Fig. 1 we show the striking similarities between the $f_0(500)$ trajectory and those of Yukawa potentials in non-relativistic scattering [8]. From the Yukawa G=2 curve in that plot, which lies closest to our result for the $f_0(500)$, we can estimate $a \simeq 0.5\,\text{GeV}^{-1}$, following [8]. This could be compared, for instance, to the S-wave $\pi\pi$ scattering length $\simeq 1.6\,\text{GeV}^{-1}$. Thus it seems that the range of a Yukawa potential that would mimic our low energy results is comparable but smaller than the $\pi\pi$ scattering length in the scalar isoscalar channel. Of course, our results are most reliable at low energies (thick continuous line) and the extrapolation should be interpreted cautiously. Nevertheless, our results suggest that the $f_0(500)$ looks more like a low-energy resonance of a short range potential, *e.g.* between pions, than a bound state of a confining force between a quark and an antiquark.

In summary, our formalism and the results for the $f_0(500)$ explains why the lightest scalar meson has to be excluded from the ordinary linear Regge fits of ordinary mesos.

"The non-ordinary Regge behavior of the $f_0(500)$ meson"

by

J. R. Pelaez, J. T. Londergan, J . Nebreda and A. Szczepaniak

arXiv:1404.6058

Conclusions

- the σ meson is once again alive and is doing well!
- for sure σ is not pure $q\bar{q}$ meson but perhaps:
  - mixture of the $q\bar{q}$ and $\pi\pi$ components,
  - something like "correlated two-pion" state?
- opens a promising area for new analyses of states crucial for the QCD:
  - $f_0(980)$: $qq\bar{q}\bar{q}$, $K\bar{K}$ state?,
  - $f_0(1500)$: lowest gg state? - look at lattice QCD predictions,
- should help end the debate about the existence of the $f_0(1370)$,
- it should help in precise determination of the CKM matrix elements and in the fight against the isobar model and old habits related with resonances