Deconfinement transition in a massive extension of the background field gauge

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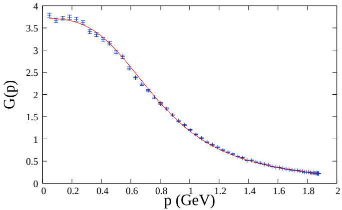
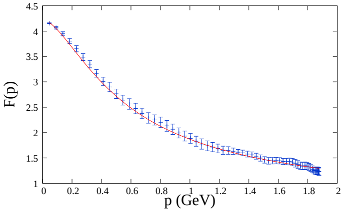
Phenomenological model I

- Ongoing investigation of the low-energy properties of QCD within a simple phenomenological model.
- Based on the observation that lattice simulations unambiguously indicate that the gluon propagator (in Landau gauge $\partial_\mu A_\mu = 0$) is massive while ghost propagator is massless.





Phenomenological model II

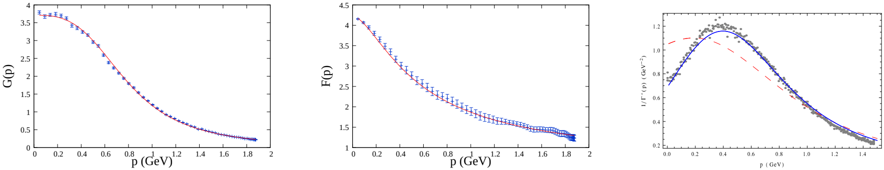
- In the Faddeev-Popov gauge-fixing action, both ghosts and gluons are massless. Several attempts to understand the dynamical generation of the mass.
- We propose to add a mass term in the action, as a phenomenological parameter. (Curcci-Ferrari model.)

$$\mathcal{L} = \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \partial_\mu \bar{c}^a \left(\partial_\mu c^a + g\, f^{abc} A^b_\mu c^c\right) + \frac{m^2}{2}\left(A^a_\mu\right)^2$$

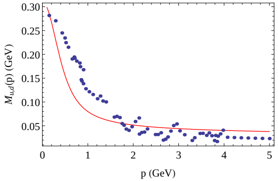
Phenomenological model III

- Only slight modification of the Feynman rules...
- ... but dramatic consequences for the low energy properties. In particular, no divergence of the running coupling constant! (no Landau pole). Opens the way to perturbative calculations in the nonperturbative regime (?!?)
- We performed one-loop calculations of several correlation functions and compared with lattice data for:
  - ghost and gluon propagators withor without quarks (10%);
  - three-point correlation functions (lattice data are noisy; qualitative agreement).
  - quark propagators (scalar part OK, vectorial part not so well);
  - Ghost and gluon propagators at finite temperature (ghost and magnetic sector OK, electric sector?)

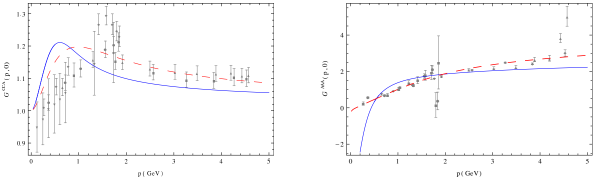
Phenomenological model IV



gluon and ghost propagators (d=3, d=4) [Cucchieri et. al, (2008)]



scalar sector of quark self energy [Bowman et. al, (2004)]



$\bar{c}cA$ and AAA 3-point correlation function [Cucchieri et. al, (2008)]

Phenomenological model V

With one phenomenological parameter, we reproduce several features of the correlation functions of QCD.
We aim at:

- justifying the presence of the mass term from first principles (our studies suggest it originates from Gribov ambiguity);
- exploring other features of QCD.

Yang-Mills theory at finite temperature

- First step toward phase diagram of QCD (with chemical potential).
- Phase transition between a confined (low T) and a deconfined (high T). Associated with a breaking of the center symmetry (non-periodic gauge transformations that preserve periodicity of the fields).
- Order parameter: Polyakov loop (Wilson loop wrapped around time direction).
- Critical temperature: roughly around 200 MeV.
- Continuous phase transition for SU(2) (Ising universality class), discontinuous for SU(3).

Extension to finite temperature

- Work in the Landau-de Witt gauge: shift the gauge field A by a background $\bar{A}$: $A = \bar{A} + a$, impose $(\bar{D}_\mu a_\mu)^a = \partial_\mu a_\mu^a + g f^{abc} \bar{A}_\mu^b a_\mu^c = 0$.
- $\bar{A}$ is like a gauge-fixing parameter.
- Generating functional of the 1PI diagrams $\Gamma[\bar{A}, a]$ depends on 2 fields.
- Choose $\bar{A}$ to be constant (translation), temporal (rotations). Up to global color rotation, $\bar{A}_0$ lies in the Cartan subalgebra.
  - SU(2): $\bar{A}_0^3$
  - SU(3): $\{\bar{A}_0^3, \bar{A}_0^8\}$
- $\langle a \rangle$ depends on $\bar{A}$. Choose $\bar{A} = \bar{A}_{\rm min}$ such that $\langle a \rangle = 0$. Actually $\bar{A}_{\rm min}$ is the minimum of $\Gamma[\bar{A}, 0]$!
- Gauge invariance after gauge fixing: $\Gamma[\bar{A}^U, UaU^\dagger] = \Gamma[\bar{A}, a]$. $\bar{A}$ is a probe for center symmetry.

Observables

There remains to compute:

- the potential $V = \frac{1}{\beta\Omega}\Gamma[\bar{A}, 0]$
- the Polyakov loop $\ell = \frac{1}{N}\mathrm{tr}\langle P\exp(ig\int_0^\beta \bar{A}_0^{\mathrm{min},a} + a_0^a)t_a\rangle$

(t^a: generators)

In a systematic expansion in g (but with arbitrary $\bar{A}$), the leading contribution is:

- $V(\bar{A}_0) = \frac{1}{\beta\Omega}(\mathrm{Tr}\log\Delta_{a,h}^{-1} - \mathrm{Tr}\log\Delta_{\bar{c},c}^{-1}) + \mathcal{O}(g^2)$
- $\ell = \exp(i\beta g\bar{A}_0^{\mathrm{min},a}t_a) + \mathcal{O}(g^2)$

SU(2)

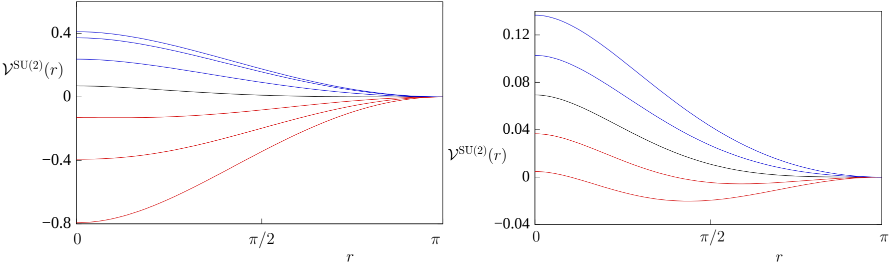
Introduce the dimensionless quantity $r_3 = \beta g \bar{A}_0^3$.

- Symmetries (in particular center symmetry) impose that the potential is 2π-periodic, $V(r_3) = V(2\pi - r_3)$.
- $\ell = \cos(r_3/2) + \mathcal{O}(g^2)$. $r_3 = \pi$ is the center-symmetric point.
- $V = T^d[(d-1)F_{\beta m}(r_3) - F_0(r_3) + \text{c.t.}] + \mathcal{O}(g^2)$ with

$$F_{\tilde{m}}(r) = \int_q \log(1 + e^{-2\sqrt{q^2+\tilde{m}^2}} - 2e^{-\sqrt{q^2+\tilde{m}^2}} \cos r)$$

- At high T, recover the Weiss potential. $r_3 = \pi$ is a max.
- At low T, $-\frac{1}{2}$ Weiss potential: $r_3 = \pi$ is a min.
- In between: phase transition.

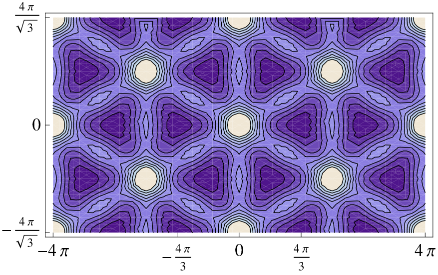
SU(2)

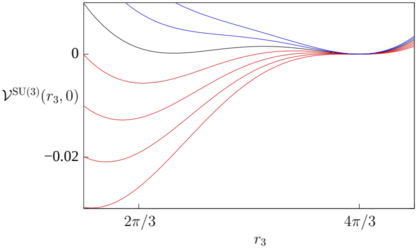


Second order transition.

SU(3)

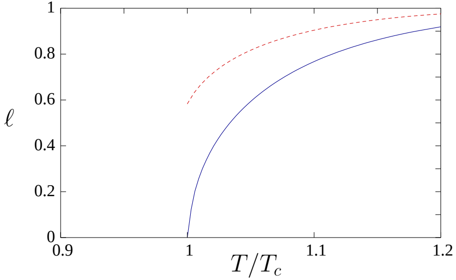
- $\ell = \frac{1}{3}[e^{-i\frac{r_8}{\sqrt{3}}} + 2e^{i\frac{r_8}{2\sqrt{3}}}\cos r_3/2] + \mathcal{O}(g^2)$
- Center symmetry: 120° rotation around $\{4\pi/3, 0\}$.
- At this "point", $\ell = 0$, center symmetric point.





- First order transition.

Polyakov loops



Clear distinction between SU(2) (continuous) and SU(3) (discontinuous).

Conclusions

- Introducing a mass to the gluons in the bare action captures lot of infrared properties of correlation functions.
- Using Landau-de Witt gauge-fixing, the leading calculation reproduces the phenomenology of finite temperature Yang-Mills theory. Determination of critical temperature $\simeq 20\%$.
- Need for a next to leading $[\mathcal{O}(g^2)]$ calculation to check convergence and improve precision (Polyakov and potential are computed, 1-loop propagators under study).
- Repeat the study with quarks, at finite (real? imaginary?) chemical potential (to appear soon...).