Holographic thermalization at strong and intermediate coupling

Aleksi Vuorinen

Quark Confinement and the Hadron Spectroscopy XI
Saint Petersburg, 11.9.2014

R. Baier, S. Stricker, O. Taanila, AV, 1205.2998 (JHEP), 1207.1116 (PRD)
D. Steineder, S. Stricker, AV, 1209.0291 (PRL), 1304.3404 (JHEP)
S. Stricker, 1307.2736 (EPJ-C)
V. Keränen, H. Nishimura, S. Stricker, O. Taanila and AV, 1405.7015 (PRD)

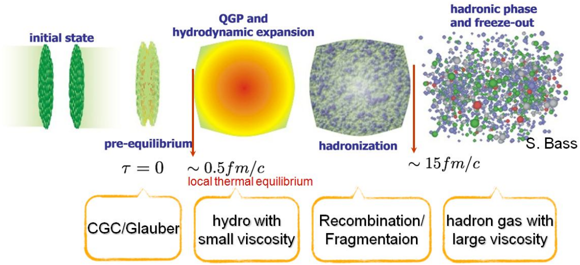
UNIVERSITY OF HELSINKI

HELSINKI INSTITUTE OF PHYSICS

Table of contents

Table of contents

Describing a heavy ion collision

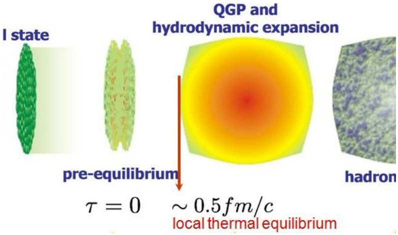


Nontrivial observation: Hydrodynamic description of fireball evolution extremely successful with few theory inputs

1. Relatively easy: Equation of state and freeze-out criterion
2. Hard: Transport coefficients of the plasma (η, ζ, ...)
3. Very hard: Initial conditions & onset time τ_{hydro}

Surprise from RHIC/LHC: Extremely fast equilibration into almost 'ideal fluid' behavior — hard to explain via weakly coupled quasiparticle picture

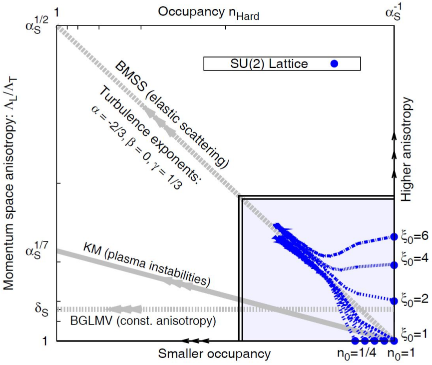
Thermalization puzzle



Major challenge for theorists: Understand the fast dynamics that take the system from complicated, far-from-equilibrium initial state to near-thermal 'hydrodynamized' plasma

Characteristic energy scales and nature of the plasma evolve fast (running coupling) $\Rightarrow$ Need to efficiently combine both perturbative and nonperturbative machinery

Thermalization in a weakly coupled plasma



Inelastic scatterings drive bottom-up thermalization

- Soft modes quickly create thermal bath
- Hard splittings lead to $q \sim Q_s$ particles being eaten by the bath

Numerical evolution of expanding SU(2) YM plasma seen to always lead to Baier-Mueller-Schiff-Son type scaling at late times (Berges et al., 1303.5650, 1311.3005)

Ongoing debate about the role of instabilities in hard interactions, argued to lead to slightly faster thermalization: $\tau_{\rm KM} \sim \alpha_s^{-5/2}$ vs. $\tau_{\rm BMSS} \sim \alpha_s^{-13/5}$

Thermalization beyond weak coupling

Recently remarkable progress for the early weak-coupling dynamics of a high energy collision (cf. talk of A. Kurkela). However, extension of the results to realistic heavy ion collision problematic:

- System clearly not asymptotically weakly coupled $\Rightarrow$ Direct use of perturbation theory requires bold extrapolation
- Dynamics classical only in an overoccupied system — classical lattice simulations work only for the early dynamics of the system
- Kinetic theory description misses important physics, e.g. effects of plasma instabilities

In absence of nonperturbative first principles techniques, clearly room for alternative approaches to tackle dynamical problems in strongly coupled quantum field theory

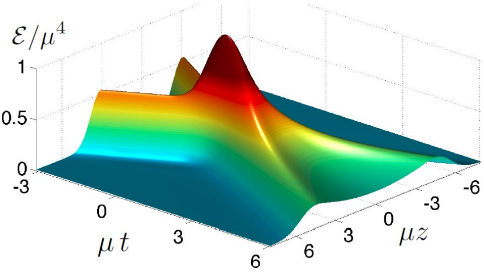
- Natural candidate: $\mathcal{N} = 4$ SYM theory at large N_c and strong 't Hooft coupling $\lambda = g^2 N_c$

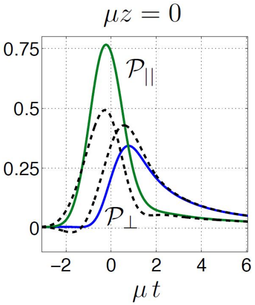
Strong coupling thermalization

Due to conformality, $\mathcal{N} = 4$ SYM very different from QCD at $T = 0$. However:

- At finite temperature, systems much more similar
  - Both describe deconfined plasmas with Debye screening, finite static correlation length,...
  - Conformality and SUSY broken due to introduction of T
- Large N_c and λ limits systematically improvable
- *Very* nontrivial field theory problems mapped to classical gravity

Strong coupling thermalization





Chesler, Yaffe, 1011.3562

Important lessons from gauge/gravity calculations at infinite coupling:

- Thermalization always of top-down type (causal argument)
- Thermalization time naturally short, $\sim 1/T$
- Hydrodynamization $\neq$ Thermalization, isotropization

Bridging the gap

Obviously, it would be valuable to bring the two limiting cases closer to each other — and to a realistic setting. Is it possible to:

- Extend weak coupling picture to lower energies, with $\alpha_s(Q) \sim 1$?
- Marry weak coupling description of the early dynamics with strong coupling evolution?
- Bring field theory used in gauge/gravity calculations closer to real QCD?
  - Finite coupling & N_c, dynamical breaking of conformal invariance,...

Bridging the gap

Obviously, it would be valuable to bring the two limiting cases closer to each other — and to a realistic setting. Is it possible to:

- Extend weak coupling picture to lower energies, with $\alpha_s(Q) \sim 1$?
- Marry weak coupling description of the early dynamics with strong coupling evolution?
- Bring field theory used in gauge/gravity calculations closer to real QCD?
  - Finite coupling & N_c, dynamical breaking of conformal invariance,...

Rest of the talk: Attempt to relax the $\lambda = \infty$ approximation in studies of holographic thermalization

Table of contents

AdS/CFT duality: $T = 0$

- Original conjecture: SU(N_c) $\mathcal{N} = 4$ SYM in $\mathbb{R}^{1,3} \leftrightarrow$ IIB ST in AdS$_5 \times$S$_5$

"center" of AdS — $r = 0$

boundary — $r = \infty$

- Pure AdS metric corresponds to vacuum state of the CFT

$$ds^2 = L^2 \Big(-r^2 dt^2 + \frac{dr^2}{r^2} + r^2 d\mathbf{x}^2 \Big)$$

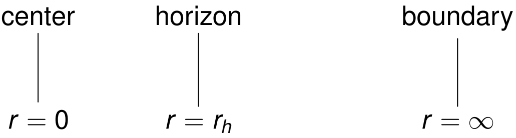
- Dictionary: CFT operators $\leftrightarrow$ bulk fields, with identification

$$(L/l_s)^4 = \lambda, \quad g_s = \lambda/(4\pi N_c)$$

$\Rightarrow$ Strongly coupled, large-N_c QFT $\leftrightarrow$ Classical sugra

AdS/CFT duality: $T \neq 0$

- Strongly coupled large-N_c SYM plasma in thermal equilibrium $\leftrightarrow$ Classical gravity in AdS black hole background



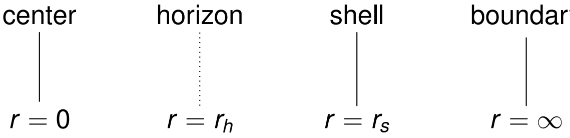
- Metric now features event horizon at $r = r_h$ ($L \equiv 1$ from now on)

$$ds^2 = -r^2(1 - r_h^4/r^4)dt^2 + \frac{dr^2}{r^2(1 - r_h^4/r^4)} + r^2 d\mathbf{x}^2$$

- Identification of field theory temperature with Hawking temperature of the black hole $\Rightarrow T = r_h/\pi$

AdS/CFT duality: Thermalizing system

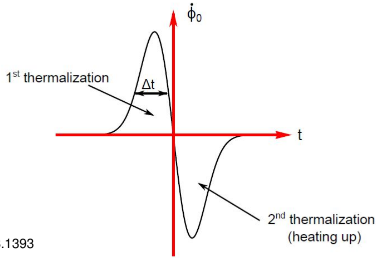
- Simplest way to take system out of equilibrium: Radial gravitational collapse of a thin shell (Danielsson, Keski-Vakkuri, Kruczenski)



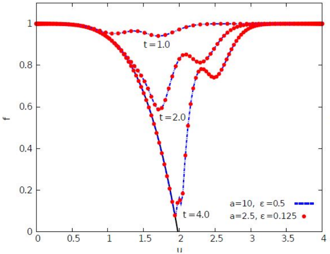
- Metric defined in a piecewise manner:

$$ds^2 = -r^2 f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 d\mathbf{x}^2, \quad f(r) = \begin{cases} f_-(r) \equiv 1\,, & \text{for } r < r_s \\ f_+(r) \equiv 1 - \frac{r_h^4}{r^4}\,, & \text{for } r > r_s \end{cases}$$

- Shell fills entire three-space ⇒ Translational and rotational invariance
- Field theory side: Rapid, spatially homogenous injection of energy at all scales



Bin Wu, 1208.1393



Shell can be realized by briefly turning on a spatially homogenous scalar source in the CFT, coupled to

- A marginal composite operator in the CFT
- The bulk metric through Einstein equations involving the corresponding bulk field

$$ds^2 = \frac{1}{u^2}\Big(-f(u,t)\, e^{-2\delta(u,t)} dt^2 + 1/f(u,t)\, du^2 + d\mathbf{x}^2 \Big), \quad u = r_h^2/r^2$$

Beyond infinite coupling: α' corrections

Recall key relation from AdS/CFT dictionary: $(L/l_s)^4 = L^4/\alpha'^2 = \lambda$, with α' the inverse string tension

- To go beyond $\lambda = \infty$ limit, need to add α' terms to the sugra action, i.e. determine the first non-trivial terms in a small-curvature expansion
- Leading order corrections $\mathcal{O}(\alpha'^3) = \mathcal{O}(\lambda^{-3/2})$

End up dealing with $\mathcal{O}(\alpha'^3)$ improved type IIB sugra

$$
\begin{aligned}
S_{IIB} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left(R_{10} - \frac{1}{2}(\partial\phi)^2 - \frac{F_5^2}{4\cdot 5!} + \gamma e^{-\frac{3}{2}\phi}(C+\mathcal{T})^4\right), \\
\mathcal{T}_{abcdef} &\equiv i\nabla_a F^+_{bcdef} + \frac{1}{16}\left(F^+_{abcmn}F^+_{def}{}^{mn} - 3F^+_{abfmn}F^+_{dec}{}^{mn}\right), \\
F^+ &\equiv \frac{1}{2}(1+*)F_5, \quad \gamma \equiv \frac{1}{8}\zeta(3)\lambda^{-3/2}
\end{aligned}
$$

$\Rightarrow \gamma$-corrected metric and EoMs for different fields

Table of contents

Holographic Green's functions

In- and off-equilibrium correlators offer useful tool for studying thermalization:

- Poles of retarded thermal Green's functions give dispersion relation of field excitations: Quasiparticle / quasinormal mode spectrum
- Time dependent off-equilibrium Green's functions probe how fast different energy (length) scales equilibrate
- Related to measurable quantities, e.g. particle production rates

Holographic Green's functions

In- and off-equilibrium correlators offer useful tool for studying thermalization:

- Poles of retarded thermal Green's functions give dispersion relation of field excitations: Quasiparticle / quasinormal mode spectrum
- Time dependent off-equilibrium Green's functions probe how fast different energy (length) scales equilibrate
- Related to measurable quantities, e.g. particle production rates

Example 1: EM current correlator $\langle J_\mu^{\text{EM}} J_\nu^{\text{EM}} \rangle$ — photon production

- Obtain by adding to the SYM theory a U(1) vector field coupled to a conserved current corresponding to a subgroup of $\text{SU}(4)_R$
- Excellent phenomenological probe of thermalization because of photons' weak coupling to plasma constituents

Holographic Green's functions

In- and off-equilibrium correlators offer useful tool for studying thermalization:

- Poles of retarded thermal Green's functions give dispersion relation of field excitations: Quasiparticle / quasinormal mode spectrum
- Time dependent off-equilibrium Green's functions probe how fast different energy (length) scales equilibrate
- Related to measurable quantities, e.g. particle production rates

Example 2: Energy momentum tensor correlators $\langle T_{\mu\nu} T_{\alpha\beta} \rangle$ related to e.g. shear and bulk viscosities and dual to metric fluctuations $h_{\mu\nu}$

- Scalar channel: h_{xy}
- Shear channel: h_{tx}, h_{zx}
- Sound channel: h_{tt}, h_{tz}, h_{zz}, h_{ii}

Quasinormal mode spectra at finite coupling

Analytic structure of retarded thermal Green's functions $\Rightarrow$ Dispersion relation of field excitations

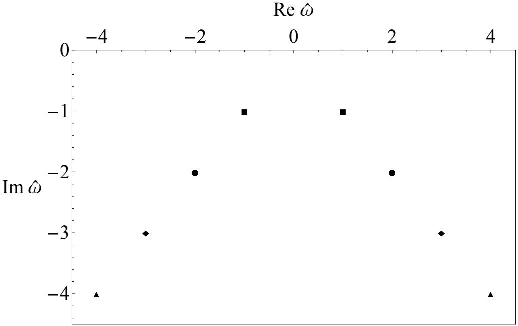
$$\omega_n(k) = E_n(k) + i\Gamma_n(k)$$

Striking difference between weakly and strongly coupled systems:

- At weak coupling (depending on operator) either *long-lived quasiparticles* with $\Gamma_n \ll E_n$ or *branch cuts*
- At strong coupling *quasinormal mode spectrum*

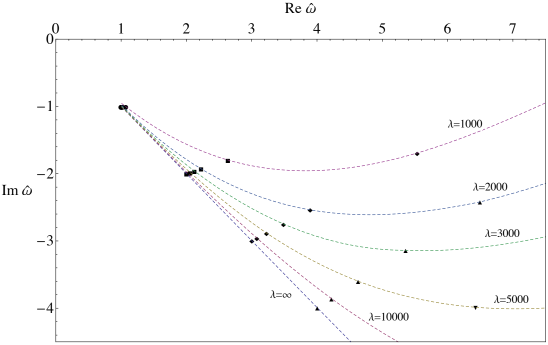
$$\hat{\omega}_n|_{k=0} = \frac{\omega_n|_{k=0}}{2\pi T} = n(\pm 1 - i)$$

QNMs at infinite coupling: Photons



Pole structure of EM current correlator displays usual quasinormal mode spectrum at $\lambda = \infty$. How about at finite coupling?

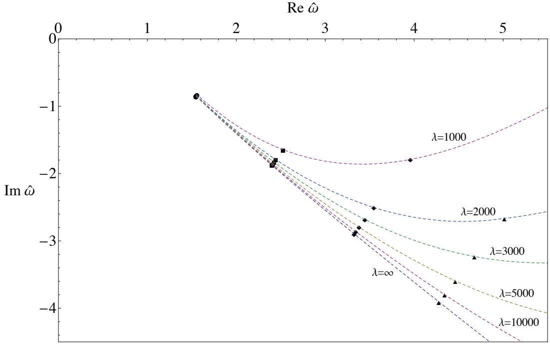
QNMs at finite coupling: Photons



Effect of decreasing λ: Widths of the excitations consistently decrease $\Rightarrow$
Modes become longer-lived

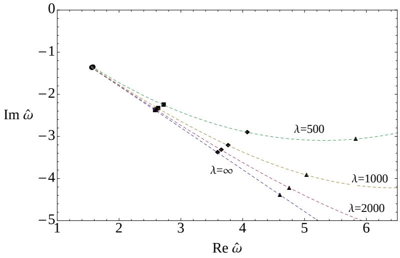
NB: Convergence of strong coupling expansion not guaranteed, when $\hat{\omega}_n|_{k=0} = n(\pm 1 - i) + \xi_n/\lambda^{3/2}$ shifted from $\lambda = \infty$ value by $\mathcal{O}(1)$ amount

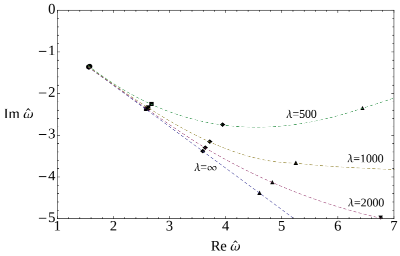
QNMs at finite coupling: Photons



Similar shift at nonzero three-momentum: $k = 2\pi T$

QNMs at finite coupling: $T_{\mu\nu}$ correlators





Same effect also in the shear (left) and sound (right) channels of energy-momentum tensor correlators (here $k = 0$)

Outside the $\lambda = \infty$ limit, the response of a strongly coupled plasma to infinitesimal perturbations appears to change, with the QNM spectrum moving towards the real axis, eventually merging into a branch cut(?)

What happens if we take the system further away from equilibrium?

Off-equilibrium Green's functions: Definitions

Natural quantities to study: Spectral density $\chi(\omega, k) \equiv \mathrm{Im}\,\Pi_\mathrm{R}(\omega, k)$ and related particle production rate (here photons)

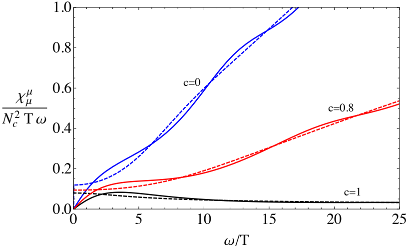
$$k^0 \frac{d\Gamma_\gamma}{d^3k} = \frac{1}{4\pi k}\frac{d\Gamma_\gamma}{dk_0} = \frac{\alpha_\mathrm{EM}}{4\pi^2}\eta^{\mu\nu}\Pi^<_{\mu\nu}(k_0 \equiv \omega, k) = \frac{\alpha_\mathrm{EM}}{4\pi^2}\eta^{\mu\nu} n_B(\omega)\chi^\mu_\mu(\omega, k)$$

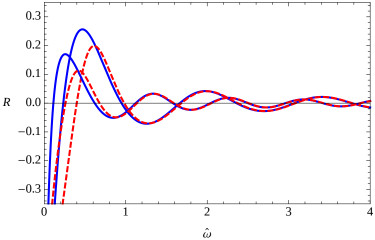
Useful measure of 'out-of-equilibriumness': Relative deviation of spectral density from the thermal limit

$$R(\omega, k) \equiv \frac{\chi(\omega, k) - \chi_\mathrm{therm}(\omega, k)}{\chi_\mathrm{therm}(\omega, k)}$$

Important consistency check: $R \to 0$, as $r_s \to r_h$

Spectral density and R at $\lambda = \infty$: Photons



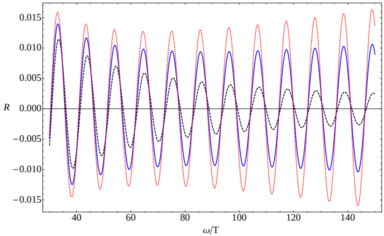


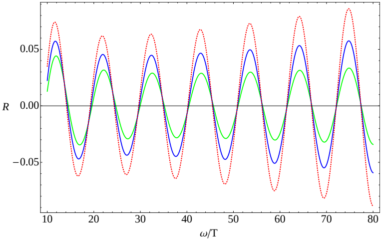
Left: Photon spectral functions for different virtualities ($c = k/\omega$) in thermal equilibrium and $r_s/r_h = 1.1$

Right: Relative deviation $R \equiv (\chi - \chi_{\text{th}})/\chi_{\text{th}}$ for dileptons ($c = 0$) with $r_s/r_h = 1.1$ and 1.01 together with analytic WKB results, valid at large ω

Note: Clear top-down thermalization pattern (as always at $\lambda = \infty$)

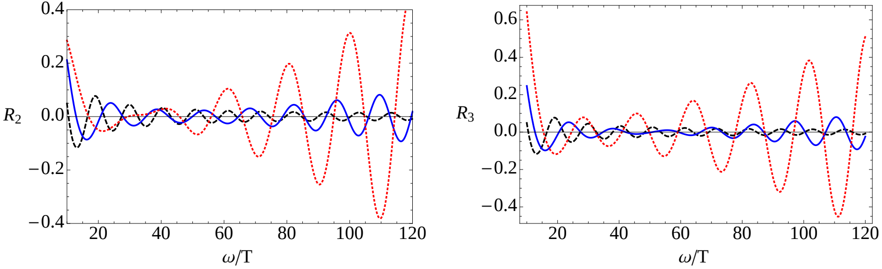
Relative deviation at finite λ: Photons





Relative deviation $R \equiv (\chi - \chi_{\text{th}})/\chi_{\text{th}}$ for on-shell photons with $r_s/r_h = 1.01$ and $\lambda = \infty$, 500, 300 (left) and 150, 100, 75 (right)

Relative deviation at finite λ: $T_{\mu\nu}$ correlators



Relative deviation $R \equiv (\chi - \chi_{\text{th}})/\chi_{\text{th}}$ in the shear and sound channels for $r_s/r_h = 1.2$, $\lambda = 100$, and $k/\omega = 0$ (black), 6/9 (blue) and 8/9 (red)

Change of pattern with decreasing λ: UV modes no longer first to thermalize. Sign of top-down behavior weakening and eventually switching to bottom-up?

Table of contents

Implications for holography

For a given quantity,

$$X(\lambda) \quad = \quad X(\lambda = \infty) \times \Big(1 + X_1/\lambda^{3/2} + \mathcal{O}(1/\lambda^3)\Big),$$

define critical coupling λ_c such that $|X_1/\lambda_c^{3/2}| = 1$. Then:

Quantity	$\lambda_\mathbf{c}$
Pressure	0.9
Transport/hydro coeffs. (η/s, τ_H, κ)	7 ± 1
Spectral densities in equilibrium	$\lambda_c(\omega = 0) = 40$, $\lambda_c(\omega \to \infty) = 0.8$, ...
Quasinormal mode n for photons / $T_{\mu\nu}$	$\lambda_c(n = 1) = 200$, $\lambda_c(n = 2) = 500$ $\lambda_c(n = 3) = 1000$,...

Lesson: What is weak/strong coupling depends strongly on the quantity.
Thermalization appears to be sensitive to strong coupling corrections, with qualitative changes in the pattern already at very large values of λ.