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Transport coefficients in superfluid neutron matter

Laura Tolós

CSIC
Consejo Superior de Investigaciones Científicas

FIAS Frankfurt Institute for Advanced Studies

Cristina Manuel, Sreemoyee Sarkar and Jaume Tarrús

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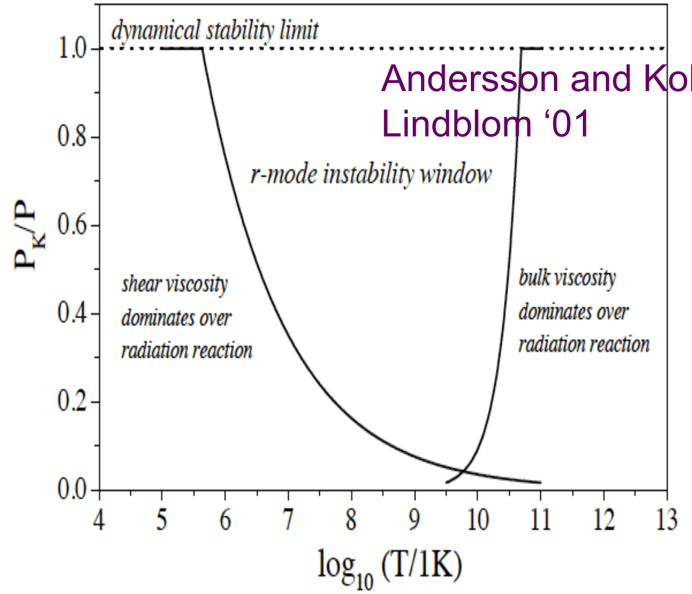
- ✧ EoS for superfluid neutron star matter
- ✧ Shear viscosity and the r-mode instability window
- ✧ Bulk viscosity
- ✧ Thermal conductivity
- ✧ Summary

Manuel and Tolos, Physical Review D 84 (2011) 123007
Manuel and Tolos, Physical Review D 88 (2013) 043001
Manuel, Tarrus and Tolos, JCAP 1307 (2013) 003
Manuel, Sarkar and Tolos, arXiv: 1407.7431 [astro-ph.SR]

Motivation

Study transport coefficients to understand the dynamics of neutron stars

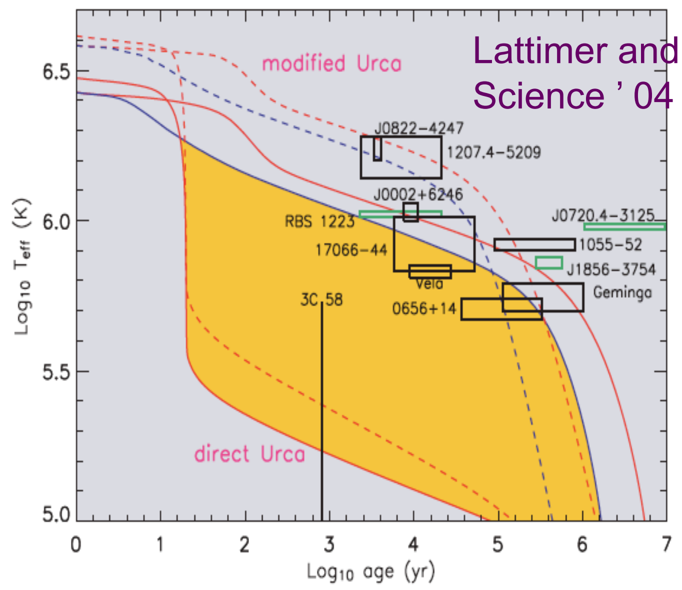
r-modes is one of the families of pulsation modes in rotating neutron stars. They are unstable via emission of gravitational waves. However, there are damping mechanisms (shear and bulk viscous processes) that may counteract the growth of an unstable r-mode



Andersson and Kokkotas ’00
Lindblom ‘01

cooling of a neutron star depends on the rate of neutrino emission, the photon emission rate, the specific heat and the thermal conductivity

Pethick ‘92
Yakovlev and Pethick ‘04



Lattimer and Prakash, Science ’ 04

EFT and superfluid phonon

Exploit the universal character of EFT at leading order by obtaining the effective Lagrangian associated to a superfluid phonon and implement the particular features of the system, associated to the coefficients of the Lagrangian, via the EoS

Son '02
Son and Wingate '06

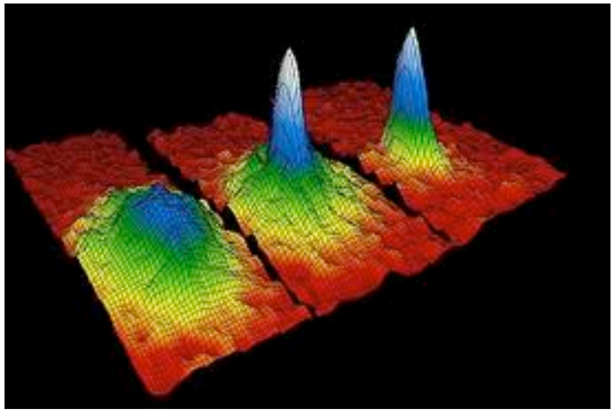
non-relativistic case

$$\mathcal{L}_{\rm LO} = P(X)$$

$$X = \mu - \partial_t \varphi - \frac{(\nabla \varphi)^2}{2m}$$

$P(\mu)$	pressure
μ	chemical potential
φ	phonon field
m	mass condense particles

Applicable in superfluid systems such as cold Fermi gas at unitary, ^{4}He or neutron stars



EoS for superfluid neutron star matter

In order to obtain the speed of sound at T=0 and the different phonon self-couplings one has to determine the EoS for neutron matter in neutron stars.

A common benchmark for nucleonic EoS is APR98
Akmal, Pandharipande and Ravenhall '98

which was later parametrized in a causal form Heiselberg and Hjorth-Jensen '00

$$E/A = \mathcal{E}_0 y \frac{y-2-\delta}{1+\delta y} + S_0 y^\beta (1-2x_p)^2$$

$$y = n/n_0 \qquad x_p = n/n_0$$

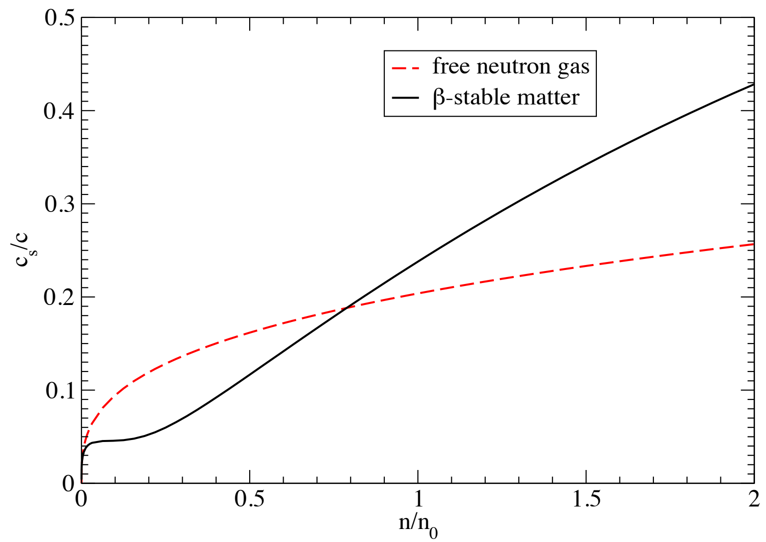
$n_0 = 0.16\,\mathrm{fm}^{-3}$

$\mathcal{E}_0 = 15.8\,\mathrm{MeV}$ $\delta = 0.2$

$S_0 = 32\,\mathrm{MeV}$ $\beta = 0.6$

For β-stable matter made up of neutrons, protons and electrons, the speed of sound at T=0 is

$$\sqrt{\frac{\partial P}{\partial \rho}} \equiv c_s$$



Effective Lagrangian for superfluid phonon at LO

$$\mathcal{L}_{\rm LO} = \frac{1}{2}((\partial_t\phi)^2 - v_{\rm ph}^2(\nabla\phi)^2) - g((\partial_t\phi)^3 - 3\eta_g\partial_t\phi(\nabla\phi)^2) + \lambda((\partial_t\phi)^4 - \eta_{\lambda,1}(\partial_t\phi)^2(\nabla\phi)^2 + \eta_{\lambda,2}(\nabla\phi)^4) + \cdots .$$

with Φ the rescaled phonon field, and where the different phonon self-couplings can be expressed in terms of the speed of sound at T=0

$$v_{ph} = \sqrt{\frac{\frac{\partial P}{\partial\mu}}{m\frac{\partial^2 P}{\partial\mu^2}}} = \sqrt{\frac{\partial P}{\partial\rho}} \equiv c_s$$

and derivatives with respect to mass density:

Escobedo and Manuel '10

$$u = \frac{\rho}{c_s}\frac{\partial c_s}{\partial\rho}, \quad w = \frac{\rho}{c_s}\frac{\partial^2 c_s}{\partial\rho^2},$$

$$g = \frac{1-2u}{6c_s\sqrt{\rho}}, \qquad \eta_g = \frac{c_s^2}{1-2u}, \qquad \lambda = \frac{1-2u(4-5u)-2w\rho}{24c_s^2\rho},$$

$$\eta_{\lambda,1} = \frac{6c_s^2(1-2u)}{1-2u(4-5u)-2w\rho}, \qquad \eta_{\lambda,2} = \frac{3c_s^4}{1-2u(4-5u)-2w\rho}$$

Results valid for neutrons pairing in $^1S_0$ channel and also valid for $^3P_2$ neutron pairing if corrections $\bar{\Delta}(^3P_2)^2/\mu_n^2$ are ignored Bedaque, Rupak and Savage '03

Including NLO corrections in the phonon dispersion law

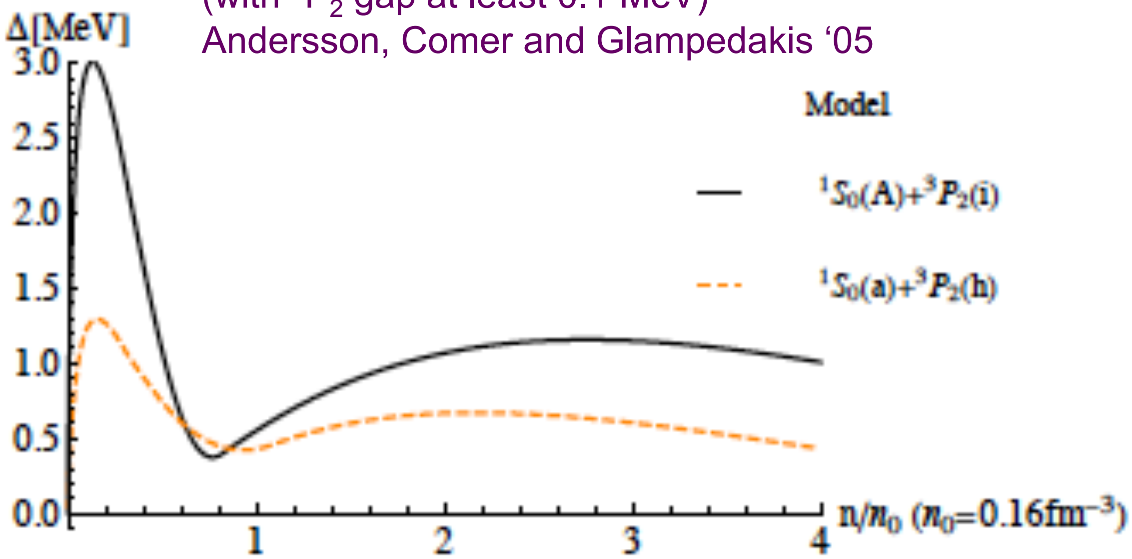
$$E_P = c_s p(1 + \gamma p^2)$$

$$\gamma = -\frac{v_F^2}{45\Delta^2}$$

v_F: Fermi velocity
Δ: gap function

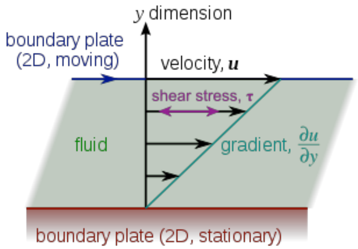
ϒ < 0: first allowed phonon scattering are binary collisions

taken from parameterizations of different models
(with $^3P_2$ gap at least 0.1 MeV)
Andersson, Comer and Glampedakis ‘05



Shear viscosity due to superfluid phonons

Shear viscosity

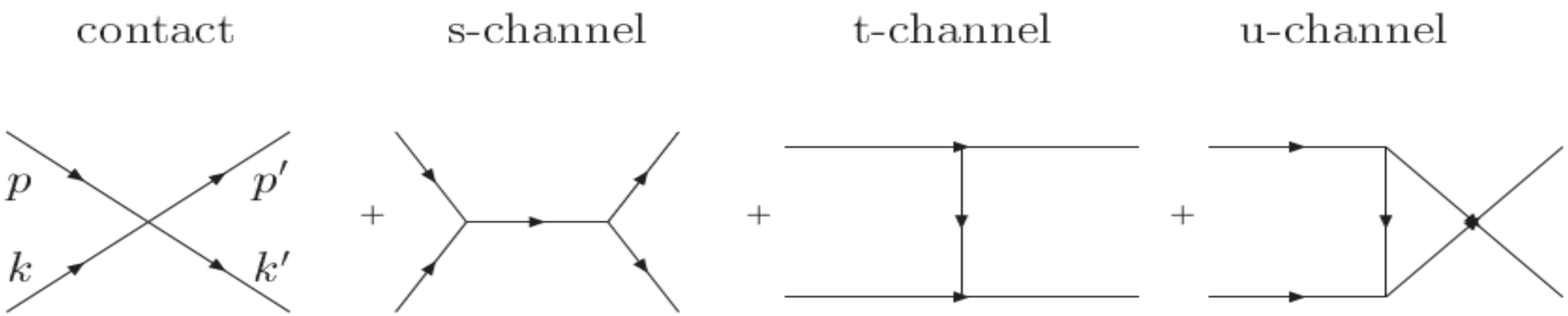


wikipedia

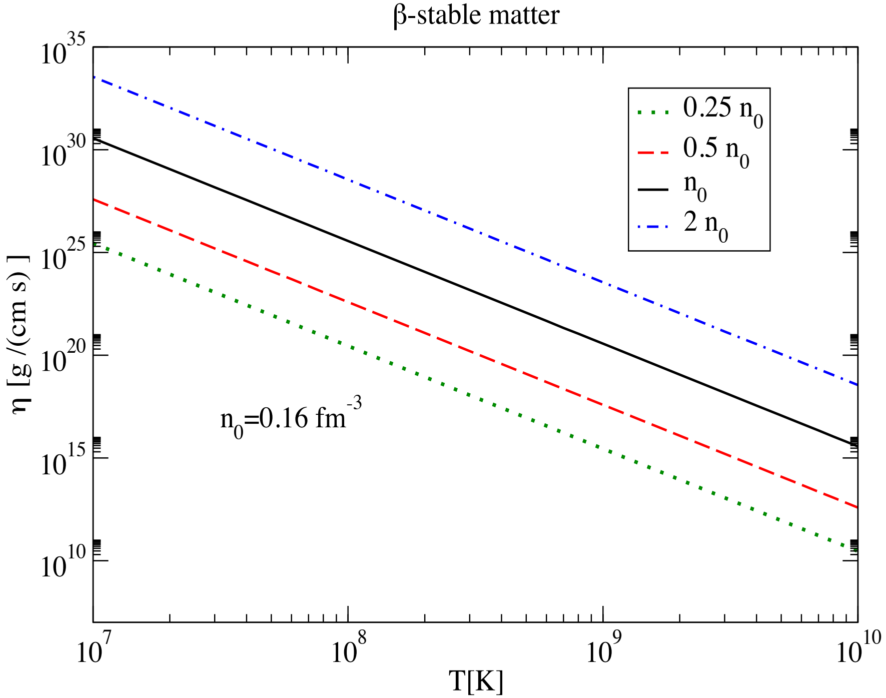
The shear viscosity is calculated using variational methods for solving the transport equation as

$$\eta = \left(\frac{2\pi}{15}\right)^4 \frac{T^8}{c_s^8} \frac{1}{M}$$

where M represents a multidimensional integral that contains the thermally weighted scattering matrix for phonons.



Shear viscosity due to binary collisions of phonons scales as $\eta \propto 1/T^5$ (also for ^{4}He and cold Fermi gas at unitary) while the coefficient depends on EoS.

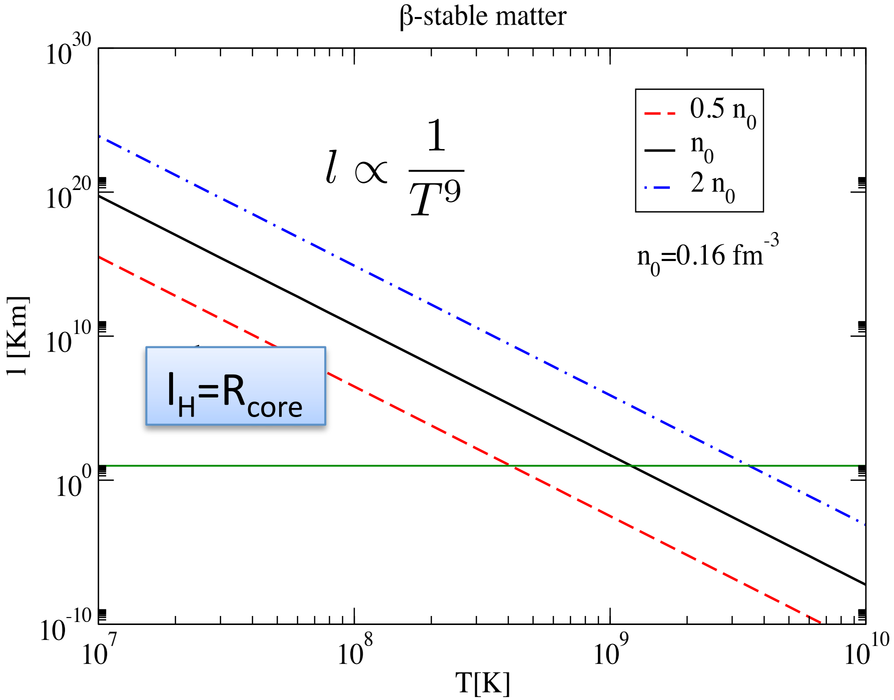


Mean free path of phonons: establish when phonons become hydrodynamic

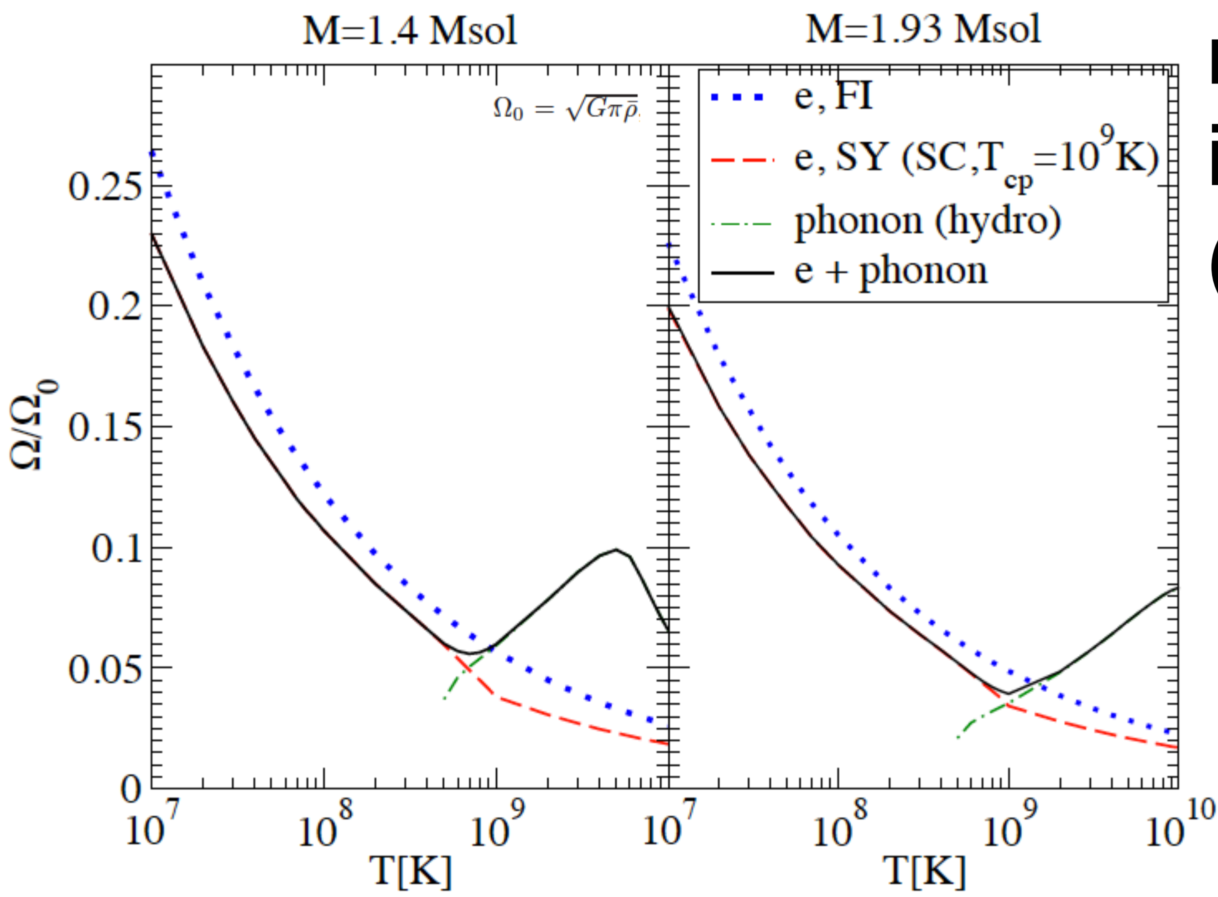
$$l = \frac{\eta}{n < p>}$$

$< p >$: thermal average

n : phonon density



Alford, Braby, and Mahmoodifar '10



r-mode instability window (only shear)

$$-\frac{1}{|\tau_{\rm GR}(\Omega)|} + \frac{1}{\tau_{\eta}(T)_{\rm hydro}} = 0$$

Dissipation due to superfluid phonons start to be relevant at T≈7 x 10^8 K for 1.4 $M_\odot$ and T≈10^9 K for 1.93 $M_\odot$

$$\frac{1}{|\tau_{\rm GR}(\Omega)|} = \frac{32\,\pi\,G\,\Omega^{2l+2}}{c^{2l+3}} \frac{(l-1)^{2l}}{((2l+1)!!)^2} \left(\frac{l+2}{l+1}\right)^{2l+2} \int_0^R \rho r^{2l+2} dr$$

$$\frac{1}{\tau_{\eta}(T)_{\rm hydro}} = (l-1)(2l+1) \int_{R_c}^{R} \eta_{\rm hydro} r^{2l} dr \left(\int_0^R \rho r^{2l+2} dr\right)^{-1}$$

l=2 (dominant)

Bulk viscosities due to superfluid phonons

The bulk viscosity coefficients are calculated from the dynamical evolution of the phonon number density* or, equivalently, by using the Boltzmann equation for phonons in the relaxation time approximation

$$\zeta_i(\omega) = \frac{1}{1+\left(\omega I_1^2 \frac{\partial\rho}{\partial n}\frac{\partial\rho}{\partial\mu}\frac{T}{\Gamma_{ph}}\right)^2}\frac{T}{\Gamma_{ph}}C_i \, , \qquad i=1,2,3,4$$

$$C_1 = C_4 = -I_1 I_2 \, , \qquad C_2 = I_2^2 \, , \qquad C_3 = I_1^2 \, ,$$

$$I_1 = \frac{60T^5}{7c_s^7\pi^2}\left(\pi^2\zeta(3)-7\zeta(5)\right)\left(c_s\frac{\partial B}{\partial\rho} - B\frac{\partial c_s}{\partial\rho}\right) \, , \qquad B = c_s\gamma$$

$$I_2 = -\frac{20T^5}{7c_s^7\pi^2}\left(\pi^2\zeta(3)-7\zeta(5)\right)\left(2Bc_s + 3\rho\left(c_s\frac{\partial B}{\partial\rho} - B\frac{\partial c_s}{\partial\rho}\right)\right) \, ,$$

NLO corrections in phonon dispersion law

Three independent coefficients: $\zeta_1 = \zeta_4 \quad \zeta_1^2 \leq \zeta_2\zeta_3 \quad \zeta_2, \zeta_3 \geq 0$

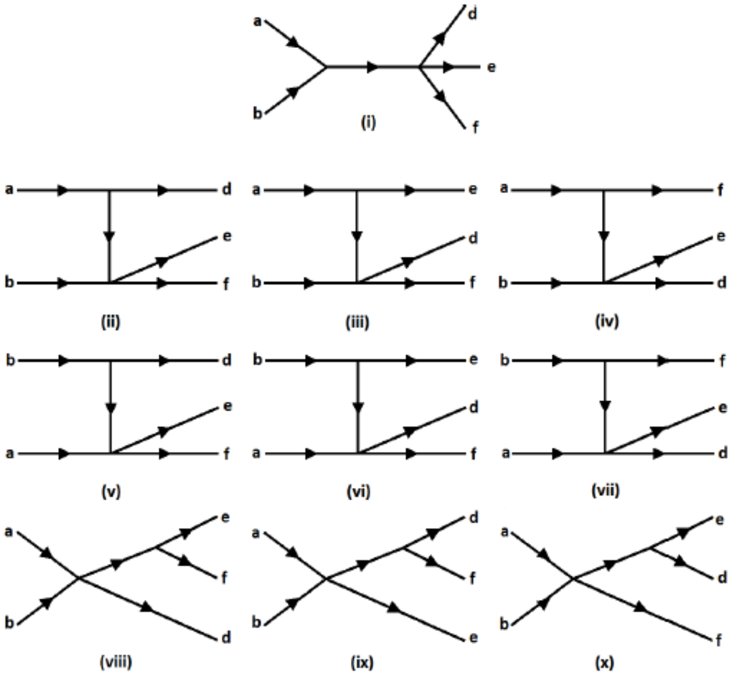
In the static limit

$$\zeta_i = \frac{T}{\Gamma_{ph}}C_i \, , \qquad i=1,2,3,4 \, ,$$

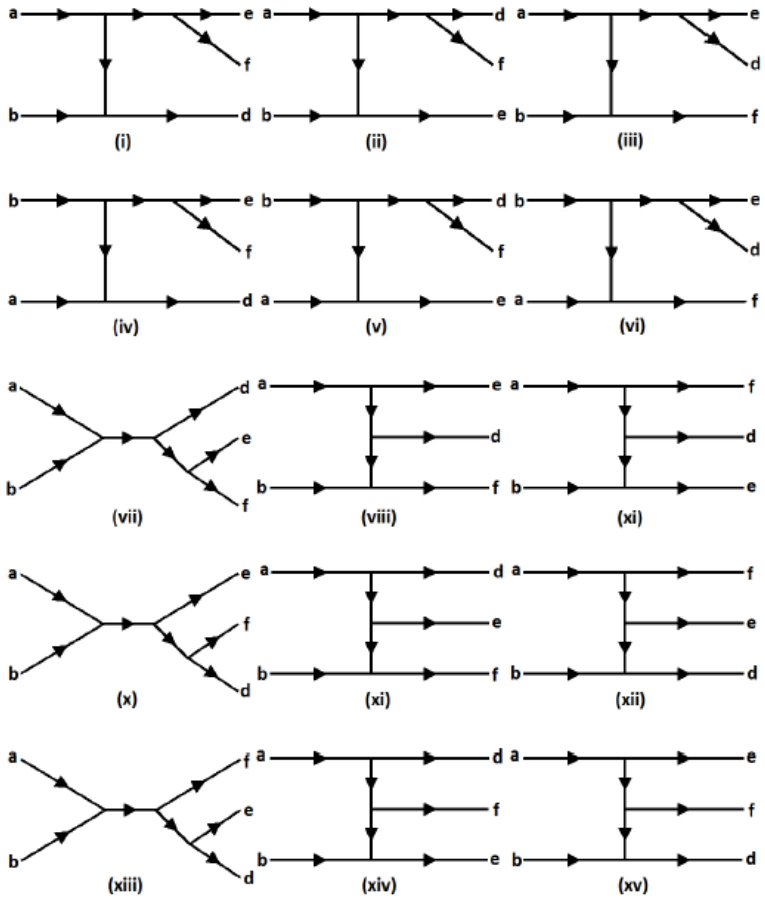
* Khalatnikov

Phonon decay rate for phonon number changing processes: 2 <-> 3

$$\Gamma_{ph} = \int d\Phi_5(p_a, p_b; p_d, p_e, p_f) \|\mathcal{A}\|^2 f(E_a) f(E_b) \left(1 + f(E_d)\right) \left(1 + f(E_e)\right) \left(1 + f(E_f)\right)$$

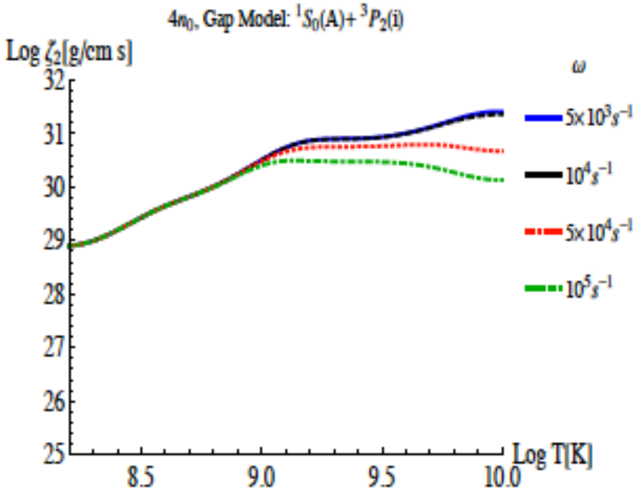


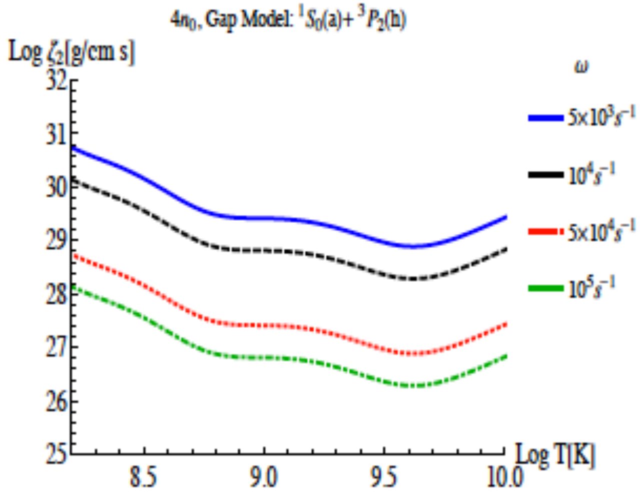
with one 4-phonon vertex
and one 3-phonon vertex



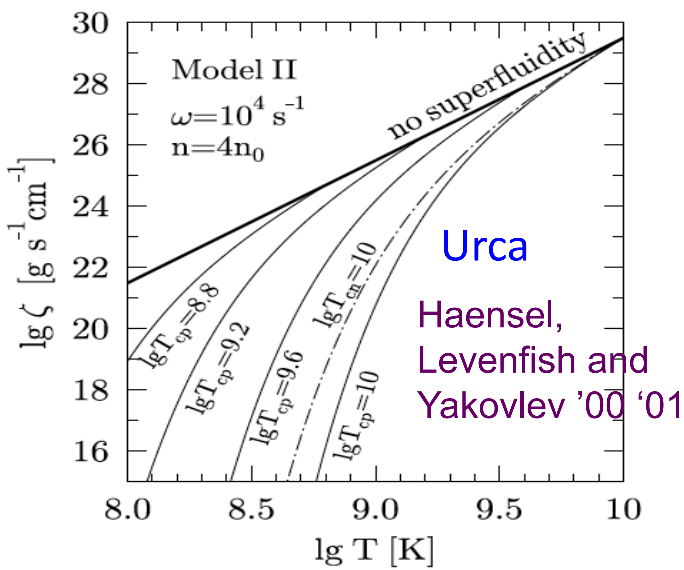
with only 3-phonon vertices

ξ_2 at $n \geq 4n_0$ is within 10% of the static value for $T \leq 10^9$ K and for the case of maximum values of the $^3P_2$ gap > 1 MeV, while, otherwise, the static solution is not valid. Bulk viscosity coefficients strongly depend on the gap.





Compared to the contribution of Urca (also modified Urca) processes to the bulk viscosities in neutron stars, those are dominated by phonon-phonon processes



Thermal conductivity due to superfluid phonons

The thermal conductivity relates the heat flux with the temperature gradient

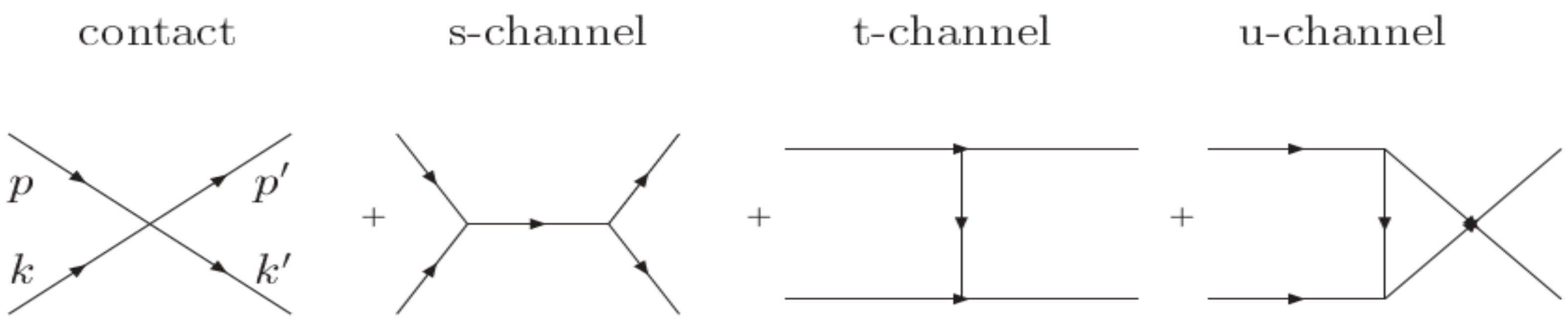
$$\mathbf{q} = -\kappa \nabla T$$

and is calculated using variational methods* for solving the transport equation as

$$\kappa \geq \left(\frac{4a_1^2}{3T^2}\right) A_1^2 M_{11}^{-1},$$

$$a_1 = \frac{4c_s^4}{15\Delta^2}\,, \qquad A_1 = \frac{256\pi^6}{245 c_s^9} T^9$$

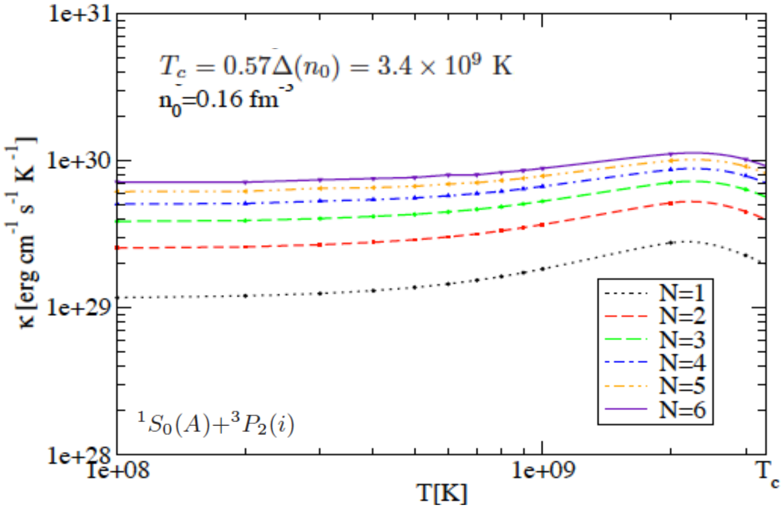
where M_{11} is the (1,1) element of a N x N matrix (N determined by convergence). Each element is a multidimensional integral that contains the thermally weighted scattering matrix for phonons:



* Braby, Chao and Schafer '10

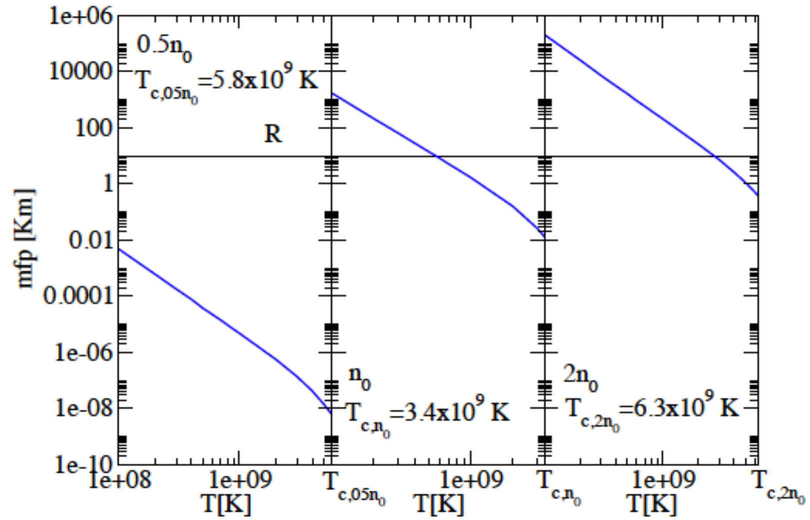
Need of NLO corrections in phonon dispersion law (seen for He^4 and CFL) Perform a variational calculation up to N=6 (deviation from N=5 ≤ 10%). We find that as in CFL

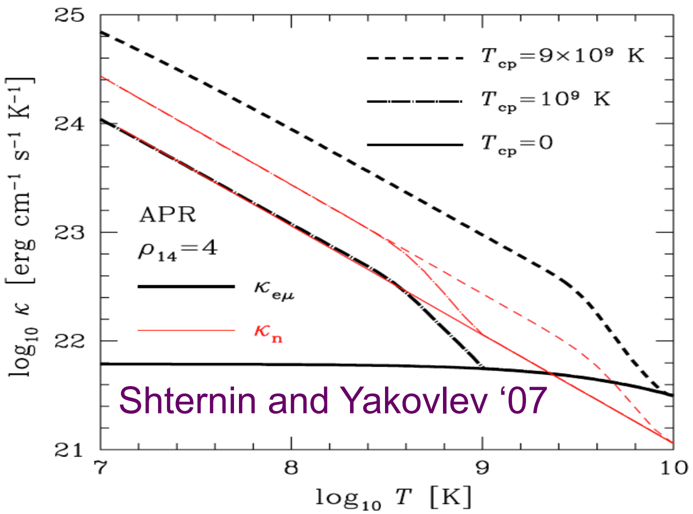
$$\kappa \propto \frac{1}{\Delta^6} \qquad T \lesssim 10^9 \text{ K}$$



Mean free path of phonons (different from shear mean free path)

$$l = \frac{\kappa}{\frac{1}{3}c_v c_s} \qquad l \propto 1/T^3$$





$$10^{25} \lesssim \kappa_{ph} \lesssim 10^{32} \text{ erg cm}^{-1} \text{ s}^{-1} \text{ K}^{-1}$$

Thermal conductivity in neutron stars is dominated by phonon-phonon collisions

Summary

Starting from a general formulation for the collisions of superfluid phonons using EFT techniques, we compute the shear and bulk viscosities as well as the thermal conductivity in terms of the EoS of the system (and the gap function)

• Binary collisions of phonons produce a shear viscosity that scales with $1/T^5$ (universal feature seen for ^{4}He and cold Fermi gas at unitary) while the coefficient depends on EoS (microscopic theory)

r-mode window modified for $T \geq (10^8\text{-}10^9)$ K due to phonon shear viscosity

• Bulk viscosity coefficients strongly depend on the gap and they are dominated by phonon-phonon collisions as compared to Urca (modified Urca) processes

• Thermal conductivity due to phonons scales as $1/\Delta^6$, the constant of proportionality depending on the EoS. As compared to electron-muon collisions, phonon-phonon collisions dominate the thermal conductivity

Future: need of an accurate analysis of electron-phonon collisions

estimates by Bedaque and Reddy'13