Excited states in Lattice QCD

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Background

- Spectrum of resonance states in QCD is poorly understood:
  - 'Missing' Baryon resonances
  - 'Exotic' States in light quark and charmonium sectors.
  - Lots of experimental effort underway (Many talks!)
- Resonances typically appear as a rapid variation in scattering phase shifts.
- We want to calculate resonance parameters from lattice QCD simulations.

- Recent algorithmic advances have improved precision and energy resolution.

- Near future: some low-lying resonance properties can become 'gold-plated'

Golden drawing room,
State Hermitage Museum,
St. Petersburg
(drawing)

Lattice QCD

Euclidean Path Integral (finite a, L, T) $\leftrightarrow$ Statistical Ensemble:

$$\langle 0|F[\hat{U}]|0\rangle_c = \int DU \; F[U] \; \frac{(\det M[U])^{N_f} \, \mathrm{e}^{-S_{YM}[U]}}{Z}$$

Generate $\{U_i\}$ using a Markov Chain (Metropolis Algorithm):

$$U_0 \to U_1 \to \ldots \to U_t \to U_{t+1}$$

Scattering Info from Lattice QCD

$$C_{ij}(t) = \sum_n \langle 0|\hat{\mathcal{O}}_i|n\rangle\langle n|\hat{\mathcal{O}}_j^\dagger|0\rangle\, \mathrm{e}^{-E_n t}$$

- Maiani-Testa no-go theorem: L. Maiani and M. Testa, `90
  - Naively, just calculate $\langle \mathbf{p}_1^f, \mathbf{p}_2^f | \mathbf{p}_1^i, \mathbf{p}_2^i \rangle$
  - In general, contains no info about on shell scattering amplitudes
- Lüscher's work-around: below inelastic thresholds

M. Lüscher, `91; M. Lüscher, U. Wolff, `90

$$\det\left[F^{-1}(q) - i\mathcal{M}(q)\right] = 0, \qquad E_{cm}^2 = 4(m^2 + q^2)$$

Generalizations of the Lüscher formalism:

- Moving frames: $\vec{P}_{tot} \neq 0$

  Gottlieb and Rummukainen '95;
  Kim, Sharpe, Sachrajda '05 ;
  Leskovec and Prelovsek '12 ; Gockeler et al '12

- Non-QCD two-hadron decays: $K \to \pi\pi$

  Lellouch, Lüscher '01
  Briceno, Hansen, Walker-Loud '14

- Coupled two-hadron decay channels: $K^* \to K\pi, K\eta$

  He, Feng, Liu `05

- Two-hadron production: $e^+e^- \to \pi\pi$

  Meyer `11

- Resonance matrix elements: $\gamma N \to N\pi$

  Bernard et al '12; Agadjanov et al '14 (talk today 18:00)

- Three-hadron decays: $\omega \to 3\pi$

  Polejaeva and Rusetsky '12;
  Sharpe and Hansen '14

1-D Quantum Mechanics:

- Recall: solutions to Schroedinger eq. in 'exterior region' with definite parity are

$$\psi_+(x) = A\cos\left(k|x| + \delta_+(k)\right)$$
$$\psi_-(x) = B\, sign(x)\, \sin\left(k|x| + \delta_-(k)\right)$$

- On a (large) circle, allowed momenta are shifted from non-interacting values to satisfy periodicity:

$$kL + 2\delta_\pm(k) = 0 \pmod{2\pi}$$

- Allowed energies will be distorted differently for each parity.

General Procedure for elastic scattering phase shifts

Example: $\rho \to \pi\pi$

1. Calculate finite-volume single-hadron energy: m_π

2. Calculate finite-volume two-hadron energies: $E_{2\pi}^{I=1}(\vec{P}_{tot})$

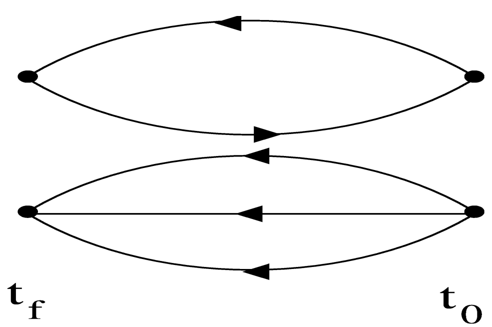
3. Obtain the center-of-mass momentum: $E^2 = 4(m^2 + q^2)$

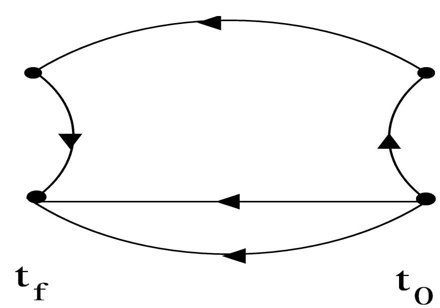
4. Use the formula

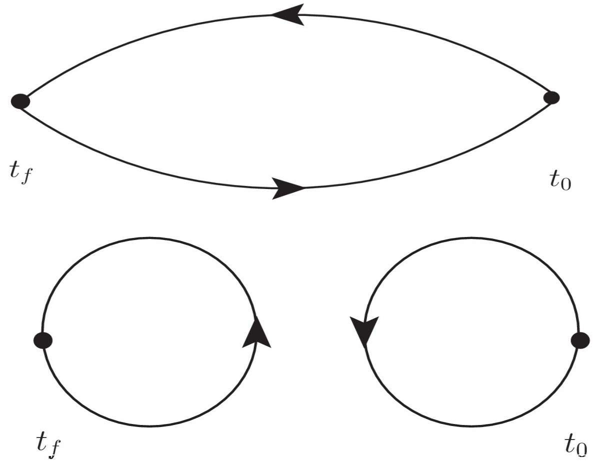
$$\det\left[F^{-1}(q) - i\mathcal{M}(q)\right] = 0$$

Algorithmic advances

- Step 2 is difficult!
- Correlators with quarks (after Wick contractions) contain $M^{-1}(x,y)$, inverse of a large ill-conditioned matrix.
- Scattering states require 'disconnected' diagrams







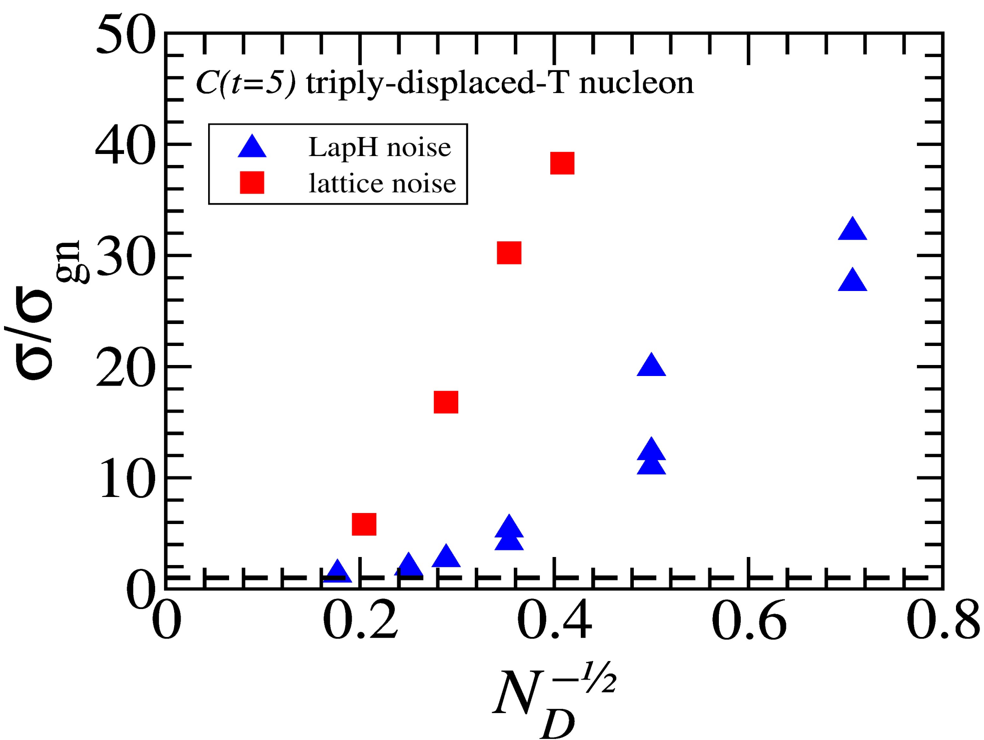
- These diagrams require the entire matrix $M^{-1}(x,y)$
- 'Distillation': matrix projected onto a subspace is tractable, but dimension grows with spatial volume.

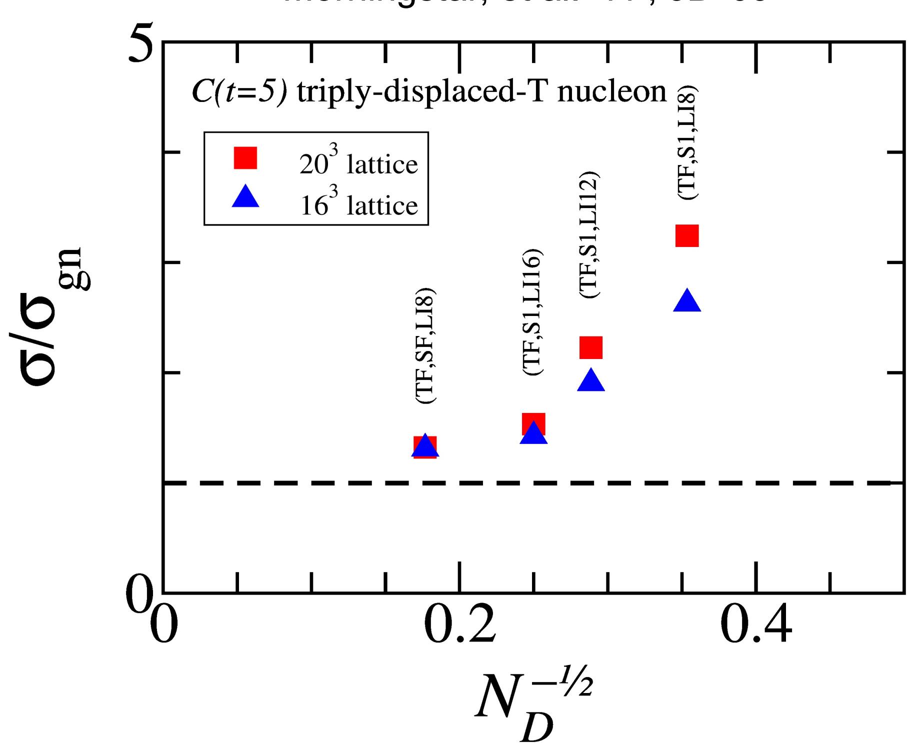
$$\nabla^2 v_n = \lambda_n v_n$$

Peardon, et al. '09

- Projection + stochastic estimation allow for efficient correlator construction. (Stochastic LapH)

Morningstar, et al. '11 ; JB '09

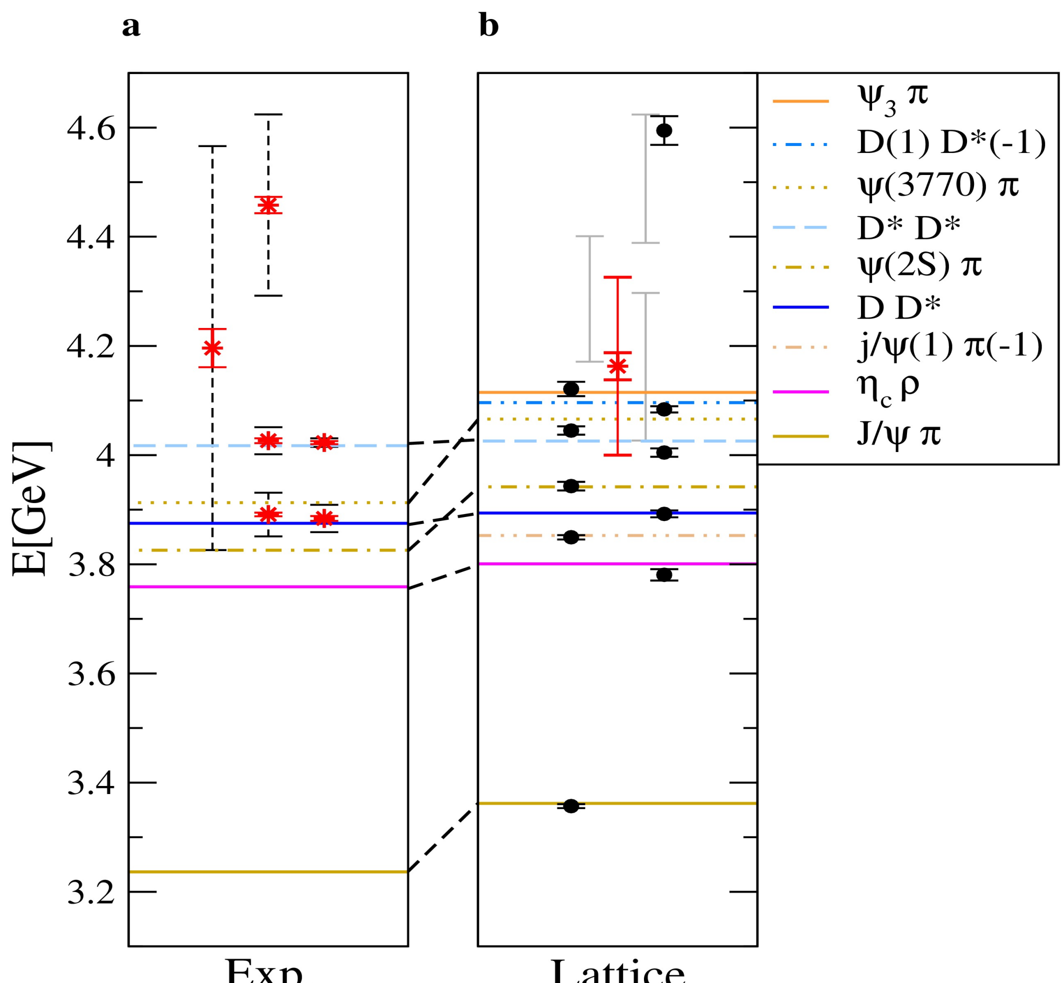




Finite Volume Spectra (2013-2014)

- Investigation of the $Z(4430)^-$ Prelovsek, et al. `14; talk today 17:10
- Lattice details: $N_f = 2, \quad a \approx 0.12\text{fm},$
  $m_\pi \approx 266\text{MeV}, \quad L \approx 2\text{fm}$
- Tetraquark and multi-hadron ops included.
- No annihilation diagrams.
- See also:

  Y. Ikeda, talk today 17:10
  T. Kawanai, talk Thursday 16:55



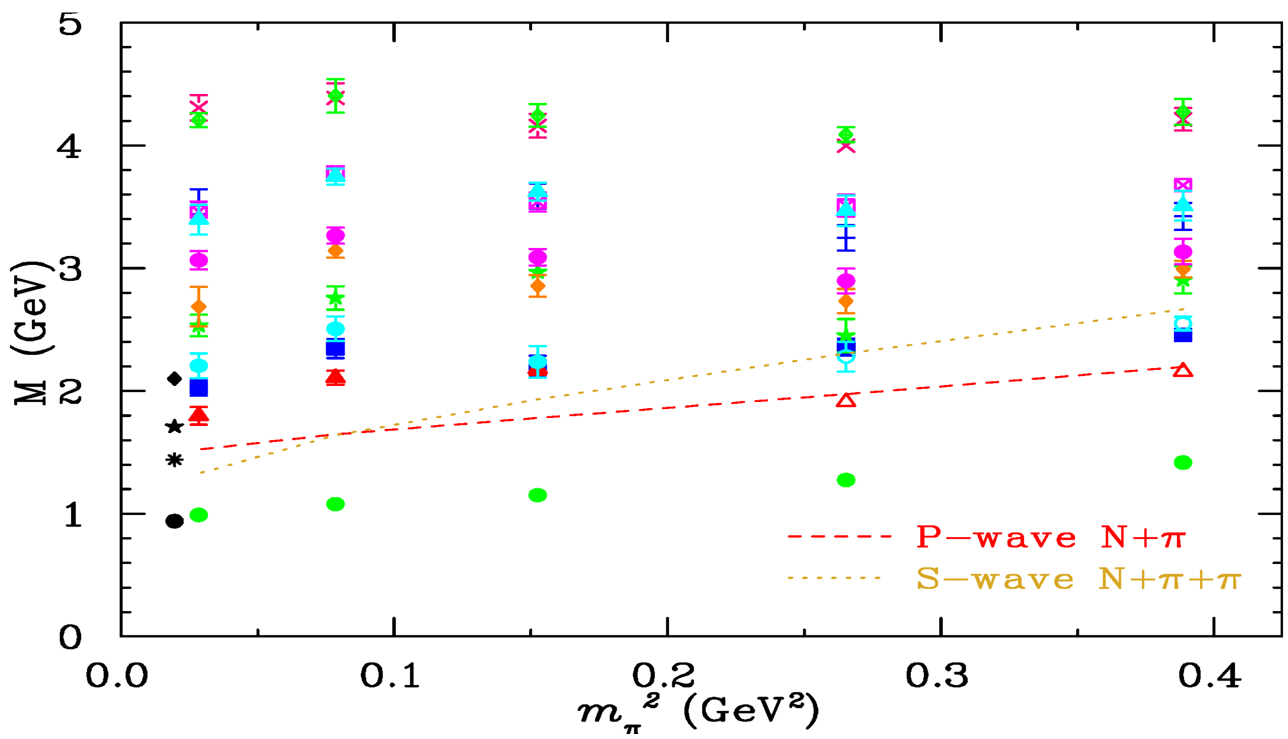
- Excited Nucleon states S. Mahbub, et al. '13

- Lattice Details:

$$N_f = 2+1, \qquad a \approx 0.09\text{fm},$$
$$m_\pi = 160 - 700\text{MeV}, \qquad L \approx 2.9\text{fm}$$

- No scattering states.

Preliminary effort:
W. Kamleh, talk Lattice2014



- Also see

N. Mathur, talk: today 14:30

- Excited isovector mesons (preliminary)

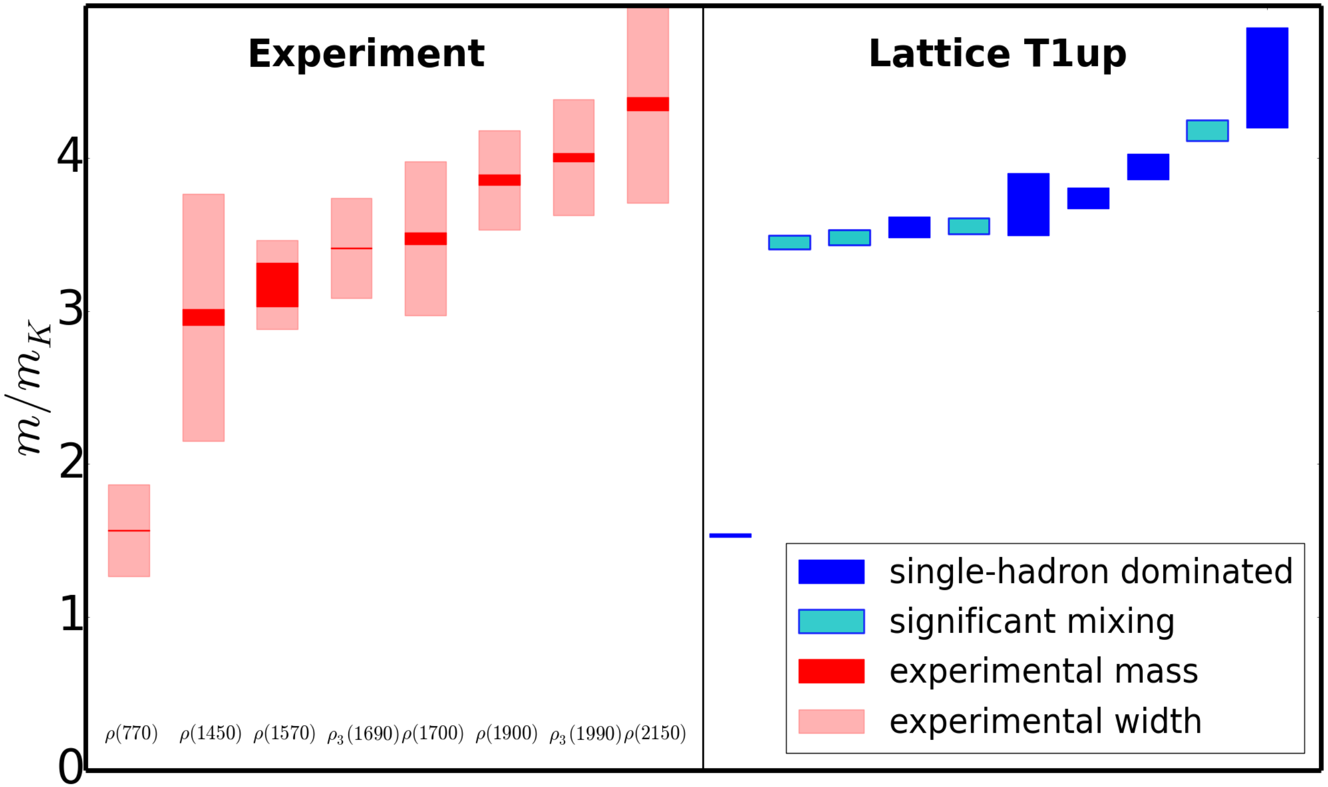
B. Fahy, C. Morningstar

- Lattice Details:

$$N_f = 2 + 1, \qquad a_s = 3.5 a_t \approx 0.12\text{fm},$$
$$m_\pi \approx 240\text{MeV}, \qquad L \approx 3.8\text{fm}$$

- Many multi-hadron ops! ~30 levels extracted

- Identify levels which overlap single-hadron operators



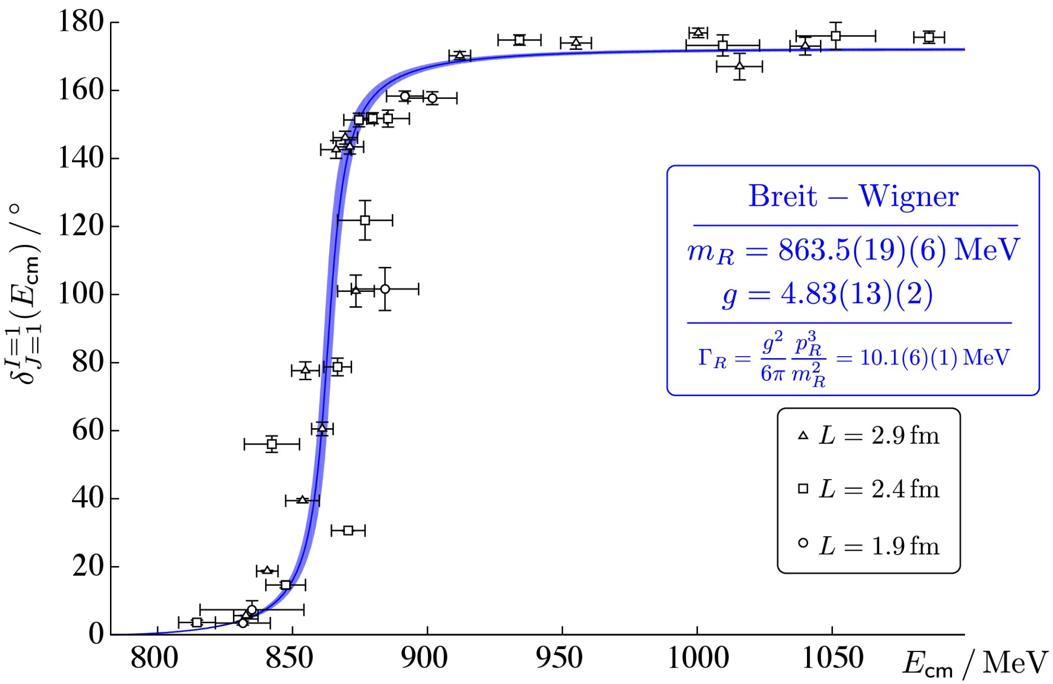
What about Step 3?

- $I = 1 \quad \pi - \pi$ scattering: 3 volumes and several frames

J. Dudek, R. Edwards, C. Thomas `13

- Lattice Details: $N_f = 2 + 1, \qquad a_s \approx 0.12\text{fm} = 3.5 a_t,$
  $m_\pi \approx 400\text{MeV}, \qquad L < 3\text{fm}$

- Distillation very costly. Impossible to go larger and lighter.

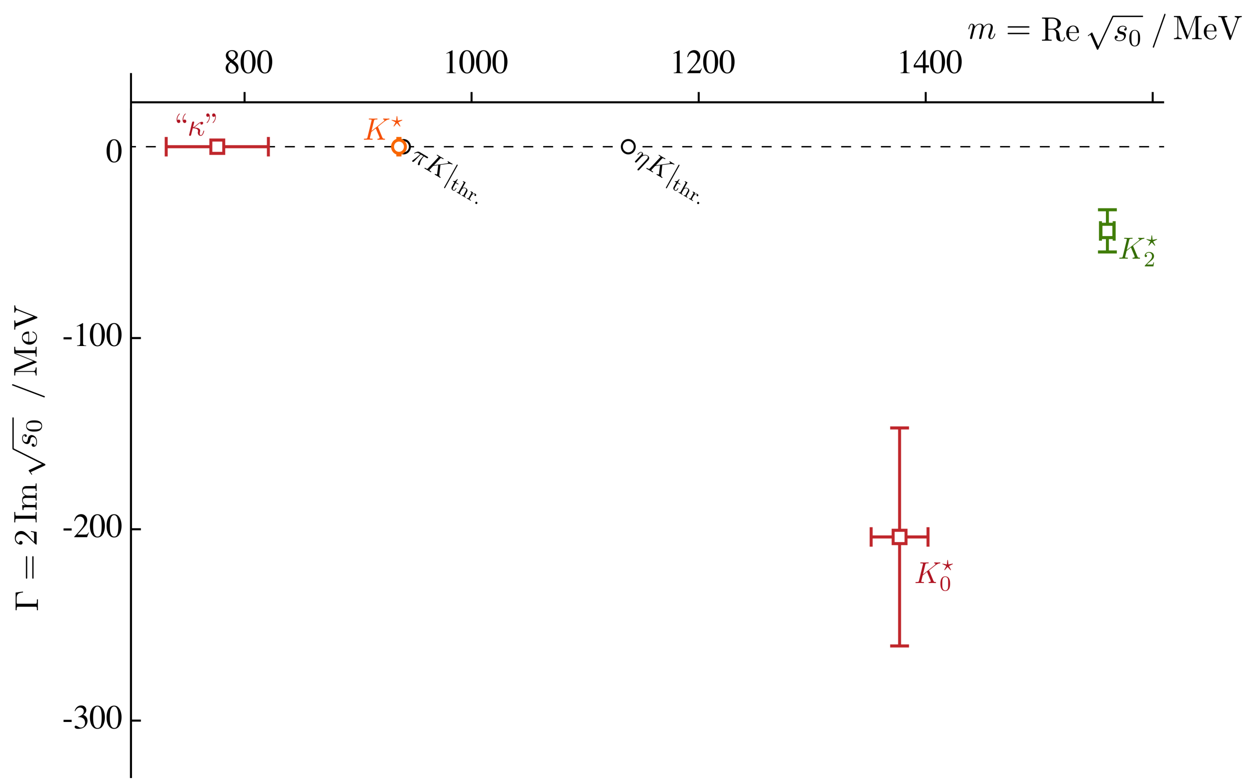


- Coupled channel analysis of $K\pi,\ K\eta$

D. Wilson et al. `14

- Coupled channel scattering amplitude parametrized using a K-matrix model with Breit-Wigner resonances.

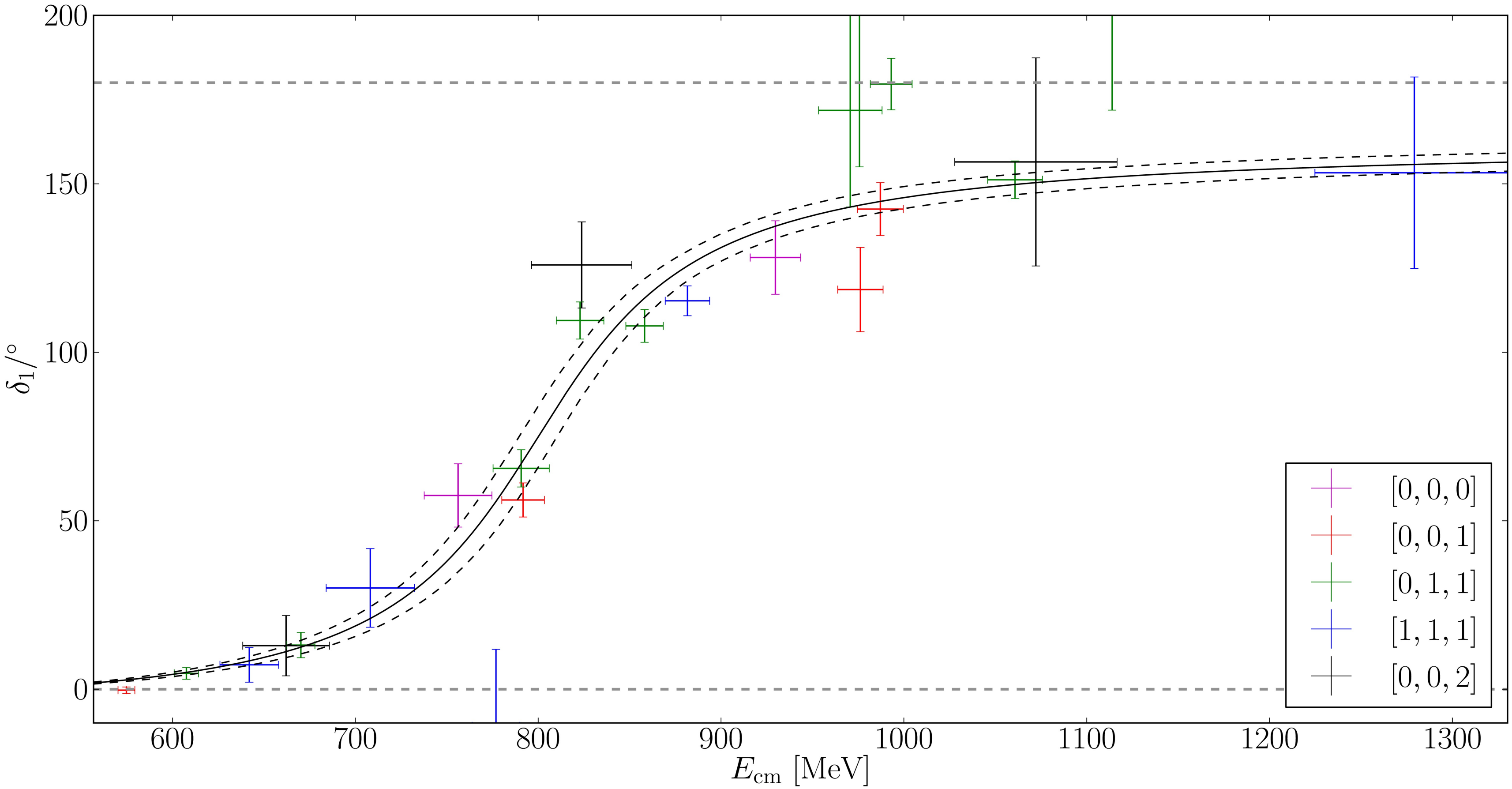
- Unfortunately, K^* is stable



Scattering phase shift (Preliminary)

B. Fahy, B. Hörz, C. Morningstar, JB

- Lattice Details: $N_f = 2+1, \quad a_s = 3.5a_t \approx 0.12\text{fm},$ $m_\pi \approx 240\text{MeV}, \quad L \approx 3.8\text{fm}$
- Single lattice, 5 moving frames, 2-3 levels each
- Width is larger! $g = 7.1(5),\ m_R = 833(10)\text{MeV}$



Conclusions

- Past few years have seen significant improvement in Lattice QCD calculations of resonances.

- Thanks to algorithmic advances, these calculations can proceed to larger volumes and lighter pion masses.

- While there has been theoretical advancement in inelastic scattering, 3 or more hadron thresholds remain a challenge.

Future Prospects

- Systematic errors will be improved:

$$a \to 0, \qquad m_\pi \to m_\pi^{phys}, \qquad m_\pi L \to \infty$$

- Additional low-hanging fruit: $D^*, \quad a_0(980), \quad f_0(600)$

- Baryon-meson scattering? $\Delta, \quad N(1440), \quad \Lambda(1405), \ldots$

- More coupled two-hadron channels.

- Resonance structure calculations. $e^+e^- \to \pi\pi, \quad F_\pi(q^2)$