Electroweak probes in the presence of resonances on the lattice

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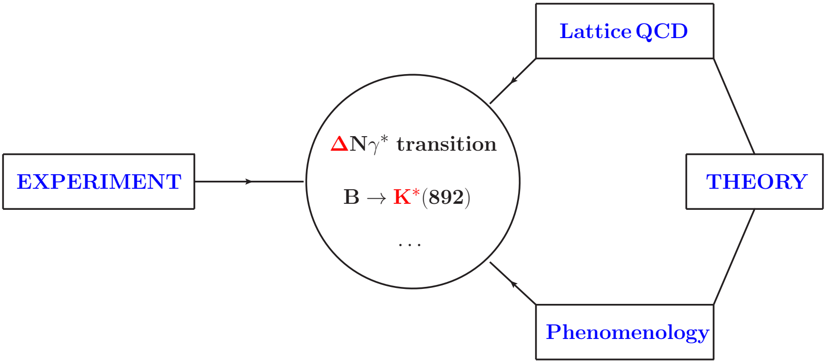
AA, V. Bernard, U.-G. Meißner, A. Rusetsky, Nucl. Phys. B 886 (2014)

Outline

- Introduction
- Motivation, background
- Goals, results
- Outlook

Introduction

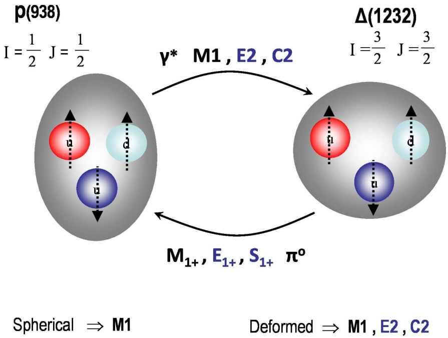


- $\gamma^* N \to \pi N$ near the $\Delta(1232)$: study of the hadron deformation
- $B \to K^* \gamma^*$, $B \to K^* l^+ l^-$ with $K^*(892) \to K\pi$: sensitive to NP

$\gamma^* N \to \Delta$

$$\gamma^* p \rightarrow \Delta^+(1232) \rightarrow p\pi^0 \quad (66\%)$$
$$\gamma^* p \rightarrow \Delta^+(1232) \rightarrow n\pi^+ \quad (33\%)$$
$$\gamma^* p \rightarrow \Delta^+(1232) \rightarrow p\gamma \quad (0.56\%)$$

$\hookrightarrow$ study of the de-excitation radiation pattern



C. Alexandrou et al., arXiv:1201.4511

- The electromagnetic transition matrix element:

$$\langle \Delta(P,\lambda)|J_\mu(0)|N(Q,\epsilon)\rangle = \left(\frac{2}{3}\right)^{1/2} \bar{u}^\sigma(P,\lambda)\mathcal{O}_{\sigma\mu}u(Q,\epsilon)\,,$$

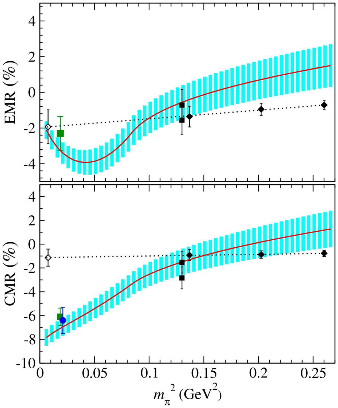
with the Lorentz-structure

$$\mathcal{O}_{\sigma\mu} = G_M(q^2)K^{M1}_{\sigma\mu} + G_E(q^2)K^{E2}_{\sigma\mu} + G_C(q^2)K^{C2}_{\sigma\mu}$$

$$\mathrm{EMR} \equiv \frac{\mathrm{Im}E^{3/2}_{1+}}{\mathrm{Im}M^{3/2}_{1+}} = -\frac{G_E(q^2)}{G_M(q^2)}, \quad \mathrm{CMR} \equiv \frac{\mathrm{Im}L^{3/2}_{1+}}{\mathrm{Im}M^{3/2}_{1+}} = -\frac{|\vec{q}|}{2m_\Delta}\frac{G_C(q^2)}{G_M(q^2)}$$

$\hookrightarrow$ extracted from experiment   calculated on the lattice $\hookleftarrow$

The values of EMR and CMR



χEFT extrapolation of lattice results to the physical pion mass

V. Pascalutsa, M. Vanderhaeghen, Phys. Rev. D **73** (2006) 034003

- The Δ is treated as a stable particle!

Motivation, background

The largest conceptual question as we enter the chiral regime in full QCD, is how to fully incorporate the physical effect of the decay of the Δ into a pion and nucleon on the transition form factors.

C. Alexandrou et al., Phys. Rev. D **77** (2008) 085012

• lattice simulations are done in a finite space $\Rightarrow$ impossible to prepare asymptotic states $\Rightarrow$ no resonances $\Rightarrow$ special theoretical treatment is needed $\Rightarrow$ Lüscher approach

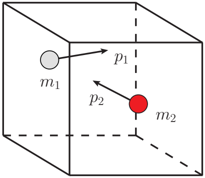
• generalization of Lüscher approach for matrix elements, involving resonances

$\hookrightarrow$ seminal work on $K \to \pi\pi$ by

L. Lellouch and M. Luscher, Commun.Math.Phys. **219**, 31 (2001)

Lüscher approach

- determine the scattering phase shift from the finite volume two particle energy spectrum



$\cot\delta_0(s) = w_{00}(\eta)$ (Lüscher equation),

$w_{lm}(\eta)$ − known function,

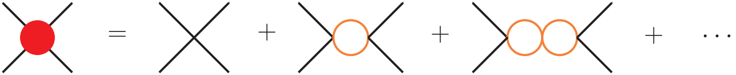
$\eta = \frac{pL}{2\pi}$, $p^2 = \lambda(s, m_1^2, m_2^2)/4s$,

$p_R \cot\delta_0(s_R) = -\frac{1}{a_0} + \frac{1}{2} r_0 p_R^2 + \cdots = -ip_R$

▷ lattice data ⇒Lüscher equation ⇒ scattering phase
▷ fit effective-range expansion parameters
▷ analytic continuation to the resonance position p_R

The framework: non-relativistic EFT

- Basic properties of the theory:
  - ▷ the total number of heavy particles is conserved
  - ▷ manifestly Lorentz-invariant formulation is possible
  - ▷ the theory is matched to the full QFT (e.g., ChPT).



Bubble-chain diagrams

$$T \propto 1 + cJ + cJ^2 + \cdots = \frac{1}{1 - cJ}$$

Goals, results

- Previous work:

  D. Hoja, U.-G. Meißner, A. Rusetsky, JHEP 1004 (2010) 050

  V. Bernard, D. Hoja, U.-G. Meißner, A. Rusetsky, JHEP 1209 (2012) 023

  $\hookrightarrow$ scalar resonance formfactor in the external scalar field (analog: $\Delta\Delta\gamma^*$)

- The $\Delta N\gamma^*$ transition:

  ▷ inclusion of *spin*;

  ▷ generalization to *transition* form factors.

- Extraction of the form factors:

  ▷ on the real energy axis $\rightarrow$ model-dependent

  ▷ at the Δ resonance pole $\rightarrow$ *process-independent*

Spin, kinematics

- The Δ is at rest $\mathbf{P} = 0$ and nucleon momentum $\mathbf{Q}$ along 3-axis.

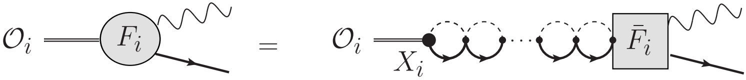
$$
\begin{aligned}
G_2 \qquad O_{3/2}(X) &= \frac{1}{2}(1+\Sigma_3)\frac{1}{2}(1+\gamma_4)\frac{1}{\sqrt{2}}\left(O^1(X) - i\Sigma_3 O^2(X)\right) \\
G_2 \qquad O_{1/2}(X) &= \frac{1}{2}(1-\Sigma_3)\frac{1}{2}(1+\gamma_4)\frac{1}{\sqrt{2}}\left(O^1(X) + i\Sigma_3 O^2(X)\right) \\
G_1 \qquad \tilde{O}_{1/2}(X) &= \frac{1}{2}(1+\Sigma_3)\frac{1}{2}(1+\gamma_4)O^3(X) \\
\bar{\psi}_{\pm 1/2}(Y) &= \bar{\psi}(Y)\frac{1}{2}(1\pm\Sigma_3)\frac{1}{2}(1+\gamma_4), \qquad \Sigma_3 = \mathrm{diag}(\sigma_3, \sigma_3)
\end{aligned}
$$

$\Rightarrow$ three $\Delta N \gamma^*$ form factors are separately projected out.

$\hookrightarrow$ similar procedure can be applied to fields with other spin

- To perform the fit (*see below*): vary p, while $|\mathbf{Q}|$ fixed.
  - ▷ twisted boundary conditions;
  - ▷ asymmetric boxes $L \times L \times L'$.

$\Delta N\gamma^*$ vertex



- The $F_i = F_i(p_n, |\mathbf{Q}|)$, i=1, 2, 3 → G_M, G_E, G_C form factors, are measured on *the lattice*.
- The $\bar{F}_i(p_n, |\mathbf{Q}|)$ are *volume-independent* irreducible amplitudes.

$$|\bar{F}_i(p_n, |\mathbf{Q}|)| = \underbrace{V^{1/2}\left(\frac{\cos^2\delta(p_n)}{|\delta'(p_n) + \phi'(\eta_n)|}\frac{p_n^2}{2\pi}\right)^{-1/2}}_{\text{LL factor}} |F_i(p_n, |\mathbf{Q}|)|$$

Real axis

- The $\bar{F}_i$ are related to the $\gamma^* N \to \pi N$ multipole amplitudes

$$\mathcal{A}_i(p, |\mathbf{Q}|) = e^{i\delta(p)} \cos\delta(p)\, \bar{F}_i(p, |\mathbf{Q}|)$$

$\hookrightarrow$ Watson's theorem

$$\mathcal{A}_i(p_n, |\mathbf{Q}|) = e^{i\delta(p_n)}\, V^{1/2} \left(\frac{1}{|\delta'(p_n) + \phi'(\eta_n)|} \frac{p_n^2}{2\pi} \right)^{-1/2} |F_i(p_n, |\mathbf{Q}|)|$$

$\hookrightarrow$ LL equation for the photoproduction amplitude in the elastic region

▷ *The narrow width approximation*:

$$|\mathrm{Im}\, \mathcal{A}_i(p_A, |\mathbf{Q}|)| = \sqrt{\frac{8\pi}{p_A \Gamma}} |F_i^A(p_A, |\mathbf{Q}|)|,$$

$p = p_A$ - Breit-Wigner pole, $F_i^A(p_A, |\mathbf{Q}|) \to \Delta N \gamma^*$ form factors.

Complex plane

- The $\Delta N\gamma^*$ matrix elements, evaluated at the pole $p = p_R$:

$$F_i^R(p_R, |\mathbf{Q}|) = Z_R^{1/2}\, \bar{F}_i(p_R, |\mathbf{Q}|),$$

$$Z_R = \left(\frac{p_R}{8\pi E_R}\right)^2 \left(\frac{16\pi p_R^3 E_R^3}{w_{1R} w_{2R}(2p_R h'(p_R^2) + 3ip_R^2)}\right), \quad w_{iR} = \sqrt{m_i^2 + p_R^2},$$

$$p^3 \cot\delta(p) \doteq h(p^2) = -\frac{1}{a} + \frac{1}{2}\, rp^2 + \cdots\,, \quad h(p_R^2) = -ip_R^3$$

▷ *The narrow width approximation* $(p_R \to p_n)$:

$$F_i^R(p_R, |\mathbf{Q}|) \to F_i^A(p_A, |\mathbf{Q}|) \quad \text{as} \quad p_R \to p_A\,!$$

$$F_i^R(p_n, |\mathbf{Q}|) = V^{1/2}\left(\frac{E_n}{2w_{1n}w_{2n}}\right)^{1/2} F_i(p_n, |\mathbf{Q}|)$$

↪ proper normalization of states

Prescription on the lattice

- Measure the $F_i(p, |\mathbf{Q}|)$ at different values of p with $|\mathbf{Q}|$ fixed.
- Real energy: extract the multipole amplitudes (*see above*).
- Extraction of the matrix elements at the resonance pole:
  1. fit the functions $p^3 \cot\delta(p)\, \bar{F}_i(p, |\mathbf{Q}|)$

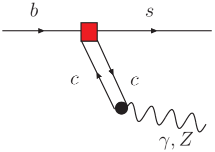
     $$p^3 \cot\delta(p)\, \bar{F}_i(p, |\mathbf{Q}|) = A_i(|\mathbf{Q}|) + p^2 B_i(|\mathbf{Q}|) + \cdots$$

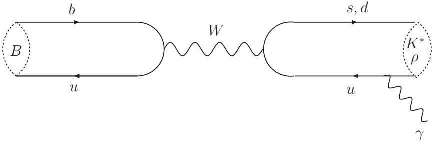
  2. evaluate the resonance matrix elements by substitution

     $$F_i^R(p_R, |\mathbf{Q}|) = i\, p_R^{-3}\, Z_R^{1/2}(A_i(|\mathbf{Q}|) + p_R^2 B_i(|\mathbf{Q}|) + \cdots)\,.$$

$B \to K^*$ transitions

- A problem: long distance contributions.





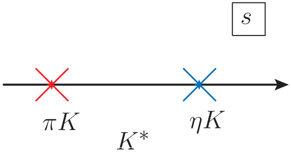
- If $q^2 > m^2_{c\bar{c}}$, then since $V_{ub}V^*_{us} << V_{tb}V^*_{ts} \Rightarrow$ we can apply our framework!

- Related work: multichannel LL equation, kinematics.

R. A. Briceño, M.T. Hansen, A. Walker-Loud, arXiv:1406.5965 (2014)

Outlook

- Rigorous physical analysis of the problem.
- Extraction of the matrix elements at the pole: two-channel case.



Thank you!