Heavy Quarkonium magnetic dipole transitions in pNRQCD

Mainly based on Phys. Rev. D87, 074024. A. Pineda and J. Segovia

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Motivation

- 1st principle computation of heavy quarkonium properties from QCD. To give model independent predictions with model independent errors.
- Determination of Standard Model parameters: m_Q, α_s, ...

Potential Non-Relativistic QCD in the weak coupling regime is ideal for this.
$m \gg mv \gg mv^2$

$$\left.\begin{array}{l}\left(i\partial_0 - \frac{\mathbf{p}^2}{2m} - V_s^{(0)}(r)\right)\Phi(\mathbf{r}) = 0 \\ +\text{corrections to the potential} \\ +\text{interaction with other low} \\ \quad\text{energy degrees of freedom}\end{array}\right\} \text{pNRQCD(Pineda, Soto)} \quad E \sim mv^2$$

In the strict weak coupling regime the starting point is
$V_s^{(0)} \simeq V^C \equiv -C_f \frac{\alpha_s(\mu)}{r}$.
Define the leading order: $\alpha_V \sim v$

$$V_s^{(0)} = -C_f \frac{\alpha_V(1/r)}{r} = -C_f \frac{\alpha_s(\mu)}{r}\left(1 + a_1 \frac{\alpha_s(\mu)}{4\pi} + a_2 \frac{\alpha_s^2(\mu)}{16\pi^2} + \cdots\right)$$

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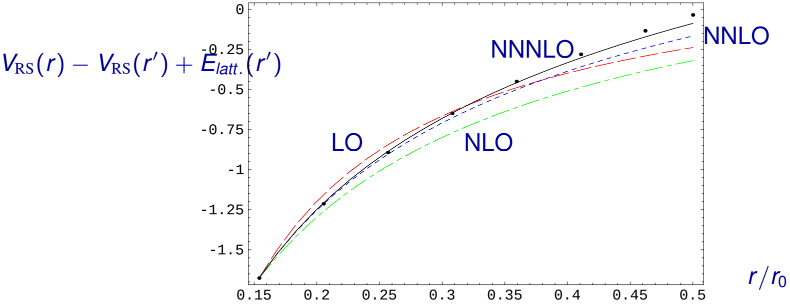
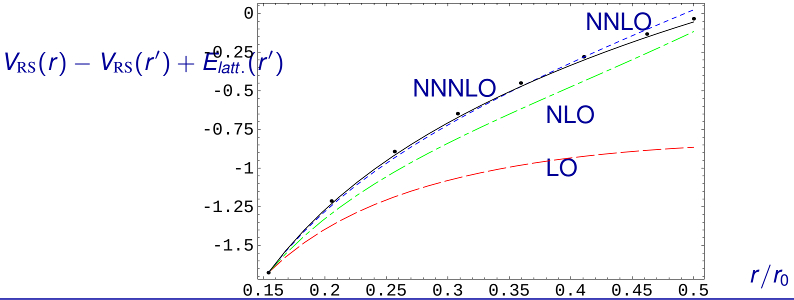

$V_{\rm RS}(r) - V_{\rm RS}(r') + E_{latt.}(r')$
0
-0.25
-0.5
-0.75
-1
-1.25
-1.5
0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5
r/r_0
NNNLO
NNLO
LO
NLO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r/r_0
NNLO
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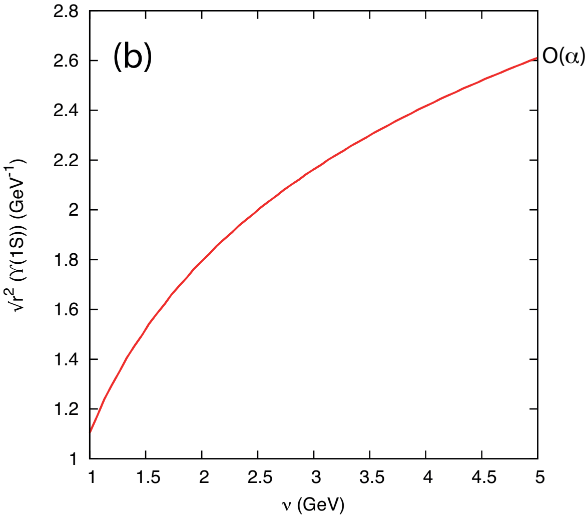
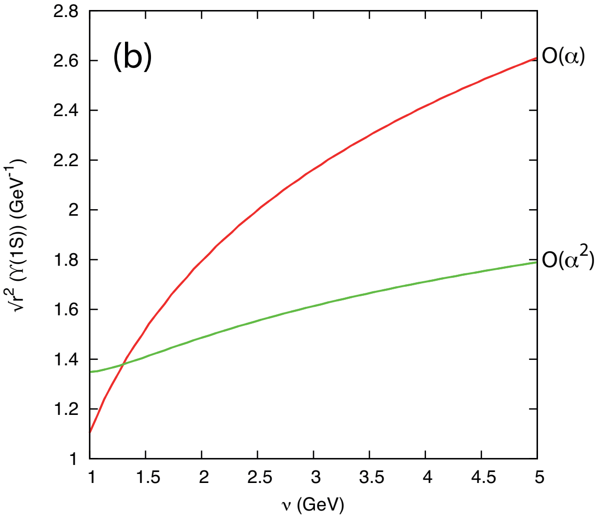
Bottomonium



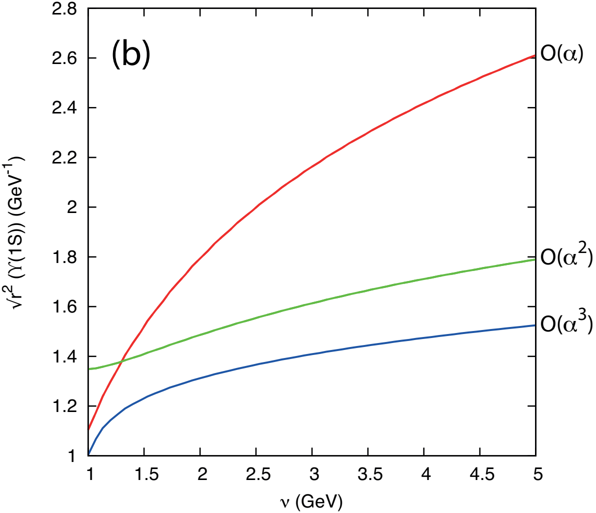
$$V_s^{(0)} = -C_f \frac{\alpha_s(\mu)}{r}$$

Bottomonium



$$V_s^{(0)} = -C_f \frac{\alpha_s(\mu)}{r} \left(1 + a_1 \frac{\alpha_s(\mu)}{4\pi}\right)$$

Bottomonium



$$V_s^{(0)} = -C_f \frac{\alpha_s(\mu)}{r}\left(1 + a_1 \frac{\alpha_s(\mu)}{4\pi} + a_2 \frac{\alpha_s^2(\mu)}{(4\pi)^2}\right)$$

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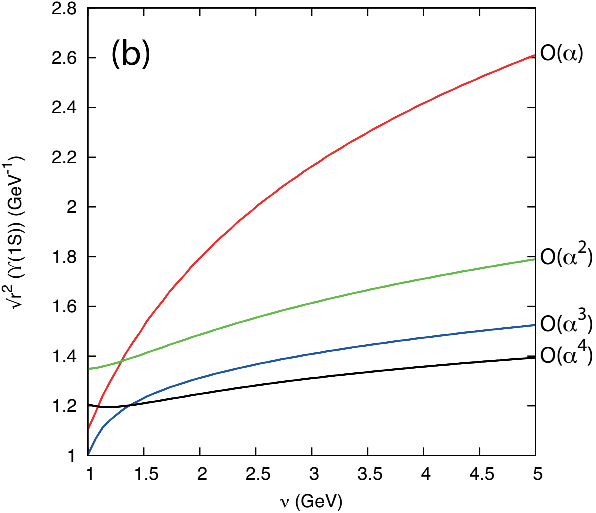
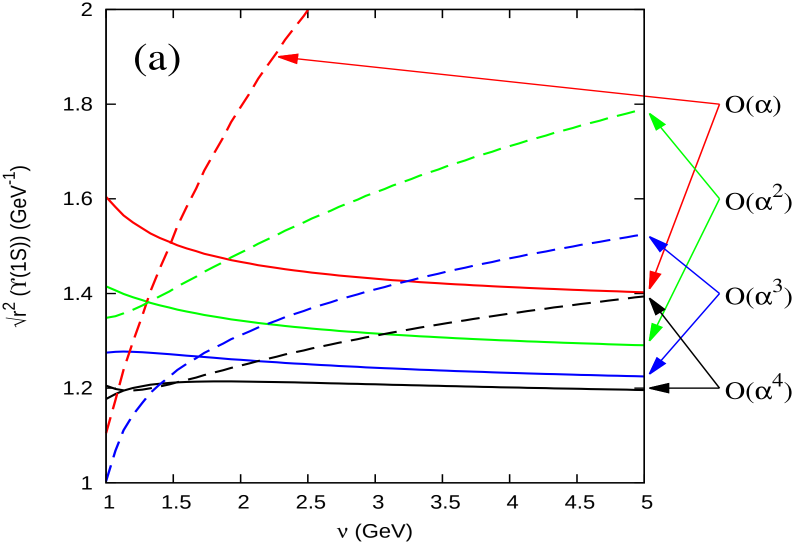


$$V_s^{(0)} = -C_f \frac{\alpha_s(\mu)}{r}\left(1 + a_1 \frac{\alpha_s(\mu)}{4\pi} + a_2 \frac{\alpha_s^2(\mu)}{(4\pi)^2} + a_3 \frac{\alpha_s^3(\mu)}{(4\pi)^3}\right)$$



$$V_s^{(0)} = -C_f \frac{\alpha_s(1/r)}{r}\left(1 + a_1 \frac{\alpha_s(1/r)}{4\pi} + a_2 \frac{\alpha_s^2(1/r)}{(4\pi)^2} + a_3 \frac{\alpha_s^3(1/r)}{(4\pi)^3}\right)$$

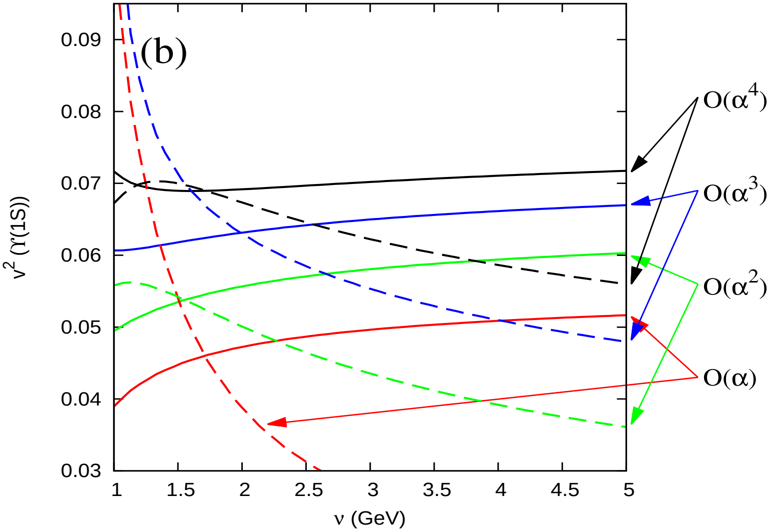


Figure: $v_{10}^2 = \langle p^2 \rangle / m^2$ using the static potential $V_{\rm RS'}^{(N)}$ at different orders in perturbation theory: $N = 0, 1, 2, 3$. Dashed lines with $\nu_r = \infty$. Continuous lines with $\nu_r = 0.7$ GeV. In both cases $\nu_f = 0.7$ GeV.

Charmonium

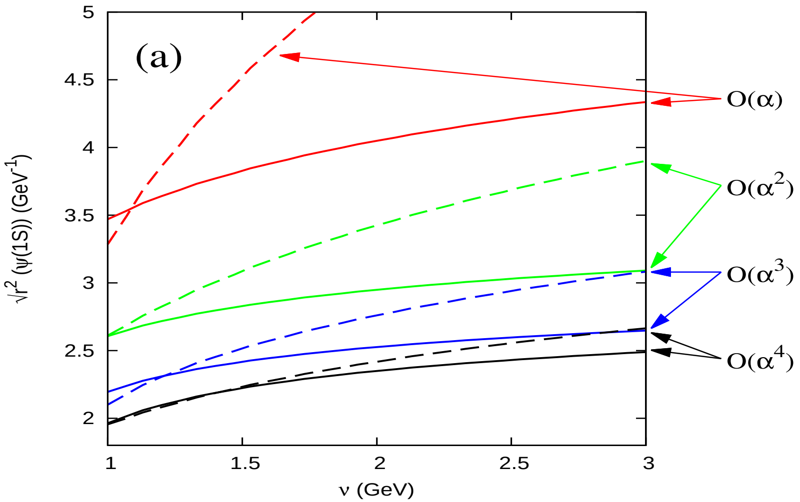
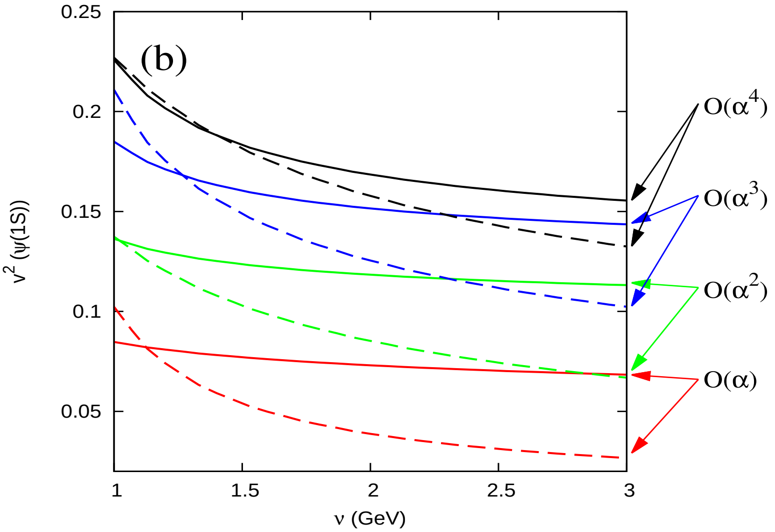


Figure: $\sqrt{\langle r^2\rangle_{10}}$ *using the static potential* $V_{\mathrm{RS}'}^{(N)}$ *at different orders in perturbation theory:* $N = 0, 1, 2, 3$*. Dashed lines with* $\nu_r = \infty$*. Continuous lines with* $\nu_r = 0.7$ *GeV. In both cases* $\nu_f = 0.7$ *GeV.*



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	$b\bar{b}(1S)$	$c\bar{c}(1S)$	$b\bar{b}(1P)$	$b\bar{b}(2S)$
v	0.26	0.43	0.25	0.24
$\sqrt{\langle r^2 \rangle}(\mathrm{GeV}^{-1})$	1.2	2.2	2.1	2.9

Table: *Estimates for* $v \equiv \sqrt{\langle p^2 \rangle / m^2}$ *and* $\sqrt{\langle r^2 \rangle}$ *for the heavy quarkonium states. For the* $b\bar{b}(2S)$ *state the number we give for* v *is quite uncertain.*

Proposal: Reorganization of perturbation theory

Strict perturbation theory

$$H^{(0)} = \frac{\mathbf{p}^2}{m} + V^C \longrightarrow E_n^C\,, \quad \phi_n^C(\mathbf{r})$$

Coulomb potential

$$V_s^C = -C_f \frac{\alpha_s(\mu)}{r}$$

Relativistic + $\delta V^{(0)}$ corrections:

$$\Delta H = \delta V^{(0)} + \frac{V^{(1)}}{m} + \frac{V^{(2)}}{m^2} + \cdots$$

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Improved perturbation theory "acceleration of perturbation theory"

$$H^{(0)} = \frac{\mathbf{p}^2}{m} + V^{(0)} \longrightarrow E_n^{(0)}, \quad \phi_n^{(0)}(\mathbf{r})$$

Keep the static potential exactly

$$V_s^{(0)} = -C_f \frac{\alpha_s(1/r)}{r}\left(1 + a_1 \frac{\alpha_s(1/r)}{4\pi} + a_2 \frac{\alpha_s^2(1/r)}{16\pi^2} + \cdots\right)$$

Relativistic corrections:

$$\Delta H = \frac{V^{(1)}}{m} + \frac{V^{(2)}}{m^2} + \cdots$$

Applied to inclusive electromagnetic decay ratios (Kiyo, Pineda, Signer).
Apply it to M1 radiative transitions.

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Allowed transitions

Strict weak coupling: Brambilla, Jia, Vairo

$$\Gamma(n^3S_1 \to n^1S_0\gamma) = \frac{4}{3}\alpha e_Q^2 \frac{k_\gamma^3}{m^2}\left[1 + 2\kappa - \frac{5}{3}\frac{\langle p^2\rangle_n^C}{m^2}\right],$$

$$\Gamma(n^3P_J \to n^1P_1\gamma) = \frac{3\Gamma(n^1P_1 \to n^3P_J\gamma)}{2J+1} = \frac{4}{3}\alpha e_Q^2 \frac{k_\gamma^3}{m^2}\left[1 + 2\kappa - d_J\frac{\langle p^2\rangle_{n1}^C}{m^2}\right],$$

where $d_0 = 1$, $d_1 = 2$, $d_2 = 8/5$,

$$k_\gamma = |\vec{k}| = \frac{M_H^2 - M_{H'}^2}{2M_H},$$

and the anomalous magnetic moment of the heavy quark reads

$$\kappa = \kappa^{(1)}\alpha_s(m)$$

Leading order= potential models

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and the anomalous magnetic moment of the heavy quark reads

$$\kappa = \kappa^{(1)}\alpha_s(m) + \kappa^{(2)}\alpha_s^2(m)$$

We also implement the renormalon.

$\Gamma_{\Upsilon(1S)\to\eta_b(1S)\gamma}$

	LO	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(v^2)$	$\alpha_s \times \mathcal{O}(\alpha_s^2)$	$v \times \mathcal{O}(v^2)$
$\delta\Gamma$ (eV)	14.87	1.29	0.73	−1.71	0.15	−0.45

Table: *The leading and subleading contributions to $\Gamma_{\Upsilon(1S)\to\eta_b(1S)\gamma}$. The last two numbers are error estimates obtained by multiplying the subleading $\mathcal{O}(\alpha_s^2)$ contribution by α_s and the subleading $\mathcal{O}(v^2)$ contribution by v.*

$$\Gamma_{\Upsilon(1S)\to\eta_b(1S)\gamma} = 15.18 \pm 0.45(\mathcal{O}(v^3))^{-0.12}_{-0.05}(N_m)^{-0.04}_{+0.03}(\alpha_s)^{-0.20}_{+0.20}(m_{\overline{\rm MS}})\ {\rm eV}\,,$$

which after combining the errors in quadrature reads

$$\Gamma_{\Upsilon(1S)\to\eta_b(1S)\gamma} = 15.18(51)\ {\rm eV}\,.$$

$\Gamma_{J/\psi(1S)\to\eta_c(1S)\gamma}$

	LO	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(v^2)$	$\alpha_s \times \mathcal{O}(\alpha_s^2)$	$v \times \mathcal{O}(v^2)$
$\delta\Gamma$ (keV)	2.34	0.33	0.16	-0.71	0.05	-0.30

Table: *The leading and subleading contributions to $\Gamma_{J/\psi(1S)\to\eta_c(1S)\gamma}$. The last two numbers are error estimates obtained by multiplying the subleading $\mathcal{O}(\alpha_s^2)$ contribution by α_s and the subleading $\mathcal{O}(v^2)$ contribution by v.*

$$\Gamma_{J/\psi(1S)\to\eta_c(1S)\gamma} = 2.12 \pm 0.30(\mathcal{O}(v^3))^{+0.21}_{-0.23}(N_m)^{-0.02}_{+0.02}(\alpha_s)^{-0.10}_{+0.11}(m_{\overline{\rm MS}})\ \mathrm{keV}\,,$$

which, after combining the errors in quadrature, reads

$$\Gamma_{J/\psi(1S)\to\eta_c(1S)\gamma} = 2.12(40)\ \mathrm{keV}\,.$$

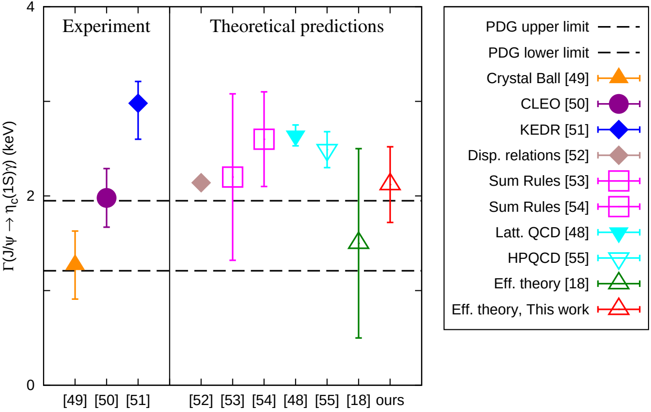

Figure: *Comparison of different theoretical and experimental predictions for* $\Gamma_{J/\psi\to\eta_c\gamma}$.

Hindered transitions. Strict weak coupling: Brambilla, Jia, Vairo

$$\Gamma(n^3S_1 \to n'^1S_0\gamma) \overset{n\neq n'}{=} \frac{4}{3}\alpha e_Q^2 \frac{k_\gamma^3}{m^2}\left[\frac{k_\gamma^2}{24}\,{}_{n'}\langle r^2\rangle_n^C + \frac{5}{6}\frac{{}_{n'}\langle p^2\rangle_n^C}{m^2} - \frac{2}{m^2}\frac{{}_{n'}\langle V_{S^2}^C(\vec{r})\rangle_n}{E_n^C - E_{n'}^C}\right]^2,$$

$$\Gamma(n^1S_0 \to n'^3S_1\gamma) \overset{n\neq n'}{=} 4\alpha e_Q^2 \frac{k_\gamma^3}{m^2}\left[\frac{k_\gamma^2}{24}\,{}_{n'}\langle r^2\rangle_n^C + \frac{5}{6}\frac{{}_{n'}\langle p^2\rangle_n^C}{m^2} + \frac{2}{m^2}\frac{{}_{n'}\langle V_{S^2}(\vec{r})\rangle_n^C}{E_n^C - E_{n'}^C}\right]^2,$$

$$V_{S^2}(\vec{r}\,) = \frac{4}{3}\pi C_f D^{(2)}_{S^2,s}(\nu)\delta^{(3)}(\vec{r}\,)$$

$$D^{(2)}_{S^2,s}(\nu) = \alpha_s(\nu)$$

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$$V_{S^2}(\vec{r}) = \frac{4}{3}\pi C_f D^{(2)}_{S^2,s}(\nu)\delta^{(3)}(\vec{r})$$

$$D^{(2)}_{S^2,s}(\nu) = \alpha_s(\nu)c_F^2(\nu) - \frac{3}{2\pi C_f}\left(d_{sv}(\nu) + C_f d_{vv}(\nu)\right)$$

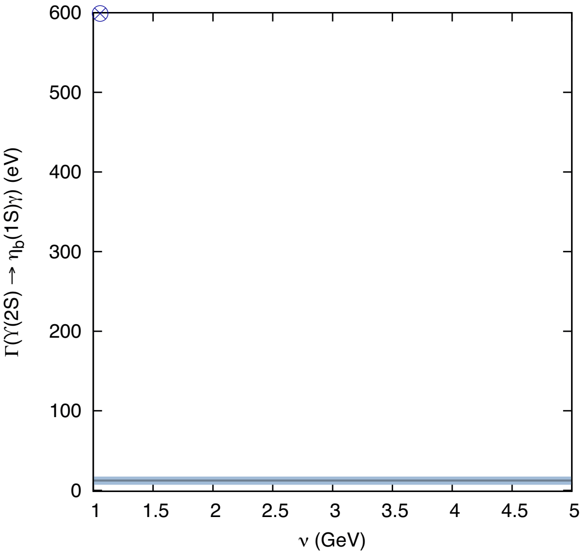
depends on the NRQCD Wilson coefficients. With LL accuracy they read

$$c_F(\nu) = z^{-C_A}, \qquad d_{sv}(\nu) = d_{sv}(m),$$

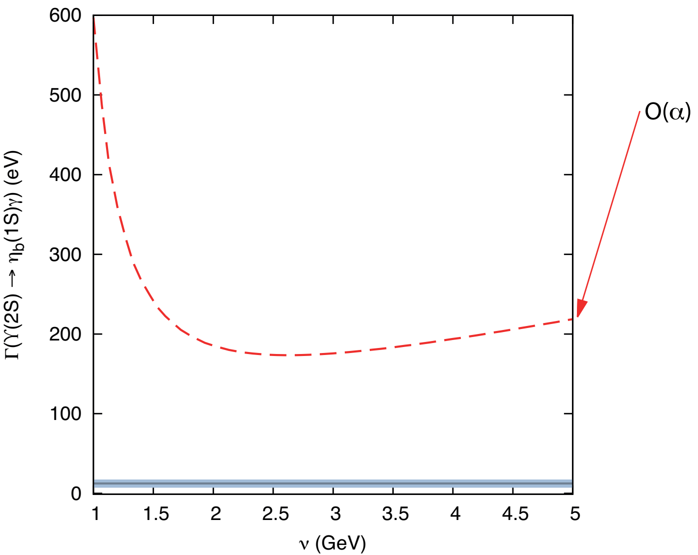
$$d_{vv}(\nu) = d_{vv}(m) + \frac{C_A}{\beta_0 - 2C_A}\pi\alpha_s(m)(z^{\beta_0 - 2C_A} - 1),$$

$$z = \left[\frac{\alpha_s(\nu)}{\alpha_s(m)}\right]^{\frac{1}{\beta_0}} \simeq 1 - \frac{1}{2\pi}\alpha_s(\nu)\ln\left(\frac{\nu}{m}\right),$$

$$d_{sv}(m) = C_f\left(C_f - \frac{C_A}{2}\right)\pi\alpha_s(m), d_{vv}(m) = -\left(C_f - \frac{C_A}{2}\right)\pi\alpha_s(m).$$



no RG , $V_s^{(0)} = -C_f \frac{\alpha_s(\mu)}{r}$, $\mu = 1$ GeV



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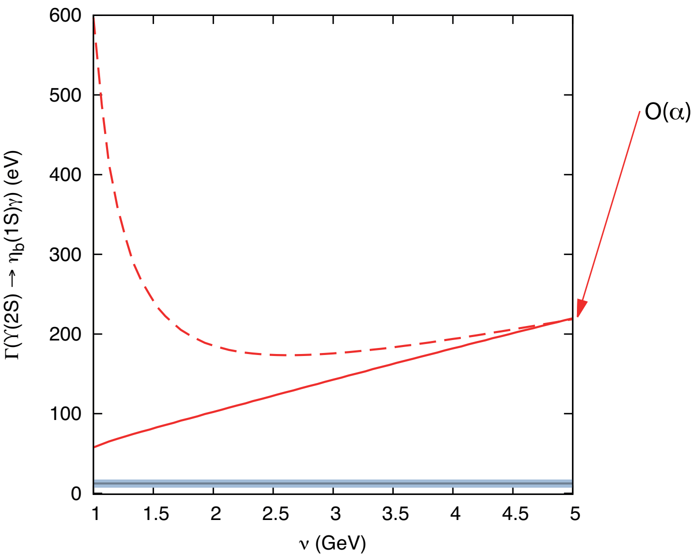

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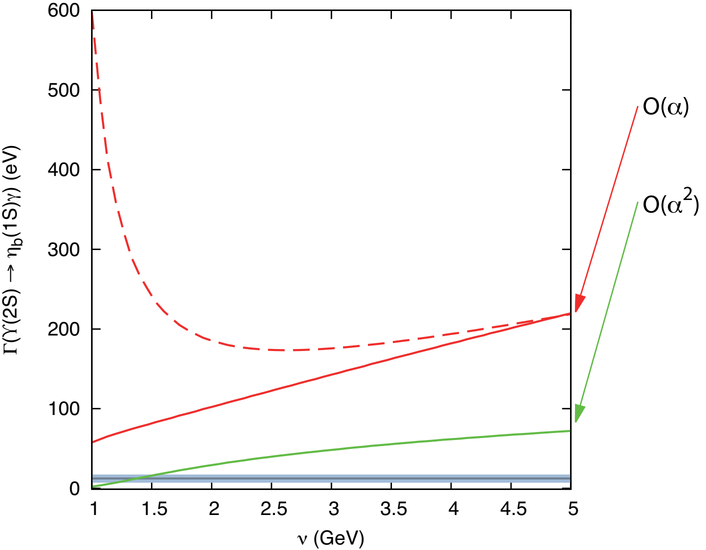

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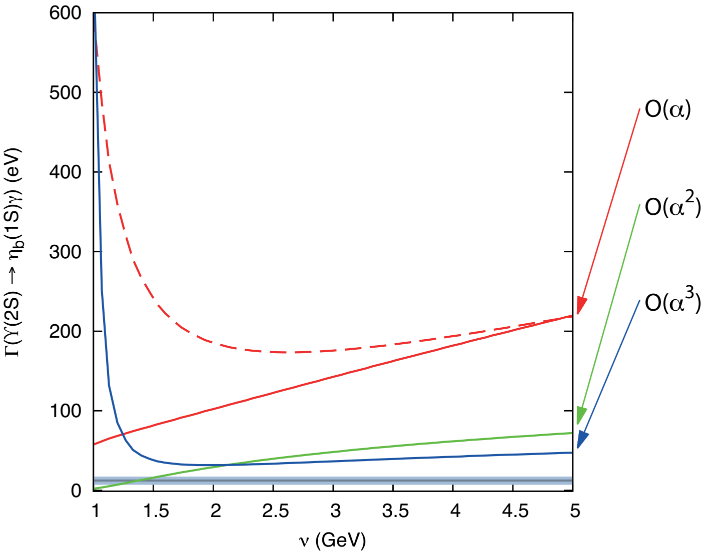

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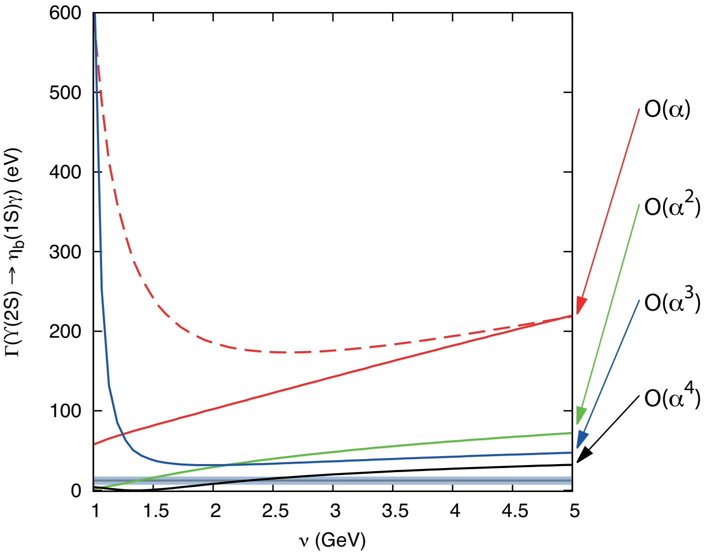

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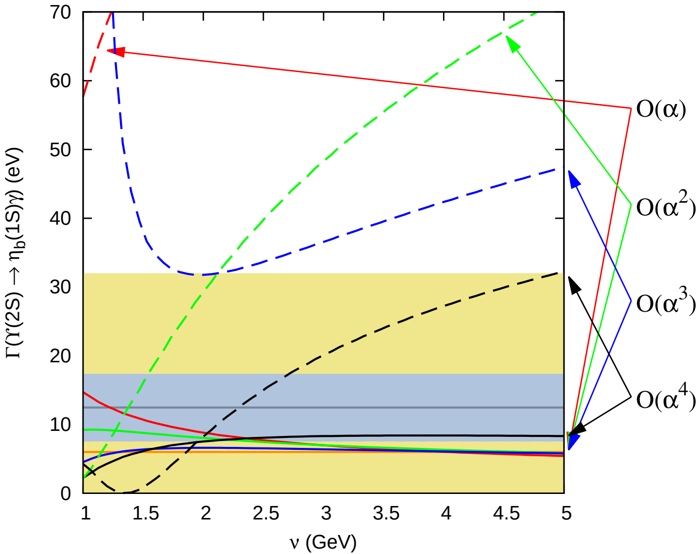

$$\text{RG}\ ,\ V_s^{(0)} = -C_f \frac{\alpha_s(1/r)}{r}\left(1 + a_1 \frac{\alpha_s(1/r)}{4\pi} + a_2 \frac{\alpha_s^2(1/r)}{(4\pi)^2} + a_3 \frac{\alpha_s^3(1/r)}{(4\pi)^3}\right)$$

Prefactor (keV)	$\mathcal{A}(r^2)$	$\mathcal{A}(\vec{p}^2)$	$\mathcal{A}(V_{S^2})$	Γ (eV)
10.3342	0.022	0.039	−0.042	6.3

Table: *The prefactor, the terms inside the brackets, and the total decay width* $\Gamma_{\Upsilon(2S)\to\eta_b(1S)\gamma}$.

$$\Gamma^{(\mathrm{th})}_{\Upsilon(2S)\to\eta_b(1S)\gamma} = 0.006 \pm 0.006(\mathcal{O}(v^5))^{+0.026}_{-0.006}(N_m)^{-0.001}_{+0.001}(\alpha_s)^{-0.000}_{+0.000}(m_{\overline{\mathrm{MS}}})\,\mathrm{keV}.$$

$$\Gamma^{(\mathrm{th})}_{\Upsilon(2S)\to\eta_b(1S)\gamma} = 6^{+26}_{-06}\,\mathrm{eV}.$$

Matrix element.
Experimental number: 0.035(7)
Ours: $0.025^{+0.031}_{-0.025}$
Lewis, Woloshyn: $\mathcal{O}(v^4) = 0.080(5)$; $\mathcal{O}(v^6) = 0.032(5)$

Both agreement but different physics. To be clarified.

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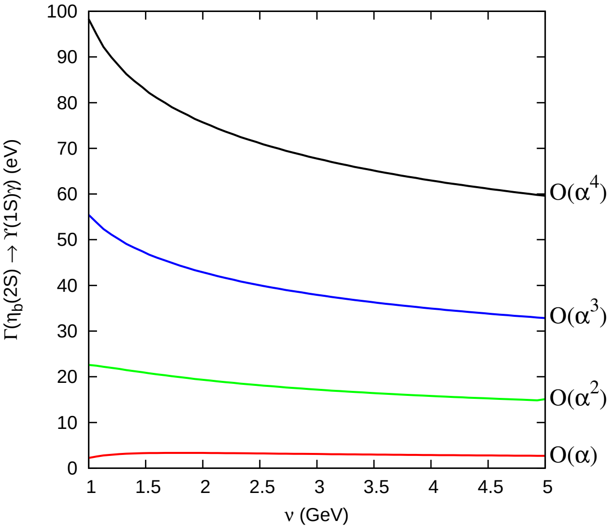

Figure: *Plot of* $\Gamma_{\eta_b(2S)\to\Upsilon(1S)\gamma}$ *using the static potential* $V^{(N)}_{\rm RS'}$ *at different orders in perturbation theory:* $N = 0, 1, 2, 3$ *with* $\nu_r = \nu_f = 0.7$ *GeV.*

CONCLUSIONS

Precision is $k_\gamma^3/m^2 \times \mathcal{O}(\alpha_s^2, v^2)$ for the allowed transitions. The convergence for the $b\bar{b}$ ground state was quite good, and also quite reasonable for the $c\bar{c}$ ground state and the $b\bar{b}$ $1P$ state.

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\begin{aligned}
\Gamma_{\Upsilon(1S)\to\eta_b(1S)\gamma} &= 15.18(51)\ \mathrm{eV}\,,\\
\Gamma_{J/\psi(1S)\to\eta_c(1S)\gamma} &= 2.12(40)\ \mathrm{keV}\,,\\
\Gamma_{h_b(1P)\to\chi_{b0}(1P)\gamma} &= 0.962(35)\ \mathrm{eV}\,,\\
\Gamma_{h_b(1P)\to\chi_{b1}(1P)\gamma} &= 8.99(55)\times 10^{-3}\ \mathrm{eV}\,,\\
\Gamma_{\chi_{b2}(1P)\to h_b(1P)\gamma} &= 0.118(6)\ \mathrm{eV}\,.
\end{aligned}
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Precision is $k_\gamma^3/m^2 \times \mathcal{O}(v^4)$ for the forbidden transitions. Large logarithms associated with the heavy quark mass scale have also been resummed.

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This number is perfectly consistent with existing data.

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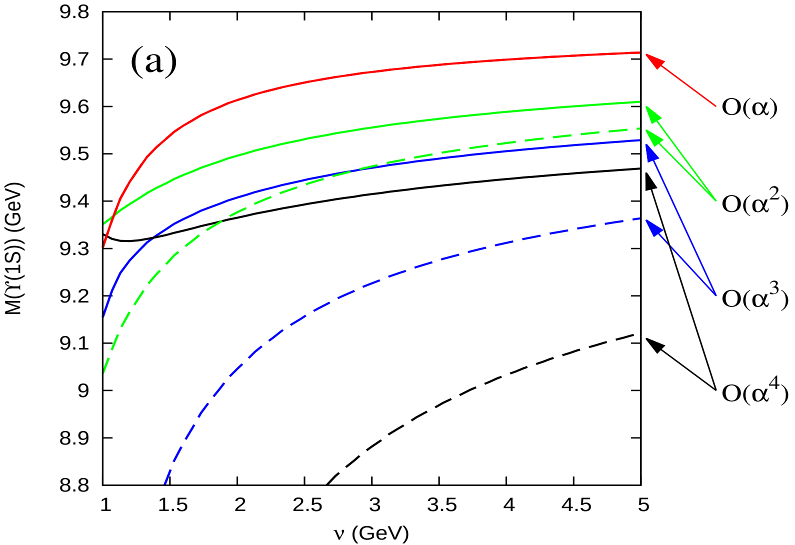

Figure: $M_{10} = 2m_{b,\mathrm{RS}'}(0.7\,\mathrm{GeV}) + E_{10}$ *using the static potential* $V^{(N)}_{\mathrm{RS}'}$ *at different orders in perturbation theory:* $N = 0, 1, 2, 3$. *Dashed lines with* $\nu_f = 0$. *Continuous lines with* $\nu_f = 0.7$ *GeV. In both cases* $\nu_r = \infty$ *GeV.*

$$V_{\mathrm{RS}'}^{(N)}(r) = \begin{cases} (V^{(N)} + 2\delta m_{\mathrm{RS}'}^{(N)})|_{\nu=\nu} \equiv \sum_{n=0}^{N} V_{RS',n}\alpha_s^{n+1}(\nu) & \text{if} \quad r > \nu_r^{-1} \\ (V^{(N)} + 2\delta m_{\mathrm{RS}'}^{(N)})|_{\nu=1/r} \equiv \sum_{n=0}^{N} V_{\mathrm{RS}',\mathrm{n}}\alpha_s^{n+1}(1/r) & \text{if} \quad r < \nu_r^{-1}. \end{cases}$$

$$\delta m_X^{(N)}(\nu_f) = \nu_f \sum_{n=0}^{N} \delta m_X^{(n)}(\frac{\nu_f}{\nu})\alpha_s^{n+1}(\nu)$$