QCD constraints on the equation of state for compact stars

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[Work done in collaboration with A. Kurkela, J. Schaffner-Bielich & A. Vuorinen]

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What is this talk about?

★ A very important and challenging question: is there deconfined quark matter in the core of compact stars?

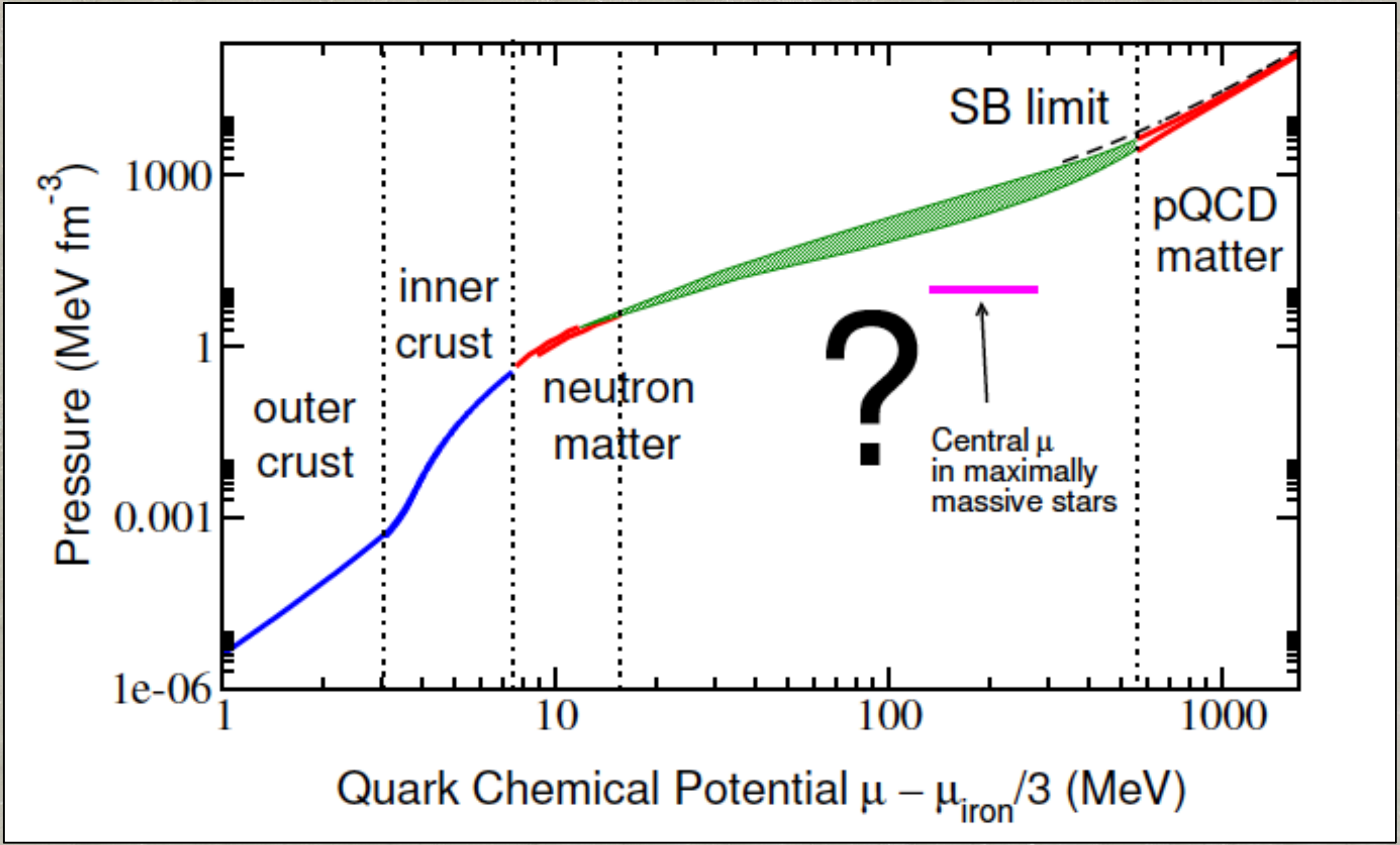
Not the point here.

★ A related but totally different question: does the fact that at asymptotically high densities one must have deconfined quark matter constrain the equation of state for compact stars?

Yes!

★ Main message: even if there is no deconfined quark matter in the core of neutron stars, the form of the QCD EoS at very large densities (which is known perturbatively) affects dramatically the EoS for compact stars!

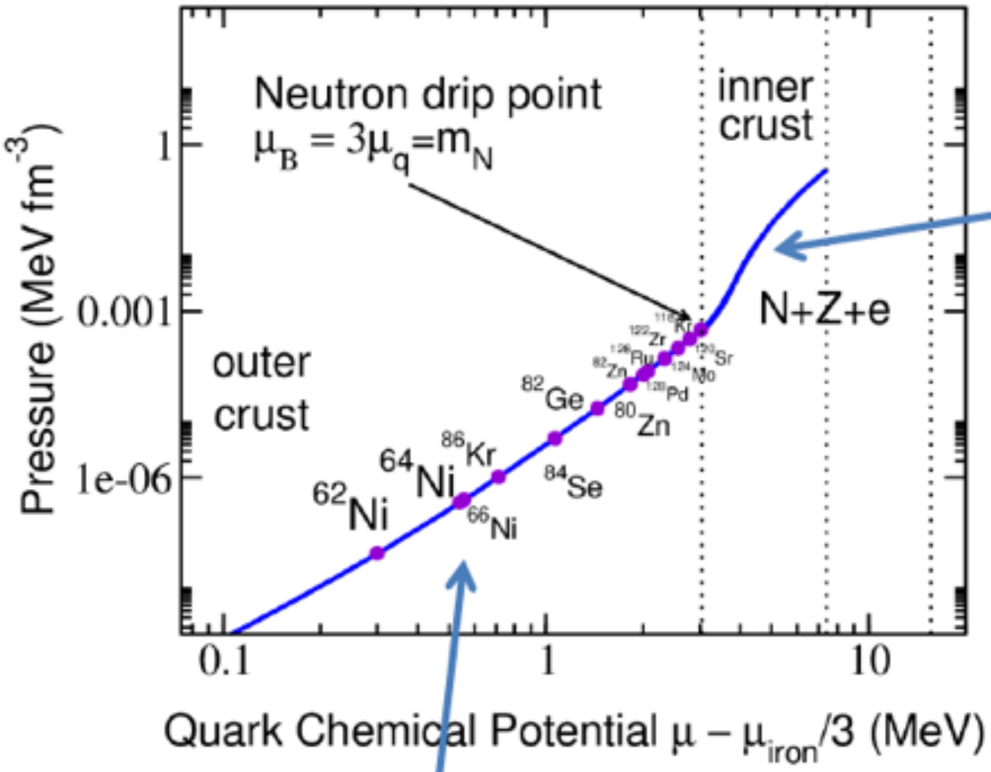
The point in this talk:



[Kurkela, ESF, Schaffner-Bielich & Vuorinen (2014)]

Low densities:

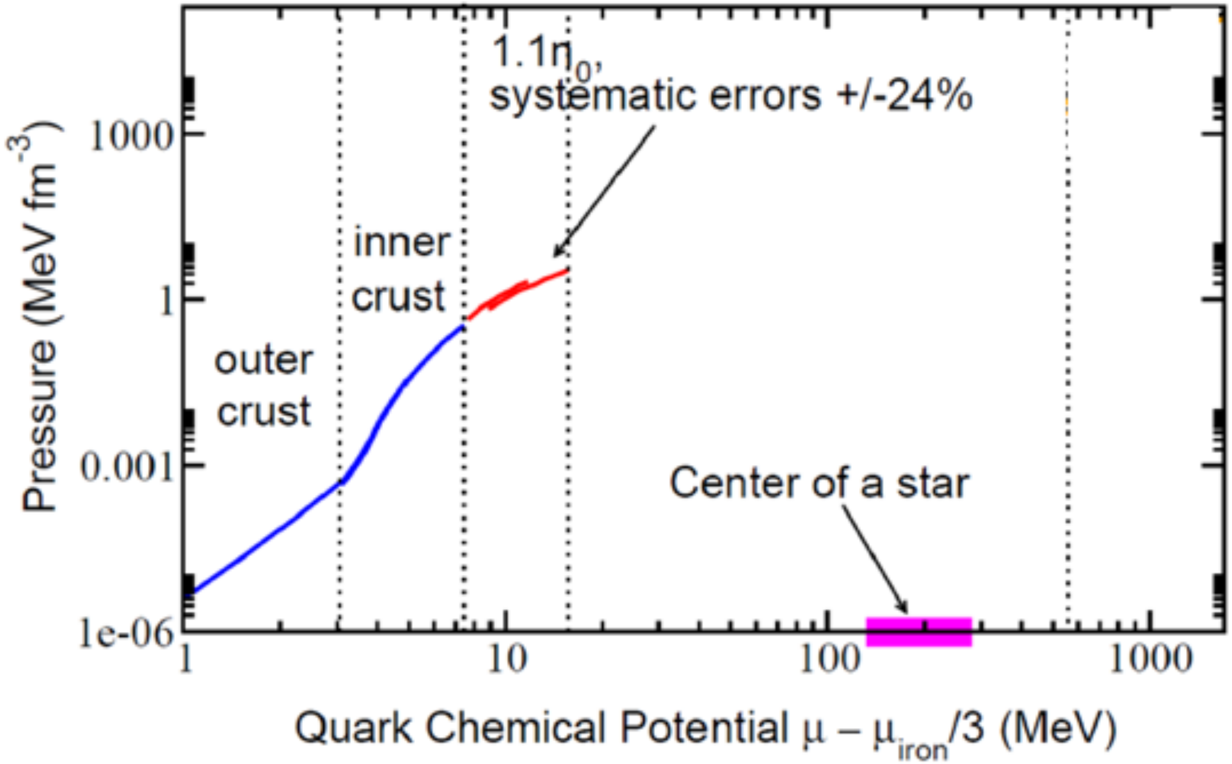
[From talks by Kurkela & Vuorinen]



- Neutron gas with nuclei and electrons
- NN interactions important for collective properties; modeled via phenom. potential models
- Eventually need 3N interactions, boost corrections,...

- Lattice of increasingly neutron rich nuclei in electron sea; pressure dominated by that of the electron gas
- At zero pressure nuclear ground state $^{56}\mathrm{Fe}$

Closer to saturation density: [Kurkela, ESF, Schaffner-Bielich & Vuorinen (2014)]



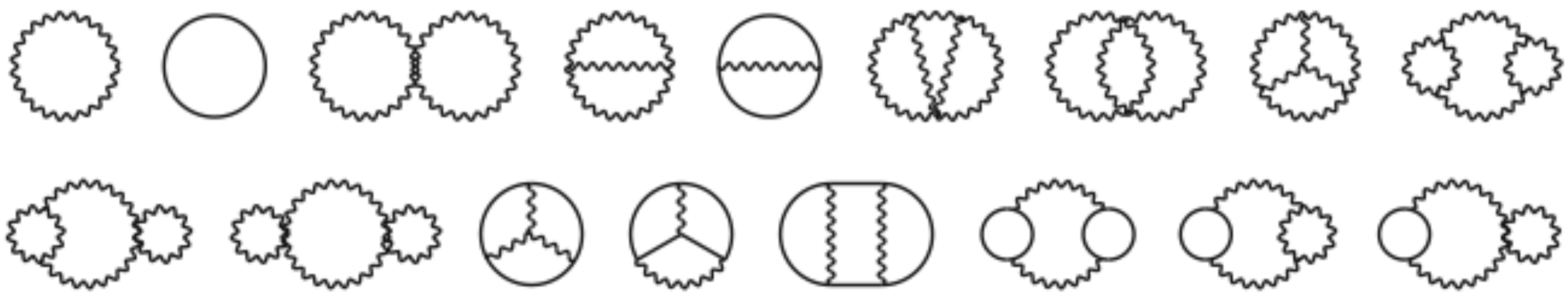
- Closer to saturation density n_s, need many-body calculations within Chiral Effective Theory, including 3N and 4N interactions
- At $1.1n_s$, errors $\pm 24\%$ - mostly due to uncertainties in effective theory parameters
- State-of-the-art NNNLO Tews et al., PRL 110 (2013), Hebeler et al., APJ 772 (2013)

Very high densities:

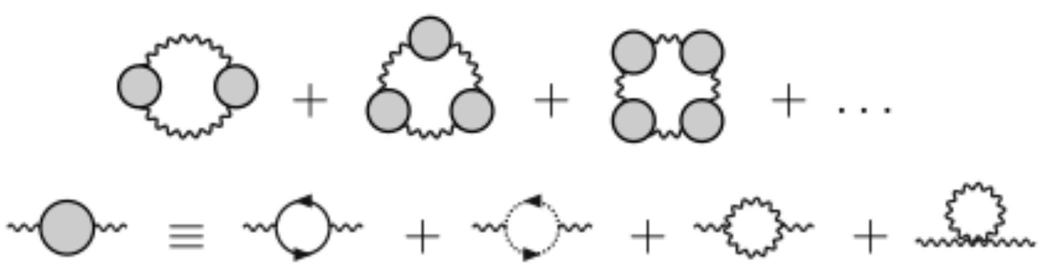
Pressure from pQCD (diagrammatically speaking...)

(Vuorinen, 2004)

Up to three loops (g^4) the diagrams to be evaluated are (without ghosts)



Also at low T the first IR problems (for the pressure) appear at 3-loop order: on top of 2PI graphs must resum



A more realistic image...

... of the “88 crazy diagrams”, although they are not really 88.

An old story...

Freedman & McLerran, 1977-1978
Baluni, 1978 ; Toimela, 1980's

Kajantie et al, 2001 ; Peshier et al, 1999-2003
Blaizot, Iancu & Rebhan, 1999-2003
ESF, Pisarski & Schaffner-Bielich, 2001
Andersen & Strickland, 2002
Rebhan & Romatschke, 2003
Vuorinen, 2004-2007

Including the strange quark mass:

Freedman & McLerran, 1977-1978; Baluni, 1978
(considered irrelevant for over 20 years...)

ESF and Romatschke, 2005
Kurkela, Romatschke & Vuorinen, 2010 → **NLO, NNLO: vacuum diagrams & ring resummation (the state of the art)**

(sorry for unavoidable omissions...)

As in other cases, after a long and subtle calculation, one realizes that the complicated result can be very well adjusted (EoS, 1st & 2nd derivatives) by a simple and compact function:

Previously: Fraga, Pisarski & Schaffner-Bielich (2001)
Alford et al. (2005)
→ effective bag model

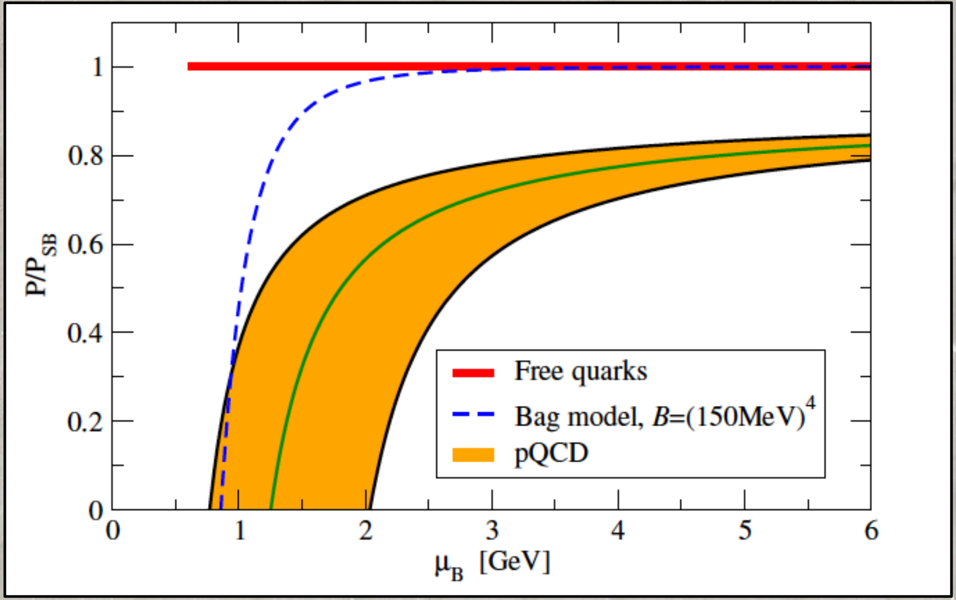
Using the complete results from Kurkela, Romatschke & Vuorinen (2010):

[ESF, Kurkela & Vuorinen (2013)]

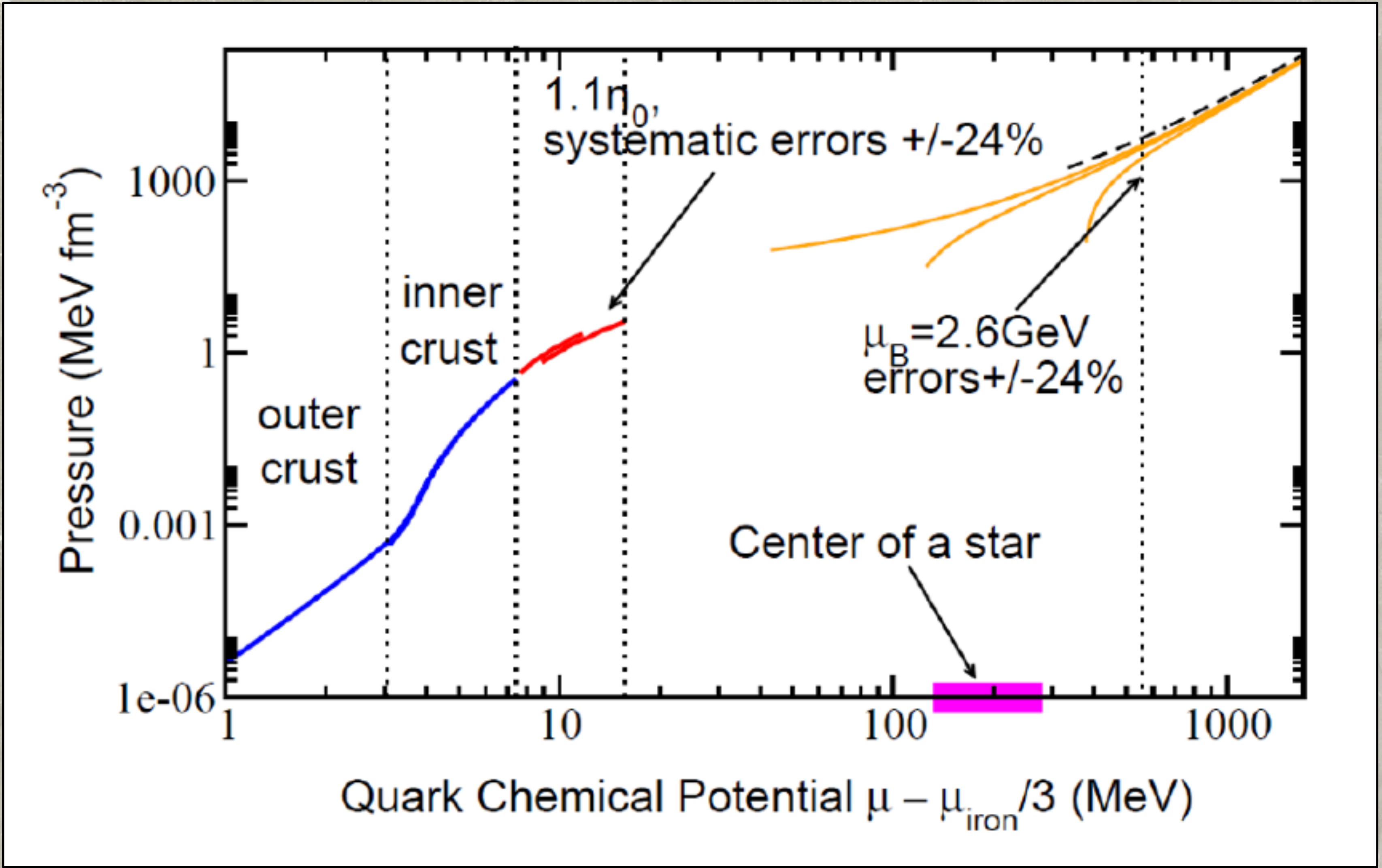
$$P_{\rm QCD}(\mu_B) = P_{\rm SB}(\mu_B)\left(c_1 - \frac{a(X)}{(\mu_B/{\rm GeV}) - b(X)}\right)$$

$$a(X) = d_1 X^{-\nu_1}, \quad b(X) = d_2 X^{-\nu_2}$$

$$P_{\rm SB} = \frac{3}{4\pi^2}(\mu_B/3)^4 \qquad X \equiv 3\bar{\Lambda}/\mu_B$$



Easy to use and play with!

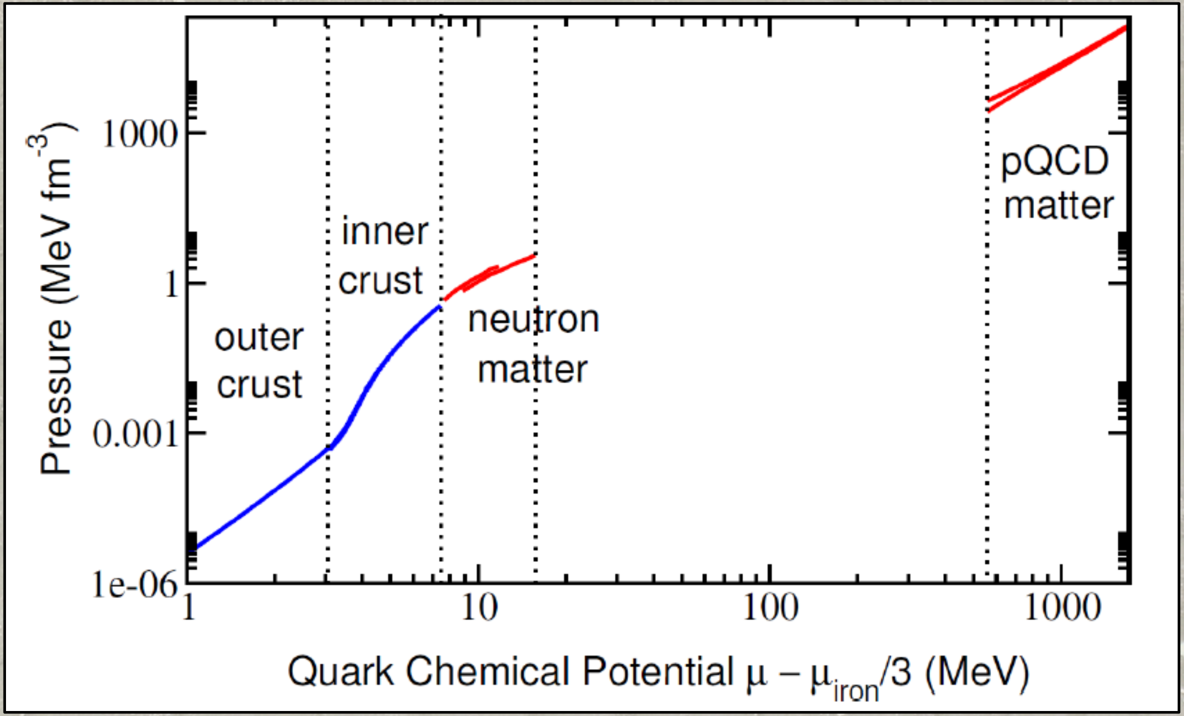


Uncertainties mostly from renormalization scale dependence, running of α_s & value of the strange quark mass.

The two limits are given:

Any interpolation in between is bound to satisfying the extrema

strong constraints!



★ One can use different phenomenological models to study this region, satisfying the "boundary" constraints.

★ We chose, instead, to use a multiple piecewise polytropic parametrization for the EoS: $p_i(n) = \kappa_i n^{\gamma_i}$

[Hebeler, Lattimer, Pethick & Schwenk (2013)]

So, by this parametrization, we quantify our ignorance by varying all parameters requiring the following:

[Kurkela, ESF, Schaffner-Bielich & Vuorinen (2014)]

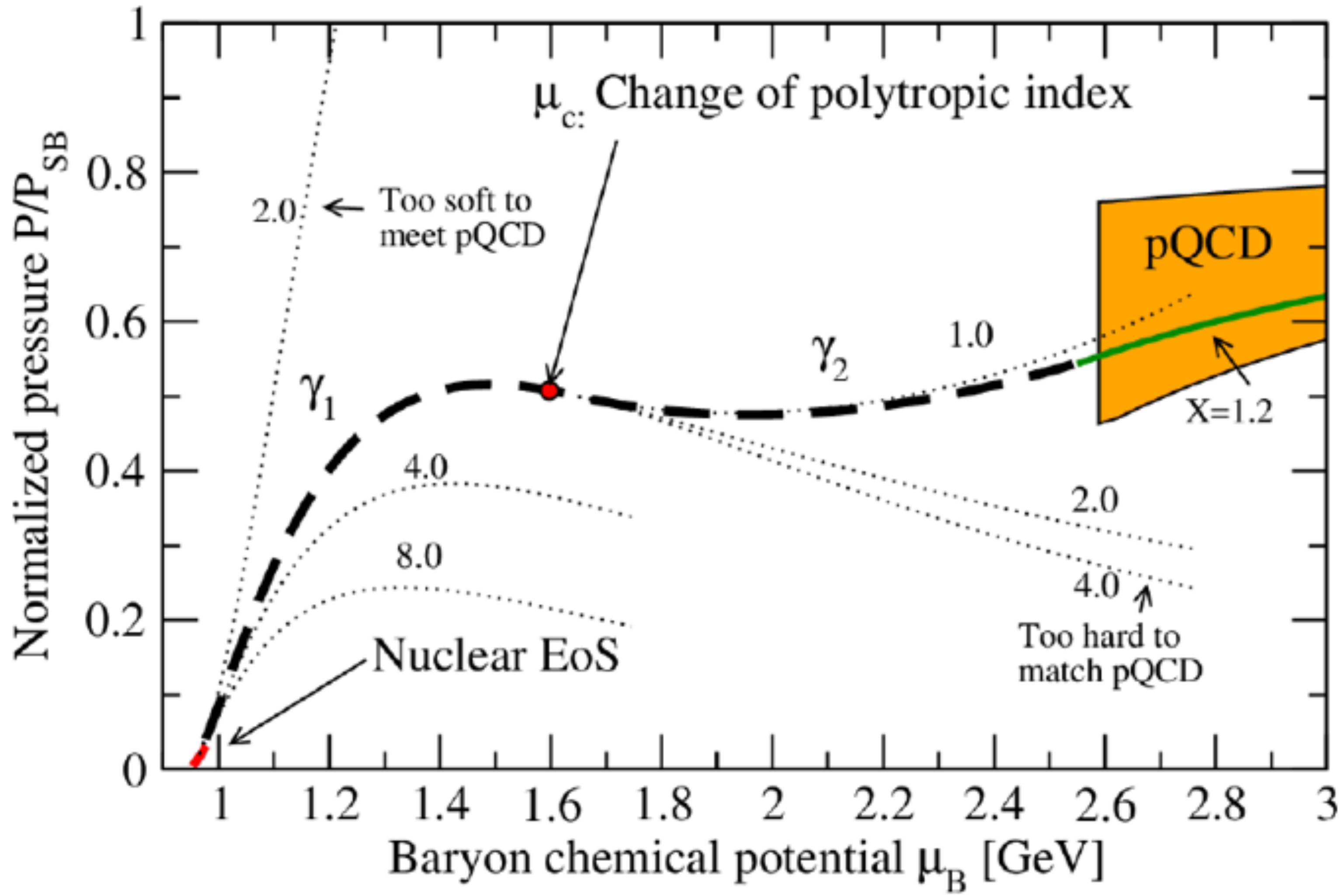
★ a smooth matching to nuclear and quark matter EoSs

★ smoothness: continuity of pressure & density when matching monotropes (can be relaxed)

★ causality: $c_s \leq 1$ (asymptotically equivalent to $\gamma \leq 2$)

★ possibility to support a two solar mass star

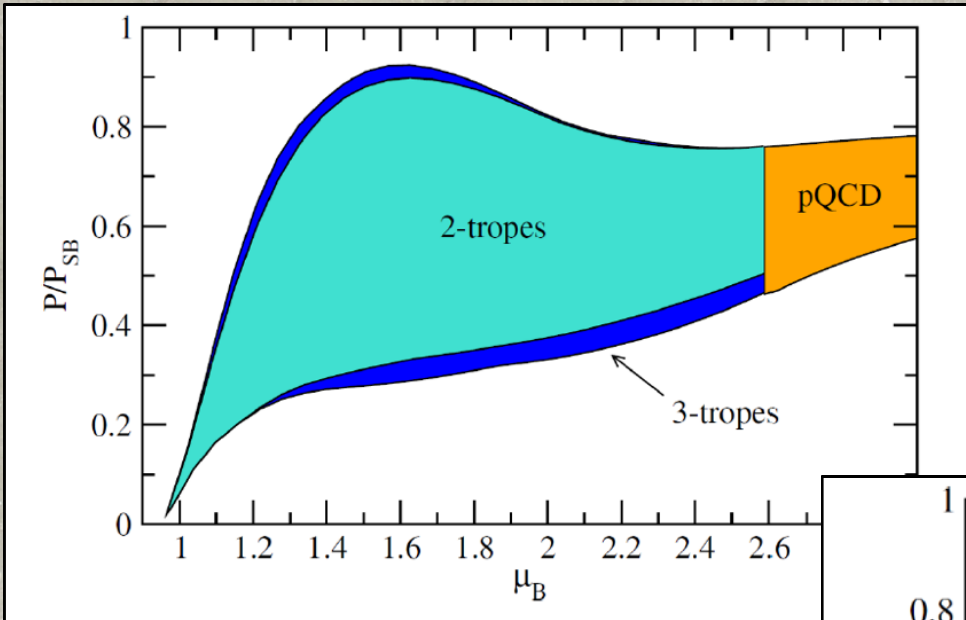
Illustration - 2 tropes:



Solutions exist for $\mu_c \in [1.08, 2.05]$ GeV, $\gamma_1 \in [2.23, 9.2]$, $\gamma_2 \in [1.0, 1.5]$

Constraining the EoS

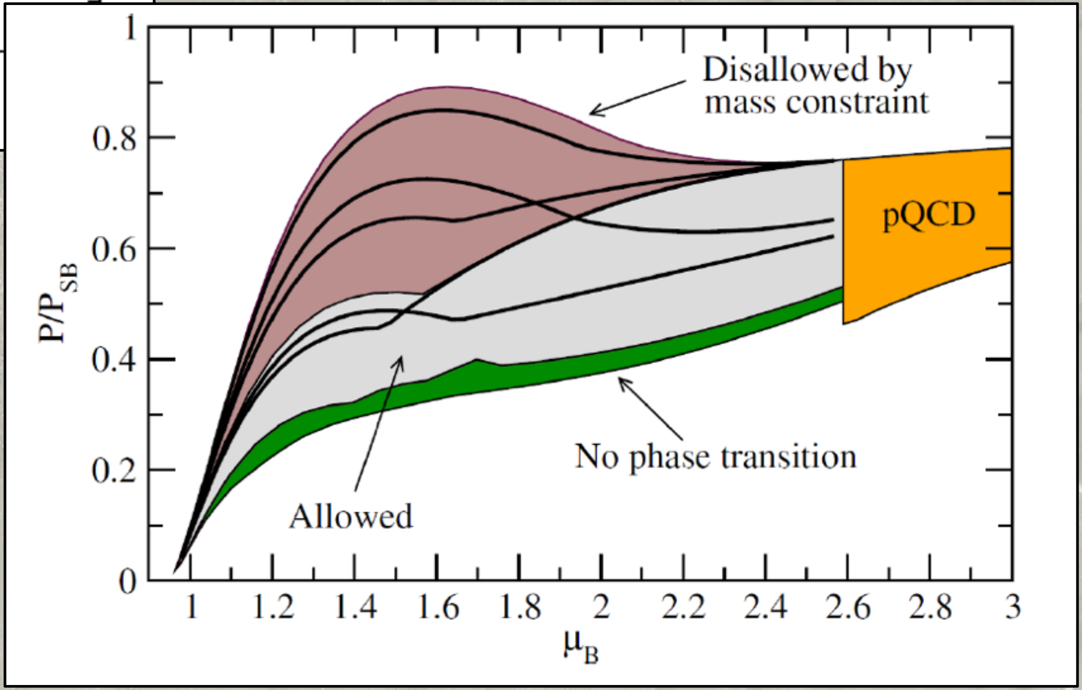
[Kurkela, ESF, Schaffner-Bielich & Vuorinen (2014)]



★ the 2-tropes case is the minimum necessary and also seems to be enough to do the job.

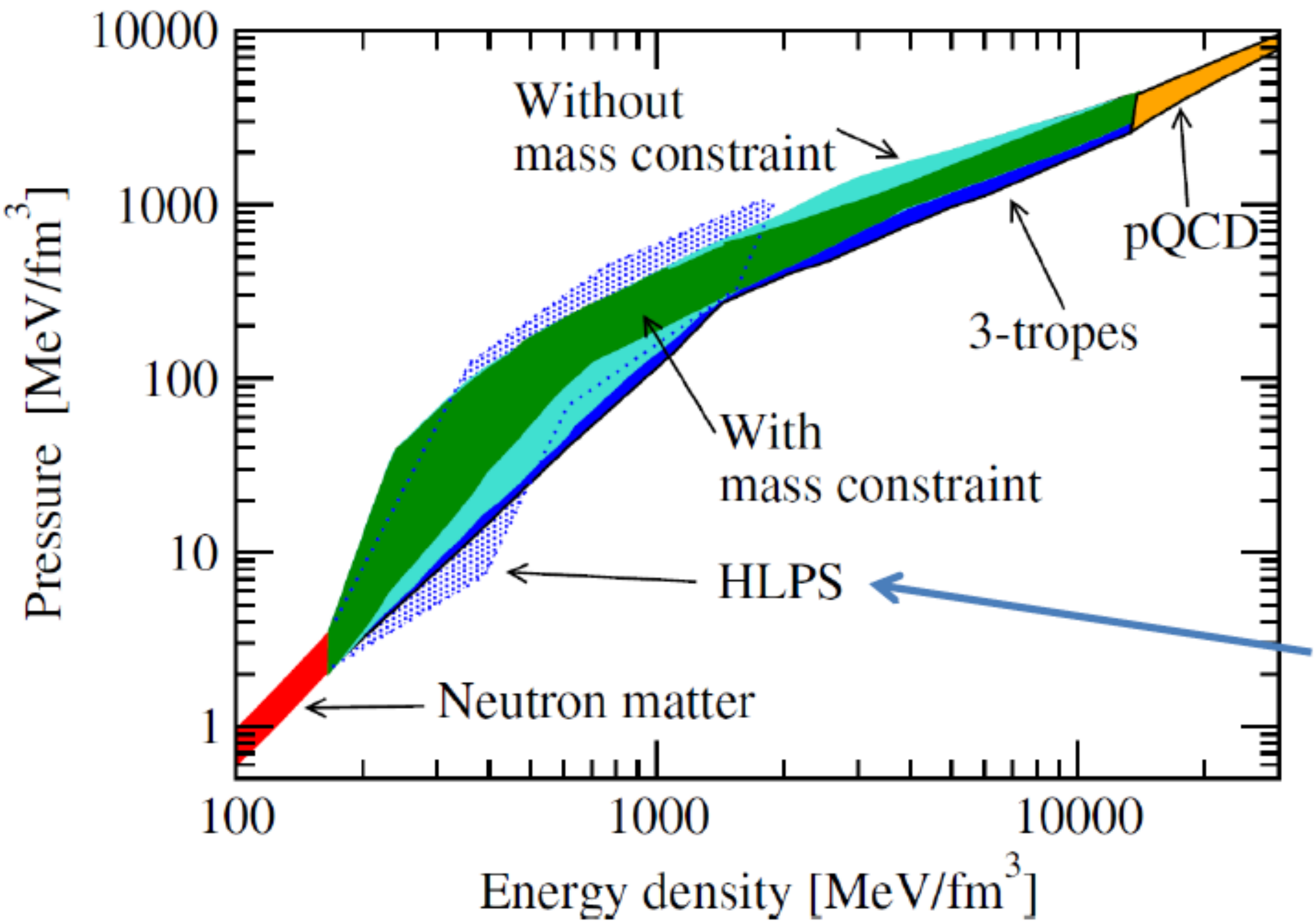
★ Allowing for $2M_{sun}$ stars constrains dramatically the band!

★ Implementing a 1st-order transition shrinks modestly the band.



The band for pressure vs. energy density

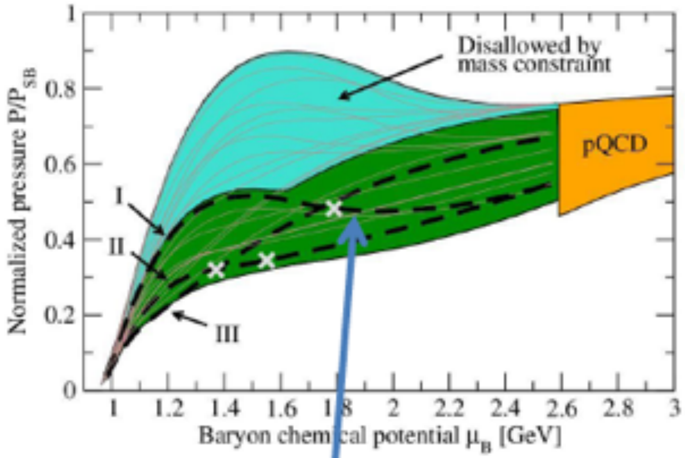
[Kurkela, ESF, Schaffner-Bielich & Vuorinen (2014)]



HLPS [Hebeler, Lattimer, Pethick, Schwenk, APJ 773 (2013)] without high-density constraint

Implications for compact stars

[Kurkela, ESF, Schaffner-Bielich & Vuorinen (2014)]



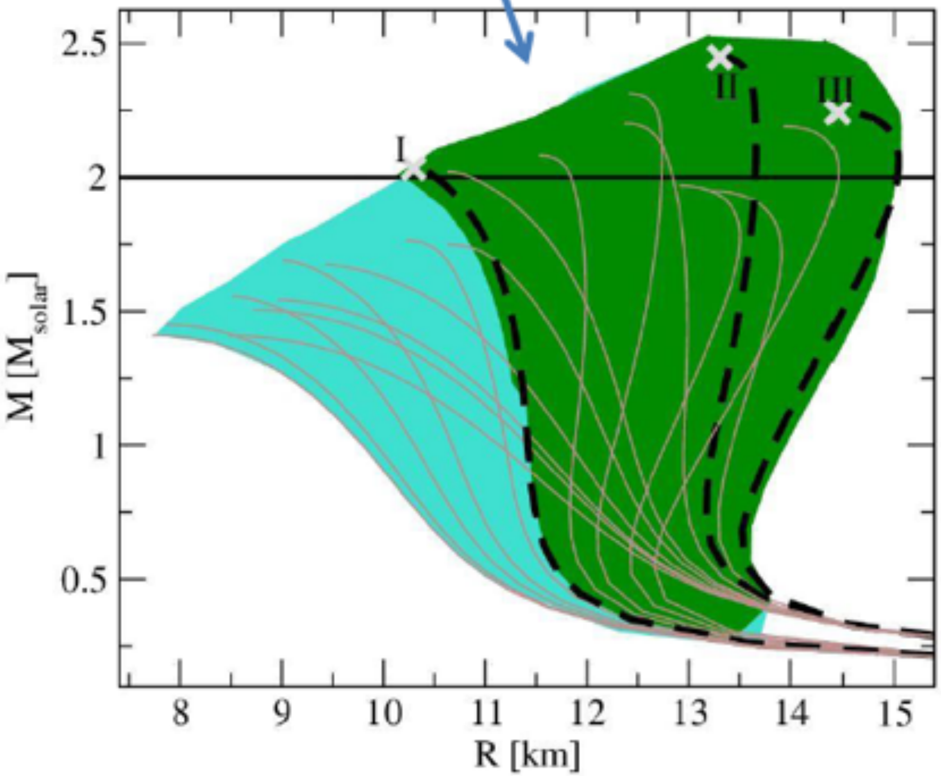
Maximal masses up to $2.5M_{\rm sun}$!

Central densities

$n_{\rm central} \in [3.7, 14.3]n_{\rm s}$

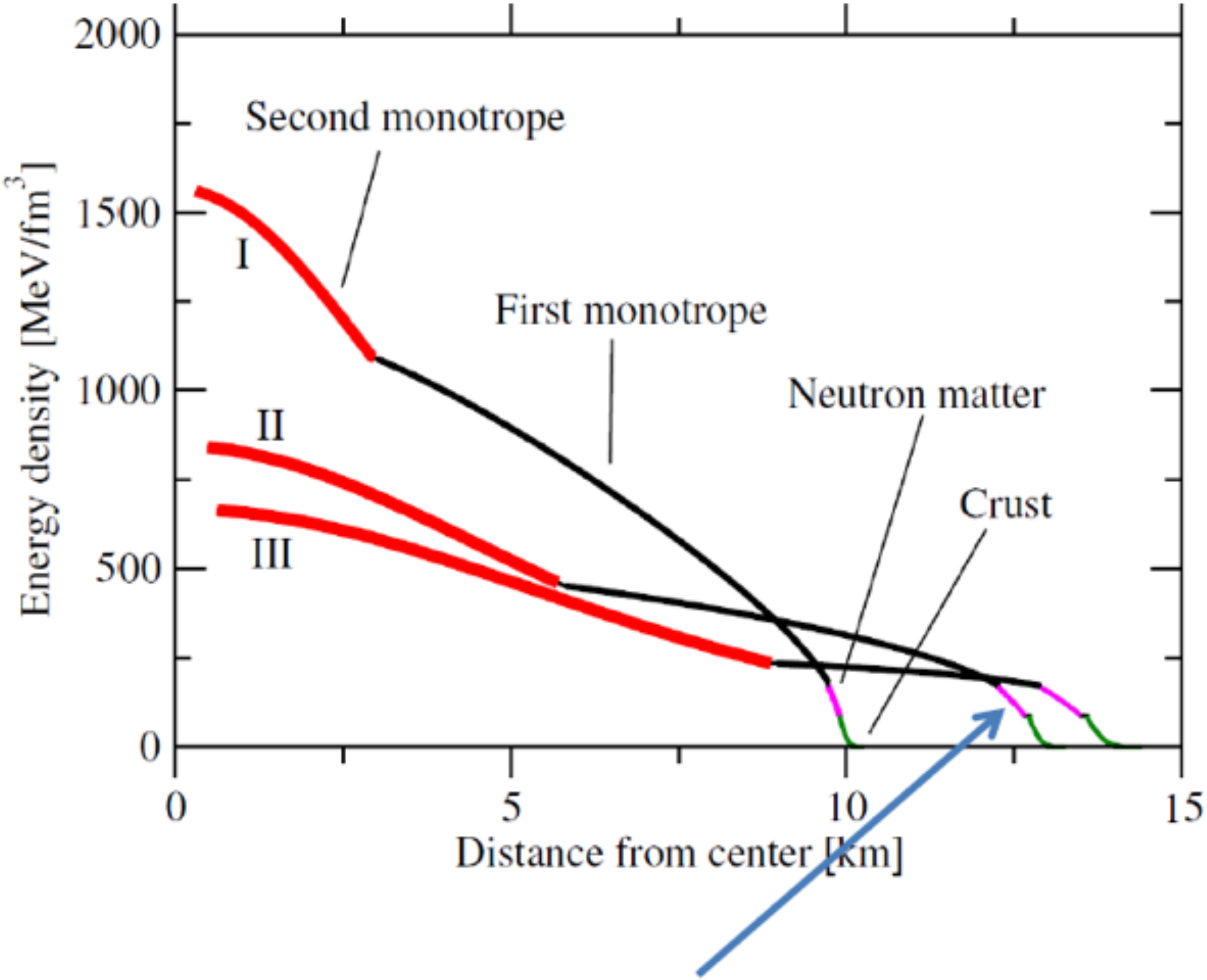
Large reduction in EoS uncertainty due to tension from mass constraint: Large stellar masses require stiff EoS, matching to pQCD soft

$\Rightarrow$ EoS uncertainty down to 30% at all densities



Energy density profile

[Kurkela, ESF, Schaffner-Bielich & Vuorinen (2014)]



Nuclear matter EoS only used very close to star surface – yet important effects from matching

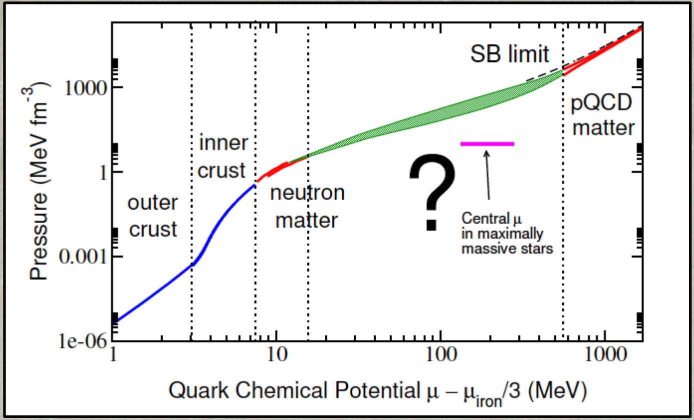
Final remarks

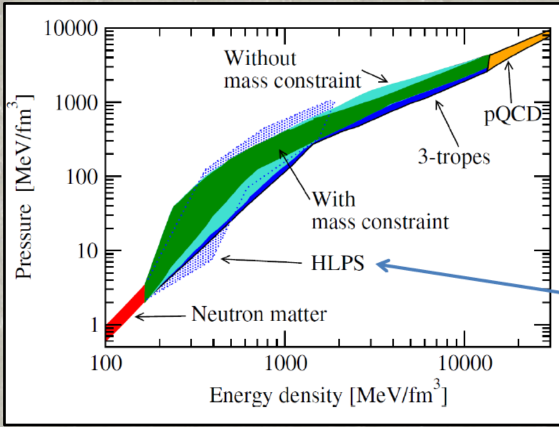
⋆ Main message: even if there is no deconfined quark matter in the core of neutron stars, the form of the QCD EoS at very large densities (which is known perturbatively) affects dramatically the EoS for compact stars!

⋆ The existence of 2 solar mass stars strongly constrains the EoS. Having a 1st-order transition or a crossover not so much.

⋆ There is a plug-and-play form (with error estimates) for the state of the art pressure:

$$P_{\mathrm{QCD}}(\mu_B) = P_{\mathrm{SB}}(\mu_B)\left(c_1 - \frac{a(X)}{(\mu_B/\mathrm{GeV}) - b(X)}\right)$$





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