Dynamical Locking of the Chiral and the Deconfinement Phase Transition in QCD at Finite Chemical Potential

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XIth Quark Confinement and the Hadron Spectrum
in Saint-Petersburg

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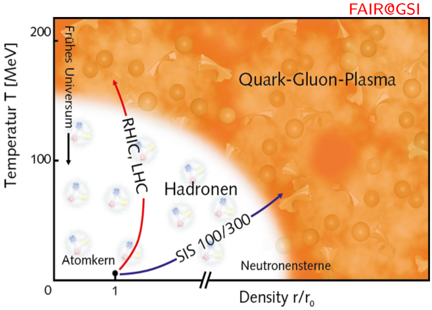
Motivation



- Confinement ⇔ **gauge degrees of freedom**
- χSB ⇔ **quark self-interactions**

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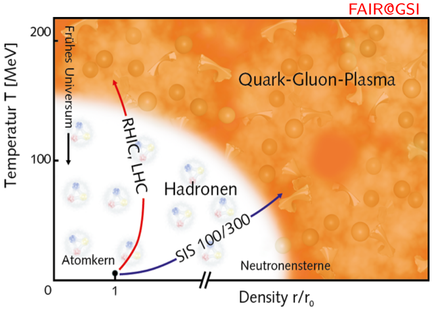

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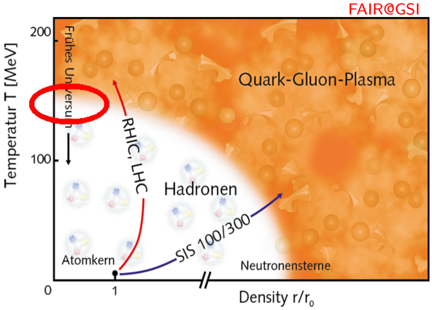

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Lattice QCD:

At $\mu = 0$ pseudo-critical temperatures are very similar for both crossovers
e.g. [Karsch et al., 2003], [Endrodi et al., 2006], [Aoki et al., 2009] etc.

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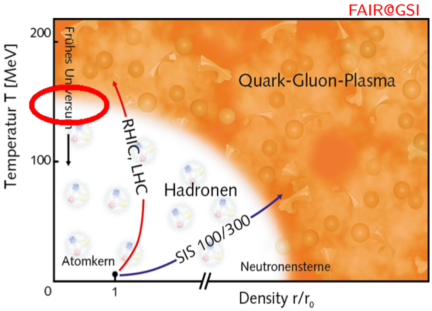

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Deeper relation between chiral and confining dynamics???

λ_ψ-deformed QCD

Real QCD: $\mathcal{L} = \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \mathrm{g} A\!\!\!/)\psi$

(chiral limit)

- Only **one** parameter: $\alpha_s(m_\tau)$ ($\Leftrightarrow \Lambda_{\mathrm{QCD}}$)

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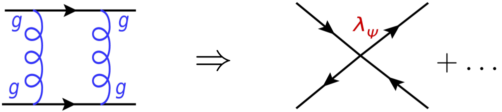
λ_ψ**-deformed** QCD:
Integrate out fluctuations:



$$\mathcal{L} = \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \mathrm{g} A\!\!\!/)\psi + \lambda_\psi(\bar{\psi}\mathcal{O}\psi)^2 + \dots$$

- Quark self-interactions are induced by the gauge fields
- **Two** parameters: $\lambda_\psi(m_\tau)$ and $\alpha_s(m_\tau)$ ($\Leftrightarrow \Lambda_{\mathrm{QCD}}$)
- **Chiral symmetry breaking** is triggered by **strong** λ_ψ

λ_ψ-deformed QCD

We investigate λ_ψ-deformed QCD (model) with two massless flavors, N_c colors and finite chemical potential:

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \gamma_0 \bar{g}\langle A_0 \rangle + \mathrm{i}\gamma_0 \mu)\psi + \frac{\bar{\lambda}_\psi}{2}[(\bar{\psi}\psi)^2 - (\bar{\psi}\vec{\tau}\gamma_5\psi)^2)]$$

- **two** parameters: $\lambda_\psi(\Lambda)$, $\langle A_0 \rangle$
- deconfinement order parameter:

$$\mathrm{Tr}_F\, L[\langle A_0 \rangle] = \frac{1}{N_c}\, \mathrm{Tr}_F[\mathcal{P}\mathrm{e}^{i\beta\bar{g}\langle A_0 \rangle}] \geq \frac{1}{N_c}\langle \mathrm{Tr}_F[\mathcal{P}\mathrm{e}^{i\bar{g}\int_0^\beta A_0}]\rangle$$

- Note: in PNJL/PQM-type model calculations:

$$\mathrm{Tr}_F\, L[\langle A_0 \rangle] = \frac{1}{N_c}\, \mathrm{Tr}_F[\mathcal{P}\mathrm{e}^{i\beta\bar{g}\langle A_0 \rangle}] \overset{!}{=} \frac{1}{N_c}\langle \mathrm{Tr}_F[\mathcal{P}\mathrm{e}^{i\bar{g}\int_0^\beta A_0}]\rangle$$

e.g. [Meisinger, Ogilvie, 1996]

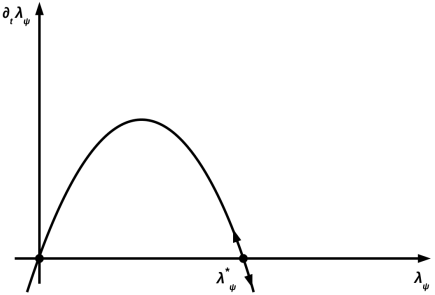
- Tool: **Wetterich flow equation** [C. Wetterich, 1993]

λ_ψ-deformed QCD: RG fixed-point analysis

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ + \qquad\qquad\qquad)\psi + \frac{\bar{\lambda}_\psi}{2}[(\bar{\psi}\psi)^2 - (\bar{\psi}\vec{\tau}\gamma_5\psi)^2)]$$

RG-flow equation:
$T = 0$, $\mu = 0$, $\langle A_0 \rangle = 0$
(k is momentum scale)

$$k\partial_k \lambda_\psi = 2\lambda_\psi - C\lambda_\psi^2$$



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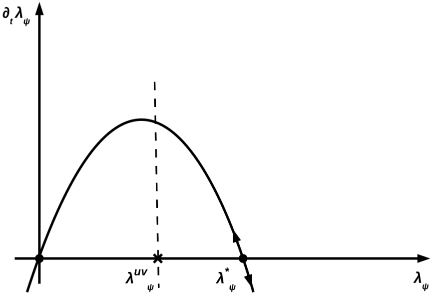

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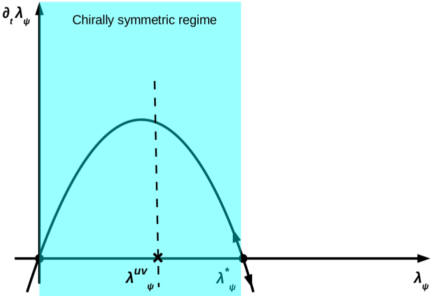

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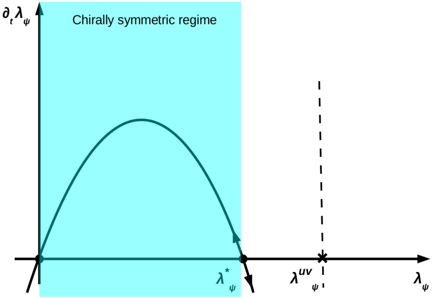

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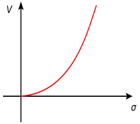
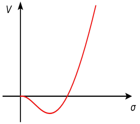
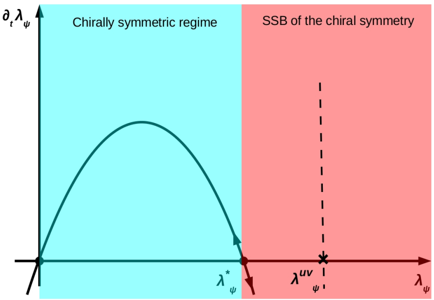


$\frac{1}{m^2} \propto \lambda_\psi \to \infty$





$$\boxed{\lambda_\psi(\Lambda) > \lambda_\psi^* \Rightarrow \chi SB}$$

λ_ψ-deformed QCD: RG fixed-point analysis

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RG-flow equation:
$T \neq 0,\ \mu \neq 0,\ \langle A_0 \rangle = 0$
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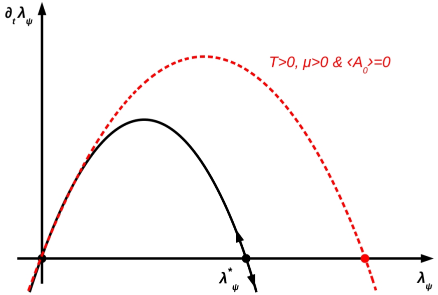
$$k\partial_k \lambda_\psi = 2\lambda_\psi - C(\tfrac{T}{k}, \tfrac{\mu}{k})\lambda_\psi^2$$



$\lambda_\psi(\Lambda) > \lambda_\psi^*$, T or (and) μ increase $\Rightarrow$ restoration of χ-Symmetry

λ_ψ-deformed QCD: RG fixed-point analysis

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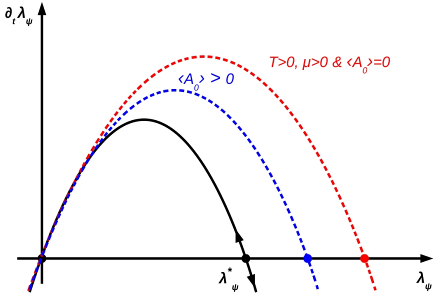
RG-flow equation:
$T \neq 0,\ \mu \neq 0,\ \langle A_0 \rangle \neq 0$
(color-confined regime)

$$k\partial_k \lambda_\psi = 2\lambda_\psi - C\left(\frac{T}{k}, \frac{\mu}{k}, \langle A_0 \rangle\right)\lambda_\psi^2$$



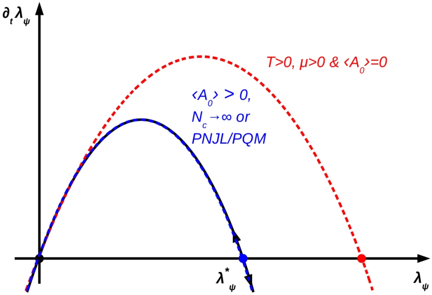
Finite $\langle A_0 \rangle \Rightarrow$ fixed point "moves" to the left

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \gamma_0 \bar{g}\langle A_0\rangle + \mathrm{i}\gamma_0\mu)\psi + \frac{\bar{\lambda}_\psi}{2}[(\bar{\psi}\psi)^2 - (\bar{\psi}\vec{\tau}\gamma_5\psi)^2)]$$

Analytical result:
as long

- $N_c \to \infty$, **or**
- $\mathrm{Tr}_F\, L[\langle A_0\rangle] \approx \langle \mathrm{Tr}_F\, L[A_0]\rangle$ (PNJL/PQM-models)

$$\Rightarrow \boxed{\lambda^*(T,\mu,\langle A_0\rangle) = \lambda^*(0,0,0)}$$



$$\boxed{T_\chi \gtrsim T_d}$$

Numerical Results

- We use the data for $\langle A_0 \rangle$ for pure $SU(N_c)$ gauge theory, i. e., we drop the back coupling of fermions to the gauge sector: T_d **is fixed!**
  [Braun, Gies, Pawlowski, 2010], [Braun, Eichhorn, Gies, Pawlowski, 2010]
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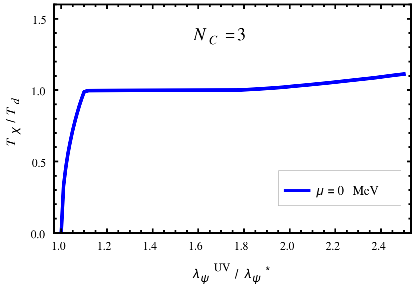

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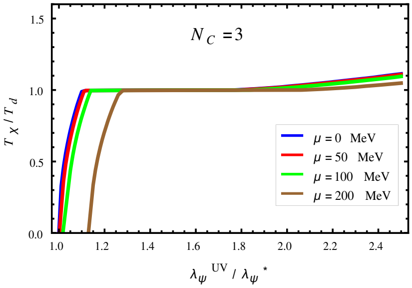

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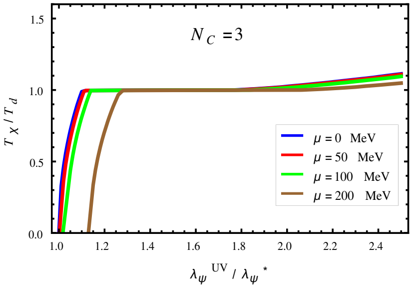

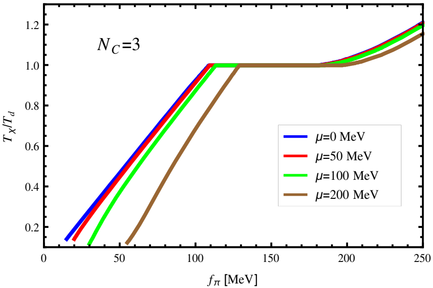

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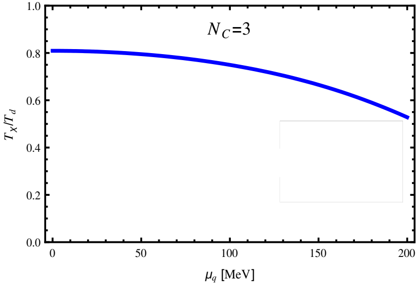

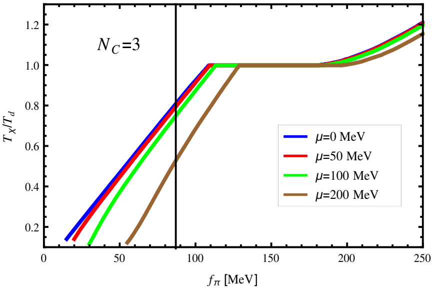

$$\kappa = -T_\chi \frac{dT_\chi(\mu^2)}{d(\mu^2)} \Big|_{\mu=0} = 0.385(5)$$

Conclusion

- **PNJL/PQM-models $\Longleftrightarrow$ Large-N_c in the coupling of the matter and gauge sector** (should not be confused with the standard large-N_c approximation, such as neglecting pion fluctuations etc.)
- $T_\chi \geq T_d$ in the phase diagram of PNJL/PQM-models
  - $\Rightarrow$ existence of quarkyonic phase in PNJL/PQM-models under debate
  - $\Rightarrow$ **Constraint on parametrization of Polyakov potential**
- Locking window in parameter space also for $N_C = 3$
  Finite μ shifts this window
- Curvature $\kappa = 0.385(5)$ is in agreement with lattice QCD estimations: $0.0032(1) \lesssim \kappa \lesssim 0.500(54)$
  [Fodor and Katz, 2004], [Allton et al., 2003], [Karsch et al., 2003], [de Forcrand and Philipsen, 2002]

Thank you for your attention!