Equation of State of hot and dense QCD: Resummed perturbation theory confronts lattice data

Jens O. Andersen [1]

NTNU – Trondheim
Norwegian University of
Science and Technology

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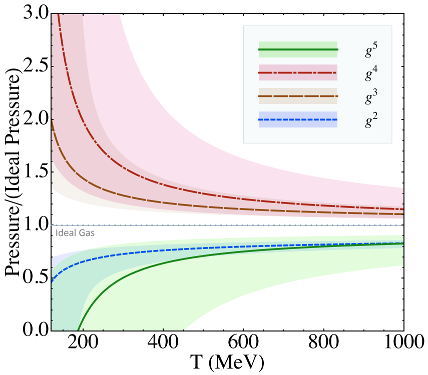
[1] In collaboration with Aritra Bandyopadhyay, Najmul Haque, Sylvain Mogliacci, Munshi Mustafa, Michael Strickland, Nan Su, and Aleksi Vuorinen, PRD 87 074003 (2013), JHEP 13 055 (2013), PRD 89 01701 (2014), JHEP 05 27 (2014)



- The weak-coupling expansion of the free energy of QCD has been calculated to order $\alpha_s^3 \log \alpha_s$ [a].
- Very poorly convergent. Generic problem in hot field theories (scalar field theory, QED).
- Goal: gauge-invariant framework with better convergence properties+able to describe dynamical properties+easy to generalize to finite μ_i.

[a] Arnold and Zhai, '94/'95, Kastening and Zhai '95, Braaten and Nieto '95, Kajantie, Laine, Rummukainen, and Schröder '02, Vuorinen '03.

Screened perturbation theory

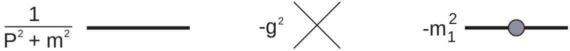
- Massless scalar field theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{g^2}{24}\phi^4 \, .$$

- Reorganization:

$$\mathcal{L}_0 = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}m^2\phi^2$$

$$\mathcal{L}_{\rm int} = -\frac{1}{2}m^2\phi^2 + \frac{g^2}{24}\phi^4 \, .$$

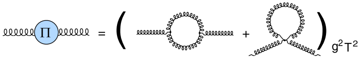


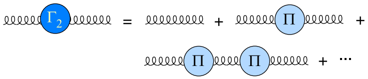
Hard-thermal-loop perturbation theory

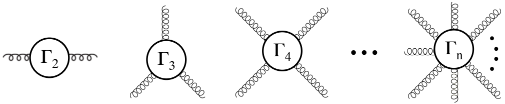
- QCD Lagrangian:

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}{\rm Tr}[G_{\mu\nu}G^{\mu\nu}] + i\bar{\psi}\gamma^{\mu}D_{\mu}\psi + \mathcal{L}_{gh} + \mathcal{L}_{gf} + \Delta\mathcal{L}_{\rm QCD}\,,$$

- For soft momenta gT, one needs effective propagators (2PI effective action: Blaizot, Iancu, and Rebhan '99, '00).





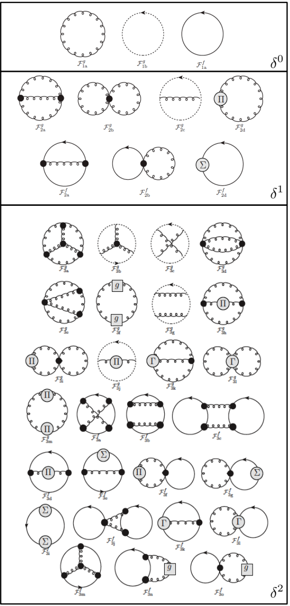


- Expansion point to gas of massive quasiparticles by adding HTL Lagrangian.

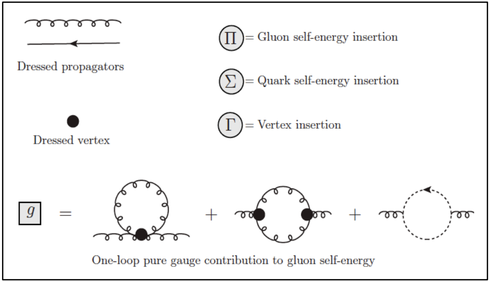
$$
\begin{aligned}
\mathcal{L}_{\mathrm{HTL}} &= -\frac{1}{2}(1-\delta)m_D^2 \mathrm{Tr}\left(G_{\mu\alpha}\left\langle \frac{y^\alpha y_\beta}{(y\cdot D)^2}\right\rangle_{\hat{\mathbf{y}}} G^{\mu\beta}\right) \\
&\quad (1-\delta) i m_q^2 \bar{\psi}\gamma^\mu \left\langle \frac{y_\mu}{y\cdot D}\right\rangle_{\hat{\mathbf{y}}} \psi \, , \\
\mathcal{L} &= \left(\mathcal{L}_{\mathrm{QCD}} + \mathcal{L}_{\mathrm{HTL}}\right)\Big|_{g\to\sqrt{\delta}g} + \Delta\mathcal{L}_{\mathrm{HTL}} \, ,
\end{aligned}
$$

- HTLpt formal expansion parameter δ and $\delta = 1$ at the end.
- Expansion generates effective propagators and vertices.

Three-loop calculation



- Compute all Feynman diagram through 3 loops using HTL propagators and vertices



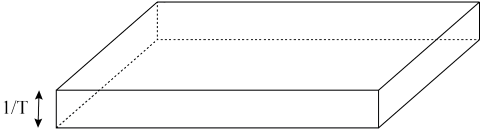
- Give a prescription for mass parameters m_D and m_q.

Thermodynamic potential

$$
\begin{aligned}
\frac{\Omega_{\rm NNLO}}{\Omega_0} = \; & \frac{7}{4}\frac{d_F}{d_A}\left(1+\frac{120}{7}\hat\mu^2+\frac{240}{7}\hat\mu^4\right)+\frac{s_F\alpha_s}{\pi}\left[-\frac{5}{8}\left(1+12\hat\mu^2\right)\left(5+12\hat\mu^2\right)+\frac{15}{2}\left(1+12\hat\mu^2\right)\hat m_D\right. \\
& \left.+\frac{15}{2}\left(2\ln\frac{\hat\Lambda_q}{2}-1-\aleph(z)\right)\hat m_D^3-90\hat m_q^2\hat m_D\right]+s_{2F}\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{15}{64}\left\{35-32\left(1-12\hat\mu^2\right)\frac{\zeta'(-1)}{\zeta(-1)}+472\hat\mu^2\right.\right. \\
& \left.\left.+1328\hat\mu^4+64\Big(-36i\hat\mu\,\aleph(2,z)+6(1+8\hat\mu^2)\aleph(1,z)+3i\hat\mu(1+4\hat\mu^2)\aleph(0,z)\Big)\right\}-\frac{45}{2}\hat m_D\left(1+12\hat\mu^2\right)\right] \\
& +\left(\frac{s_F\alpha_s}{\pi}\right)^2\left[\frac{5}{4\hat m_D}\left(1+12\hat\mu^2\right)^2+30\left(1+12\hat\mu^2\right)\frac{\hat m_q^2}{\hat m_D}+\frac{25}{12}\left\{\left(1+\frac{72}{5}\hat\mu^2+\frac{144}{5}\hat\mu^4\right)\ln\frac{\hat\Lambda_q}{2}\right.\right. \\
& +\frac{1}{20}\left(1+168\hat\mu^2+2064\hat\mu^4\right)+\frac{3}{5}\left(1+12\hat\mu^2\right)^2\gamma_E-\frac{8}{5}\left(1+12\hat\mu^2\right)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{34}{25}\frac{\zeta'(-3)}{\zeta(-3)} \\
& -\frac{72}{5}\Big[8\aleph(3,z)+3\aleph(3,2z)-12\hat\mu^2\aleph(1,2z)+12i\hat\mu\left(\aleph(2,z)+\aleph(2,2z)\right)-i\hat\mu\left(1+12\hat\mu^2\right)\aleph(0,z) \\
& \left.\left.-2(1+8\hat\mu^2)\aleph(1,z)\Big]\right\}-\frac{15}{2}\left(1+12\hat\mu^2\right)\left(2\ln\frac{\hat\Lambda_q}{2}-1-\aleph(z)\right)\hat m_D\right] \\
& +\left(\frac{c_A\alpha_s}{3\pi}\right)\left(\frac{s_F\alpha_s}{\pi}\right)\left[\frac{15}{2\hat m_D}\left(1+12\hat\mu^2\right)-\frac{235}{16}\left\{\left(1+\frac{792}{47}\hat\mu^2+\frac{1584}{47}\hat\mu^4\right)\ln\frac{\hat\Lambda_q}{2}-\frac{144}{47}\left(1+12\hat\mu^2\right)\ln\hat m_D\right.\right. \\
& +\frac{319}{940}\left(1+\frac{2040}{319}\hat\mu^2+\frac{38640}{319}\hat\mu^4\right)-\frac{24\gamma_E}{47}\left(1+12\hat\mu^2\right)-\frac{44}{47}\left(1+\frac{156}{11}\hat\mu^2\right)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{268}{235}\frac{\zeta'(-3)}{\zeta(-3)} \\
& \left.-\frac{72}{47}\Big[4i\hat\mu\,\aleph(0,z)+\left(5-92\hat\mu^2\right)\aleph(1,z)+144i\hat\mu\,\aleph(2,z)+52\aleph(3,z)\Big]\right\}+90\frac{\hat m_q^2}{\hat m_D}+\frac{315}{4}\left\{\left(1+\frac{132}{7}\hat\mu^2\right)\ln\frac{\hat\Lambda_q}{2}\right. \\
& \left.\left.+\frac{11}{7}\left(1+12\hat\mu^2\right)\gamma_E+\frac{9}{14}\left(1+\frac{132}{9}\hat\mu^2\right)+\frac{2}{7}\aleph(z)\right\}\hat m_D\right]+\frac{\Omega^{\rm YM}_{\rm NNLO}(\Lambda_g)}{\Omega_0}\,.
\end{aligned}
$$

Dimensional reduction

- Three momentum scales: Hard scale T, soft scale gT, and supersoft scale g^2T.
- Integrate out hard scale T perturbatively to obtain an effective three-dimensional theory (Electrostatic QCD). [2]



$$\mathcal{L}_{\rm eff} = \frac{1}{2}{\rm Tr}[G_{ij}^2] + {\rm Tr}[(D_iA_0)^2] + m_D^2{\rm Tr}[A_0^2] + \ldots$$

- Parameters depend on T, μ_f, g...

[2] Braaten and Nieto '96, Kajantie, Laine, K. Rummukainen, and Shaposhnikov '96, Vuorinen '03.

Dimensional reduction

- Integrate out scale gT i.e. A_0 to obtain a second effective field theory (magnetostatic QCD).

$$\mathcal{P} = \mathcal{P}_{\rm hard} + \mathcal{P}_{\rm soft} + \mathcal{P}_{\rm supersoft}$$

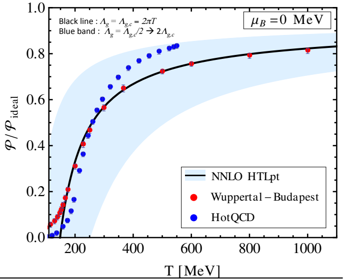
- Hard scale T contributions from dimensional reduction.
- Soft scale gT contributions from calculations using EQCD.
- MQCD contributes first at order g^6.
- As in HTLpt, keep full g-dependence of m_D to resum.

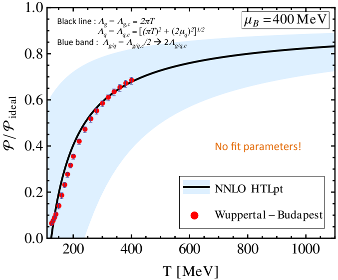
Results - pressure for $\mu_B = 0$ and $\mu_B = 400$ MeV

- Use one-loop running with $\alpha_s(1.5\ \mathrm{GeV}) = 0.326$ [3]
- Mass prescription: use $m_f = 0$ and m_D from EQCD [4]

$$
\hat{m}_D^2 = \frac{\alpha_s}{3\pi}\left\{c_A + \frac{c_A^2\alpha_s}{12\pi}\left(5 + 22\gamma_E + 22\ln\frac{\hat{\Lambda}_g}{2}\right) + \frac{1}{N_f}\sum_f\left[s_F\left(1+12\hat{\mu}_f^2\right) + \frac{c_A s_F\alpha_s}{12\pi}\left(\left(9+132\hat{\mu}_f^2\right) + 22\left(1+12\hat{\mu}_f^2\right)\gamma_E \right.\right.\right.
$$

$$
\left.\left.\left. + 2\left(7+132\hat{\mu}_f^2\right) + 4\aleph(z_f)\right) + \frac{s_F^2\alpha_s}{3\pi}\left(1+12\hat{\mu}_f^2\right)\left(1 - 2 + \aleph(z_f)\right) - \frac{3}{2}\frac{s_{2F}\alpha_s}{\pi}\left(1+12\hat{\mu}_f^2\right)\right]\right\}
$$





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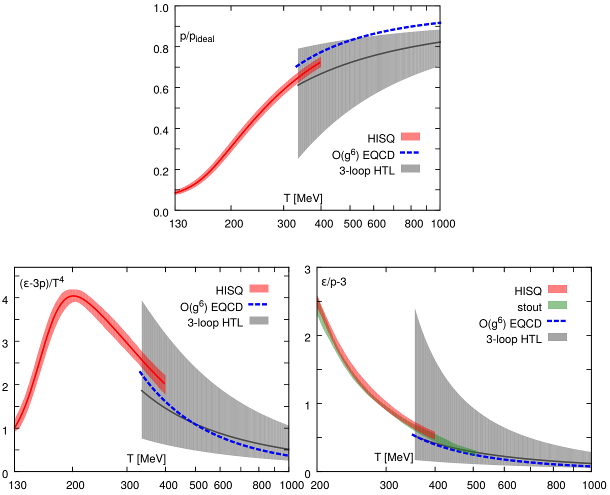
[3] Bazavov et al (2012)

[4] Vuorinen '03.

[5] Borsanyi et al '10 and '12, Bazavov et al '09.

More results on the pressure



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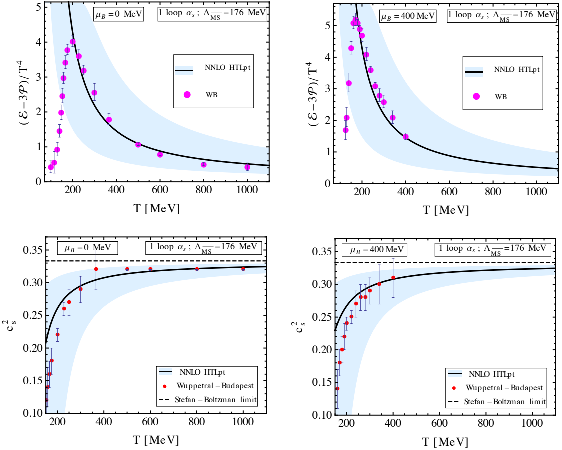
6 HotQCD Bazavov et al, arXiv:1407.6387.

Results - interaction measure and speed of sound

$$c_s^2 = \frac{\partial \mathcal{P}}{\partial \mathcal{E}} .$$



Susceptibilities

- Pressure

$$\frac{\mathcal{P}}{T^4} = \frac{\mathcal{P}_0}{T^4} + \sum_{ijk} \frac{1}{i!j!k!} \chi_{ijk} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k$$

- Results - quark - and baryon-number susceptibilities

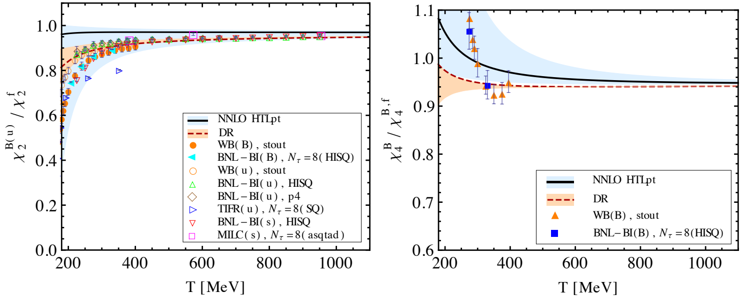
$$\chi_{ijk\ldots} = \left.\frac{\partial^{i+j+k+\ldots}\mathcal{P}(T,\boldsymbol{\mu})}{\partial\mu_u^i\partial\mu_d^j\partial\mu_s^k}\right|_{\boldsymbol{\mu}=0} .$$

$$\chi_n^B = \left.\frac{\partial^n\mathcal{P}}{\partial\mu_B^n}\right|_{\mu_B=0} .$$

$$\chi_2^B = \frac{1}{9}\left[\chi_2^{uu} + \chi_2^{dd} + \chi_2^{ss} + 2\chi_2^{ud} + 2\chi_2^{us} + 2\chi_2^{ds}\right] ,$$

$$\chi_2^{uu} = \chi_{200} , \qquad \text{etc.}$$

Results - susceptibilities



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7 Borsanyi et al '10 and '12, Bazavov et al '13.

Summary and Outlook

- Hard-thermal-loop perturbation theory represents a gauge-invariant reorganization of the perturbative series.
- Analytic result for the three-loop QCD thermodynamic potential at finite T and μ.
- Agreement with lattice data for a number of variables is good, in particular considering that there are no fit parameters.
- HTLpt is formulated in Minkowski space and can therefore be applied to real-time quantities as well.
- Resummed DR also in good agreement with lattice.