Lattice NRQCD study on in-medium modification of $b\bar{b}$ spectra using a novel Bayesian reconstruction

Alexander Rothkopf
Institute for Theoretical Physics
Heidelberg University

in collaboration with

S. Kim (Sejong-U.) and P. Petreczky (BNL)

PoS LATTICE 2013 (169) & in preparation

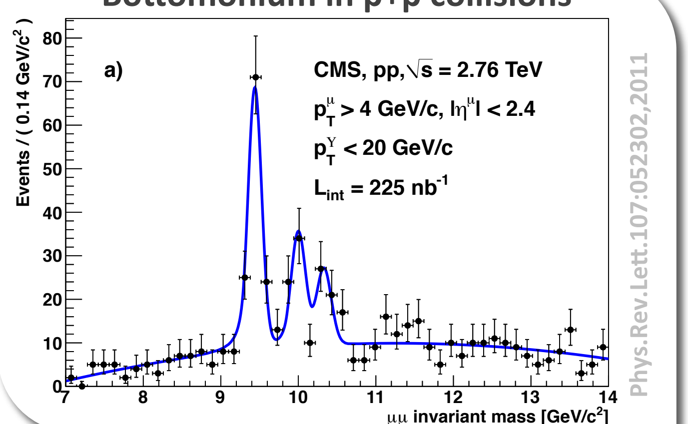
Motivation: Heavy-Ion Collisions

- At RHIC and LHC: Precision era of relativistic heavy-ion collision experiments
- Good probes: susceptible to medium but distinguishable from it $Q_{probe} >> T_{med}$

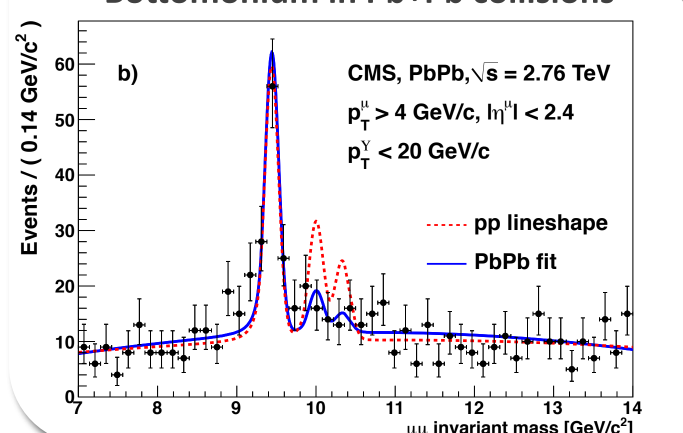
Bound states of $c\bar{c}$ or $b\bar{b}$: **Heavy quarkonium** $M_Q >> T_{med}$

- Extract properties of the QGP from observed vector channel Bottomonium yields

Bottomonium in p+p collisions



Bottomonium in Pb+Pb collisions



- How to understand $b\bar{b}$ suppression in a strongly coupled QGP: Lattice QCD

A Lattice QCD Challenge

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- PRACTICAL: High cost if light and heavy d.o.f share the same spacetime grid

$$a \ll \frac{1}{2m_b} \approx 0.02\mathrm{fm} \qquad \frac{1}{T} = N_\tau a \sim 1\mathrm{fm}$$

➡ Turn the separation of scales into an advantage: effective field theory NRQCD
Thacker, Lepage Phys.Rev. D43 (1991) 196-208

- CONCEPTUAL: Simulations in Euclidean time, no direct access to spectral information
  - Analytic continuation from a finite and noisy dataset necessary: ill-defined problem
    M. Jarrell, J. Gubernatis, , Physics Reports 269 (3) (1996)

➡ New Bayesian spectral reconstruction: improving on the Maximum Entropy Method
Y.Burnier, A.R. PRL 111 (2013) 18, 182003
M. Asakawa, T. Hatsuda and Y. Nakahara, Prog. Part. Nucl. Phys. 46, 459 (2001)

Effective Field Theory: Lattice NRQCD

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

$$L_{NRQCD} = \psi^\dagger\left(iD_t + \frac{D_i^2}{2M_Q} + \dots\right)\psi + \xi^\dagger\left(\dots\right)\xi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{q}\left(\dots\right)q$$

Lepage et.al, Phys.Rev. D46 (1992) 4052-4067
Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

Hevay quark ψ and antiquark ξ as separate non- relativistic Pauli spinors

Light medium d.o.f. from a fully relativistic lattice simulation

- Separation of scales $T/M_Q << 1$, $\Lambda_{QCD}/M_Q<<1$, $p/M_Q<< 1$: systematic expansion in $1/M_Qa$
- Individual Q or anti-Q in a medium background: Initial value problem $G(\tau)=<\psi(\tau)\psi^\dagger(0)>$

$$G(\mathbf{x}, \tau + a) = U_4^\dagger(\mathbf{x}, \tau)\left(1 - \frac{\mathbf{p}_{lat}^2}{4M_Q a} + \dots\right)G(\mathbf{x}, \tau)$$

well behaved if $M_Qa > 1.5$

Davies, Thacker Phys.Rev. D45 (1992)

- **$^3S_1$ (ϒ)** and **$^3P_1$ (χ_{b1})** channel correlators D(τ) from products of heavy quark propagators G(τ)

$$D(\tau) = \sum_{\mathbf{x}} \langle O(\mathbf{x}, \tau)G_{\mathbf{x}\tau}O^\dagger(\mathbf{x}_0, \tau_0)G^\dagger_{\mathbf{x}\tau}\rangle_{med} \qquad O(^3S_1; x, \tau) = \sigma_i, \quad O(^3P_1; x, \tau) = \overleftrightarrow{\Delta}_i\sigma_j - \overleftrightarrow{\Delta}_j\sigma_i$$

Thacker, Lepage Phys.Rev. D43 (1991)

A Medium With Nf=2+1 Light HISQ Flavors

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- Light d.o.f. (gluons, u d s quarks) represented by HotQCD configurations

  A. Bazavov et. al., Phys. Rev. D 85 (2012) 054503

  - 48^3x12 with relatively light pions M_π~ 161MeV and a T_C=159±3MeV

HotQCD HISQ/tree action $48^3 \times N_\tau$ $m_{u,d}/m_s = 0.05$

β	6.664	6.700	6.740	6.770	6.800	6.840	6.880
a[fm]	0.1169	0.1130	0.1087	0.1057	0.1027	0.09893	0.09528
$M_b a$	2.759	2.667	2.566	2.495	2.424	2.335	2.249
$T/T_C(N_\tau = 12)$	0.911	0.944	0.980	1.008	1.038	1.078	1.119
β	6.910	6.950	6.990	7.030	7.100	7.150	7.280
a[fm]	0.09264	0.08925	0.086	0.08288	0.07772	0.07426	0.06603
$M_b a$	2.187	2.107	2.030	1.956	1.835	1.753	1.559
$T/T_C(N_\tau = 12)$	1.151	1.194	1.240	1.286	1.371	1.436	1.614

- Important property for the use with lattice NRQCD: $2.759 > M_b a > 1.559 > 1.5$

- Temperature changed by variation of the lattice spacing 140MeV < T < 249MeV

  For a study based on the fixed scale approach see: FASTSUM G. Aarts et. al. JHEP 1407 (2014) 097, JHEP 1111 (2011) 103

- Low temperature configurations available at b=6.664, 6.8, 6.95, 7.28

Spectral Functions In NRQCD

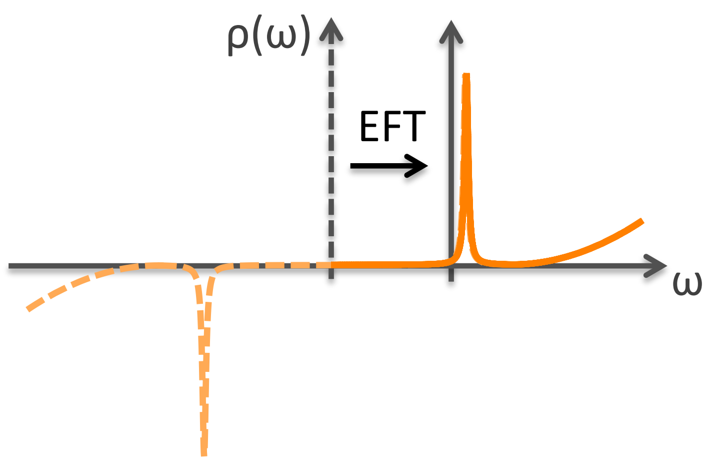
UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

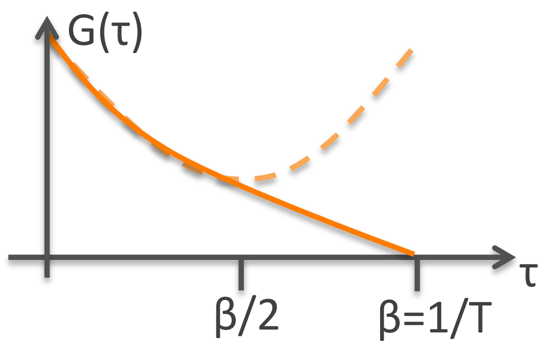
- "Integrating out M_b" in setting up NRQCD introduces a scale dependent frequency shift

Drawback: setting absolute frequency scale at T>0 requires additional T=0 calibration

Advantage: Correlator not periodic in 1/T and linked to spectra via simple T=0 Kernel,

$$D(\tau) = \int_{-2M_Q}^{\infty} d\omega e^{-\tau\omega} \rho(\omega)$$





Novel Bayesian Spectral Reconstruction

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- Inversion of Laplace transform required to obtain spectra from correlators

$$D_i = \sum_{l=1}^{N_\omega} \exp[-\omega_l \tau_i]\, \rho_l\, \Delta\omega_l$$

1. N_ω parameters ρ_l >> N_τ datapoints
2. data D_i has finite precision

- Give meaning to problem by incorporating prior knowledge: Bayesian approach

M. Jarrell, J. Gubernatis, Physics Reports 269 (3) (1996)

  - Bayes theorem: Regularize the naïve χ^2 functional P[D|ρ] through a prior P[ρ|I]

$$P[\rho|D, I] \propto P[D|\rho]\; P[\rho|I]$$

  - New prior enforces: ρ positive definite, smoothness of ρ, result independent of units

$$P[\rho|I] \propto e^S \qquad S = \alpha \sum_{l=1}^{N_\omega} \Delta\omega_l \left(1 - \frac{\rho_l}{m_l} + \log\left[\frac{\rho_l}{m_l}\right]\right)$$

Y.Burnier, A.R. PRL 111 (2013) 18, 182003

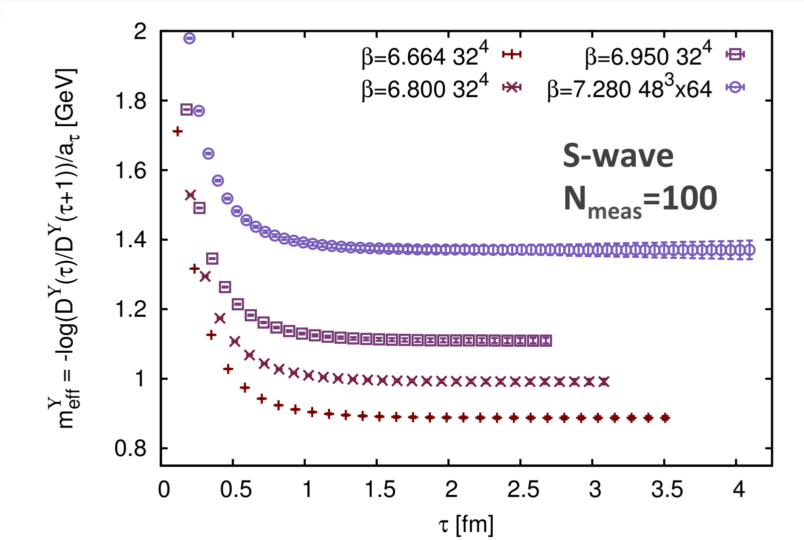
  - **Different from Maximum Entropy Method**: S not entropy, no more flat directions

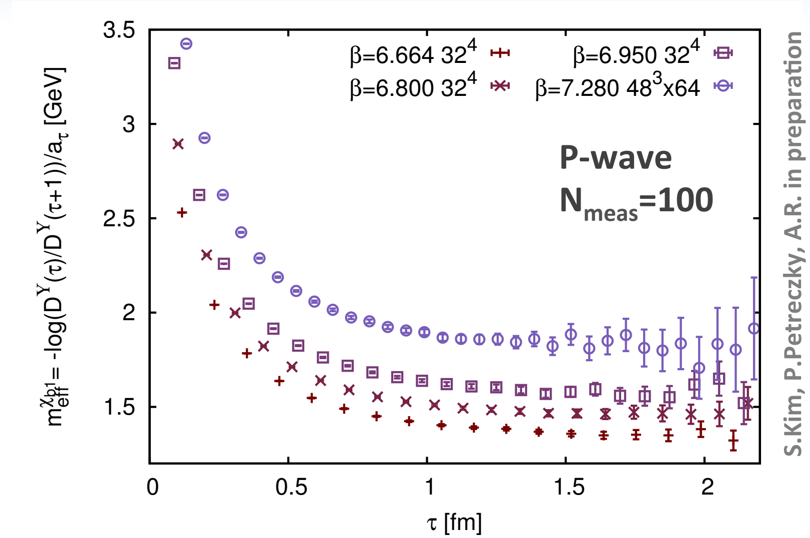
$$\frac{\delta}{\delta\rho} P[\rho|D, I]\Big|_{\rho=\rho^{BR}} = 0$$

  - No apriori restriction on the search space
  - Convergence to unique global extremum

Bottomonium Correlators Close To T=0

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386



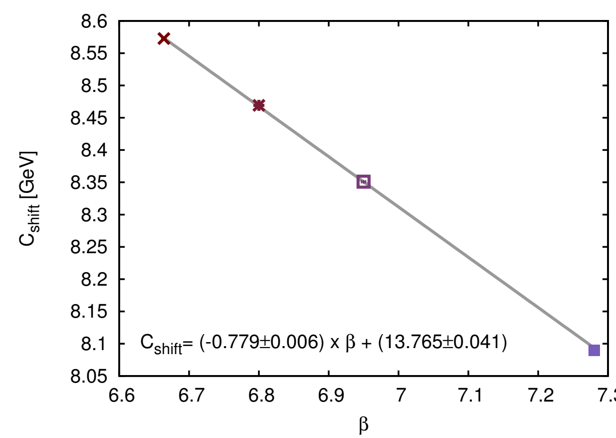


- Set absolute scale by comparison to experiment

$$M^{\text{exp}}_{\Upsilon(1S)} = M^{\text{NRQCD}}_{\Upsilon(1S)} + 2(Z_{M_b} M_b - E_0)$$

$$2(Z_{M_b} M_b - E_0) \rightarrow C_{\text{shift}}(\beta)$$

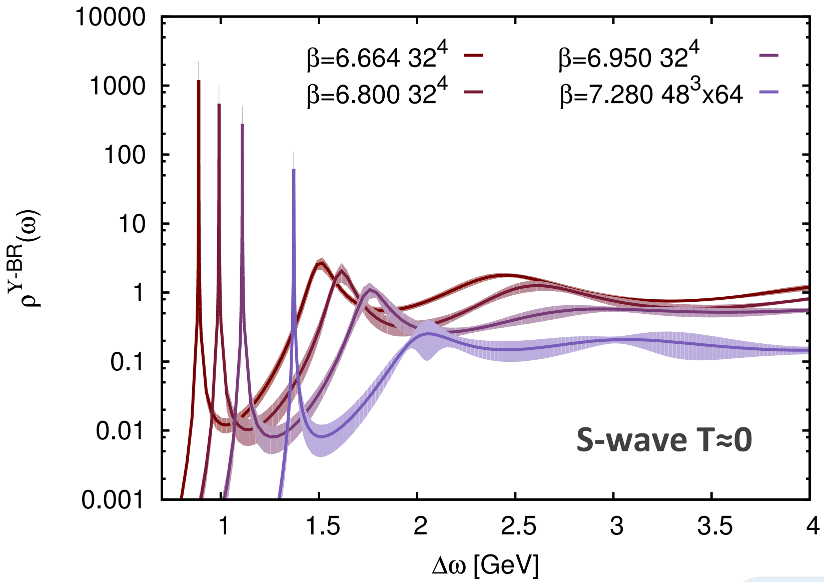
$$M^{\text{exp}}_{\Upsilon(1S)} = 9.46030(26)\,\text{GeV}$$

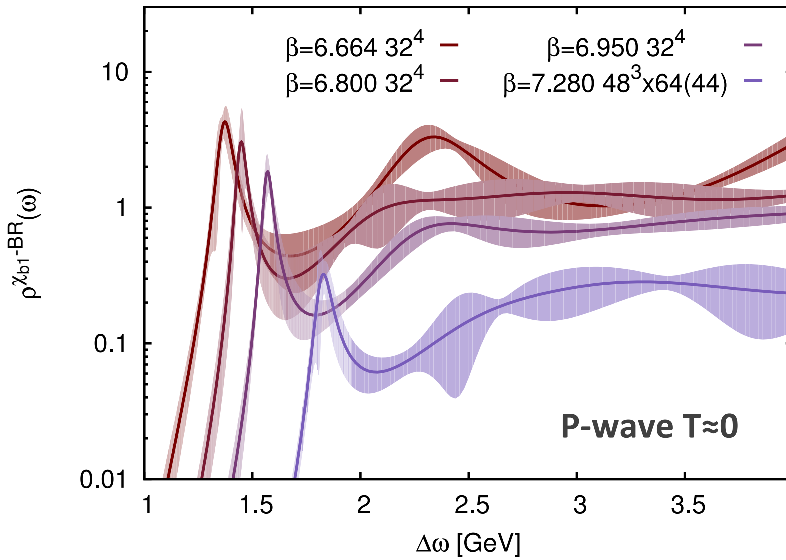


- Linear dependence: interpolated values to calibrate mass shift at intermediate β

Spectral Functions Close To T=0

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386





S.Kim, P.Petreczky, A.R. in preparation

- Bayesian reconstruction:

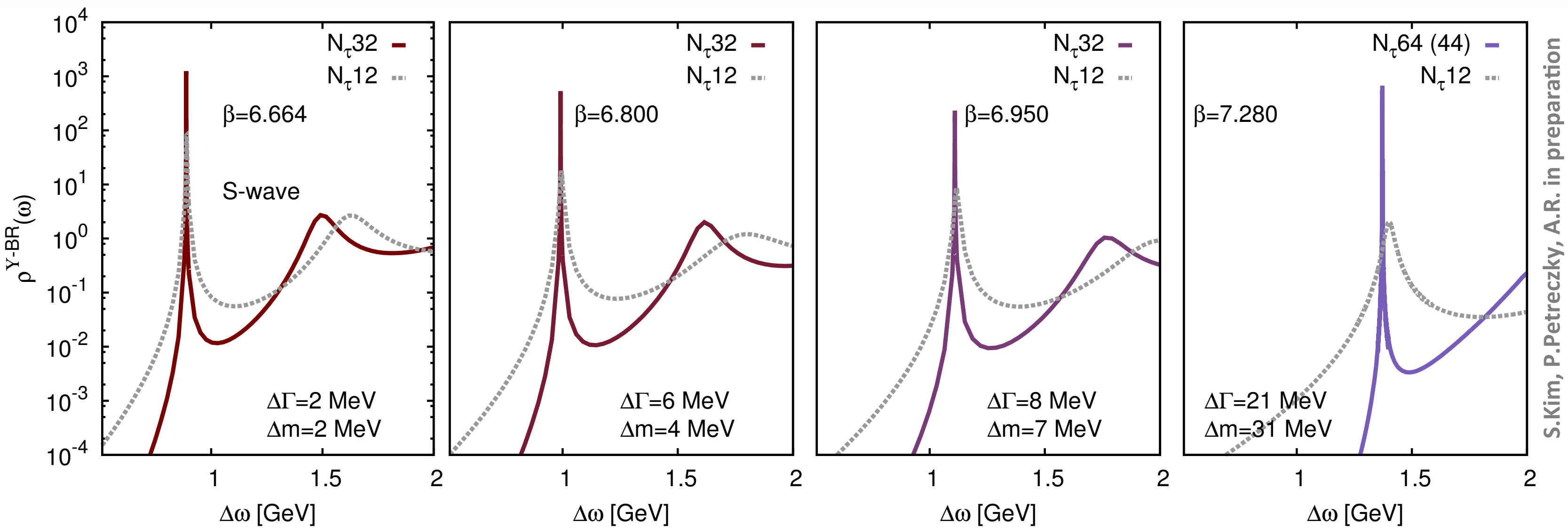
N_ω=1200 I_ω=[-0.5,30] β^{num}=20 N_{jack}=10

m_l=const, 512 bit precision, Δtol= 10^{-60}

- S-wave ground state peak very well resolved, next peak mostly from Υ(2S)
- P-wave ground state broader: worse s/n ratio and smaller physical peak size

$$M_{\chi_{b1}(1P)} = M^{\rm NRQCD}_{\chi_{b1}} + C(\beta) = 9.917(3)\,{\rm GeV} \;>\; M^{\rm exp}_{\chi_{b1}(1P)} = 9.89278(26)(31)\,{\rm GeV}$$

Reconstruction Accuracy: S-wave



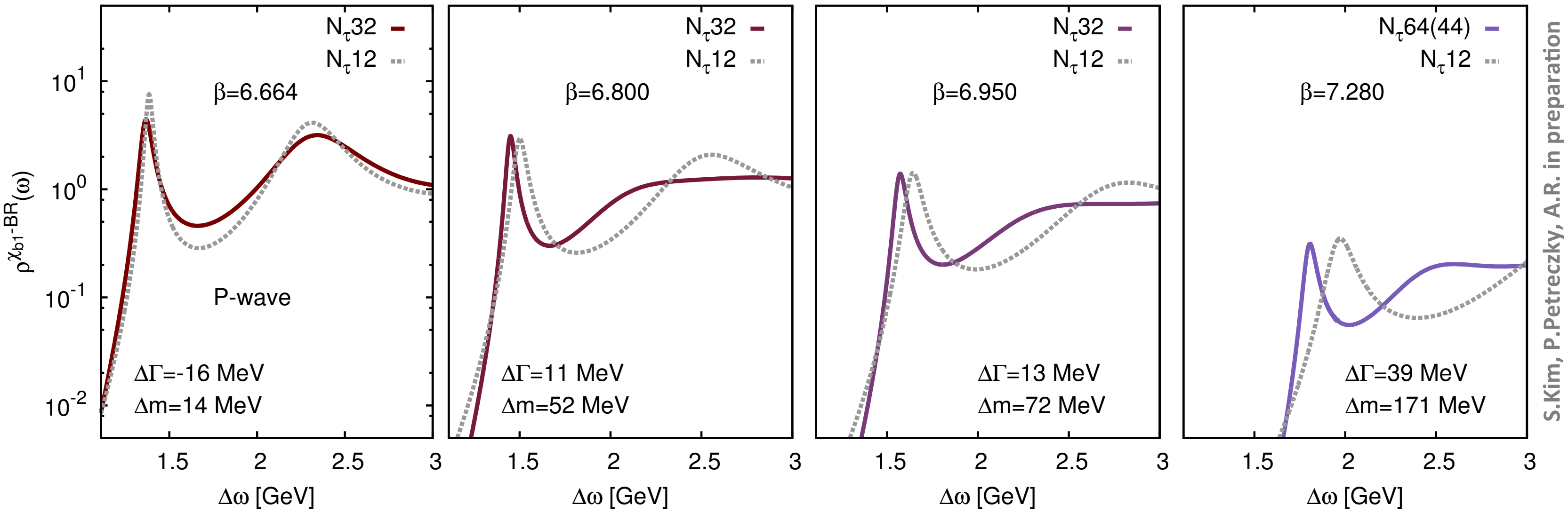
- High precision of the improved Bayesian reconstruction (narrow width resolved)
- How does accuracy suffer from limited available information at T>0 (Nτ=12) ?
- One of the tests we ran: truncate T=0 dataset (Nτ=32/64) to Nτ=12

Overall Limits:

$$\beta = 6.664: \quad \Delta m_T < 2\text{MeV}, \quad \Delta\Gamma_T < 5\text{MeV}$$
$$\beta = 7.280: \quad \Delta m_T < 40\text{MeV}, \quad \Delta\Gamma_T < 21\text{MeV}$$

Reconstruction Accuracy: P-wave

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386



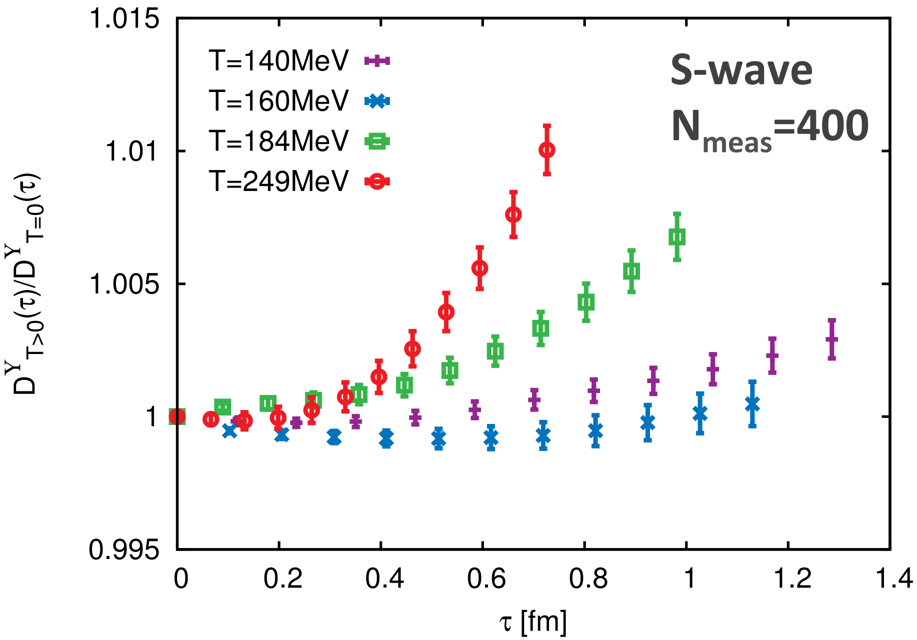
- Estimate systematics: truncate T=0 dataset (Nτ=32/64) to Nτ=12
- Due to a worse signal-to noise ratio, effect in P-wave is larger than for S-wave

Overall Limits:

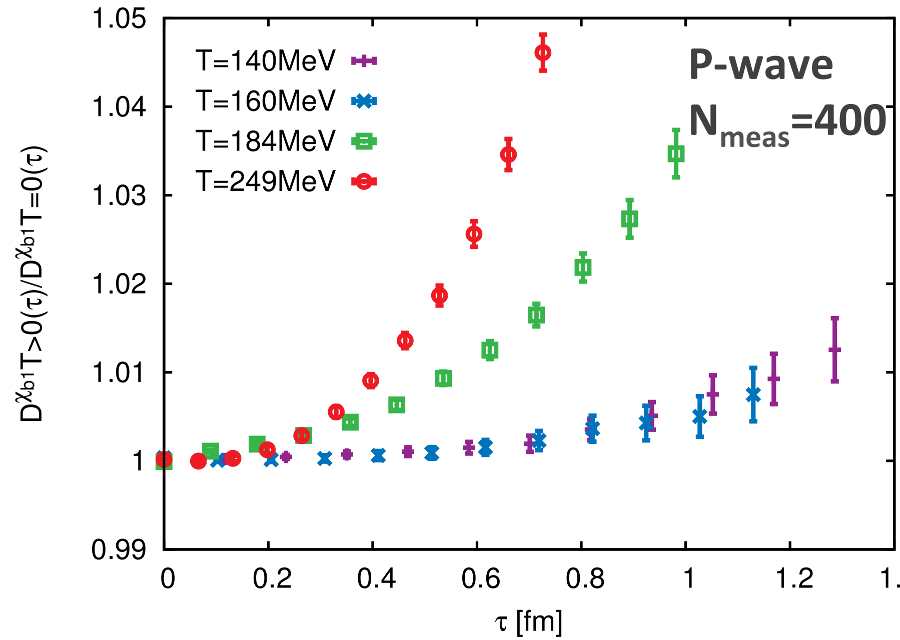
$$\beta = 6.664: \quad \Delta m_T < 60\text{MeV}, \quad \Delta\Gamma_T < 20\text{MeV}$$
$$\beta = 7.280: \quad \Delta m_T < 200\text{MeV}, \quad \Delta\Gamma_T < 40\text{MeV}$$

Bottomonium Correlators At Finite T

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386



S-wave at most 1% change



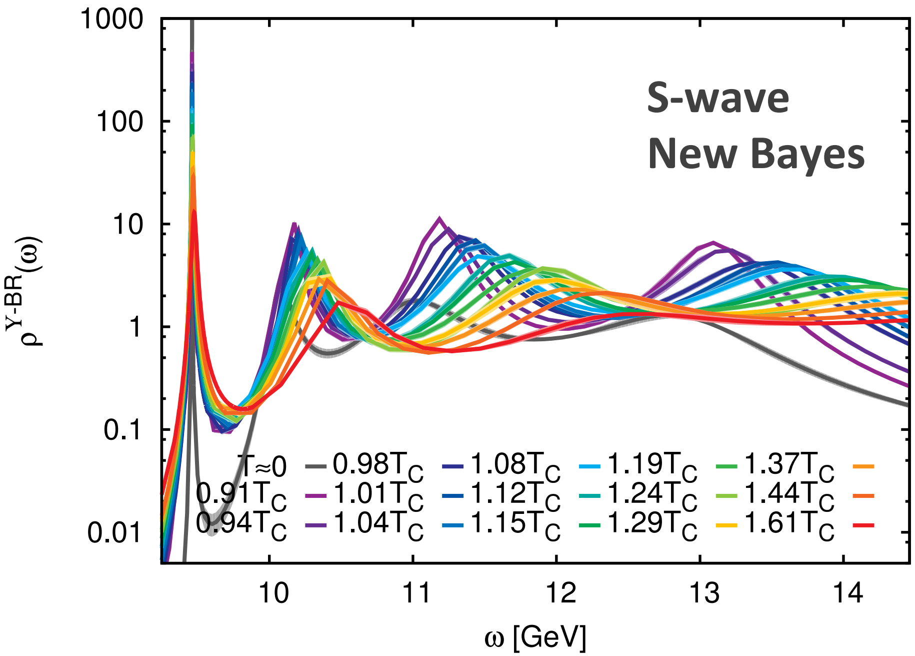
P-wave at most 5% change

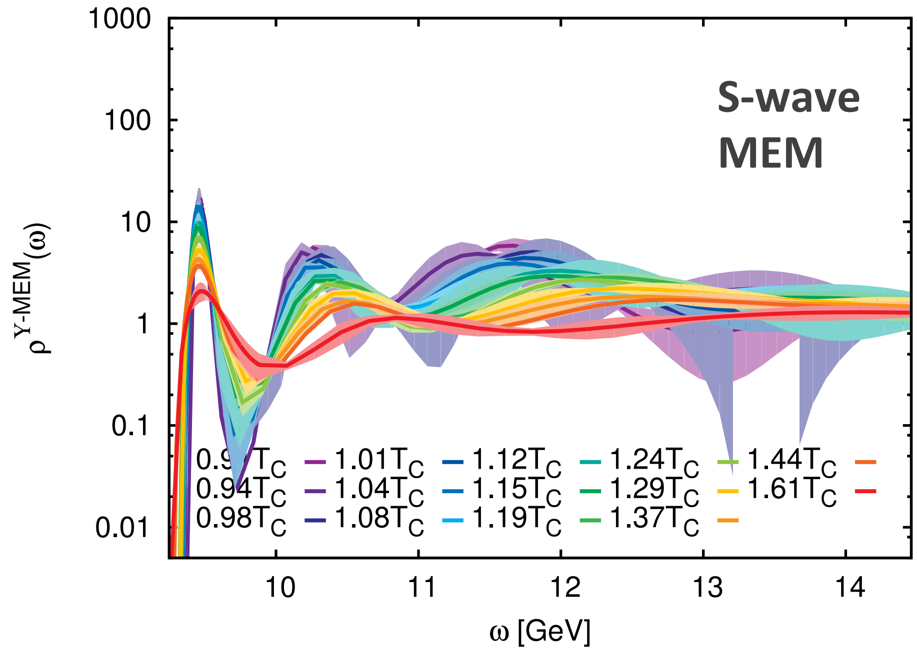
S.Kim, P.Petreczky, A.R. in preparation

- Statistically significant in-medium modification above T=160MeV
- Side remark: similar qualitative and quantitative behavior for η_b and h_b (scalar)

S-wave Spectral Functions At T>0

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386





S.Kim, P.Petreczky, A.R. in preparation

- Bayesian reconstruction:

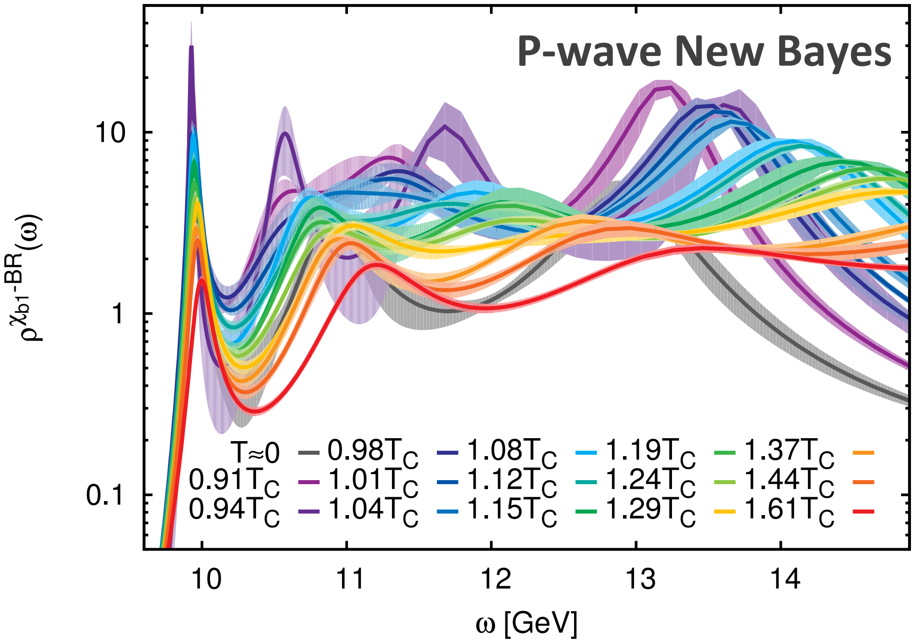
N_ω=1200 I_ω=[-1,25] β^{num}=20 N_{jack}=10

m_l=const 512 bit precision, Δtol=10^{-60}

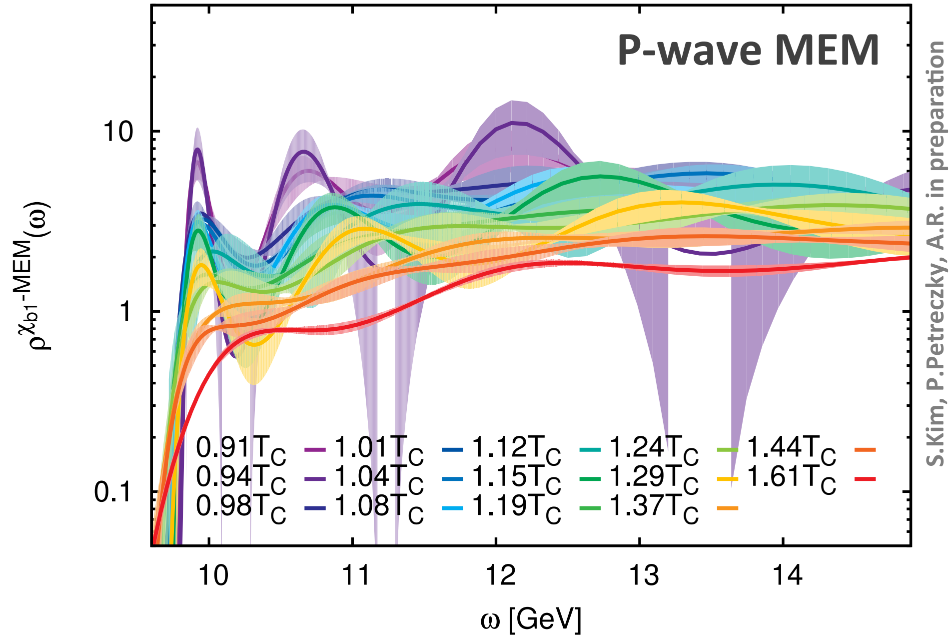
- New Bayesian method resolves peaks much better than MEM
  - observed broadening and peak shifts at finite T smaller than accuracy limits
- Well defined **ground state peak present up to 1.61T_C**

P-wave Spectral Functions At T>0

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386



Ground state peak well defined up to $T=1.61T_C$



Ground state peak disappears for $T>1.29T_C$

- Worse signal to noise ratio leads to larger Jackknife errors than for S-wave
  - observed broadening and peak shifts also smaller than accuracy limits
- New approach finds well defined peak up to highest T investigated 249 MeV

MEM result similar to FASTSUM G. Aarts et. al. JHEP 1407 (2014) 097

How To Verify Survival Of A Bound State?

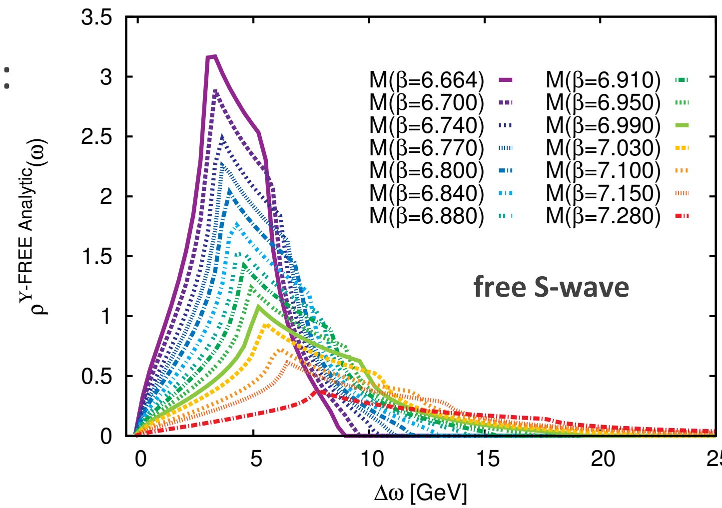
UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- Inspection by eye insufficient: systematic comparison to non-interacting spectra
  - **Analytically**: From free NRQCD dispersion relation:

$$a_\tau E_{\mathbf{p}} = -\log\left(1 - \frac{\mathbf{p}^2_{\mathrm{lat}}}{8 M_b a_s}\right)$$

$$\rho_S(\omega) = \frac{4\pi N_c}{N_s^2}\sum_{\mathbf{p}} \delta(\omega - 2E_{\mathbf{p}}) \quad \rho_P(\omega) = \frac{4\pi N_c}{N_s^2}\sum_{\mathbf{p}} \mathbf{p}^2\delta(\omega - 2E_{\mathbf{p}})$$

G.Aarts et. al., JHEP 1111 (2011) 103



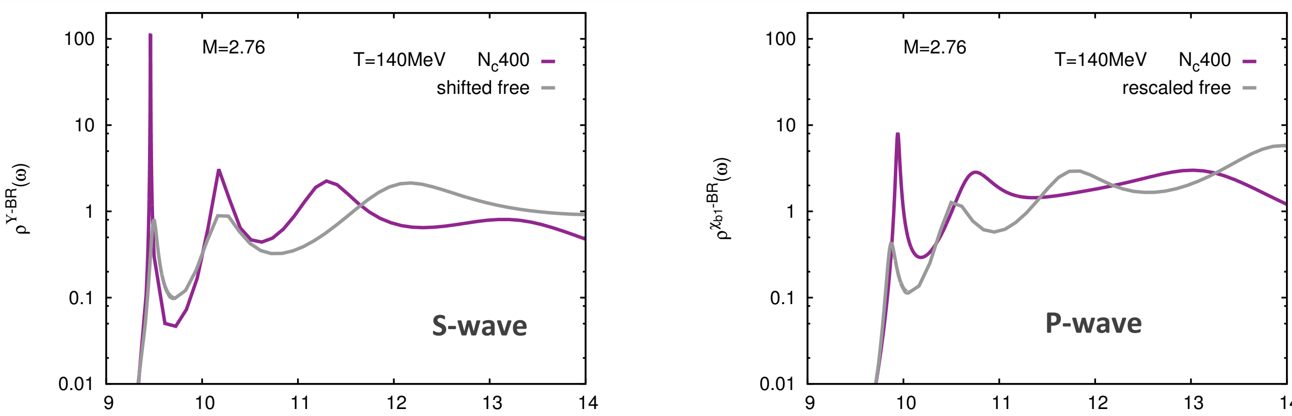
S.Kim, P.Petreczky, A.R. in preparation

  - **Numerically**: Reconstruct from free NRQCD correlator (U_μ=1)

  - Expectation: Presence of peaked features due to numerical **Gibbs ringing**

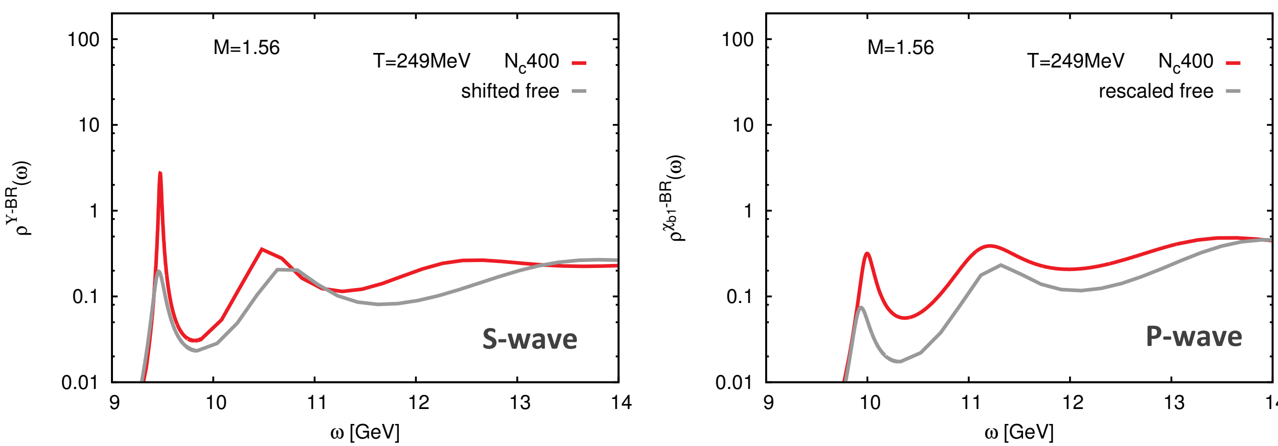
S-wave And P-wave Survival At T=249MeV

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386



S.Kim, P.Petreczky, A.R. in preparation

- At T=140MeV clear difference between ground state peak and numerical ringing



- At T=249 MeV: Ground state peak still stronger than numerical ringing

Conclusion

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

- Lattice NRQCD: efficient non-perturbative treatment of Bottomonium at T>0
- Improved Bayesian approach to spectral function reconstruction is promising
  - Outperforms MEM consistently: higher resolution on same datasets
  - No restricted search space: accuracy suffers from loss of information alone
- On HotQCD lattices with N_f=2+1 light HISQ flavors (48^3x12, T_C=159±3MeV)
  - In-medium modification of correlators above T=160MeV [up to 1% (ϒ) and 5% (χ_{b1})]
  - S-wave and P-wave ground state spectral peak well defined up to 249MeV
  - N_τ=12 datapoints allow us to set upper bounds on in-medium modification
  - A systematic comparison between free and interacting spectra show:

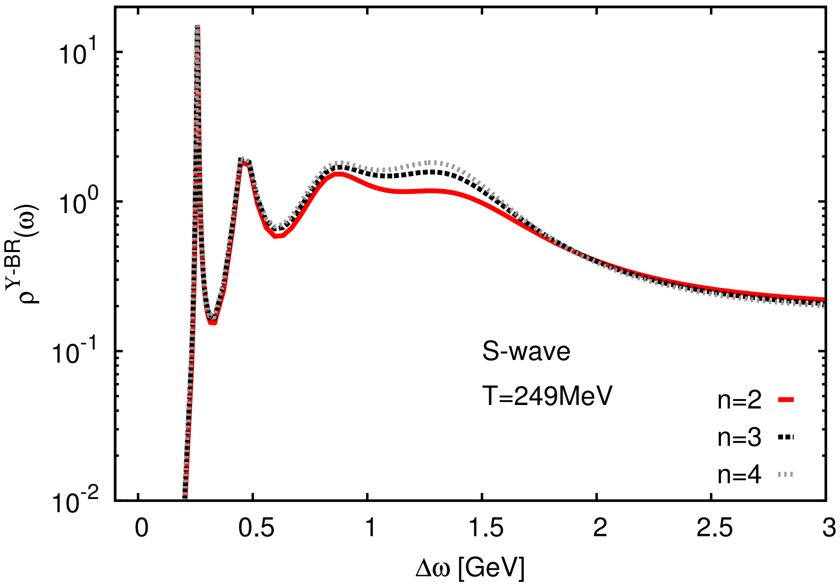
S-wave and P-wave ground state survive up to at least T=249MeV

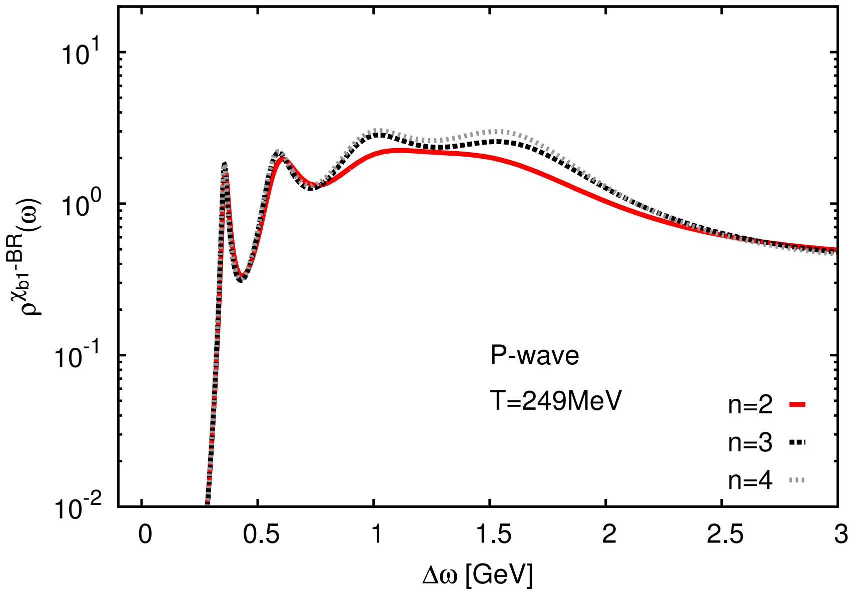
Благодарю вас за внимание - Thank you for your attention

Dependence On The NRQCD Discretization

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- Reduce the effective temporal step size for NRQCD propagator E.O.M.

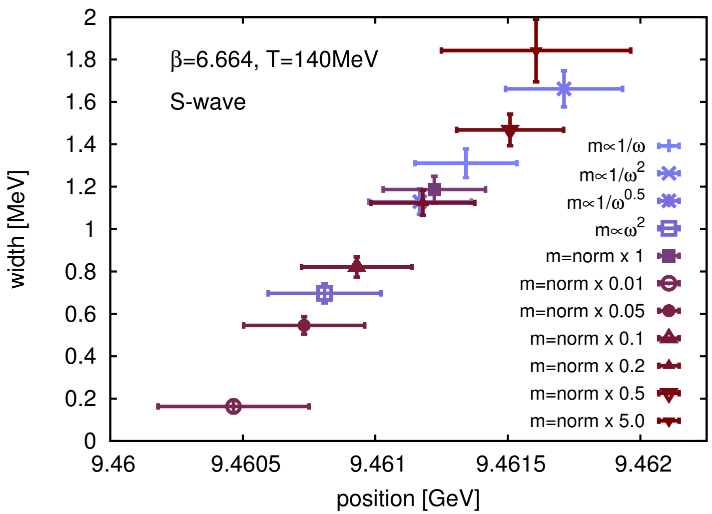


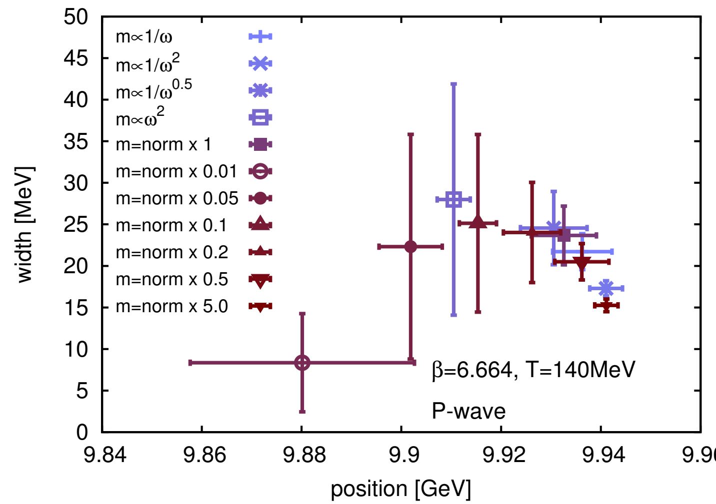


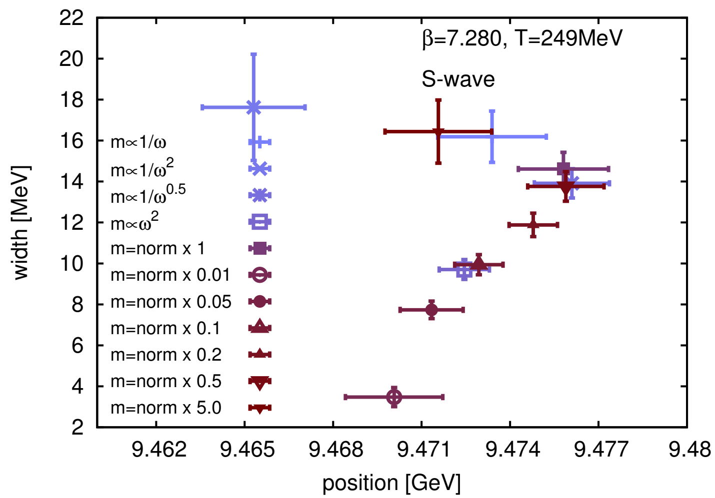
- As expected: high momentum behavior changes but IR unaffected

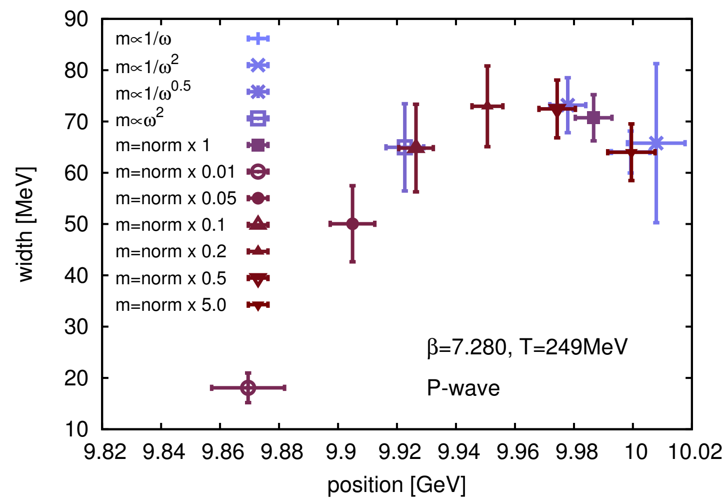
UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

Default Model Dependence









UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

Free Spectra: Default Model Dependence