# Precise lattice calculation of nucleon form factor with all-mode-averaging

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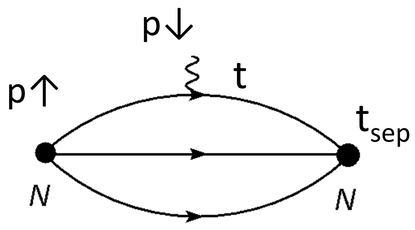
## 1. Introduction and motivation

Lattice computation of nucleon EM form factor ($G_{EM}$) and axial charge ($G_A$) has still 10 times larger uncertainty than experimental result. The main issue is how to control excited state contamination




$$\sim \quad e^{-E_N t - m_N(t_{\rm sep}-t)} \times \left[\{G_X, g_A\} + c_1 e^{-\Delta(t_{\rm sep}-t)} + c_2 e^{-\Delta' t}\right]$$

($\Delta$ and $\Delta'$ denote exited state mass difference.)

At $t_{sep}$ >> 1 and $t_{sep}$-t >>1, excited state is suppressed, but signal-to-noise ratio becomes worse as $\mathrm{S/N}(t) \sim N^{1/2}\exp[-(m_N - 3m_\pi/2)t]$

This study is, to figure out what range of ground state dominance, we demonstrate all-mode-averaging technique (AMA) in form factor computation. By using AMA, statistical error can be reduced to 40% and less.

## 2. All-mode-averaging (AMA)

All-mode-averaging technique [1] is the recent idea to reduce the statistical error of correlation function in Monte-Carlo simulation. The AMA improved estimator is defined as

$$O^{(\rm imp)} = O^{(\rm rest)} + \frac{1}{N_G}\sum_{g\in G} O^{(\rm appx),g},\ O^{(\rm rest)} = O - O^{(\rm appx)}$$

where we compute $N_G$ approximation $O^{(appx)}$ whose cost is much smaller than O. The lattice transformation $g$ of the symmetry $G$ is used for average of $O^{(appx)}$. The statistical error is reduced to

$$\frac{\sigma^{\rm imp}}{\sigma} \simeq \sqrt{\frac{1}{N_G} + 2\Delta r + R^{\rm corr}},\quad R^{\rm corr} = \frac{1}{N_g^2}\sum_{g\neq g'} r_{gg'},\quad \Delta r = 1 - r$$

In this equation, the correlations are important factors,

$$r = \frac{\langle \Delta O \Delta O^{(\rm appx)}\rangle}{\sigma\sigma^{(\rm appx)}}$$ : Correlation between $O^{(appx)}$ and O

$$r_{gg'} = \frac{\langle \Delta O^{(\rm appx),g} \Delta O^{(\rm appx),g'}\rangle}{\sigma^{(\rm appx),g}\sigma^{(\rm appx),g'}}$$ : Correlation between different g

with $\Delta O = O - \langle O\rangle$. For error reduction, we need to search the approximation having small 1−r and $r_{gg'}$. Here we deal with

- G is translational invariance.
- $O^{(appx)}$ is constructed from fixed iter. in SAP + deflation + GCR [2]
- 3-parameters:
  - Domain-size in SAP
  - Number of deflation field
  - Number of GCR iteration

control the quality of approximation and computational cost.

## 3. Lattice setting

Two-flavor dynamical Wilson-clover fermions configurations generated by CLS group is used in this simulation.

| | Lattice | $a$ (fm) | $m_\pi$ (GeV) | $N_G$ | $t_{sep}$ (fm) | #conf | #meas |
|---|---|---|---|---|---|---|---|
| B6 | $96\times48^3$ | 0.079 | 0.267 | 64 | 0.79, 1.11, 1.26 | ~50 | ~3200 |
| | | | | 112 | 1.42 | 48 | 5376 |
| F7 | $96\times48^3$ | 0.063 | 0.277 | 64 | 0.82, 0.95, 1.07 | ~100 | ~6400 |
| | | | | 64 | 1.32 | 198 | 12672 |
| | | | | 128 | 1.51 | 193 | 25344 |
| N6 | $96\times48^3$ | 0.05 | 0.332 | 64 | 0.9 | 17 | 1088 |
| | | | | 32 | 1.1 | 40 | 1280 |
| | | | | 32 | 1.3, 1.5 | ~180 | 5760 |
| G8 | $128\times64^3$ | 0.063 | 0.193 | 64 | 0.88, 1.07 | ~100 | 6400 |
| | | | | 120 | 1.26 | 98 | 7680 |

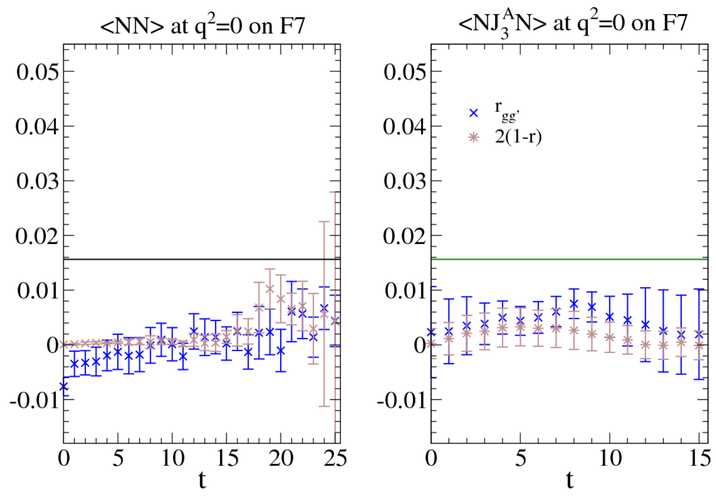
## 4. AMA setting

| | SAP domain size | $N_\lambda$ | GCR iter. (2pt:3pt) | $x_{min}$ (fm) | Speed-up |
|---|---|---|---|---|---|
| B6 | 6x6x6x6 | 40 | 4:3 | 0.95 | 4--5 |
| F7 | 6x6x6x6 | 30 | 4:3 | 0.76 | 6--7 |
| N6 | 6x6x6x6 | 30 | 4:3 | 1.20 | 4--5 |
| G8 | 8x8x8x4 | 40 | 4:3 | 1.01 | 5--6 |

- $N_\lambda$: # of deflation
- $x_{min}$: Distance of different $O^{(appx),g}$
- Speed-up is just for solver part.

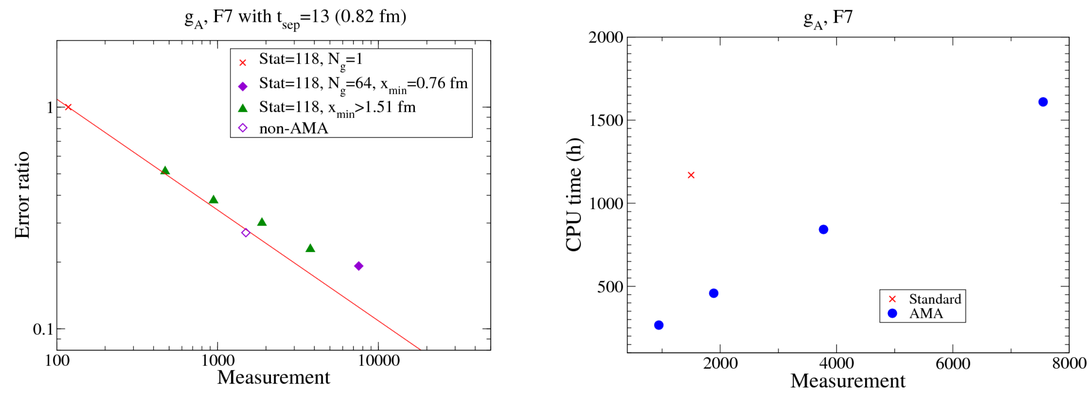
## 5. Performance of AMA

- r and $r_{gg'}$




- 2(1-r) and $r_{gg'}$ are below $1/N_G$ (solid line).
- Correlation is enough small rather than $1/N_G$, and then the statistical error is able to be controlled by $N_G$

- Error scaling and computational cost

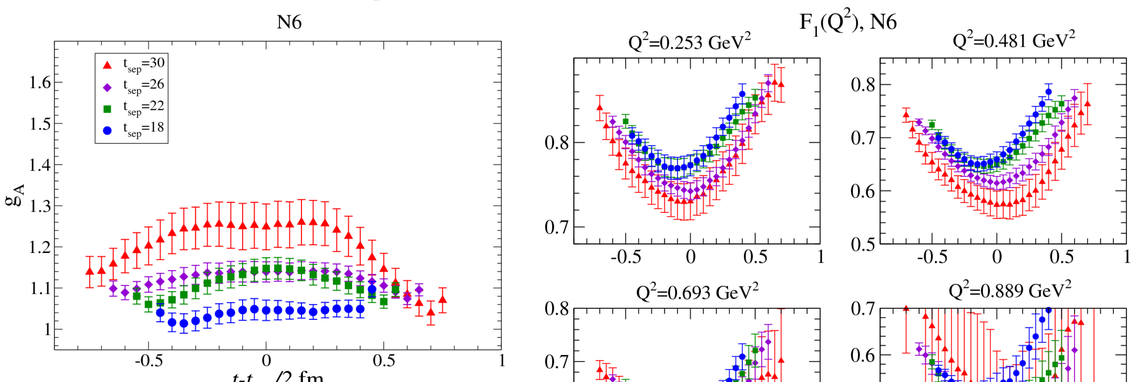



- Error ratio of $g_A$ : $\Delta g_A/g_A$[AMA] / ($\Delta g_A/g_A$[118 stat, w/o AMA])
- Error scaling depends on [meas]$^{1/2}$ if there is no correlation

## 6. Time-slice dependence

The ratio of 3pt and 2pt is to factor out the leading exponent,

$$R(t,t_{\rm sep}) = \frac{\langle N J_\mu \bar{N}\rangle(t,t_{\rm sep}|\vec{p})}{\langle N\bar{N}\rangle_{\rm sm}(t_{\rm sep}|\vec{0})} R_c(t,t_{\rm sep}|\vec{p}) \simeq \{G_X, g_A\} + c_1 e^{-\Delta(t_{\rm sep}-t)} + c_2 e^{-\Delta' t}$$




- Significant $t_{sep}$ dependence appears, due to excited state.

## 7. Comparison of two extraction methods

Plateau method:

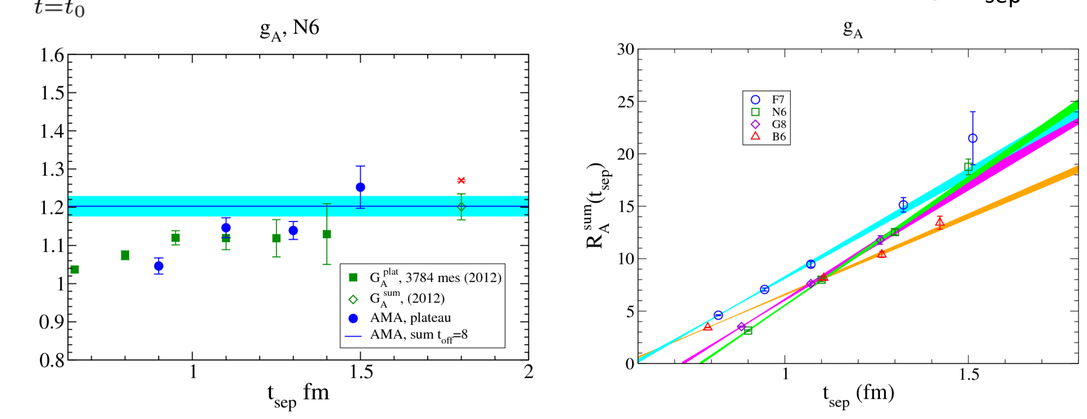
$$R(t,t_{\rm sep}) \simeq \{G_X, g_A\} + \mathcal{O}(e^{-\Delta(t_{\rm sep}-t)}, e^{-\Delta' t})$$

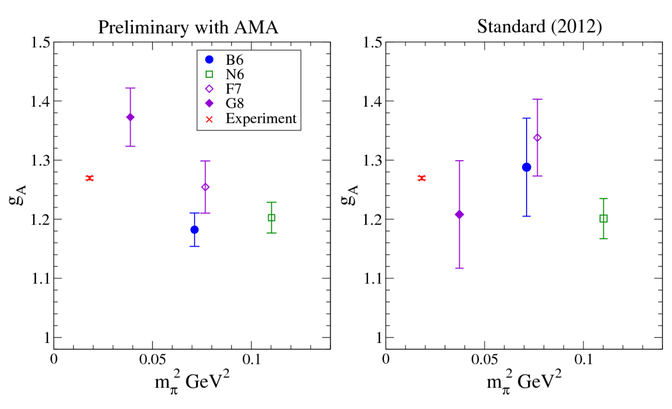
Constant fit in large $t_{sep}$-t and t

Summation method [3,4]:

$$\sum_{t=t_0}^{t_1} R(t,t_{\rm sep}) \simeq {\rm cost} + (t_1 - t_0)\left(\{G_X, g_A\} + \mathcal{O}(e^{-\Delta t_{\rm sep}})\right)$$

Linear fit in large $t_{sep}$




- The excited state contamination is not negligible below $t_{sep}$ =1 fm.
- $t_{sep}$ > 1.3 fm needs to control excited state.
- Statistical error is reduced as expected, but there still needs careful analysis of sys. error.

**References:** [1] T. Blum, T. Izubuchi, E. Shintani, PRD88.094503 (2013), 1402.0244 [hep-lat]. [2] M. Luscher, Comp.Phys.Comm.156,209 (2004). JHEP 07,081(2007). [3] Capitani et al. PRD86, 074502 (2012) [4] B. Jäger, T.D. Rae, et al, 1311.5804v2 [hep-lat]