Combined analysis of the decays $\tau^- \to K_S \pi^- \nu_\tau$ and $\tau^- \to K^- \eta \nu_\tau$

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in collaboration with Rafel Escribano, Matthias Jamin and Pablo Roig
(JHEP 09 (2014) 042 or ArXiv:1407.6590)

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Institut de Física d'Altes Energies IFAE

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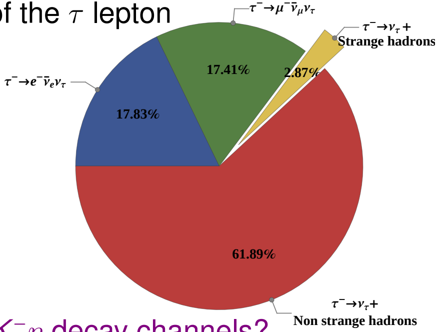
Decay Spectrum of the τ lepton



Why $K_S\pi^-$ and $K^-\eta$ decay channels?

- Sensitive to vector resonances such as $K^{*-}(892)$ and $K^{*-}(1410)$
- Available experimental data from the Belle Collaboration

Why a joint fit if...

- $\tau^- \to K_S\pi^-\nu_\tau$ has already been studied in detail
  (Jamin-Pich-Portolés Phys.Lett. B640 (2006), Boito-Escribano-Jamin JHEP 1009 (2010) 031)
- $\tau^- \to K^-\eta\nu_\tau$ has been analyzed in detail (Escribano-Gonzàlez-Solís-Roig JHEP 10 (2013) 039)

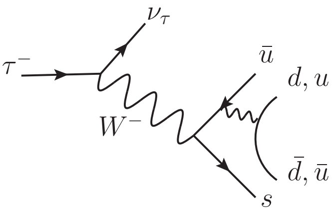
Purpose

To constraint the mass and the width of the $K^{*-}(1410)$ resonance

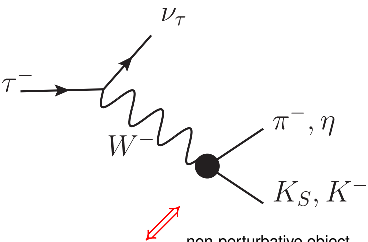
Outline

1. **Hadronic Matrix Element**
2. **Form Factors**
   - Vector Form Factor
   - Scalar Form Factor
3. **Experimental data analysis**
   - $\tau^- \to K_S \pi^- \nu_\tau$
   - $\tau^- \to K^- \eta \nu_\tau$
4. **Fit results**
5. **Conclusions**

@ quark level



@ meson level



$$\mathcal{M} \overset{q^2 << M_W^2}{=} \frac{g^2}{8M_W^2} V_{us} \bar{u}(p_{\nu_\tau})\gamma^\mu(1-\gamma^5)u(p_\tau)\langle P^- P^0|\bar{s}\gamma^\mu(1 - \underbrace{\cancel{\gamma^5}}_{0^-,1^+ \nrightarrow 0^+,1^-})u|0\rangle$$

$P^- P^0 = K_S\pi^-, K^-\eta$

The hadronic matrix element is generally parametrized as

$$\langle P^- P^0|\bar{s}\gamma^\mu u|0\rangle = C^V_{P^-P^0}\left[(p_{P^0} - p_{P^-})^\mu F_+^{P^-P^0}(s) - (p_{P^0} + p_{P^-})^\mu F_-^{P^-P^0}(s)\right]$$

with $C^V_{K_S\pi^-} = -1$ and $C^V_{K^-\eta^{(\prime)}} = -\sqrt{3/2}$.

Taking the divergence we obtain on one hand

$$\langle 0|\partial_\mu(\bar{s}\gamma^\mu u)|P^+P^0\rangle = i(m_s - m_u)\langle 0|\bar{s}u|P^+P^0\rangle = i\Delta_{K\pi} C^S_{P^-P^0} F_0^{P^-P^0}(s) \quad (1)$$

where $\Delta_{K\pi} = M_K^2 - M_\pi^2$, $C^S_{K_S\pi^-} = -1/\sqrt{6}$ and $C^S_{K^-\eta} = \frac{2}{\sqrt{3}}$

and on the other hand we get (where $q_\mu = (p_{P^0} + p_{P^-})_\mu$ and $q^2 = s$)

$$iq_\mu\langle P^-P^0|\bar{s}\gamma^\mu u|0\rangle = iC^V_{P^-P^0}\left[(m^2_{P^0} - m^2_{P^-})F_+^{P^-P^0}(s) - sF_-^{P^-P^0}(s)\right] \quad (2)$$

$\Rightarrow$ Vector current not conserved

Equating Eq.(1) and Eq.(2) allows us to relate $F_-^{P^-P^0}(s)$ with $F_0^{P^-P^0}(s)$ as

$$F_-^{P^-P^0}(s) = -\frac{\Delta_{P^-P^0}}{s}\left[\frac{C^S_{P^-P^0}}{C^V_{P^-P^0}}\frac{\Delta_{K\pi}}{\Delta_{P^-P^0}}F_0^{P^-P^0}(s) + F_+^{P^-P^0}(s)\right] \quad (3)$$

The vectorial hadronic matrix element finally reads

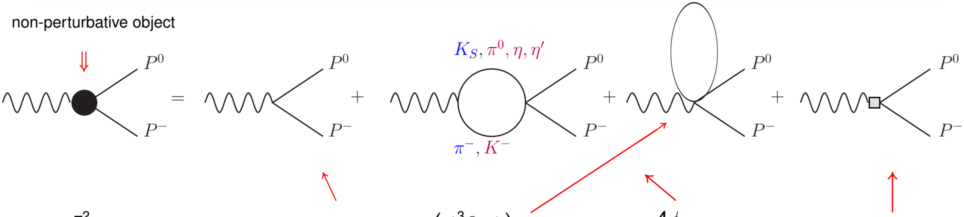
$$\langle P^-P^0|\bar{s}\gamma^\mu u|0\rangle = \left[(p_{P^0} - p_{P^-})^\mu + \frac{\Delta_{P^-P^0}}{s}q^\mu\right]C^V_{P^-P^0}F_+^{P^-P^0}(s) + \frac{\Delta_{K\pi}}{s}q^\mu C^S_{P^-P^0}F_0^{P^-P^0}(s)$$

Advantages of this decomposition:

$F_0^{P^-P^0}(s)$ corresponds to the S-wave projection of the final state, whereas $F_+^{P^-P^0}(s)$ is the P-wave component.

Chiral Perturbation Theory

non-perturbative object



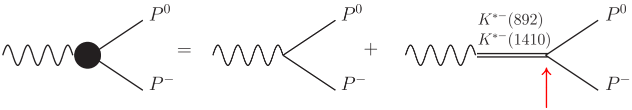
$$J_V^\mu = i\frac{F_\pi^2}{2}\left(D_\mu U^\dagger U + D_\mu U U^\dagger\right) = -i\phi D_\mu \phi + \mathcal{O}\left(\frac{\phi^3\partial_\mu\phi}{F_\pi^2}\right) + \ldots, \qquad \mathcal{L}_{p^2}^{4\phi} \qquad \phi^r, \mathcal{L}_{p^4}$$

$$U = \exp(i\sqrt{2}\phi/F_\pi) \qquad \phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 \end{pmatrix}$$

$$F_+^{K_S\pi^-}(s) = 1 + \frac{2}{F_\pi^2}L_9^r(\mu) + \frac{3}{2}\left(\widetilde{H}_{K\pi}(s) + \cos\theta\widetilde{H}_{K\eta}(s) + \sin\theta\widetilde{H}_{K\eta'}(s)\right)$$

$$F_+^{K^-\eta}(s) = \cos\theta F_+^{K_S\pi^-}(s)$$

Resonance Chiral Theory



$$\mathcal{L}_V = i\frac{G_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle = i\frac{G_V}{\sqrt{2}F_\pi^2}\langle V_{\mu\nu}[(\partial^\mu\phi)(\partial^\nu\phi) - (\partial^\nu\phi)(\partial^\mu\phi)]\rangle$$

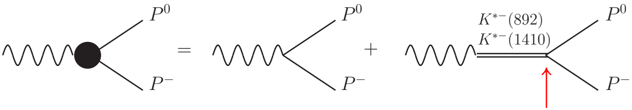
$$u^\mu = iu^\dagger D^\mu U u^\dagger$$

$$u^2 = U$$

$$V_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 & \rho^+ & K^{\star+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 & K^{\star 0} \\ K^{\star-} & \bar{K}^{\star 0} & -\frac{2}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 \end{pmatrix}$$

$$F_+^{K_S\pi^-}(s) = 1 + \frac{F_V G_V}{F_\pi^2}\frac{s}{M_{K^\star}^2 - s} + \frac{F_V' G_V'}{F_\pi^2}\frac{s}{M_{K^{\star\prime}}^2 - s} \qquad F_+^{K^-\eta}(s) = \cos\theta\left(1 + \frac{F_V G_V}{F_\pi^2}\frac{s}{M_{K^\star}^2 - s} + \frac{F_V' G_V'}{F_\pi^2}\frac{s}{M_{K^{\star\prime}}^2 - s}\right)$$

Resonance Chiral Theory



$$\mathcal{L}_V = i\frac{G_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle = i\frac{G_V}{\sqrt{2}F_\pi^2}\langle V_{\mu\nu}[(\partial^\mu\phi)(\partial^\nu\phi) - (\partial^\nu\phi)(\partial^\mu\phi)]\rangle$$

$$u^\mu = iu^\dagger D^\mu U u^\dagger \qquad u^2 = U$$

$$V_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 & \rho^+ & K^{\star +} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 & K^{\star 0} \\ K^{\star -} & \bar{K}^{\star 0} & -\frac{2}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 \end{pmatrix}$$

$$F_+^{K_S\pi^-}(s) = 1 + \frac{F_V G_V}{F_\pi^2}\frac{s}{M_{K^\star}^2 - s} + \frac{F_V' G_V'}{F_\pi^2}\frac{s}{M_{K^{\star\prime}}^2 - s} \qquad F_+^{K^-\eta}(s) = \cos\theta F_+^{K_S\pi^-}(s)$$

Requirement: $F_+^{K\pi}(s)$ vanish for $s \to \infty \Rightarrow F_V G_V + F_V' G_V' = F_\pi^2$

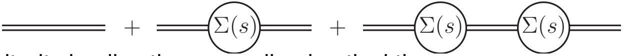
$$F_+^{K\pi}(s) = \frac{M_{K^\star}^2 + \gamma s}{M_{K^\star}^2 - s} - \frac{\gamma s}{M_{K^{\star\prime}}^2 - s} \qquad \gamma = -\frac{F_V' G_V'}{F_\pi^2} = \frac{F_V G_V}{F_\pi^2} - 1$$

Treatment of unstable particles (resonances) and final state interactions

$$F_+^{K\pi}(s) = \frac{M_{K^\star}^2 + \gamma s}{M_{K^\star}^2 - s} - \frac{\gamma s}{M_{K^{\star\prime}}^2 - s}$$

- **Limitation**: Breaks down when $s = M^2_{K^{\star(\prime)}}$ (resonance on-shell)
- **Remedy**: To resumme self-energy insertions in the propagator

The resonance propagator to all-orders can be expressed as



$$+ \ldots = \frac{i}{s - M^2_{K^\star} + \Sigma(s)}$$

Unitarity implies the generalized optical theorem

$$\underbrace{\mathcal{M}(i \to f) - \mathcal{M}^\dagger(f \to i)}_{loop-level} = -i\Sigma_X \int \Pi^X_{LIPS}(2\pi)^4 \delta^4(p_i - p_X) \underbrace{\mathcal{M}(i \to X)\mathcal{M}^\dagger(X \to f)}_{tree-level}$$

$$\mathrm{Im}\mathcal{M}(A \to A) = iM_A \Sigma_X \Gamma(A \to X); \quad (\mathcal{M} \sim \Sigma(s) = H_{K\pi}(s)|_{ChPT})$$

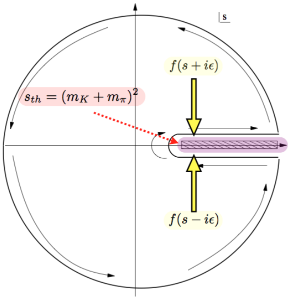
Breit-Wigner-like parameterization:

$$F_+^{K\pi}(s) = \frac{M_{K^\star}^2 + \gamma s}{M_{K^\star}^2 - s - iM_{K^\star}\Gamma^{K\pi}_{K^\star}(s)} - \frac{\gamma s}{M_{K^{\star\prime}}^2 - s - iM_{K^{\star\prime}}\Gamma^{K\pi}_{K^{\star\prime}}(s)}; \quad \star \ \mathrm{Re}\widetilde{H}_{K\pi}(s) \text{ neglected}$$

$$\Gamma^{K\pi}_{K^{\star(\prime)}}(s) = \frac{2}{3}\Gamma^{K^0\pi^-} + \frac{1}{3}\Gamma^{K^-\pi^0}; \quad \Gamma^{P^0P^-}_{K^{\star(\prime)}}(s) = \Gamma_{K^{\star(\prime)}} \frac{s}{M^2_{K^{\star(\prime)}}} \frac{\sigma^3_{P^0P^-}(s)}{\sigma^3_{P^0P^-}(M^2_{K^{\star(\prime)}})}$$

Dispersive representation of the FF (Boito-Escribano-Jamin JHEP 1009 (2010) 031)

$$F_+^{K\pi}(s) = \frac{M_{K^\star}^2 + \gamma s}{M_{K^\star}^2 - s - \frac{3}{2} M_{K^\star}^2 \mathrm{Re}\widetilde{H}_{K\pi}(s) - i M_{K^\star}\Gamma_{K^\star}^{K\pi}(s)} - \frac{\gamma s}{M_{K^{\star\prime}}^2 - s - \frac{3}{2} M_{K^\star}^2 \mathrm{Re}\widetilde{H}_{K\pi}(s) - i M_{K^{\star\prime}}\Gamma_{K^{\star\prime}}^{K\pi}(s)}$$



Analyticity through dispersion relation

$$F_+^{K\pi}(s) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im}F(s')}{s'(s'-s-i\epsilon)}$$

$\Downarrow$ *Elastic unitarity+Watson's theorem*

$$\mathrm{Im}F(s') = |F(s')|\sin\delta_1^{1/2}(s') = \tan\delta_1^{1/2}(s')\mathrm{Re}F(s')$$

$\Downarrow$ *Omnès solution*

$$F_+^{K\pi}(s) = P(s)\exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\delta_1^{1/2}(s')}{s'(s'-s-i0)}\right]$$

$$\widetilde{f}_+^{K\pi}(s) = \exp\left[\alpha_1 \frac{s}{m_{\pi^-}^2} + \frac{1}{2}\alpha_2 \frac{s^2}{m_{\pi^-}^4} + \frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{cut}} ds' \frac{\delta(s')}{(s')^3(s'-s-i0)}\right]; \quad \delta(s) = \tan^{-1}\left[\frac{\mathrm{Im}F_+(s)}{\mathrm{Re}F_+(s)}\right]$$

Comments:

- Three-times subtracted dispersion relations → helps the convergence of the form factor
- The higher-energy region of the FF (which is less know) is suppressed

Dispersive representation

$$\widetilde{f}_+^{K\pi}(s) = \exp\left[\alpha_1 \frac{s}{m_{\pi^-}^2} + \frac{1}{2}\alpha_2 \frac{s^2}{m_{\pi^-}^4} + \frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{cut}} ds' \frac{\delta(s')}{(s')^3(s'-s-i0)}\right]$$

$$\alpha_1 = \lambda'_+ \quad \alpha_2 + \alpha_1^2 = \lambda''_+$$

λ'_+ and λ''_+ are important parameters for describing the the $K\pi$ form factor at low energies

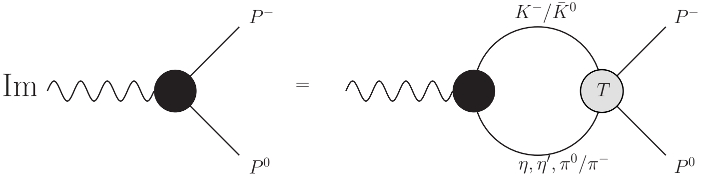
e.g. $K_{\ell 3}$ decay is typically described by (in the regime $m_\ell < t < (m_K - m_\pi) << M_{K^\star}^2$, the FF is real)

$$f_+^{K\pi}(t) = \frac{M_{K^\star}^2}{M_{K^\star}^2 - t} \xrightarrow{\text{Taylor expansion}} f_+^{K\pi}(t) = 1 + \frac{\lambda'_+}{M_{\pi^-}^2} t + \frac{1}{2}\frac{\lambda''_+}{M_{\pi^-}^4} t^2$$

Both λ'_+ and λ''_+ will be fitted as well

Scalar form factor through dispersion relation

$$F_0^{P^-P^0}(s) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im}F(s')}{s'(s'-s-i\epsilon)}$$

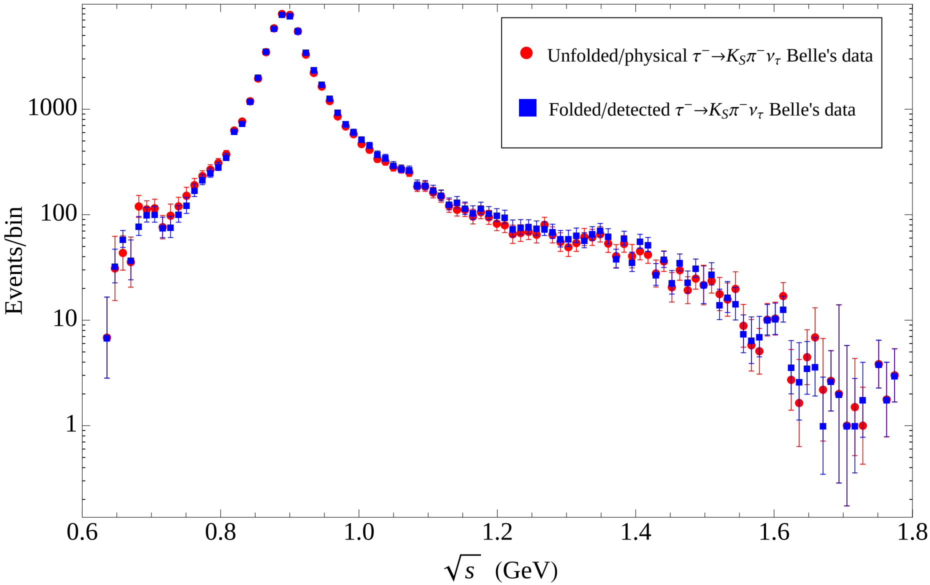


$$F_0^i(s) = \frac{1}{\pi}\sum_{j=1}^{3}\int_{s_i}^{\infty} ds' \frac{\sigma_j(s')F_0^j(s')T_0^{i\to j}(s')^{\star}}{(s'-s-i0)}$$

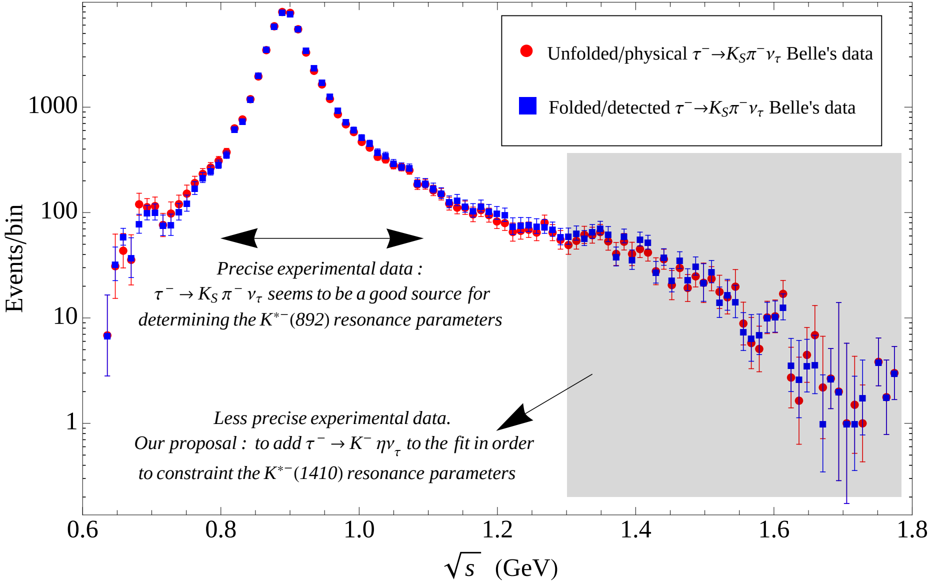
Analytic and Unitary ✓

(Jamin-Oller-Pich: Nucl.Phys. B622 (2002))

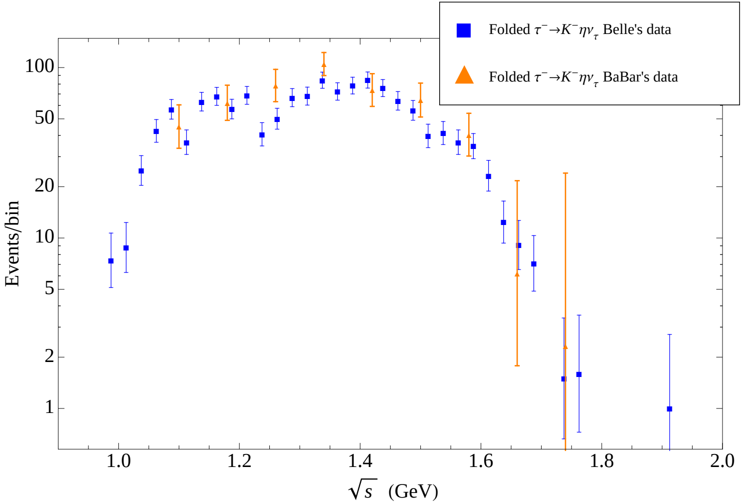
$\tau^- \to K_S \pi^- \nu_\tau$ Belle's data Phys. Lett. **B 654 (2007) 65** [arXiv:0706.2231]



$\tau^- \to K_S \pi^- \nu_\tau$ Belle's data Phys. Lett. **B 654 (2007) 65** [arXiv:0706.2231]

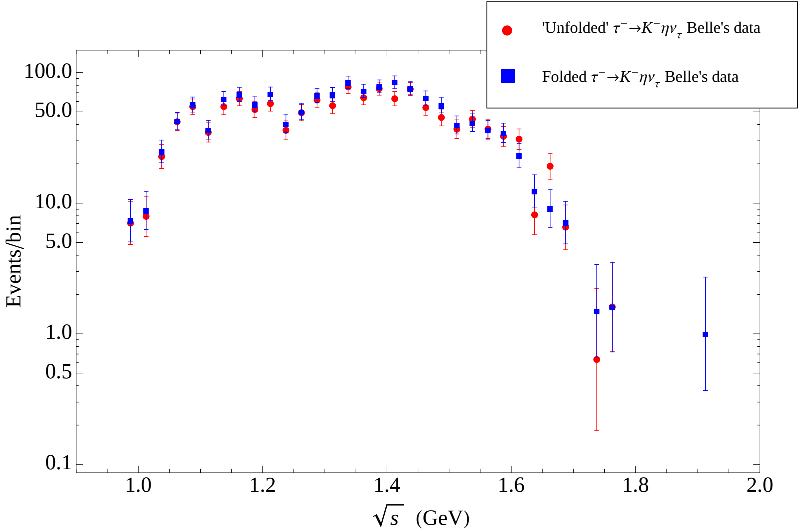


$\tau^- \to K^- \eta \nu_\tau$ Belle's data Phys. Lett. **B 672 (2009) 109** [arXiv:0811.0088]



No unfolded/physical $\tau^- \to K^- \eta \nu_\tau$ data available $\Rightarrow$ To 'unfold' $\tau^- \to K^- \eta \nu_\tau$ data

Unfolding $\tau^- \to K^- \eta \nu_\tau$ Belle's data through an "unfolding" function from $\tau^- \to K_S \pi^- \nu_\tau$



- Experimentalist: To provide unfolded data would be really useful ☺
- Theorists: To provide theoretical models to be fitted by experimentalists

We relate the experimental data with the differential decay distribution from theory through

$$\frac{dN_{events}}{d\sqrt{s}} = N_{events}\Delta_{bin}\frac{1}{\Gamma_\tau BR(\tau \to P^- P^0 \nu_\tau)}\frac{d\Gamma\left(\tau^- \to P^- P^0 \nu_\tau\right)}{d\sqrt{s}} \quad (4)$$

$$\frac{d\Gamma\left(\tau^- \to P^- P^0 \nu_\tau\right)}{d\sqrt{s}} = \frac{G_F^2 M_\tau^3}{32\pi^3 s} S_{EW} |V_{us} F_+^{P^- P^0}(0)|^2 \left(1 - \frac{s}{M_\tau^2}\right)^2$$
$$\times \left\{\left(1 + \frac{2s}{M_\tau^2}\right) q^3_{P^- P^0}(s) |\widetilde{F}_+^{P^- P^0}(s)|^2 + \frac{3\Delta^2_{P^- P^0}}{4s} q_{P^- P^0}(s) |\widetilde{F}_0^{P^- P^0}(s)|^2\right\}$$

where

$$q_{P^- P^0}(s) = \frac{\sqrt{s^2 - 2s\Sigma_{P^- P^0} + \Delta^2_{P^- P^0}}}{2\sqrt{s}}, \quad \Sigma_{P^- P^0} = m^2_{P^-} + m^2_{P^0}, \quad \Delta_{P^- P^0} = m^2_{P^-} - m^2_{P^0}$$

and

$$\widetilde{F}_{+,0}^{P^- P^0}(s) = \frac{F_{+,0}^{P^- P^0}(s)}{F_{+,0}^{P^- P^0}(0)}$$

with $\theta = (-13.3 \pm 1.0)^\circ$ being the $\eta - \eta'$ mixing angle and $V_{us} \cdot F_+^{K^- \pi^0}(0) = 0.2163 \pm 0.0005$ from $K_{l3}^{0,\pm}$

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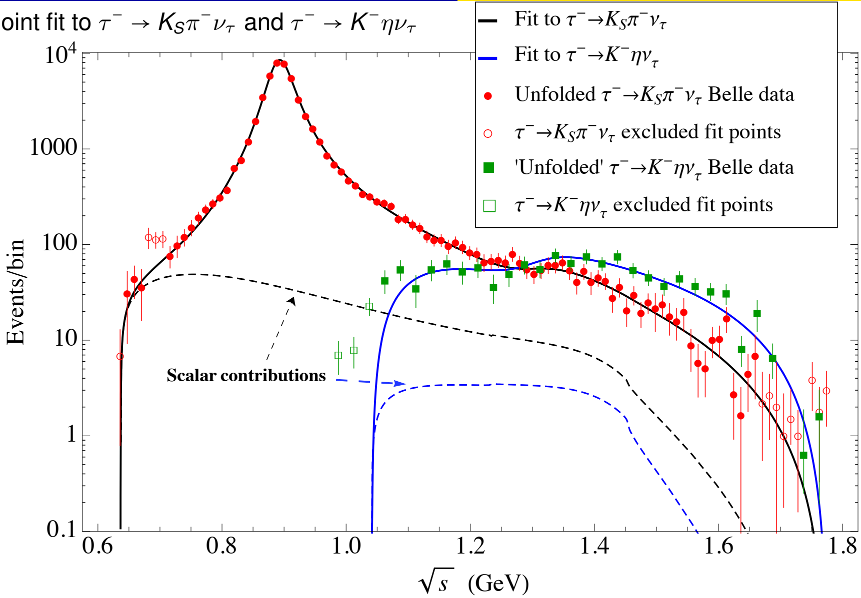
$$\times \left\{ \left(1 + \frac{2s}{M_\tau^2}\right) q^3_{P^- P^0}(s) |\widetilde{F}_+^{P^- P^0}(s)|^2 + \frac{3\Delta^2_{P^- P^0}}{4s} q_{P^- P^0}(s) |\widetilde{F}_0^{P^- P^0}(s)|^2 \right\}$$

- $P^- P^0 = K_S \pi^- \to BR^{Belle}_{exp} = 0.404\%$ $N_{events} = 53113$ $\Delta_{bin} = 0.0115$ GeV/bin
- $P^- P^0 = K^- \eta \to BR^{Belle}_{exp} = 1.58 \cdot 10^{-4}$ $N_{events} = 1271$ $\Delta_{bin} = 0.025$ GeV/bin
- $\Gamma_\tau = 2.265 \cdot 10^{-12}$
- Function minimised in our fit

$$\chi^2 = \sum_{bin}\left(\frac{\mathcal{N}^{th} - \mathcal{N}^{exp}}{\sigma_{\mathcal{N}^{exp}}}\right)^2 + \sum_{K_S\pi^-, K^-\eta}\left(\frac{\bar{B}^{th} - \bar{B}^{exp}}{\sigma_{\bar{B}^{exp}}}\right)^2$$

Joint fit to $\tau^- \to K_S \pi^- \nu_\tau$ and $\tau^- \to K^- \eta \nu_\tau$



How to determine the physical parameters of the resonances?

To look for the zero's of the denominator of the propagator in the complex plane through $s_{\rm pole} = (M_{\rm phys} - \frac{\rm i}{2}\Gamma_{\rm phys})^2$

$$M_{K^\star}^2 - s_{\rm pole} - \frac{3}{2}M_{K^\star}^2 {\rm Re}\widetilde{H}_{K\pi}(s) - iM_{K^\star}\Gamma_{K^\star}(s) = 0,$$

where $M_{K^\star}$ and $\Gamma_{K^\star}$ are model parameters

Obtained parameters from a joint fit to $\tau^- \to K_S \pi^- \nu_\tau$ and $\tau^- \to K^- \eta \nu_\tau$

$M_{K^{*-}(892)} = 892.03 \pm 0.19$ MeV
$\Gamma_{K^{*-}(892)} = 46.18 \pm 0.44$ MeV
} no gain

$M_{K^{*-}(1410)} = 1304 \pm 17$ MeV
$\Gamma_{K^{*-}(1410)} = 171 \pm 62$ MeV
} improvement

$\gamma_{K\pi} = \gamma_{K\eta} = -3.4^{+1.2}_{-1.4} \cdot 10^{-2}$

$\bar{B}_{K\pi} = (0.0404 \pm 0.012)\%$

$\bar{B}_{K\eta} = (1.58 \pm 0.10) \cdot 10^{-4}$

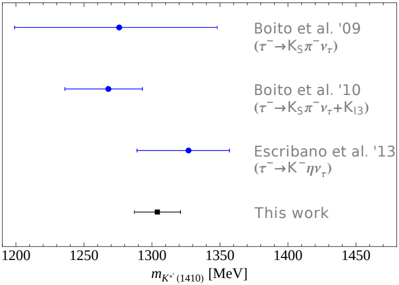
$\lambda'_{K\pi} = (23.9 \pm 0.9) \cdot 10^{-3}$
$\lambda'_{K\eta} = (20.9 \pm 2.7) \cdot 10^{-3}$
} isospin violation?

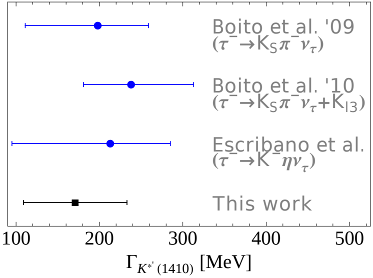
$\lambda''_{K\pi} = (11.8 \pm 0.2) \cdot 10^{-4}$
$\lambda''_{K\eta} = (11.1 \pm 0.5) \cdot 10^{-4}$
} isospin violation?

$\Uparrow$

$\tau^- \to K^- \pi^0 \nu_\tau \& K_{\ell 3}$

$\chi^2/d.o.f = 108.1/105 = 1.03$





Conclusions

- A good description of the vector form factor (by analyticity+unitarity arguments) is crucial to unveil the parameters of the intermediate resonances which drive the decays
- Fitting both decay spectra together we have considerable improved the determination of the $K^{*-}(1410)$ mass while we slightly reduced the uncertainty of the width
- Call for (an unfolded) analysis of $\tau^- \to K^-\pi^0\nu_\tau$ for unveiling possible isospin violations on the low-energy parameters $\lambda^{\prime('')}$
- Agreement between theory and Belle's data

Extra slides

- The ChPT loop functions $\widetilde{H}_{PQ}$ from page 6 can be found in Gasser-Leutwyler Annals Phys. **158** (1984) 142