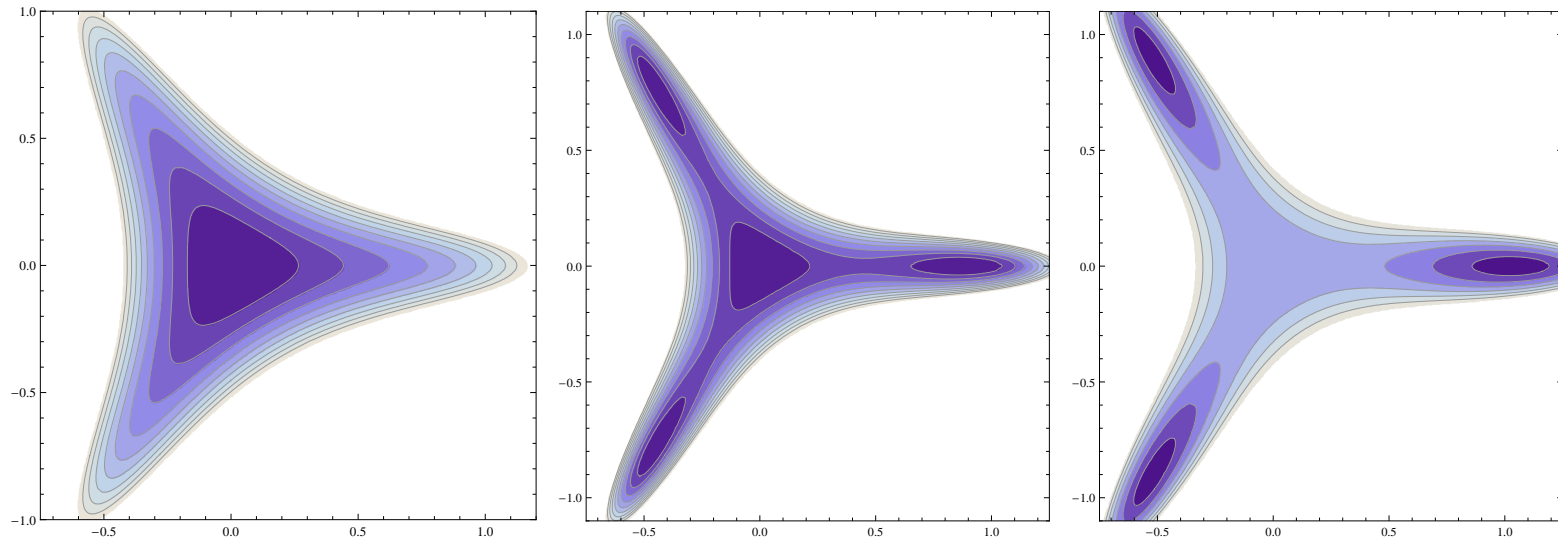


Continuity of the Deconfinement Transition in (Super) Yang Mills Theory

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with Mithat Ünsal and Erich Poppitz

arXiv:1205.0290 & arxiv:1212.1238

Dedicated to the memory of



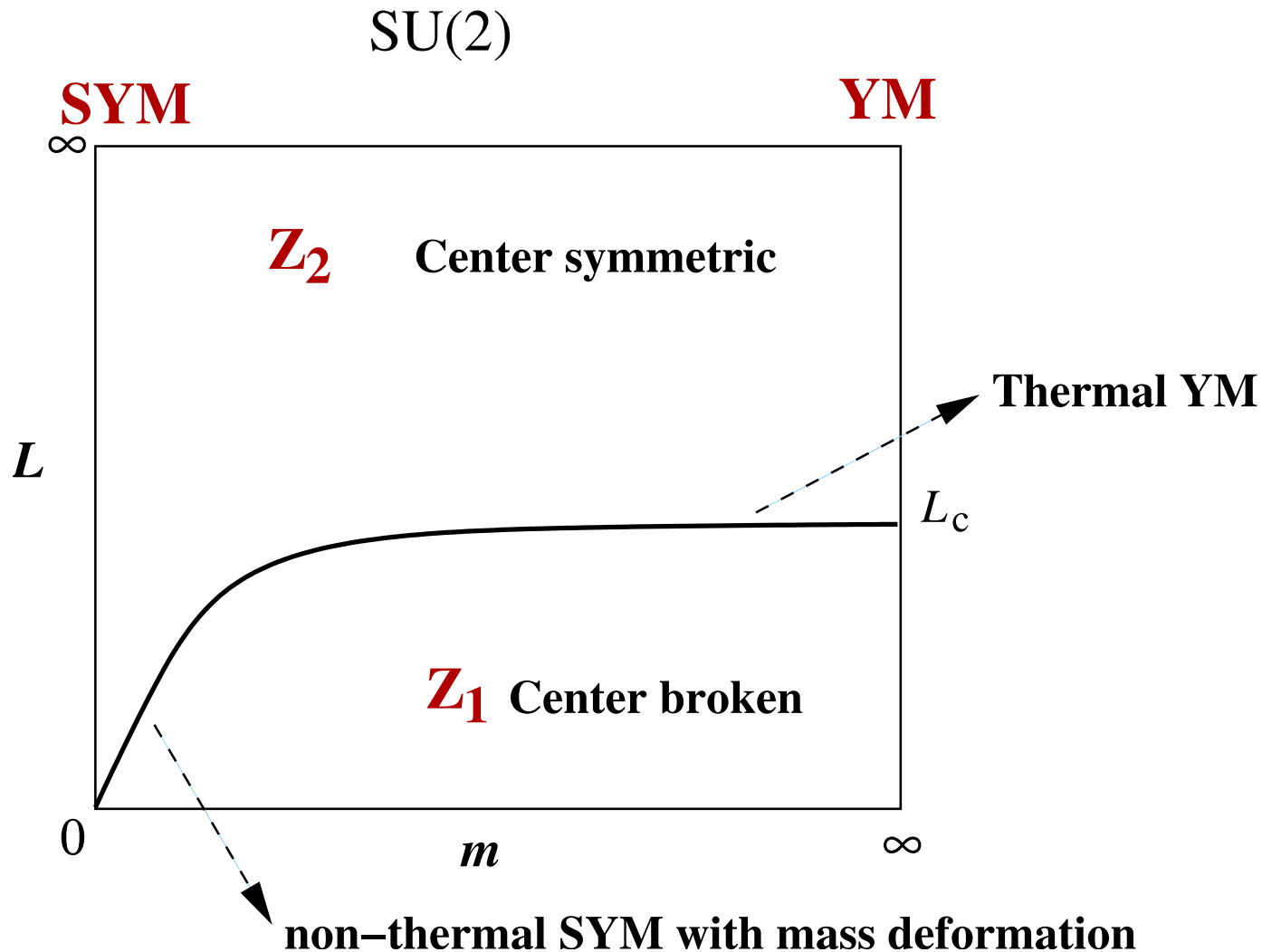
Pierre van Baal



Mitya Diakonov

$SU(2)$ YM with $n_f^{adj} = 1$ Weyl fermions on $R^3 \times S_1$

Phase diagram in L - m plane



Ingredients

- $R^3 \times S_1$ circle-compactified gauge theory.
- Small S_1 : Effective 3d theory involving holonomy and (dual) photon.
- Double expansion: Perturbative and non-perturbative effects (monopoles, topological molecules).
- Topological molecules: supersymmetry versus BZJ.
- Competition: Center stabilizing molecules, center breaking perturbative (and monopole) effects.

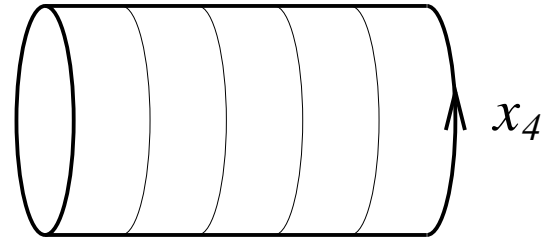
Gauge theory on $R^3 \times S_1$

SU(2) gauge theory, $n_f = 1$ adjoint Weyl fermion

$$\mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu}^a F^{a\mu\nu} - \frac{i}{g^2} \lambda^a \sigma \cdot D^{ab} \lambda^b + \frac{m}{g^2} \lambda^a \lambda^a$$

$$A_\mu^a(0) = A_\mu^a(L)$$

$$\lambda^a(0) = \lambda^a(L)$$



Vacua labeled by Polyakov line

$$\Omega = \exp \left[i \int A_4 dx_4 \right]$$

Center symmetry $\Omega \rightarrow z\Omega \quad z \in Z_2$

Small S_1 : Effective Theory

Consider small S_1 : Effective theory in 3d

$\Omega \neq 1$: A_4^3 is a Higgs field, theory abelianizes $SU(2) \rightarrow U(1)$.

Light bosonic modes: (dual) “photon” σ and holonomy b

$$\mathcal{L} = \frac{g^2}{32\pi^2 L} [(\partial_i b)^2 + (\partial_i \sigma)^2] + V(\sigma, b)$$

$$\Omega = \begin{pmatrix} e^{i\alpha_s b/2} & 0 \\ 0 & e^{-i\alpha_s b/2} \end{pmatrix} \quad \epsilon_{ijk} \partial_k \sigma = \frac{4\pi L}{g^2} F_{ij}$$

holonomy b

dual photon σ

Note: $m = 0$ effective theory can be super-symmetrized

$$\Phi = b + i\sigma + \sqrt{2}\theta^\alpha \lambda^\alpha$$

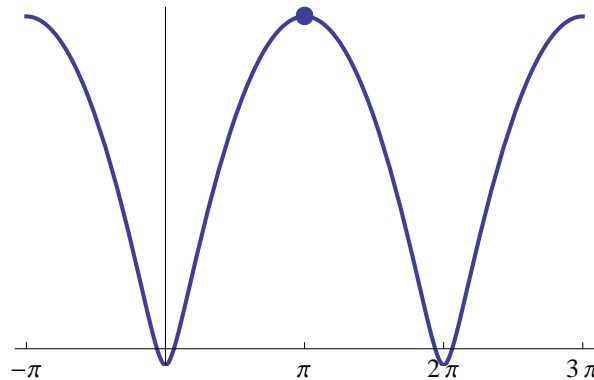
Perturbation Theory

Perturbative potential for holonomy (Gross, Pisarski, Yaffe, 1981)

$$V(\Omega) = -\frac{m^2}{2\pi^2 L^2} \sum_{n=1}^{\infty} \frac{1}{n^2} |\text{tr } \Omega^n|^2 = -\frac{m^2}{L^2} B_2 \left(\frac{\Delta\theta}{2\pi} \right)$$

$m = 0$: Bosonic and fermionic terms cancel.

$m \neq 0$: Center symmetric vacuum $\text{tr}(\Omega) = 0$ unstable.



Non-perturbative effects

Topological classification on $R^3 \times S_1$ (GPY)

1. Topological charge

$$Q_{top} = \frac{1}{16\pi^2} \int d^4x F \tilde{F}$$

2. Holonomy (eigenvalues q^α of Polyakov line at spatial infinity)

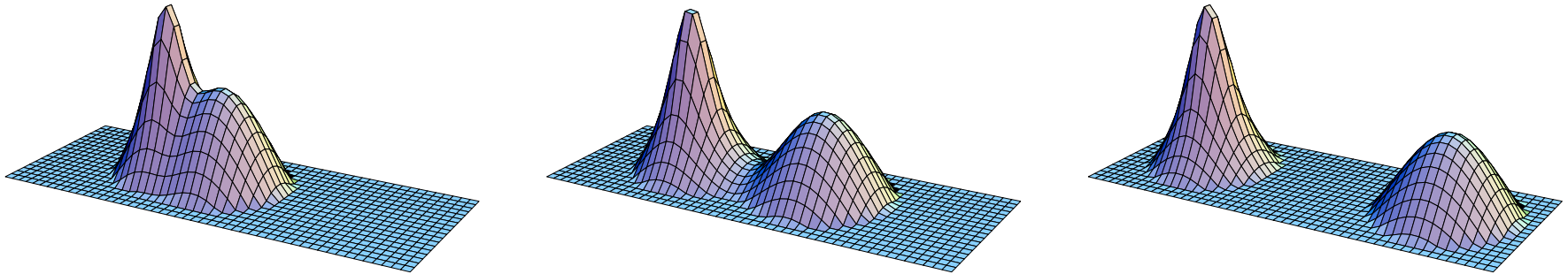
$$\langle \Omega(\vec{x}) \rangle = \left\langle \text{Tr} \exp \left[i \int_0^\beta A_4 dx_4 \right] \right\rangle$$

3. Magnetic charges

$$Q_M^\alpha = \frac{1}{4\pi} \int d^2S \text{Tr} [P^\alpha B]$$

Calorons at finite holonomy: monopole constituents

KvBLL (1998) construct calorons with non-trivial holonomy



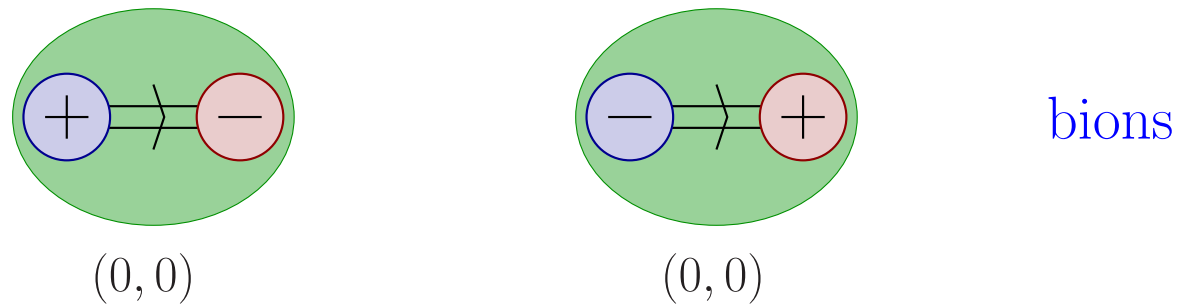
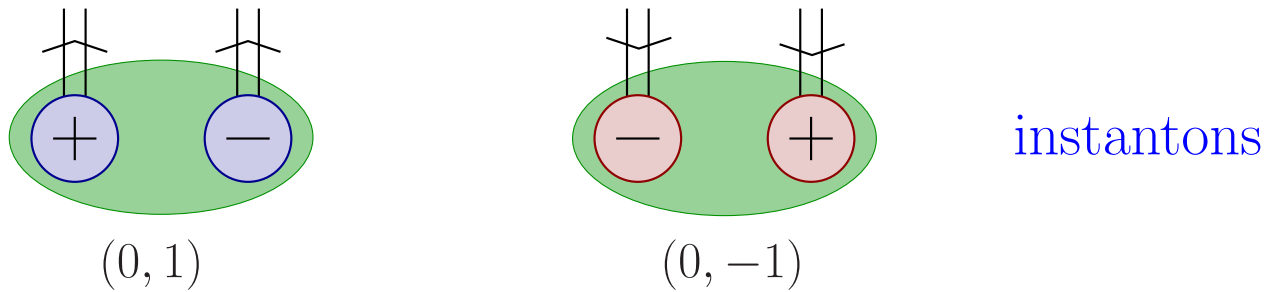
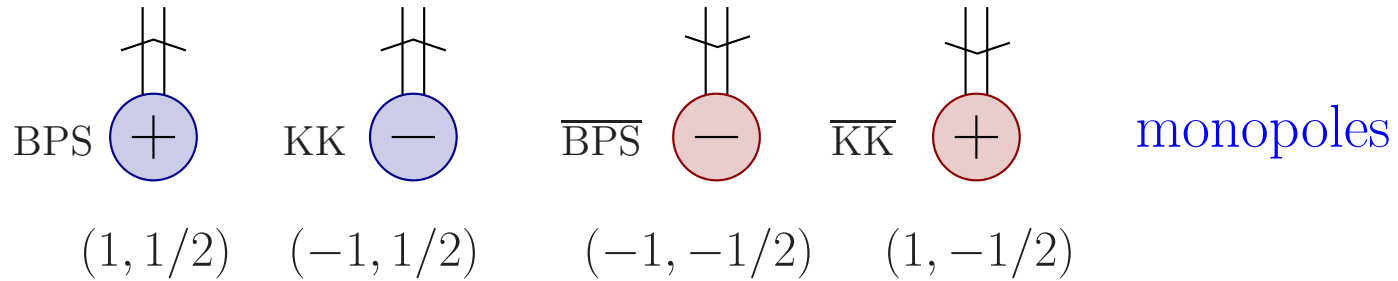
BPS and KK monopole constituents. Fractional topological charge, $1/2$ at center symmetric point.

$2 \times (3 + 1) = 8$ bosonic zero modes, 2×2 fermionic ZM.

$$\int d\phi_1 \int d^3x_1 \int d^2\zeta e^{-S_1} \int d\phi_2 \int d^3x_2 \int d^2\xi e^{-S_2}$$

Topological objects

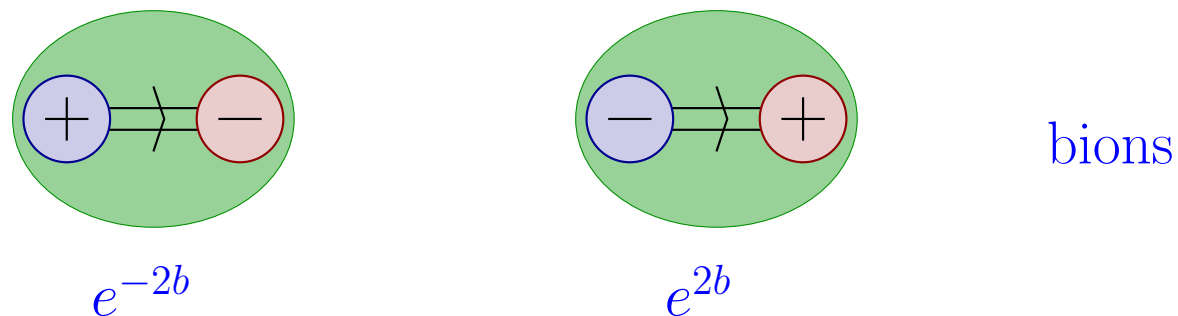
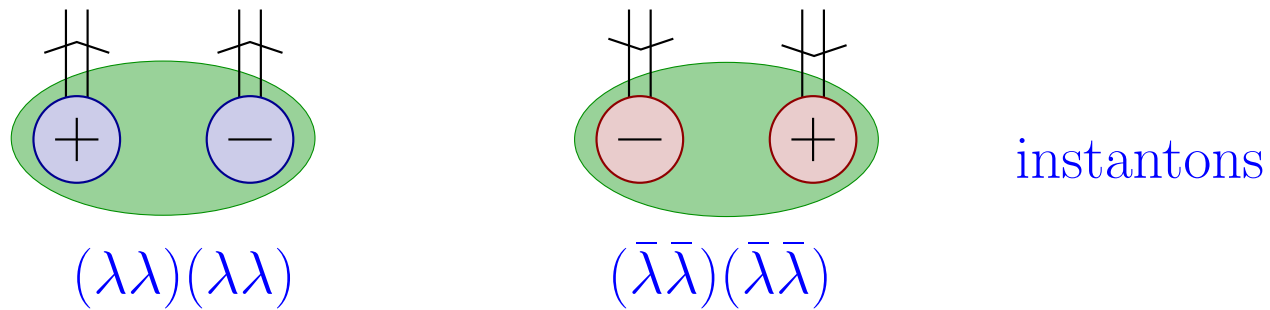
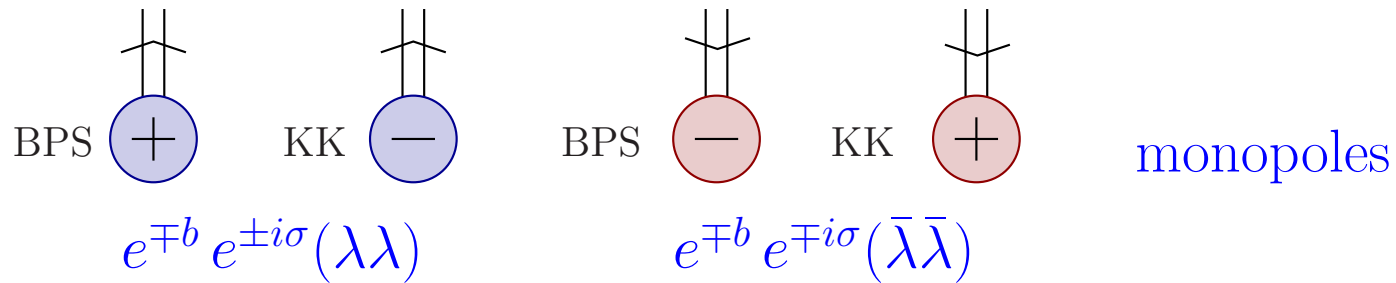
$$(Q_M, Q_{top}) = \left(\int_{S_2} B \cdot d\Sigma, \int_{R^3 \times S_1} F \tilde{F} \right)$$



Note: BPS/KK topological charges in Z_2 symmetric vacuum. Also have $(2, 0)$ (magnetic) bions.

Topological objects: Coupling to low energy fields

$$(Q_M, Q_{top}) = \left(\int_{S_2} B \cdot d\Sigma, \int_{R^3 \times S_1} F \tilde{F} \right)$$



Non-perturbative effects at $m = 0$ from supersymmetry

Monopoles contribute to superpotential: $(\lambda\lambda)e^{-b+i\sigma} \sim \int d^2\theta e^{-B}$

$$\mathcal{W} = \frac{M_{PV}^3 L}{g^2} (e^{-B} + e^{-2S_0} e^B)$$

Scalar potential

$$V(b, \sigma) \sim \left| \frac{\partial \mathcal{W}}{\partial B} \right|^2 \sim \frac{M_{PV}^6 L^3 e^{-2S_0}}{g^6} \left[\cosh \left(\frac{8\pi}{g^2} (\Delta\theta - \pi) \right) - \cos(2\sigma) \right]$$

Center symmetric vacuum $\text{tr}(\Omega) = 0$ preferred

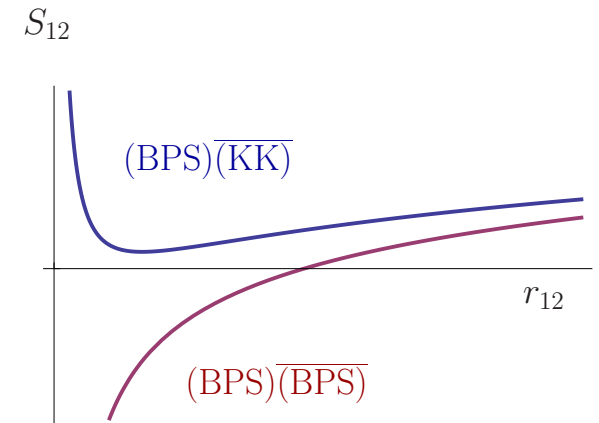
Mass gap for dual photon $m_\sigma^2 > 0$ (\rightarrow confinement)

Non-perturbative effects at $m = 0$ from BZJ

Consider magnetically neutral topological molecules. Integrate over near zero-mode:

$$V_{BPS, \overline{BPS}} \sim e^{-2b} e^{-2S_0} \int d^3r e^{-S_{12}(r)}$$

$$S_{12}(r) = \frac{4\pi L}{g^2 r} (q_m^1 q_m^2 - q_b^1 q_b^2) + 4 \log(r)$$



Saddle point integral after analytic continuation $g^2 \rightarrow -g^2$ (BZJ)

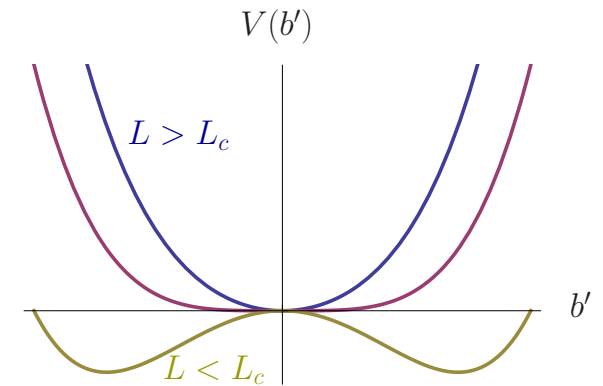
$$V(b, \sigma) \sim \frac{M_{PV}^6 L^3 e^{-2S_0}}{g^6} \cosh \left(\frac{8\pi}{g^2} (\Delta\theta - \pi) \right)$$

Same for magnetically charged molecules: $V \sim \cos(2\sigma)$.

Effective potential for $m \neq 0$

Effective potential: molecules, monopoles, perturbation theory

$$\begin{aligned} \tilde{V} &= \cosh 2b' - \cos 2\sigma \\ &+ \frac{\tilde{m}}{2\tilde{L}^2} \cos \sigma \left(\cosh b' - \frac{b' \sinh b'}{3 \log \tilde{L}^{-1}} \right) \\ &- \frac{1}{1728} \left(\frac{\tilde{m}}{\tilde{L}^2} \right)^2 \frac{1}{\log^3 \tilde{L}^{-1}} (b')^2 . \end{aligned}$$



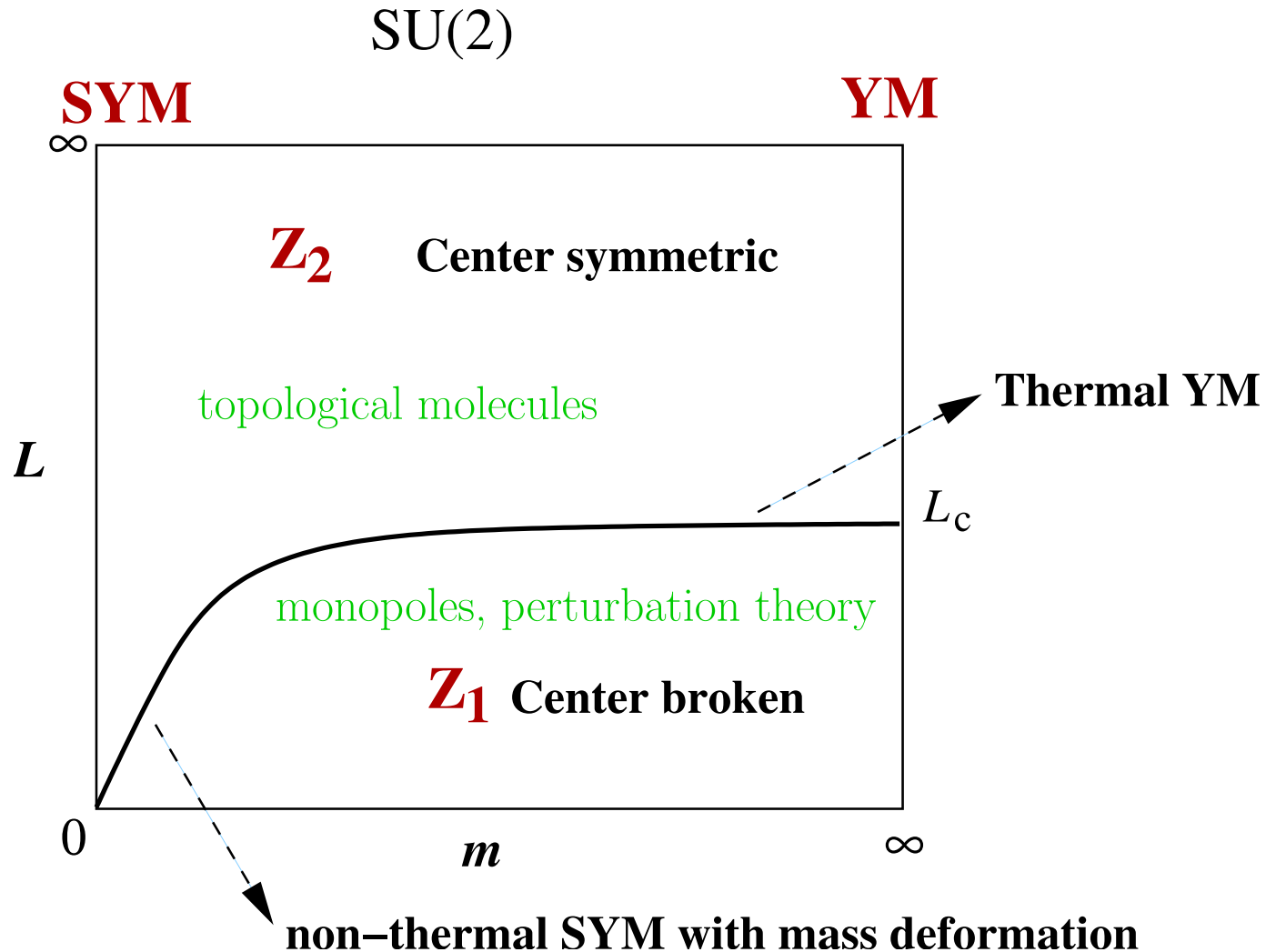
$$\tilde{L} = L\Lambda, \tilde{m} = m/\Lambda, b' = \frac{4\pi}{g^2} (\Delta\theta - \pi)$$

Critical S_1 size $\tilde{L}_c^2 = \frac{\tilde{m}}{8} \left[1 + \mathcal{O} \left(\frac{1}{\log \tilde{L}}, \frac{\tilde{m}}{\tilde{L}^2} \right) \right],$

Corresponds to $T_c = \sqrt{\frac{8}{\tilde{m}}} \Lambda_{QCD}$

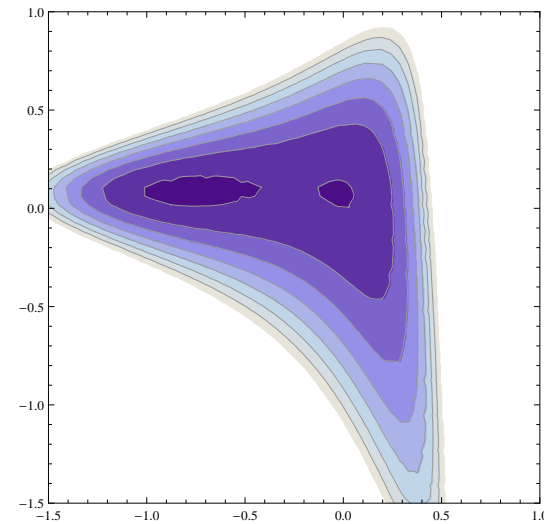
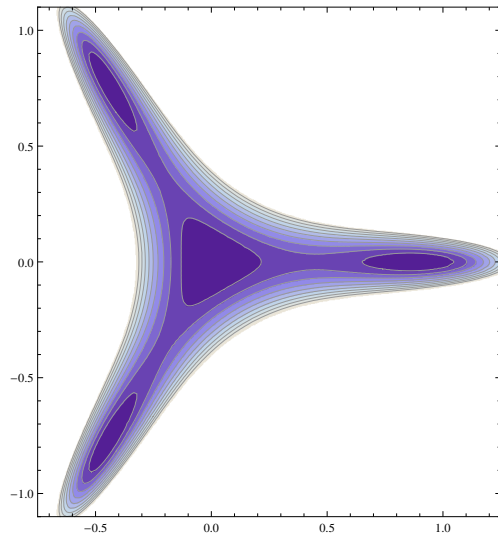
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Outlook: higher rank gauge groups, θ dependence, pure gauge

- $SU(3)$ and G_2 : First order transition with/wo change of symmetry



- G_2 : First order transition without change of symmetry.
- $\theta \neq 0$: Get $V \sim \cos\left(\frac{2\pi k + \theta}{N}\right)$, $k = 1, \dots, N - 1$.
- Pure gauge theory: Find center stabilizing molecules from BZJ.
But: Semi-classical approximation not reliable.