Four-pomeron interaction

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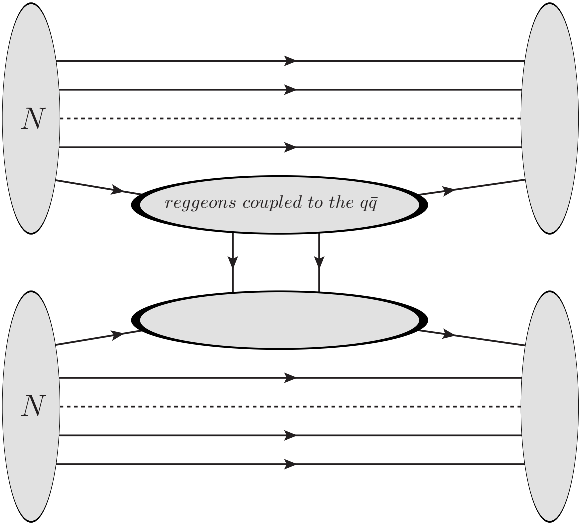
Outline

1. Introduction, dipole model
2. Formalism developed by L. Lipatov
3. J.Bartels formalism and L. Lipatov formalism
4. Formalism developed by J.Bartels
5. Conclusion

Introduction

We study the processes of nucleus-nucleus interaction.



Each pomeron in the dipole model can also split into two ones through a certain known triple pomeron vertex

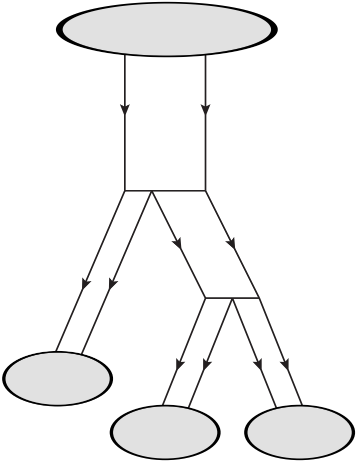


Figure: Pomeron fan diagram

Our immediate problem is to take into account the four-pomeron interaction.

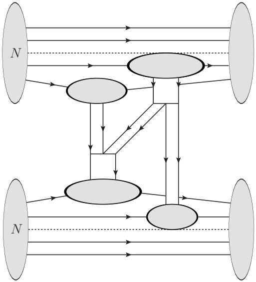


Figure: An example it's four-pomeron interaction in nucleus-nucleus

Interaction of gluons with 4 quarks inside two heavy nuclei with atomic numbers A and B provides factor F^2 where

$$F = (AB)^{2/3}(N_c\alpha_s)^2$$

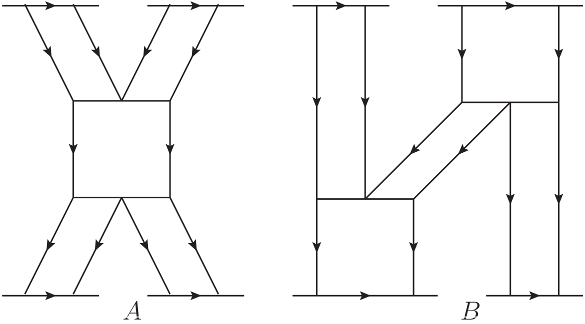
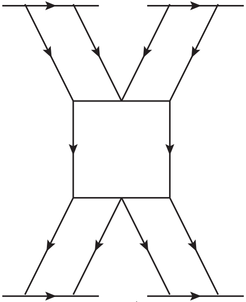


Figure: Simplest diagrams generated by the triple-pomeron interaction

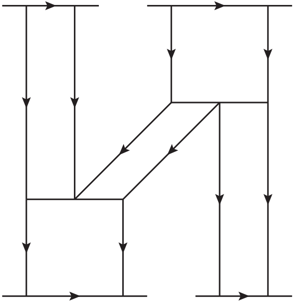
$$C_{1A} = f^{ae_1e_2} f^{e_2e_3a} f^{e_3e_4b} f^{e_4e_5b} f^{e_5e_6d} f^{e_6e_7d} f^{e_7e_8c} f^{e_8e_1c} = N_c^4(N_c^2 - 1)$$

and

$$C_{1B} = f^{ace_1} f^{e_1e_2a} f^{e_2e_3e_4} f^{e_3ce_5} f^{e_4e_6b} f^{e_6e_5e_7} f^{e_7de_8} f^{e_8db} = N_c^4(N_c^2 - 1)$$



A

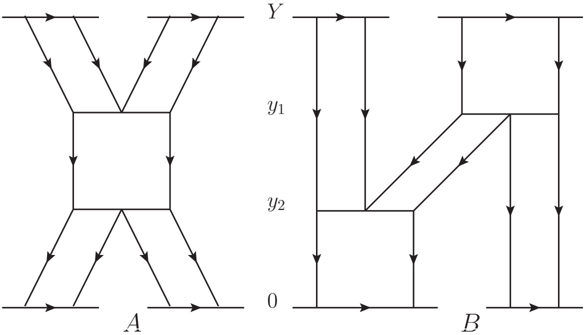


B

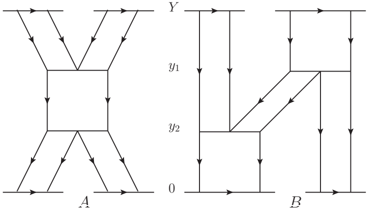
Of these N_c^4 come from external participants which leaves N_c^2. So we find

$$(1A) \sim (1B) \sim F^2 \alpha_s^4 N_c^2$$

as it should be.



To estimate the factor coming from integrations over intermediate rapidities we shall include the BFKL evolution and crudely assume that the pomeron propagator behaves as $e^{\Delta y}$, where $\Delta \sim N_c \alpha_s$ and $\Delta \geq 0$. Let the total rapidity be Y.



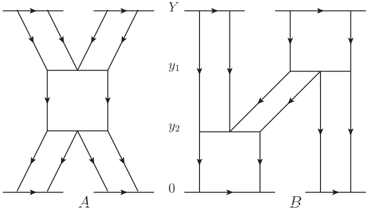
Then two integrations in the diagram in Fig. 1 A give

$$I_{1A} = \int_0^Y dy_1 \int_0^{y_1} dy_2 e^{2\Delta(Y-y_1)} e^{\Delta(y_1-y_2)} e^{2\Delta y_2} = \frac{1}{\Delta} e^{2\Delta Y} \left[Y - \frac{1}{\Delta} \left(1 - e^{-\Delta Y} \right) \right]$$

and in the diagram in Fig. 1 B

$$I_{1B} = \int_0^Y dy_1 \int_0^{y_1} dy_2 e^{\Delta(Y-y_1)} e^{\Delta(Y-y_2)} e^{\Delta(y_1-y_2)} e^{\Delta y_1} e^{\Delta y_2} =$$

$$= \frac{1}{\Delta^2} e^{2\Delta Y} \left(e^{\Delta Y} - 1 - \Delta Y \right) \quad (1)$$



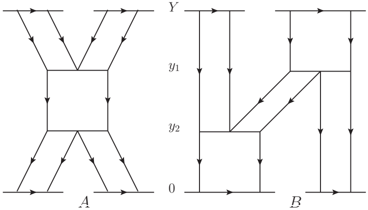
If we assumed that $Y \sim 1/(N_c\alpha_s) >> 1$, so that $N_c\alpha_s Y \sim 1$.

$$I_{1A} = \frac{1}{\Delta} e^{2\Delta Y}\Big[Y - \frac{1}{\Delta}\Big(1 - e^{-\Delta Y}\Big)\Big] \sim 1/(N_c\alpha_s)^2$$

and

$$I_{1B} = \frac{1}{\Delta^2} e^{2\Delta Y}\Big(e^{\Delta Y} - 1 - \Delta Y\Big) \sim 1/(N_c\alpha_s)^2$$

As a result, in this region both contributions from Fig. 1 have the total order $F^2\alpha_s^2$.



If we assumed that $\exp \Delta Y >> 1$:

$$(1A) \sim F^2 \alpha_s^4 N_c^2 I_{1A} = F^2 \alpha_s^4 N_c^2 \frac{1}{\Delta} e^{2\Delta Y} \left[Y - \frac{1}{\Delta}\left(1 - e^{-\Delta Y}\right)\right] \sim F^2 \alpha_s^2 (N_c \alpha_s Y) e^{2\Delta Y}$$

$$(1B) \sim F^2 \alpha_s^4 N_c^2 I_{1B} = F^2 \alpha_s^4 N_c^2 \frac{1}{\Delta^2} e^{2\Delta Y} \left(e^{\Delta Y} - 1 - \Delta Y \right) \sim F^2 \alpha_s^2 e^{3\Delta Y}$$

We see that the diagram in Fig. 1 B dominates and it can be of the same order as the single BFKL exchange even for light nuclei provided

$$F \alpha^2 e^{2\Delta Y} \sim 1 \tag{2}$$

It is the same order if $F\alpha^2 e^{2\Delta Y} \sim 1$

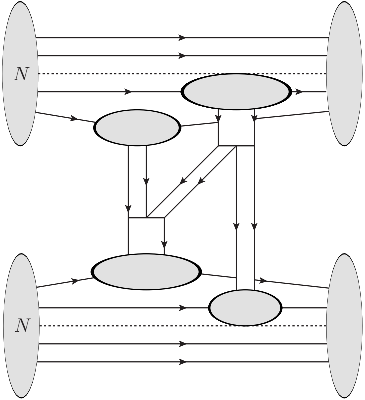


Figure: Four-pomeron interaction

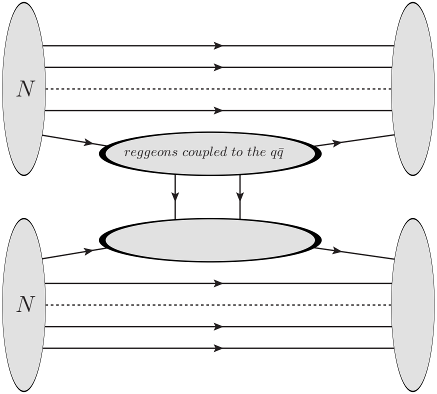


Figure: Single BFKL exchange

Lipatov effective action formalism

The effective Lagrangian describes interaction of the quark ψ and gluon $V_\mu = -iT^a V_\mu^a$ fields and their interaction with the independent reggeon field $A_\mu = -iT^b A_\mu^b$ with $A_\perp = 0$:

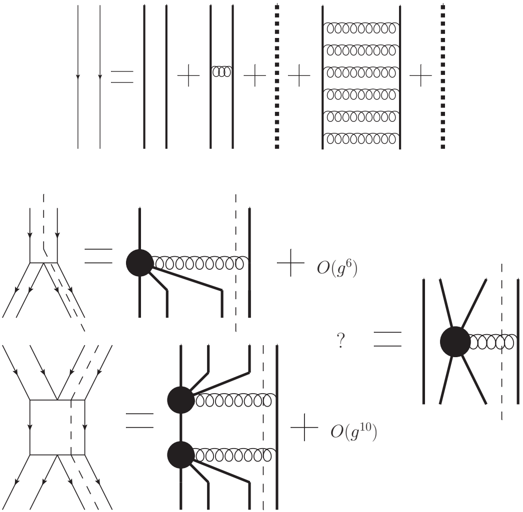
$$\mathcal{L}_{eff} = \mathcal{L}_{YM}(v_\mu) + \bar{\psi}\Big(i\hat{\partial} + ig\hat{v} - M\Big)\psi + \\ + \mathrm{Tr}\Big((\mathcal{A}_+(v_+) - A_+)\partial_\perp^2 A_-\Big) + \mathrm{Tr}\Big((\mathcal{A}_-(v_-) - A_-)\partial_\perp^2 A_+\Big), \quad (3)$$

where $\mathcal{L}_{YM}$ is the standard Yang-Mills Lagrangian,

$$\mathcal{A}_\pm(v_\pm) = -\frac{1}{g}\partial_\pm \frac{1}{D_\pm}\partial_\pm * 1 = \sum_{n=0}^{\infty}(-g)^n v_\pm(\partial_\pm^{-1} v_\pm)^n = \\ = v_\pm - g v_\pm \partial_\pm^{-1} v_\pm + g^2 v_\pm \partial_\pm^{-1} v_\pm \partial_\pm^{-1} v_\pm + \ldots \quad (4)$$

and the shift of the field variable $v_\mu = V_\mu + A_\mu$. The reggeon fields are assumed to be subject to kinematic conditions $\partial_- A_+ = \partial_+ A_- = 0$

We have connection between dipole model and Lipatov formalism



M.A.Braun et al., arXiv: 1209.2490

Bartels formalism and Lipatovs Formalism

Communication

Strictly speaking Bartels formalism - a restoration of the amplitude of multiple jump when consistently dissected all s-channel gluons.
"Lipatovs Formalism", or just Lipatovs action, allows us to calculate not only discontinuity, but any of the Green function and amplitude, but we have to deal with virtual particles with propagators that depend on the longitudinal momentum components which are not present in the formalism Bartels.

In Bartels formalism

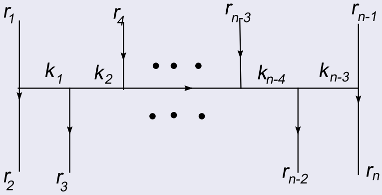
According to [1] the general vertex for the transition of l gluons into m gluons with $l + m = n$ and subsequent reggeons going in turn to the projectile and target can be written in the form

$$K_{l\to m}(r_1, r_2, ...r_n) = \left\langle V_1 \middle| T(k_2)T(k_3)\cdots T(k_{n-4}) \middle| V_n \right\rangle, \qquad (5)$$

where $V_1 = r_1 - k_1\frac{r_1^2}{k_1^2} = -r_1^2 L(k_1, r_2)$,

$$V_n = r_{n-1} + k_{n-3}\frac{r_{n-1}^2}{k_{n-3}^2} = -r_{n-1}^2 L(-k_{n-3}, r_n)$$

$$T_{ij}(k) = \delta_{ij} - 2\frac{k_i k_j}{k^2} \qquad (6)$$



[1] J.Bartels, Nucl. Phys. B 175 (1980) 365.

However there are two problems related to this expression.

1. Diagrams do not include contributions from 4-gluon interaction
2. There are problems with factorization

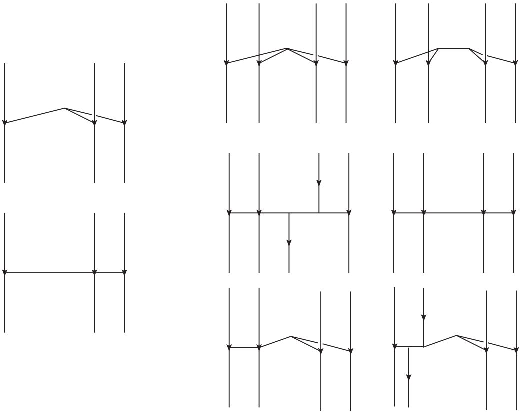


Figure: Diagrams for $3 \to 3$ (left) and $4 \to 4$ (right) transitions, which are not included in Bartels formalism

Take the $2 \to 3$ transition

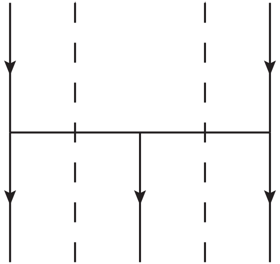
$$K(2 \to 3) = r_1^2 r_4^2 L(k_1, r_2) L(-k_2, r_5) \quad (7)$$

where $k_1 - k_2 = r_3$. So one can present (7) in two equivalent forms

$$K(2 \to 3) = r_1^2 r_4^3 L(k_1, r_2) L(-k_1 + r_3, r_5) = r_1^2 r_4^2 L(k_1, r_2) B(-k_1, r_3, r_5) \quad (8)$$

or

$$K(2 \to 3) = r_1^2 r_4^2 L(k_2 + r_3, r_2) L(-k_2, r_5) = r_1^2 r_4^2 B(k_2, r_3, r_2) L(-k_2, r_5) \quad (9)$$



Now take $3 \to 3$ transition

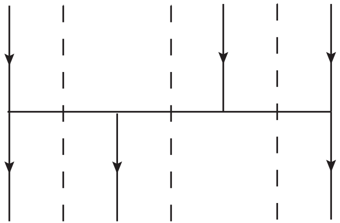
$$K(3 \to 3) = r_1^2 r_5^2 \Big[L(k_1, r_2) L(-k_3, r_6) - 2\frac{2}{k_2^2} \Big(k_2 L(k_1, r_2) \Big) \Big(k_2 L(-k_3, r_6) \Big) \Big] \quad (10)$$

We can factorize it as

$$K(3 \to 3) = r_1^2 r_5^2 L(k_1, r_2) \Big[L(-k_3, r_6) - 2\frac{k_2}{k_2^2} \Big(k_2 L(-k_3, r_6) \Big) \Big] \quad (11)$$

or as

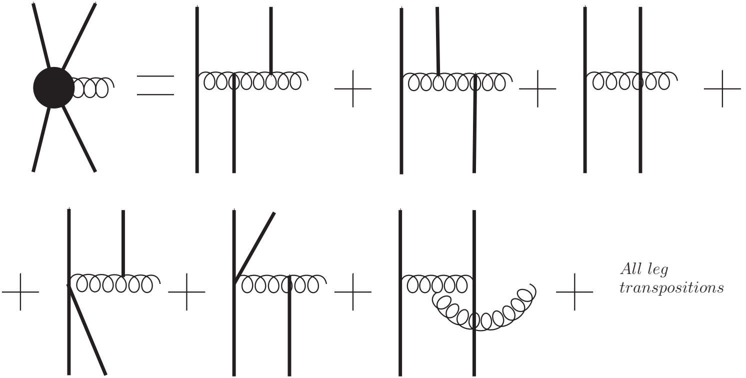
$$K(3 \to 3) = r_1^2 r_5^2 \Big[L(k_1, r_2) - 2\frac{k_2}{k_2^2} \Big(k_2 L(k_1, r_2) \Big) \Big] L(-k_3, r_6) \quad (12)$$



Conclusion

These are problems to be solved by calculating vertex

$2R \rightarrow 2R + P$



Conclusion

Thank you for your attention