Magnetohydrodynamics, charged currents and directed flow in heavy ion collisions

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with D. Kharzeev and K. Rajagopal
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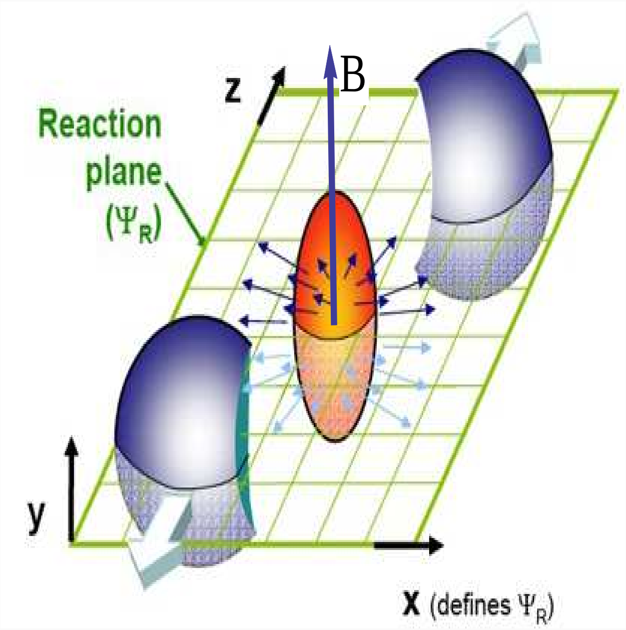
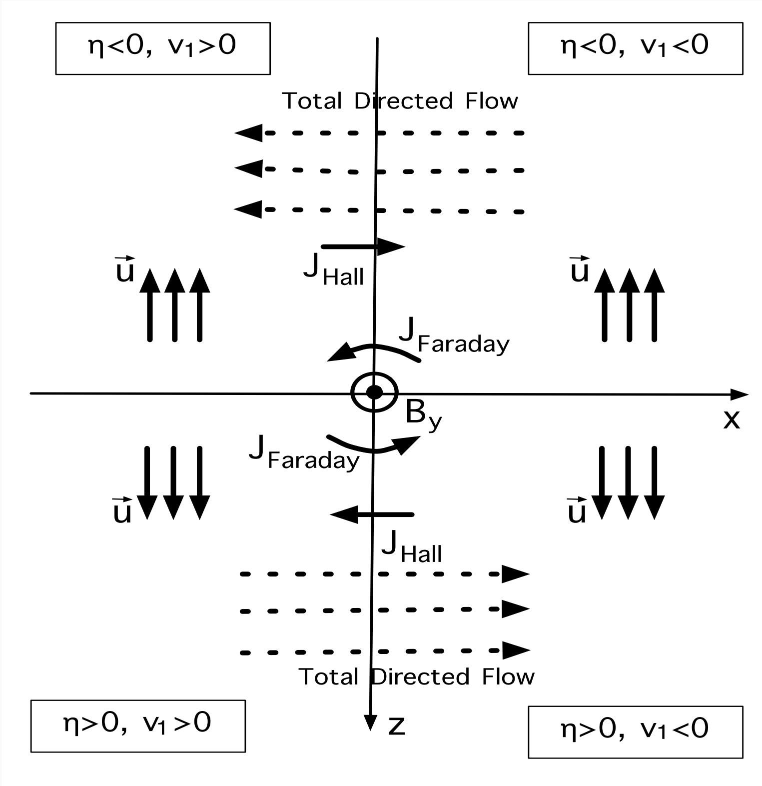
Heavy ion collisions and magnetic fields



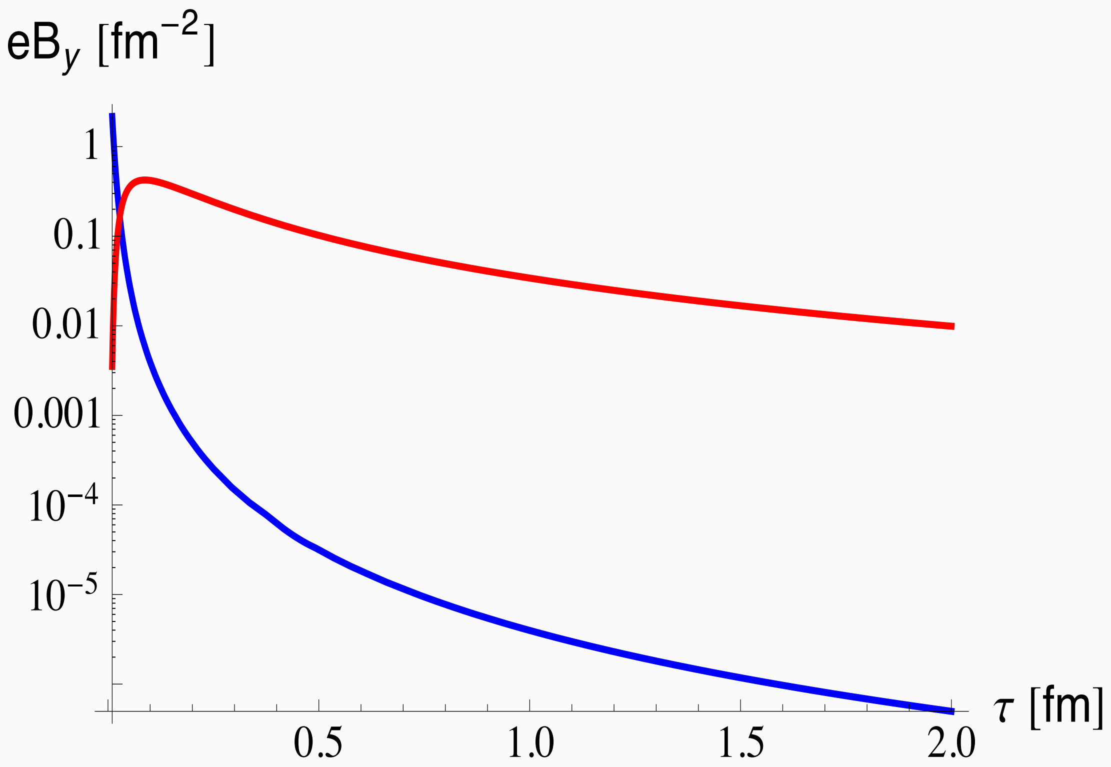
- Initial magnitude of B
- Bio-Savart: $B_0 \sim \gamma Z e \frac{b}{R^3} \Rightarrow$ $eB \approx 5 - 15 \times m_\pi^2$ at RHIC (LHC).
- In this talk $b = 7$fm and $R = 7$fm.
- Motivation: find observables that are directly tied to the presence of B



"Classical" currents in charged and expanding medium:

- Faraday currents $\vec{J}_F \sim \sigma \vec{E}_F$ with $\nabla \times \vec{E}_F = -\frac{\partial \vec{B}}{\partial t}$
- Hall currents $\vec{J}_H \sim \sigma \vec{E}_H$ with $\vec{E}_H = \vec{u} \times \vec{B}$

Time profile of B at LHC



with $\sigma = 0.023\text{fm}^{-1}$ and with $\sigma = 0$

- Simplifying assumption hard-sphere distribution for spectators and participants
- For participants empirical distribution over Y: Kharzeev et al. 2007
  $f(Y_b) = (4\sinh(Y_0/2))^{-1}\, e^{Y_b/2}, \qquad -Y_0 \leq Y_b \leq Y_0$

Perturbative Magnetohydrodynamics

- Suppose $u^\mu(x)$ with no back reaction of electromagnetic fields and $\vec{E}$, $\vec{B}$ are known
- Go to the comoving frame by $\Lambda(-\vec{u})$ e.g. $F'_{\mu\nu} = (\Lambda \cdot F \cdot F)_{\mu\nu}$
- Compute the stationary velocity:

$$m\frac{d\langle\vec{v_B}\rangle}{dt} = q\langle\vec{v_B}\rangle \times \vec{B}' + q\vec{E}' - \mu m\langle\vec{v_B}\rangle = 0\,,$$

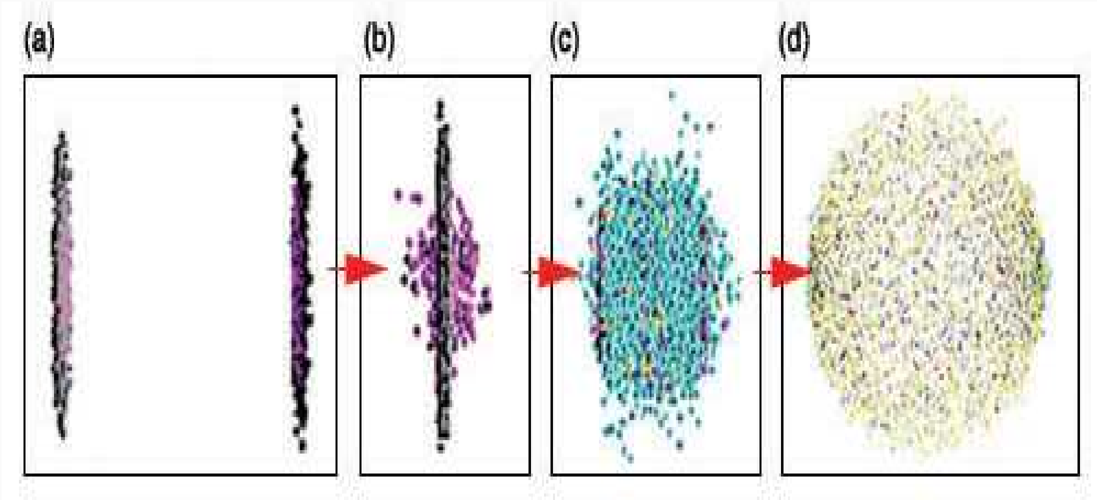
μm the drag coefficient; e.g. from AdS/CFT: $\frac{\pi\sqrt{\lambda}}{2}T^2$

- Go back to the center of mass frame: by $V^\mu = \Lambda(\vec{u})^\mu_\nu v^\nu_B$
- V^μ contains both u^μ and v^μ_B
  $\Rightarrow$ construct observables from V

Assumptions

- Validity of Perturbative magnetohydro $\Rightarrow |\vec{v}_B| \ll |\vec{u}|$
- Validity of classical force equation $\Rightarrow$ magnetic energy $E_B \ll \frac{2\pi h}{\lambda}$ with $\lambda \approx R = 7$ fm.
- Both are checked in our set-up
- First may be violated in reality

Constructing u^μ for the expanding fluid



- Start from the Bjorken flow: Bjorken '83
  1. Boost invariance along z: $\xi = z\partial_t + t\partial_x$
  2. Rotation around z: $\xi = x\partial_y - y\partial_x$
  3. Translations in transverse plane: $\xi = \partial_x$ and $\xi = \partial_y$
- Solution to $[\xi, u] = 0$ is $u = \partial_\tau$ $(ds^2 = -d\tau^2 + \tau^2 d\eta^2 + dx_\perp^2 + x_\perp^2 d\phi^2)$
- Fine except transverse translations

Gubser's flow solution

Gubser '10

- Begin by Bjorken's flow
- Replace $\xi_i = \partial_x,\, \partial_y$ with $\xi_i = \partial_i + q^2\left[2x^i x^\mu \partial_\mu - x^\mu x_\mu \partial_i\right]$
- Solution to $[\xi, u] = 0$ is $u = \cosh\kappa\partial_\tau + \sinh\kappa\partial_\perp$ with $\kappa = \frac{2q^2\tau x_\perp}{1+q^2\tau^2+q^2x_\perp^2}$
- Solution to Hydrodynamics: $\nabla_\mu T^{\mu\nu} = 0$ with $\epsilon = \frac{\hat{\epsilon}_0}{\tau^{4/3}} \frac{(2q)^{8/3}}{\left[1+2q^2(\tau^2+x_\perp^2)+q^4(\tau^2-x_\perp^2)^2\right]^{4/3}}$
- Also analytic dissipative correction with η/S.
- Two parameters to fix: Initial energy $\hat{\epsilon}_0$ and "system size" $1/q$

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- How is this consistent with non-zero B then?

- Gubser's solution has axial symmetry around the beam axis
- How is this consistent with non-zero B then?
- This is actually precisely what we want:
- Adding electromagnetism on an axial plasma $\Rightarrow$ extract effects on only charge identified hadrons!
- e.g. v_1, v_2, etc. flow coefficients will be only for charged constituents, neutral background subtracted.
- We will see clearly: $v_1 = v_2 = \cdots = 0$ for Gubser flow u^μ but non-zero for Gubser + electrodynamics $V^\mu = u^\mu + v_B^\mu$

How to test Gubser's flow?

- Hadron spectrum from hydrodynamic flow: Cooper-Frye:

$$S_i = p^0 \frac{dN_i}{dp^3} = -\frac{g_i}{(2\pi)^3} \int d\Sigma_\mu p^\mu F\left(\frac{p^\mu V_\mu}{T_f}\right)$$

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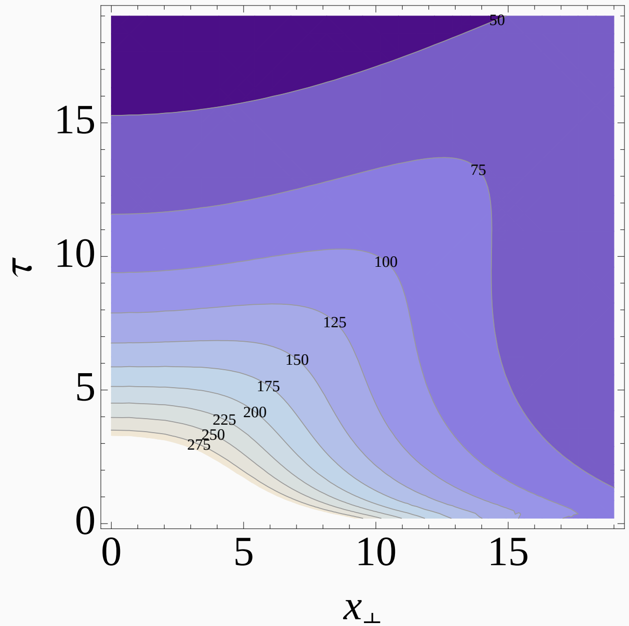

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- T_f is the freezout temperature, $T_f \approx 130$ MeV
- Assume Boltzmann distribution: $F(x) = e^x$

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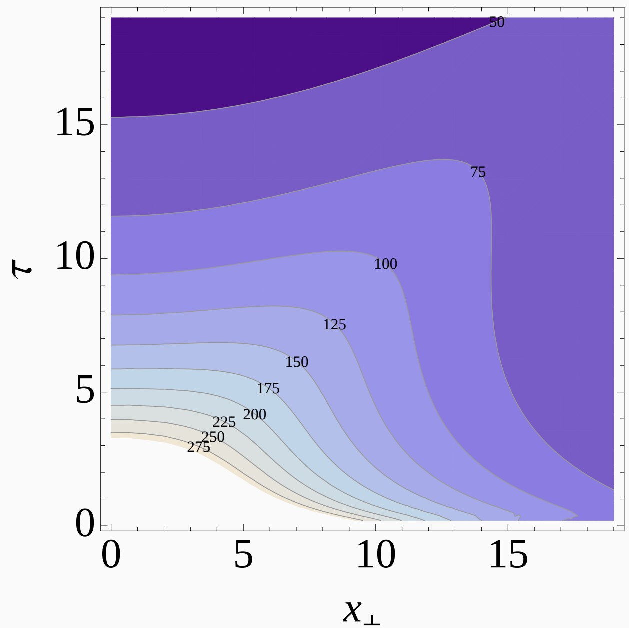

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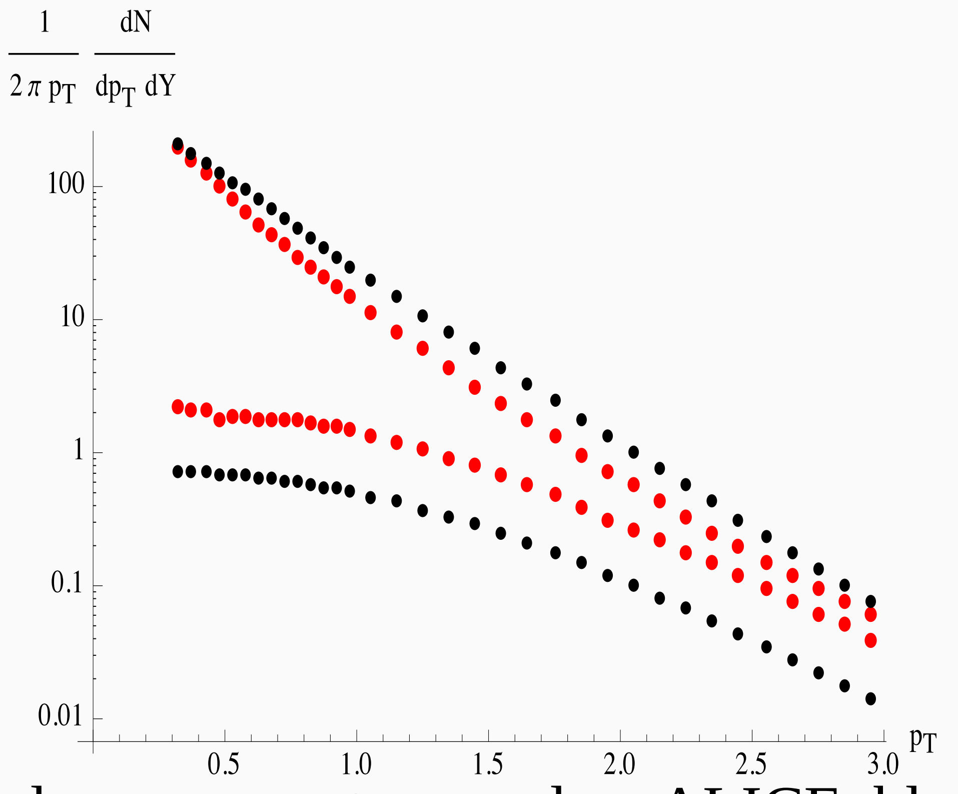
- Assume Boltzmann distribution: $F(x) = e^x$

- $S_i(p_T) =$
  $\frac{g_i}{2\pi^2} \int dx_\perp x_\perp \tau_f \left\{ K_1(\frac{m_T u^\tau}{T_f}) I_0(\frac{p_T u^\perp}{T_f}) - \tau_f' p_T K_0(\frac{m_T u^\tau}{T_f}) I_1(\frac{p_T u^\perp}{T_f}) \right\}$
- Gubser's flow is independent of Φ_p and Y $\Rightarrow v_n = 0$

Cooper-Frye and parameter fixing



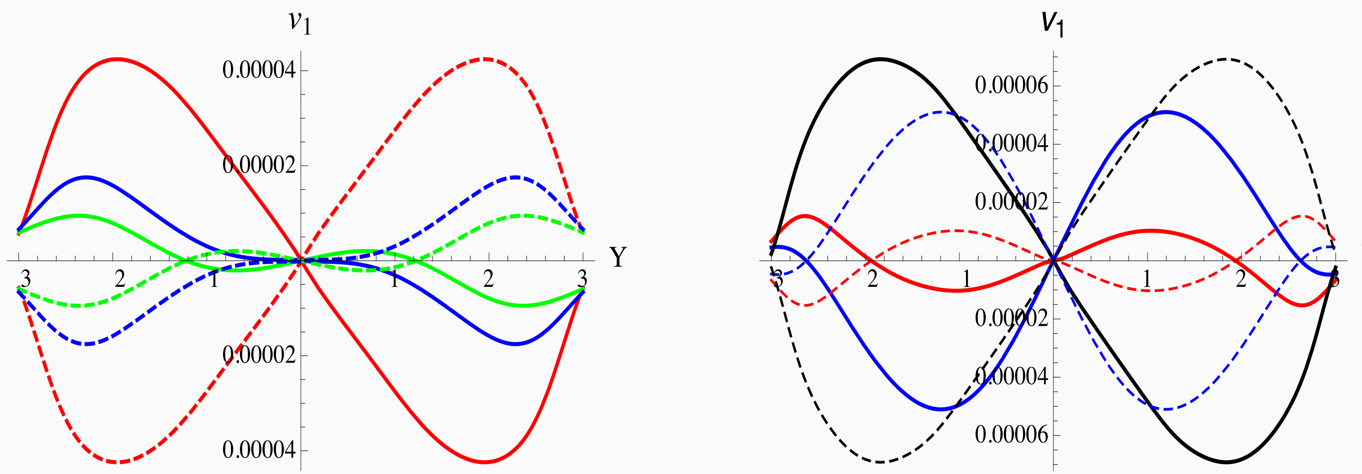
Upper ⇒ pions, lower ⇒ protons; red ⇒ ALICE, black ⇒ hydro

- Demand realistic comparison with ALICE data for pions and protons and reasonable hydronization temperature $T_h \approx 400 - 550$ MeV
- Optimal solution $q^{-1} = 6.5$ fm and $\hat{\epsilon}_0 = (8.7)^4$.

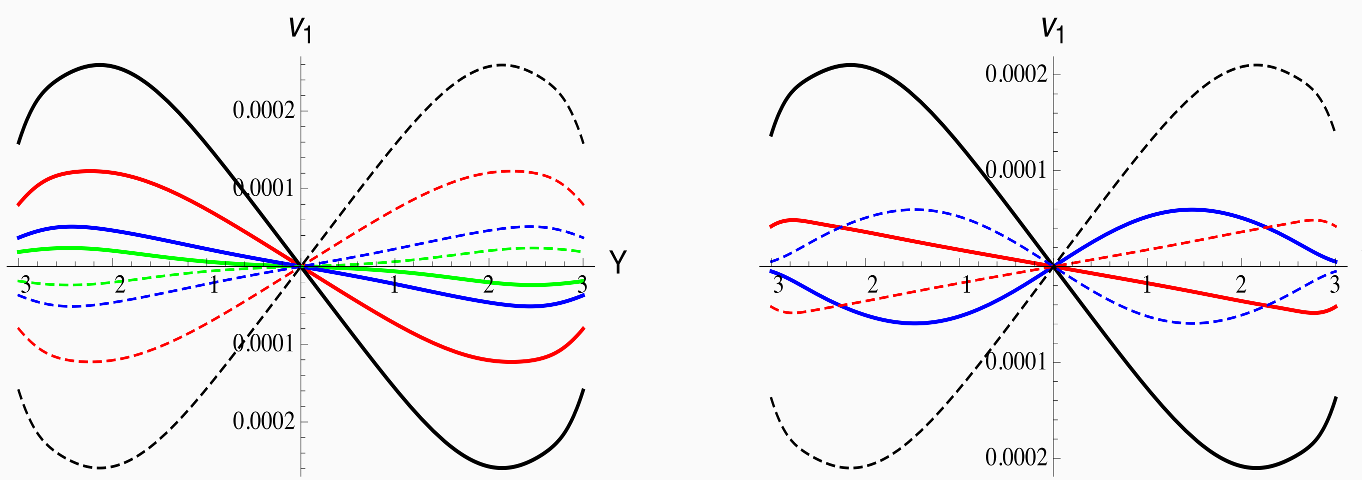
Results

- Calculated B
- Fixed Gubser's flow parameters $\Rightarrow u^{\mu}$
- Solve classical force equation electromagnetic force = drag
- Do Cooper-Frye to calculate v_n
- The simplest and most direct effect: directed flow v_1:

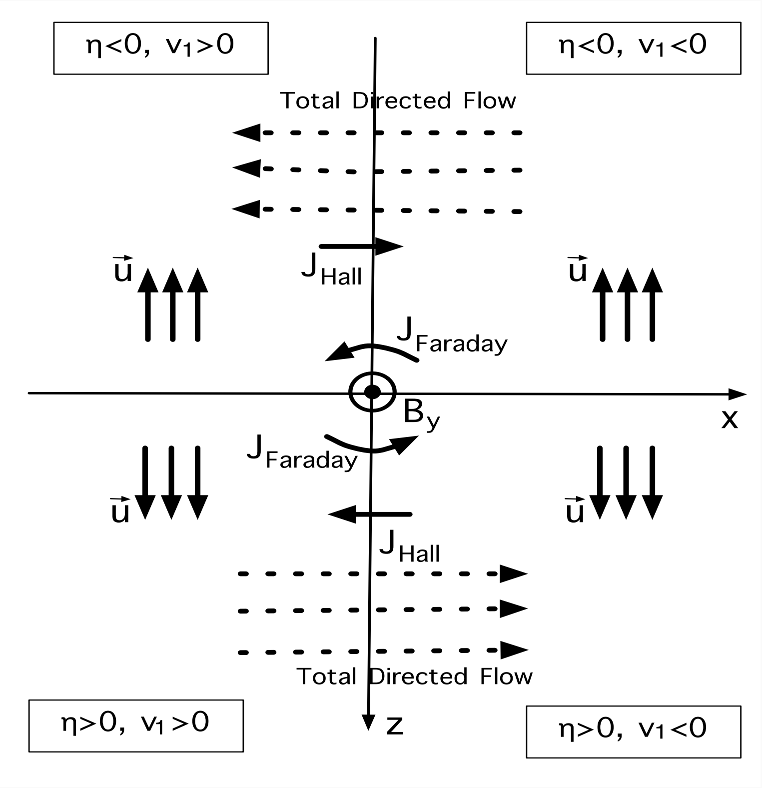
- Pions and protons at LHC



- Pions and protons at RHIC



Color coding: p_T = 0.25 (green), 0.5 (blue), 1 (red), 2 (black) GeV



- Effect is small: $\sim 5 \times 10^{-5}$ at LHC, $\sim 2 \times 10^{-4}$ at RHIC.
- Specific features for detection: $v_1(-Y) = -v_1(Y)$, $v_1^+ = -v_1^-$

Proposal for observables

- Define $A_1^{+-}(Y_1, Y_2) = v_1^+(Y_1) - v_1^-(Y_2)$,
  $A_1^{++}(Y_1, Y_2) = v_1^+(Y_1) - v_1^+(Y_2)$, etc.
  to eliminate charge independent contributions to v_1 produced in event-by-event fluctuations
- Look at quadratic observables
  $C_1^{+-,+-}(Y,Y) = \langle A_1^{+-}(Y,Y) A_1^{+-}(Y,Y)\rangle = 4\langle v_1^+(Y) v_1^+(Y)\rangle$
  to eliminate event-by-event fluctuations in direction of B.
- To be compared with data ...

Summary:

- Calculated the contribution of the time-varying B in an expanding plasma, using a perturbative approach to magnetohydrodynamics.
- Effect odd under charge and rapidity.
- Competition between Faraday and “Hall” effects.
- However the magnitude is small.

Summary:

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Outlook:

- Time dependence of σ, μ, T etc.
- More realistic hydrodynamics.
- Backreaction of EM on hydro $\Rightarrow$ full magnetohydrodynamics
- More realistic distributions for the sources
- Compute charge identified v_n for $n \geq 2$.

THANK YOU !