Quark–gluon plasma phenomenology from anisotropic lattice QCD

Jon-Ivar Skullerud

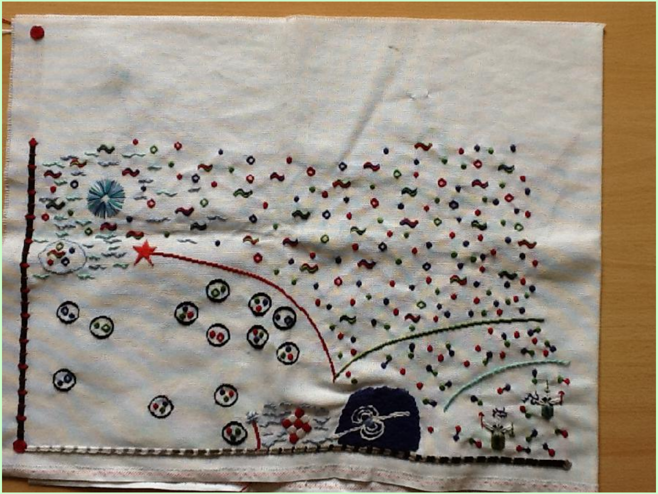
Collaborators:
Gert Aarts Chris Allton Alessandro Amato Wynne Evans
Simon Hands Tim Harris Aoife Kelly Dhagash Mehta
Maria-Paola Lombardo Seyong Kim Buğra Oktay Sinéad Ryan
Don Sinclair
+ Yannis Burnier Alexander Rothkopf

Maynooth University, FASTSUM

Confinement XI, St Petersburg, 8 Oct 2014

Outline

Background



Background

- Quark–gluon plasma is created in heavy-ion collisions at RHIC and LHC
- No direct observation of QGP — must infer from "fallout"
- Dynamical medium: expanding, cooling fireball
  - $\rightarrow$ transport coefficients are crucial in understanding
  - $\rightarrow$ most perfect liquid known to humankind?
- Hard probes may carry information from early stages
- Sequential suppression $\longrightarrow$ quarkonia as QGP thermometers?

Lattice simulations

- QGP near crossover is strongly interacting: nonperturbative methods required
- Equilibrium thermal field theory formulated in euclidean space — suitable for Monte Carlo simulations

$$\langle \mathcal{O} \rangle = \int \mathcal{D}[\Phi]\mathcal{O}[\Phi]e^{-S[\Phi]}$$

- Temperature $T = \frac{1}{L_\tau} = (N_\tau a_\tau)^{-1}$
- Real-time quantities may be determined from spectral function

$$G_E(\tau; T) = \int_0^\infty d\omega K(\omega, \tau; T)\rho(\omega; T)$$

- 2+1 active light flavours required for quantitative predictions!

Dynamical anisotropic lattices

- A large number of points in time direction required to extract spectral information
- For $T = 2T_c$, $\mathcal{O}(10)$ points $\Longrightarrow a_t \sim 0.025$ fm
- Far too expensive with isotropic lattices $a_s = a_t$!
- Fixed-scale approach
  - $\rightarrow$ vary T by varying N_τ (not a)
  - $\rightarrow$ need only 1 $T = 0$ calculation for renormalisation
  - $\rightarrow$ independent handle on temperature

- Introduces 2 additional parameters
- Non-trivial tuning problem
  [PRD **74** 014505 (2006); HadSpec Collab, PRD **79** 034502 (2009)]

Simulation parameters

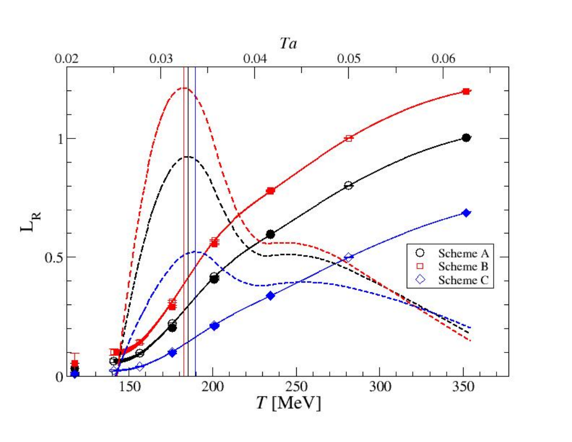
Gen	N_f	ξ	a_s (fm)	a_τ^{-1} (GeV)	m_π/m_ρ	N_s	L_s (fm)
1	2	6.0	0.162	7.35	0.54	12	1.94
2	2+1	3.5	0.123	5.63	0.45	24	2.94
						32	3.94

Gen 1			Gen 2		
N_τ	T (MeV)	T/T_c	N_τ	T (MeV)	T/T_c
80	92	0.42	128	44	0.24
			48	117	0.63
32	230	1.05	40	141	0.76
28	263	1.20	36	156	0.84
24	306	1.40	32	176	0.95
20	368	1.68	28	201	1.09
18	408	1.86	24	235	1.27
16	459	2.09	20	281	1.52
			16	352	1.90

Deconfinement transition

Renormalised Polyakov loop

$$L_R = e^{-F_q/T} = e^{-(F_0+\Delta F)/T} = (e^{\Delta F})^{\frac{1}{T}} e^{-F_0/T} = z_L^{N_\tau} L_0$$



Scheme A:
$L_R(T = \frac{1}{16a}) = 1$

Scheme B:
$L_R(T = \frac{1}{20a}) = 1$

Scheme C:
$L_R(T = \frac{1}{20a}) = \frac{1}{2}$

Spectral functions

- $\rho_\Gamma(\omega, \vec{p})$ related to euclidean correlator $G_\Gamma(\tau, \vec{p})$

$$G_\Gamma(\tau, \vec{p}) = \int \rho_\Gamma(\omega, \vec{p}) \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)} d\omega$$

  [for mesonic correlators in thermal equilibrium]
- an ill-posed problem
- use Maximum Entropy Method to determine most likely $\rho(\omega)$
- requires a large number of time slices to have any chance of a reliable determination
- must introduce model function $m(\omega)$
- in absence of data, MEM will reproduce model function
- parametrise $\rho(\omega) = m(\omega) \exp[\sum c_k u_k(\omega)]$

Transport coefficients

Transport coefficients can be related to spectral functions through Kubo relations

$$\kappa = c_\kappa \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

$\rho(\omega)$ is the spectral function of the relevant conserved current. Conductivity and diffusion coefficients are both determined from the vector current correlator

$$G_{ij}(\tau, \vec{p}) = \int d^3x e^{i\vec{p}\cdot\vec{x}} \langle V_i(\tau, \vec{x}) V_j(0, \vec{0}) \rangle$$

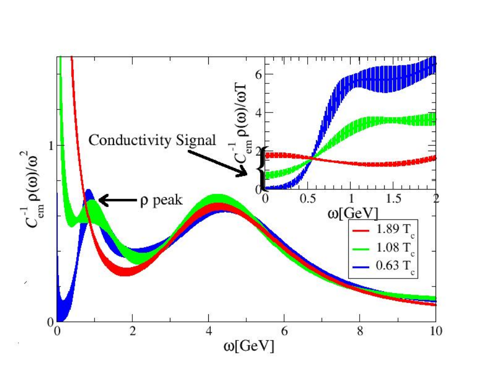
- Sensitive to long-distance, nearly constant modes
- Very high precision data required
- Model function must allow $\rho(\omega)/\omega$ finite as $\omega \to 0$

Conductivity

[PRL **111** 172001 (2013)] — 2nd generation

$$\sigma = \lim_{\omega\to 0} \frac{\rho^{em}_{ii}(\omega)}{6\omega}$$

Used conserved vector current: no renormalisation required

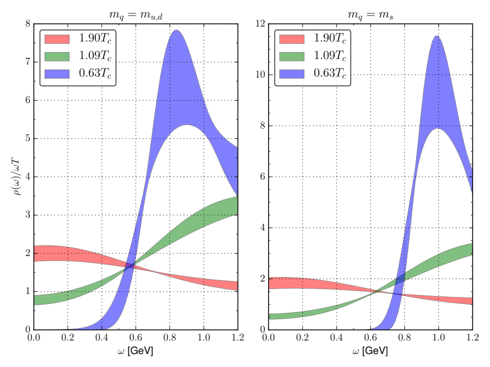


Conductivity

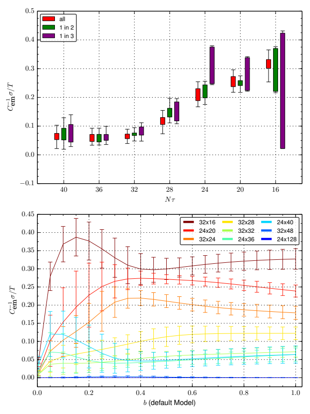
[PRL **111** 172001 (2013)] — 2nd generation

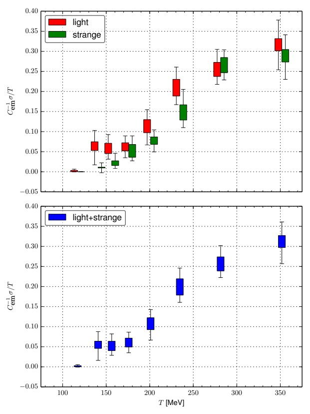
$$\sigma = \lim_{\omega \to 0} \frac{\rho_{ii}^{em}(\omega)}{6\omega}$$

Used conserved vector current: no renormalisation required



Conductivity results





[Note that narrow transport peak at low T cannot be ruled out]

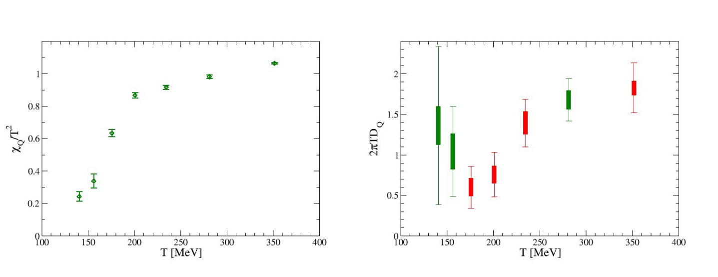
Charge susceptibility and diffusion

Electric charge susceptibility is given by

$$\chi_Q = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial \mu_Q^2} = \sum_{i,j=u,d,s} q_i q_j \chi_{ij}$$

Analogous definitions for isospin, baryon number susceptibilites
Related to event-by-event fluctuations in heavy-ion collisions

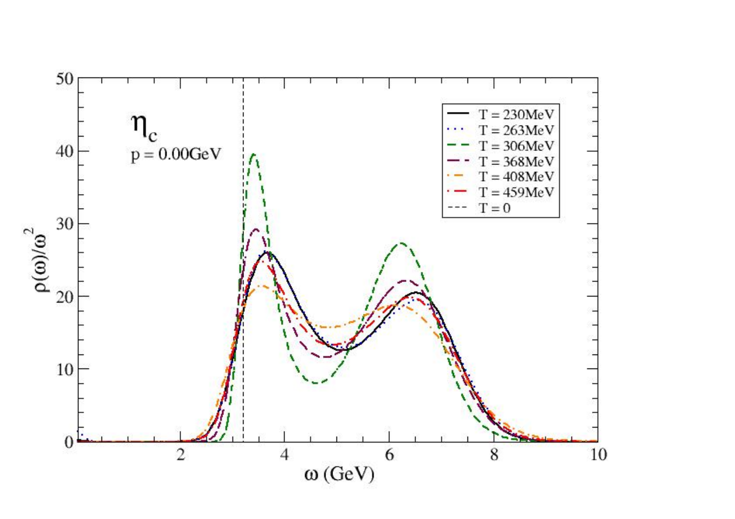
Charge diffusion related to conductivity, susceptibility: $D_Q = \sigma/\chi_Q$



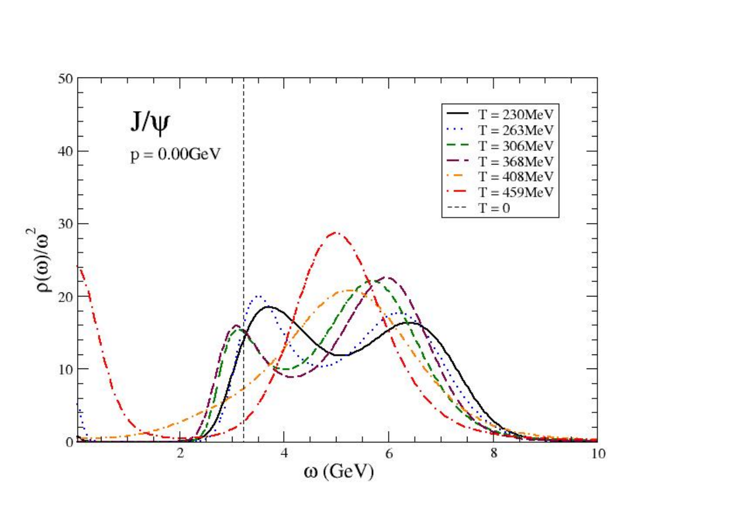
Charm

- J/ψ suppression — a probe of the quark–gluon plasma? [Matsui & Satz 1986]
- c quarks created in primordial collisions, hard probes?
- To what extent do c quarks thermalise?
- How reliable are quenched lattice simulations?
- Are potential models valid?

S-wave T dependence (η_c)

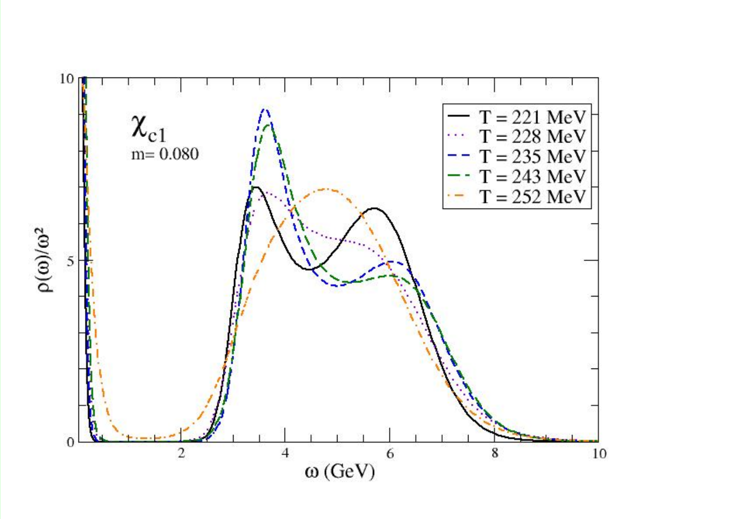


S-wave T dependence (J/ψ)



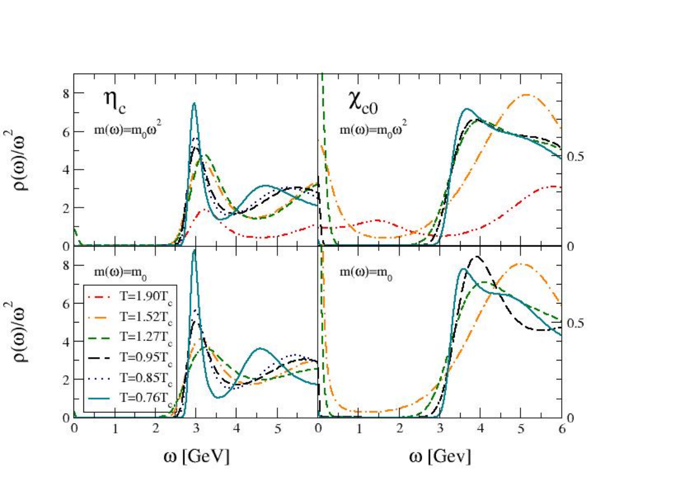
J/ψ (S-wave) melts at $T \sim 370 - 400$ MeV or $1.7 - 1.9T_c$?

P-waves



P-waves melt at $T < 250$ MeV or $1.2T_c$?

Second generation [Preliminary]



Consistent with 1st generation results!

Nonzero momentum

[With MB Oktay, arXiv:1005.1209; A Kelly et al, in progress]

- Charmonium is produced at nonzero momentum
- Transverse momentum (and rapidity) distributions important to distinguish between models
- Momentum dependent binding?
- Gives an additional window to transport properties
- Related to screening masses

Reconstructed correlators

Reconstructed correlator is defined as

$$G_r(\tau; T, T_r) = \int_0^\infty \rho(\omega; T_r) K(\tau, \omega, T) d\omega$$

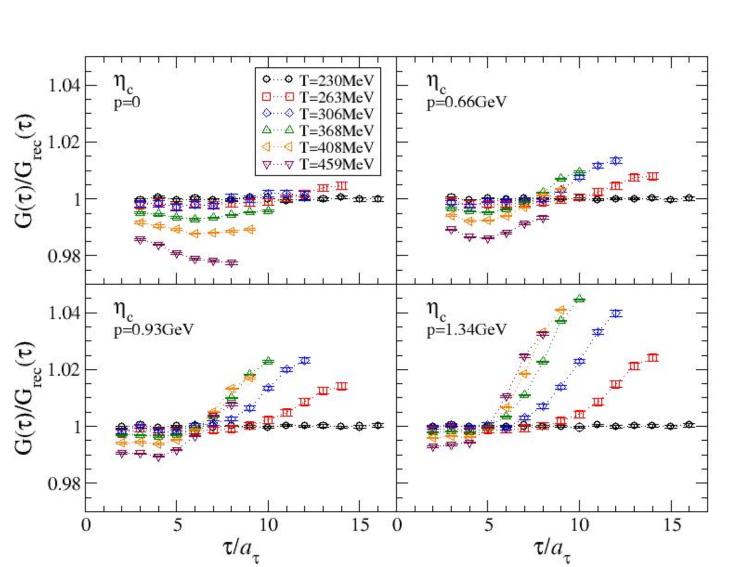
where K is the kernel

$$K(\tau, \omega, T) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

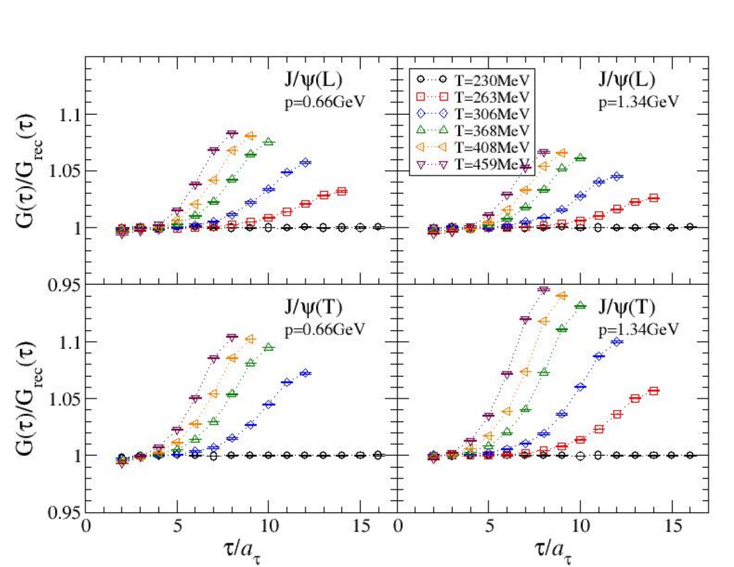
If $\rho(\omega; T) = \rho(\omega; T_r)$ then $G_r(\tau; T, T_r) = G(\tau; T)$

We use $N_\tau = 32$ as our reference temperature for Gen1 since the spectral function is most reliably determined there

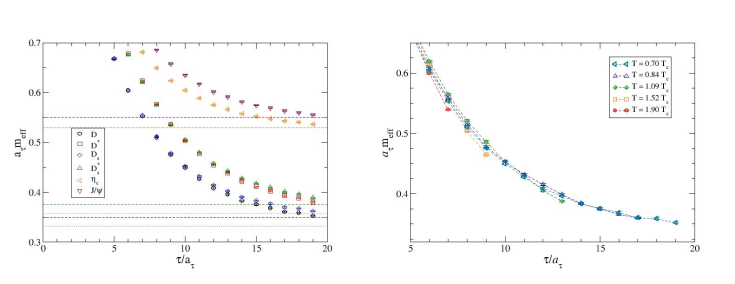
Reconstructed correlators



Reconstructed correlators



D mesons [Preliminary!]



- Small but significant thermal effects in D correlators
- Apparently similar in magnitude to charmonium system
- Spectral analysis in progress

Charmonium potential

[PWM Evans, CR Allton, JIS, PRD **89** 071502 (2014); arXiv:1309.3415]

- Potential models are widely used to study quarkonia
- At $T = 0$ the potential between two infinitely heavy quarks can be uniquely determined from the euclidean Wilson loop
- At $T > 0$ this gives the free energy $F_{qq} = U_{qq} - TS$ — no direct relation to potential in Schrödinger equation!
- Potential may be derived in effective theories [Laine, Brambilla, Petrezcky et al] $\longrightarrow$ real and imaginary parts
- Recent progress in determining potential nonperturbatively from spectral functions of Wilson loops [Rothkopf&Burnier]

Charmonium potential

[PWM Evans, CR Allton, JIS, PRD **89** 071502 (2014); arXiv:1309.3415]

- Potential models are widely used to study quarkonia
- At $T = 0$ the potential between two infinitely heavy quarks can be uniquely determined from the euclidean Wilson loop
- At $T > 0$ this gives the free energy $F_{qq} = U_{qq} - TS$ — no direct relation to potential in Schrödinger equation!
- Potential may be derived in effective theories [Laine, Brambilla, Petrezcky et al] $\longrightarrow$ real and imaginary parts
- Recent progress in determining potential nonperturbatively from spectral functions of Wilson loops [Rothkopf&Burnier]

Almost no results for finite quark masses

Reverse-engineering Schrödinger equation

Define point-split correlators

$$C_\Gamma(\mathbf{r},\tau) = \sum_{\mathbf{x}} \langle \bar{q}(\mathbf{x},\tau)\Gamma U(x,x+\mathbf{r})q(\mathbf{x}+\mathbf{r},\tau)\ \bar{q}(0)\bar{\Gamma}q(0)\rangle$$
$$= \sum_j \frac{\psi_j^*(\mathbf{0})\psi_j(\mathbf{r})}{2E_j}\ \left(e^{-E_j\tau} + e^{-E_j(N_\tau-\tau)}\right),$$

Schrödinger equation for NBS wavefunctions $\psi_j(\mathbf{r})$:

$$\left[-\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + V_\Gamma(r)\right]\psi_j(r) = E_j\psi(r), \qquad \mu = \frac{m_c}{2} \approx \frac{M_{J/\psi}}{4}$$

Ignoring the backward mover, we see that

$$\frac{\partial C_\Gamma(r,\tau)}{\partial\tau} = \sum_j \left(\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} - V_\Gamma(r)\right)\frac{\psi_j^*(0)\psi_j(r)}{2E_j}e^{-E_j\tau}$$
$$= \left(\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} - V_\Gamma(r)\right)C_\Gamma(r,\tau).$$

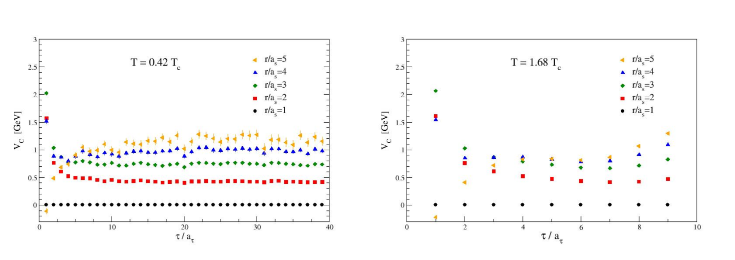
Central and spin-dependent potential

In general, for S-waves

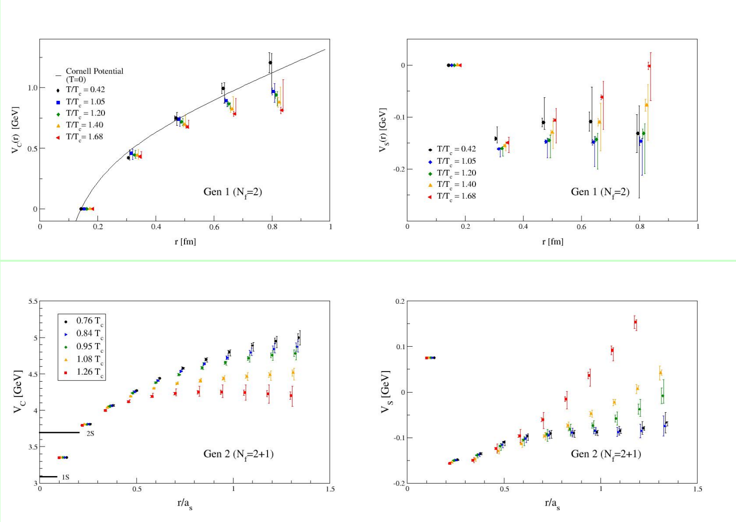
$$V_\Gamma(r) = V_C(r) + \mathbf{s}_1 \cdot \mathbf{s}_2 \; V_S(r)$$

$$\Longrightarrow \quad V_C(r) = \frac{3}{4} V_V(r) + V_{PS}(r) \,, \qquad V_S(r) = V_V(r) - V_{PS}(r)$$

1st generation results

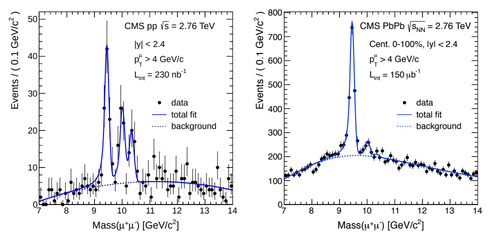


Potential results



Beauty (and the beast?)

- Many b quarks are produced at LHC
- Cold nuclear matter effects, recombination less important
  $\rightarrow$ cleaner probes?
- $T_d^{\Upsilon} \sim 3-5T_c$ — hard to do on the lattice
- $\chi_b, \Upsilon(2S)$ melt at $T_d' \lesssim 1.2T_c$?
- Sequential suppression observed at CMS (+ ATLAS, STAR)?



NRQCD

Scale separation $M_Q \gg T, M_Q v$

Integrate out hard scales $\longrightarrow$ Effective theory

Expand in orders of heavy quark velocity $\mathbf{v}$; we use $\mathcal{O}(\mathbf{v}^4)$ action

Advantages

- No temperature-dependent kernel, $G(\tau) = \int \rho(\omega) e^{-\omega\tau} \frac{d\omega}{2\pi}$
- No zero-modes
- Longer euclidean time range
- Appropriate for probes not in thermal equilibrium

Disadvantages

- Not renormalisable, requires $Ma_s \gtrsim 1$
- Does not incorporate transport properties

Correlators

Bound state

$$G(\tau) \sim e^{-\Delta E \tau}$$

Effective mass $a_\tau m_{\text{eff}}(\tau) = \log(G(\tau - a_\tau)/G(\tau))$

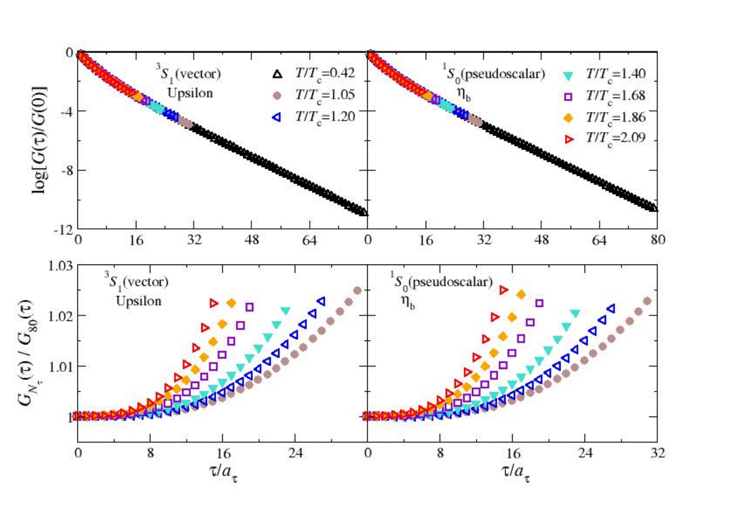
Noninteracting quarks

S-waves: $G_S(\tau) \sim e^{-\omega_0 \tau} \tau^{-3/2}$

P-waves: $G_P(\tau) \sim e^{-\omega_0 \tau} \tau^{-5/2}$

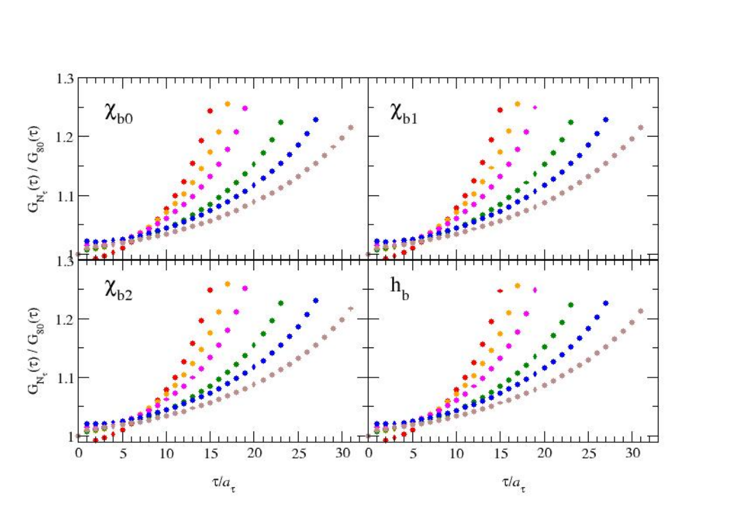
ω_0 is threshold representing additive energy shift.
Effective power $\alpha_{\text{eff}}(\tau) = -\tau G'(\tau)/G(\tau)$

Correlator ratios (S-waves)



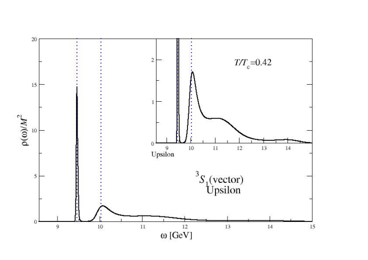
Note: Changes are entirely due to changes in spectral density

Correlator ratios (P-waves)

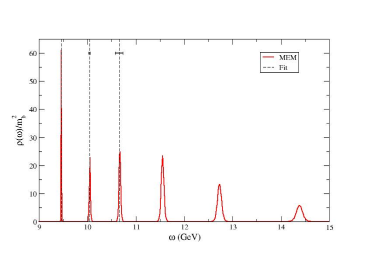


Spectral functions — $T = 0$

1st generation [JHEP **1111** 103 (2011)]

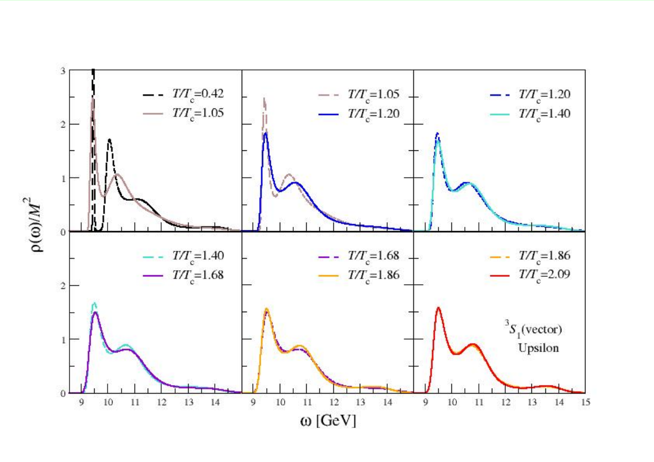


2nd generation [JHEP **1407** 097 (2014)]]

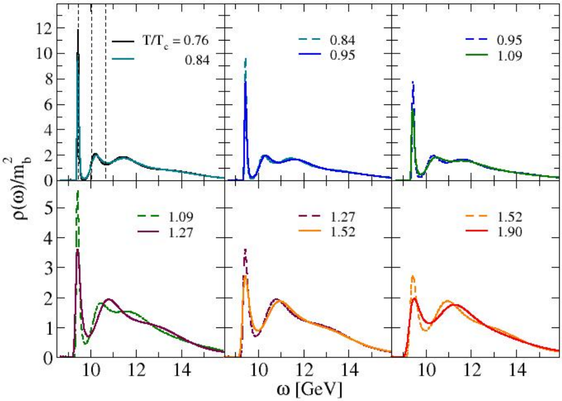


ϒ (1S), ϒ (2S) clearly identified
[3rd peak does not coincide with physical ϒ (3S)]

Spectral functions — First generation



Spectral functions — Second generation

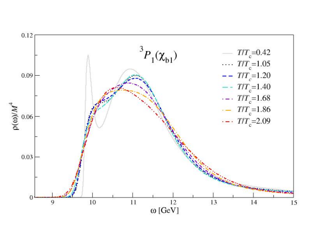


ϒ (2S) melts, but ground state remains robust

P-waves

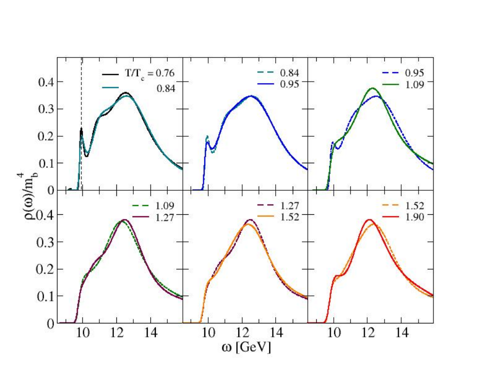
1st generation

[JHEP **1312** 064 (2013)]



2nd generation

[JHEP **1407** 097 (2014)]



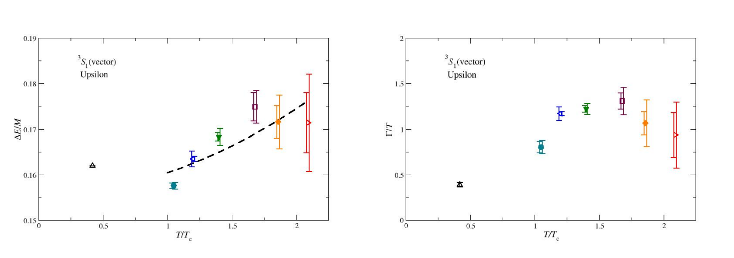
P-waves dissociate close to T_c

Mass shift and width

Fit (left side of) peaks to gaussian

$\longrightarrow$ determine peak position (mass) and width

Width is upper bound



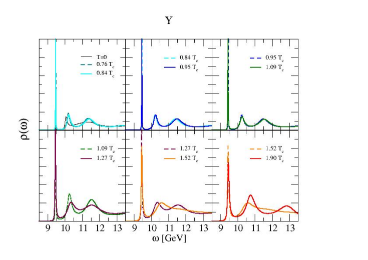
Results are consistent with perturbation theory,

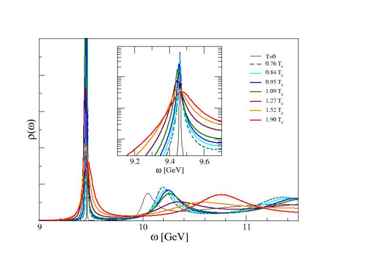
$$\frac{\Gamma}{T} = \frac{1156}{81}\alpha_s^3\,, \qquad \frac{\delta E}{M} = \frac{17\pi}{9}\alpha_s T^2 M^2\,,$$

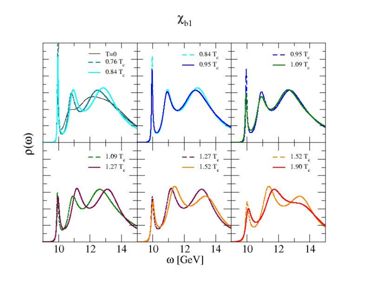
with $\alpha_s \sim 0.4$.

Burnier–Rothkopf method [Preliminary!]

[See also talk by Alexander Rothkopf, Thu 1430]







P-wave appears to survive to higher T?
Analysis of mass shift and width in progress

Summary

- Anisotropic lattices provide a wealth of information on QGP phenomenology

Summary

- Anisotropic lattices provide a wealth of information on QGP phenomenology
- Conductivity increasing with T across the transition
- Charge diffusion has dip near T_c

Summary

- Anisotropic lattices provide a wealth of information on QGP phenomenology
- Conductivity increasing with T across the transition
- Charge diffusion has dip near T_c
- Charmonium S-waves survive to $T \sim 1.6 - 2T_c$
- P-waves melt at $T < 1.3T_c$
- Significant momentum dependence in reconstructed corrs
- Charmonium potential is screened at high T

Summary

- Anisotropic lattices provide a wealth of information on QGP phenomenology
- Conductivity increasing with T across the transition
- Charge diffusion has dip near T_c
- Charmonium S-waves survive to $T \sim 1.6 - 2T_c$
- P-waves melt at $T < 1.3T_c$
- Significant momentum dependence in reconstructed corrs
- Charmonium potential is screened at high T
- Beautonium S-wave ground states survive up to $T \gtrsim 2T_c$
- Mass shift and width consistent with perturbation theory
- P-waves and excited states dissociate close to T_c?

Summary

- Anisotropic lattices provide a wealth of information on QGP phenomenology
- Conductivity increasing with T across the transition
- Charge diffusion has dip near T_c
- Charmonium S-waves survive to $T \sim 1.6 - 2T_c$
- P-waves melt at $T < 1.3T_c$
- Significant momentum dependence in reconstructed corrs
- Charmonium potential is screened at high T
- Beautonium S-wave ground states survive up to $T \gtrsim 2T_c$
- Mass shift and width consistent with perturbation theory
- P-waves and excited states dissociate close to T_c?
- 2+1 flavours with larger anisotropy planned $\rightarrow$ higher T
- Further analysis of spectral reconstruction systematics in progress