Nonuniform phases in the 't Hooft extended Nambu–Jona-Lasinio Model

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QCHS XI

Outline

1. Introduction and formalism
   - Nambu–Jona-Lasinio Model
   - Nonuniform phase ansatz
   - Thermodynamic potential
   - Regularization
2. Results
   - NJL case
   - NJLH case
3. Conclusions
   - Final slide

Nambu–Jona-Lasinio Model

QCD: the very successfull Theory of **Strong Interactions**

- Non-perturbative low energy regime
- **Dynamical Chiral Symmetry Breaking (DχSB)** plays an important role in low energy phenomenology

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NJL: effective model for the non-perturbative low energy regime of **QCD** with Dynamical Chiral Symmetry Breaking (**DχSB**)

- **NJL** shares the global symmetries with **QCD**
- Dynamical generation of the **constituent mass**
- Light pseudoscalar as (quasi) **Nambu-Goldstone boson**
- **Quark condensates** as order parameter
- No gluons (no confinement/deconfinement)
- Local and non renormalizable

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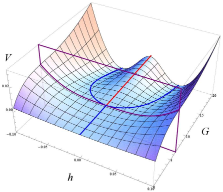
- **Nambu–Jona-Lasinio** (4 q)

$$\mathcal{L}_{NJL} = \frac{G}{2}\left[\left(\overline{q}\lambda_a q\right)^2 + \left(\overline{q}\imath\gamma_5\lambda_a q\right)^2\right]$$



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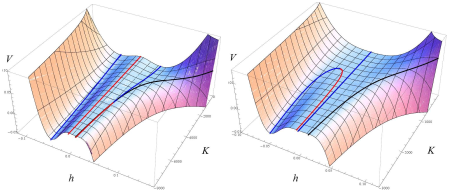
- $\mathcal{L}_{NJL} = \frac{G}{2}\left[(\overline{q}\lambda_a q)^2 + (\overline{q}\imath\gamma_5\lambda_a q)^2\right]$
- **'t Hooft determinant** (6 q)

$$\mathcal{L}_H = K\left(\det\overline{q}P_L q + \det\overline{q}P_R q\right)$$



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OZI violation in $\mathcal{L}_H$.

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  - Light $\langle\overline{\psi_l}\psi_l\rangle = \frac{h_l}{2}\cos(\boldsymbol{q}\cdot\boldsymbol{r}), \quad \langle\overline{\psi_l}i\gamma_5\tau_3\psi_l\rangle = \frac{h_l}{2}\sin(\boldsymbol{q}\cdot\boldsymbol{r})$
  - Strange $\langle\overline{\psi_s}\psi_s\rangle = \frac{h_s}{2}, \quad \langle\overline{\psi_s}i\gamma_5\psi_s\rangle = 0$
- Energy spectrum
  - Light $E^{\pm} = \sqrt{M^2 + p^2 + \frac{q^2}{4} \pm \sqrt{(\boldsymbol{p}\cdot\boldsymbol{q})^2 + M^2q^2}}$
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  - Strange $E = \sqrt{M^2 + p^2}$
- Nontrivial interplay due to flavor mixing

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Thermodynamic potential and stationary phase conditions[1]

Integrating the gap equations we get:

$$\Omega = V_{st} + \frac{N_c}{8\pi^2}\left(J_{-1}(M_u, \mu_u, q) + J_{-1}(M_d, \mu_d, q) + J_{-1}(M_s, \mu_s, 0)\right)$$

$$V_{st} = \frac{1}{16}\left(4G\left(h_u^2 + h_d^2 + h_s^2\right) + \kappa h_u h_d h_s\right)\Big|_0^{M_i}$$

$$\begin{cases} m_u - M_u = G h_u + \frac{\kappa}{16} h_d h_s \\ m_d - M_d = G h_d + \frac{\kappa}{16} h_u h_s \\ m_s - M_s = G h_s + \frac{\kappa}{16} h_u h_d \end{cases}$$

[1] For details see: Phys. Rev. D 89, 036009 (2014); 1312.4942 [hep-ph]

Pauli-Villars regulator: $\hat{\rho} = 1 - (1 + s\Lambda^2)\exp(-s\Lambda^2)$

$$J_{-1} = J_{-1}^{vac} + J_{-1}^{med}$$

$$J_{-1}^{vac} = \int \frac{\mathrm{d}^4 p_E}{(2\pi)^4} \int_0^\infty \frac{\mathrm{d}s}{s} \hat{\rho}\, 8\pi^2 e^{-s\left(p_{0\,E}^2 + p_\perp^2\right)} \left(e^{-s\left(\frac{q}{2} + \sqrt{M^2 + p_z^2}\right)^2} + e^{-s\left(\frac{q}{2} - \sqrt{M^2 + p_z^2}\right)^2} \right) \Bigg|_{0,0}^{M,q}$$

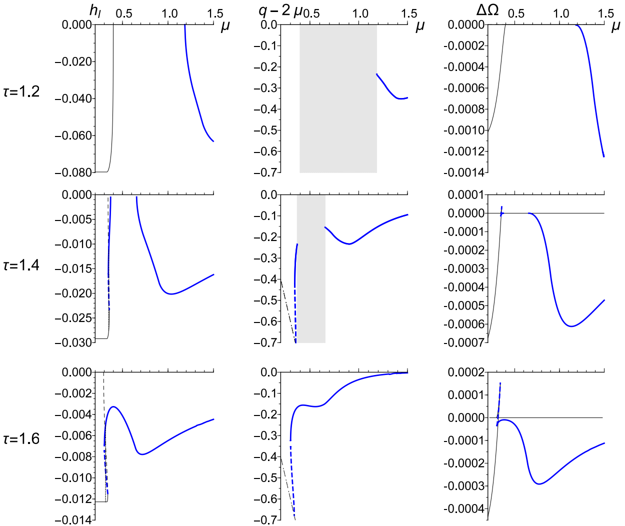
$$J_{-1}^{med} = -\int \frac{\mathrm{d}^3 p}{(2\pi)^3} 8\pi^2 T \left(\mathcal{Z}_+^+ + \mathcal{Z}_-^+ + \mathcal{Z}_+^- + \mathcal{Z}_-^- \right) \Big|_{0,0}^{M,q} + C(T,\mu)$$

$$\mathcal{Z}_\pm^\pm = \log\left(1 + e^{-\frac{E^\pm \mp \mu}{T}}\right) - \log\left(1 + e^{-\frac{E_\Lambda^\pm \mp \mu}{T}}\right) - \frac{\Lambda^2}{2TE_\Lambda^\pm} \frac{e^{-\frac{E_\Lambda^\pm \mp \mu}{T}}}{1 + e^{-\frac{E_\Lambda^\pm \mp \mu}{T}}}$$

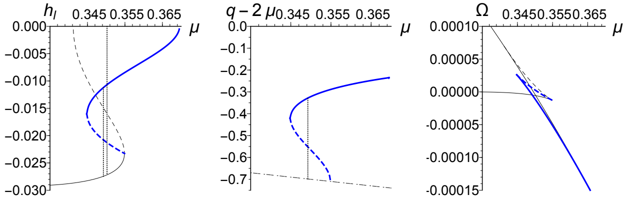
$$C(T,\mu) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} 16\pi^2 T \log\left(\left(1 + e^{-\frac{|\mathbf{p}| - \mu}{T}}\right) \left(1 + e^{-\frac{|\mathbf{p}| + \mu}{T}}\right) \right)$$

- $E_\Lambda^\pm = \sqrt{(E^\pm)^2 + \Lambda^2}$
- $|_{0,0}^{M,q}$: subtraction of the same quantity evaluated for $M = 0$ and $q = 0$
- $q \to 0$ recovers uniform case integrals

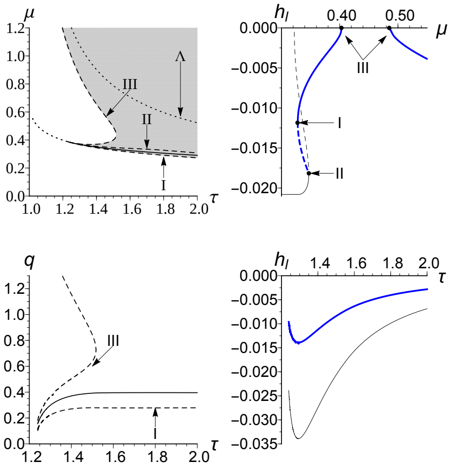
NJL: gap solutions for several values of $\tau = \frac{N_c}{2\pi^2} G\Lambda^2$



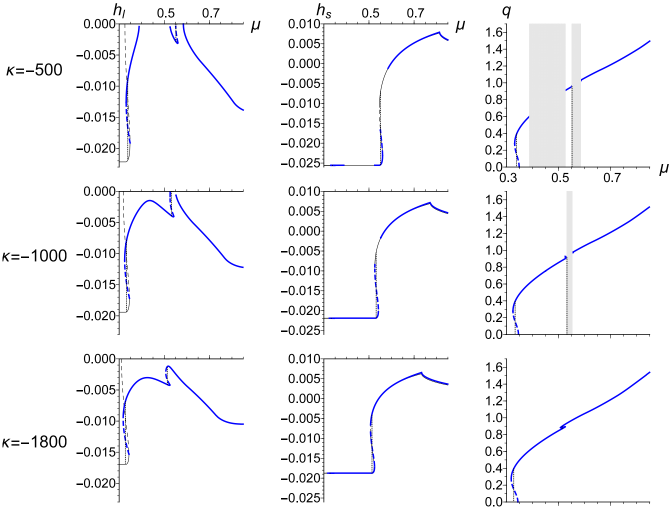
NJL: gap solutions (zoom for $\tau = 1.4$)



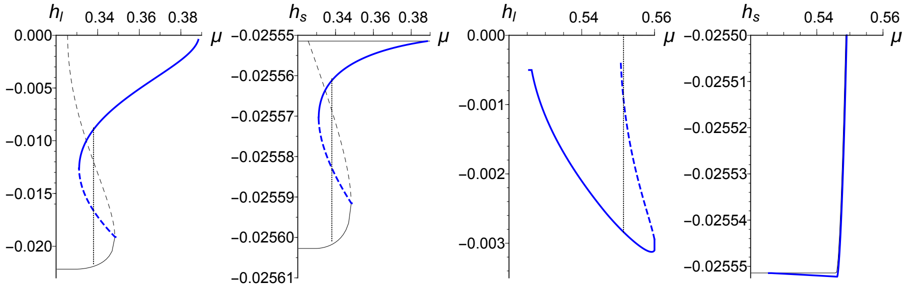
NJL: μ_{crit}


μ
1.2
1.0
0.8
0.6
0.4
0.2
0.0
1.0 1.2 1.4 1.6 1.8 2.0
τ
Λ
III
II
I
h_l
0.40 0.50
μ
0.000
−0.005
−0.010
−0.015
−0.020
III
I
II
q
1.2
1.0
0.8
0.6
0.4
0.2
0.0
1.2 1.4 1.6 1.8 2.0
τ
III
I
h_l
1.4 1.6 1.8 2.0
τ
0.000
−0.005
−0.010
−0.015
−0.020
−0.025
−0.030
−0.035

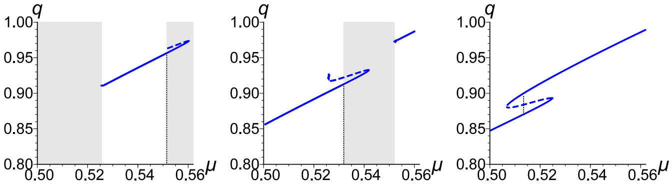
NJLH ($m_s = 186$ MeV$, \tau = 1.4$): gap solutions



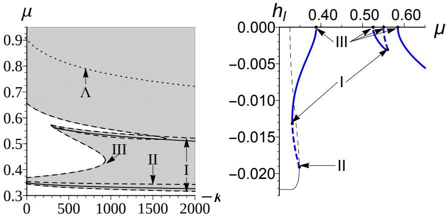
NJLH ($m_s = 186$ MeV, $\tau = 1.4$, $\kappa = -500$ GeV^{-5}): condensate solutions zoom



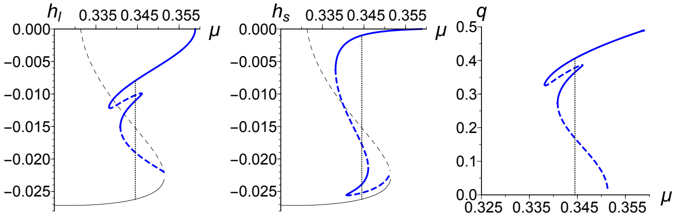
NJLH ($m_s = 186$ MeV, $\tau = 1.4$, $\kappa = -500$, -1000, -1800 GeV^{-5}):q solutions zoom



NJLH ($m_s = 186$ MeV, $\tau = 1.4$): μ_{crit}



NJLH chiral limit ($\tau = 1.4$): gap solutions



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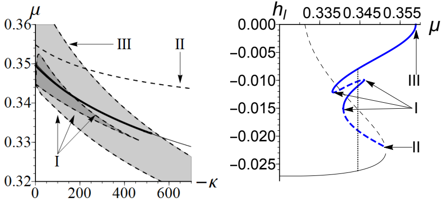

Conclusions

- New window for inhomogeneous solutions in the vicinity of M_s
- ’t Hooft coupling can change the position and extent of this window
- A third one exists at yet higher chemical potential (not new)
- In the asymptotic regime $q \to 2\mu$
- For $\kappa > \kappa_c$, there is a 1st order transition $q_1 \to q_2$
- In the chiral limit the branches overlap with the possibility of a window where $h_s < h_l$

For more information:

- J. Moreira, B. Hiller, W. Broniowski, A. A. Osipov, A.H. Blin, *Nonuniform phases in a three-flavor Nambu–Jona-Lasinio model*, Phys. Rev. D 89, 036009 (2014); 1312.4942 [hep-ph]