From Jet Quenching to Wave Turbulence

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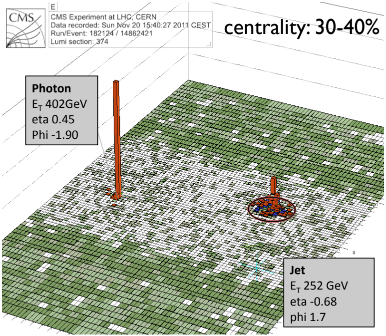
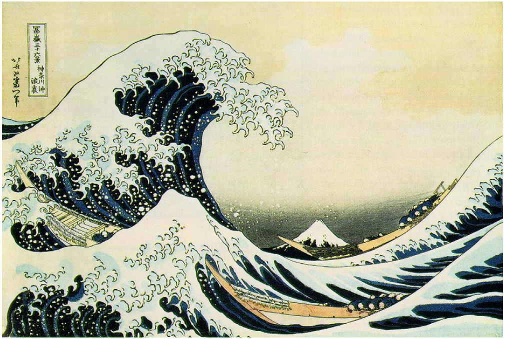
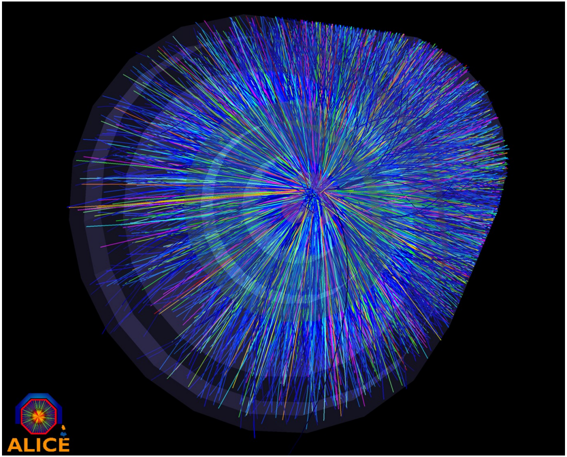
IPhT Saclay & CNRS





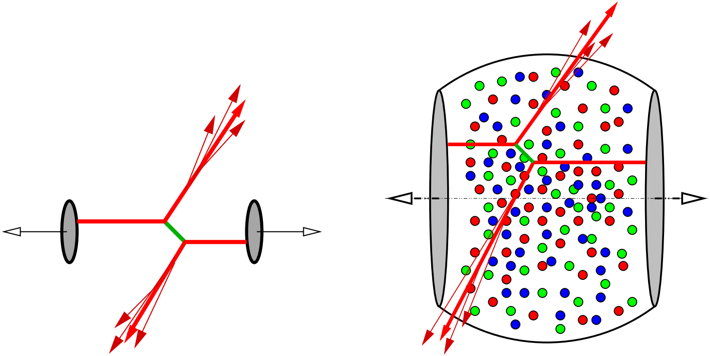
Heavy ion collisions @ the LHC



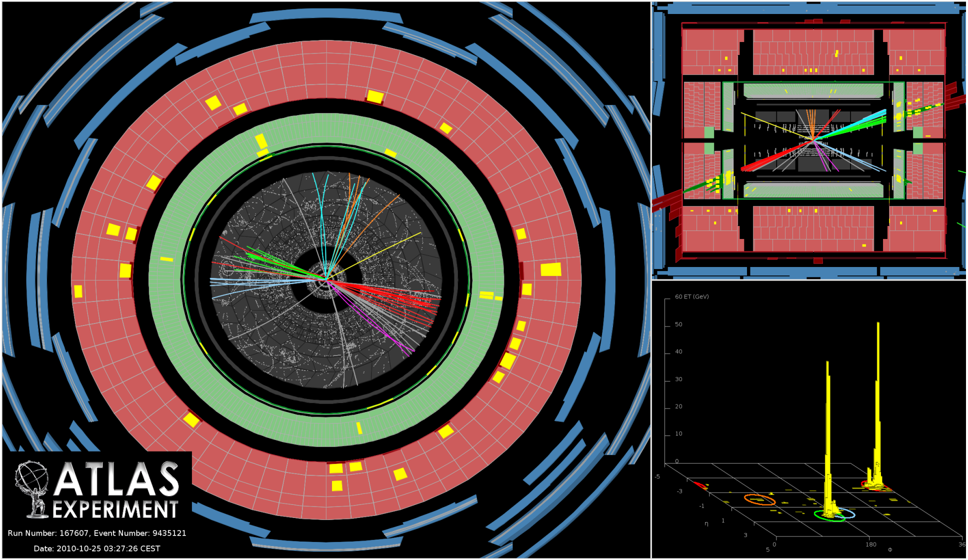
- Pb+Pb collisions at the LHC: ~ 1600 hadrons at central rapidity
- They carry information about the early partonic stages
- Best appreciated if one uses p+p as a benchmark

Hard probes

- Hard partons are typically created in pairs which propagate back–to–back in the transverse plane
- Particle production can be modified by the surrounding medium



- The ensemble of these modifications : 'jet quenching'
- 'Jet': 'leading particle' + 'products of fragmentation'


ATLAS
EXPERIMENT
Run Number: 167607, Event Number: 9435121
Date: 2010-10-25 03:27:26 CEST
60 ET (GeV)
50
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-5
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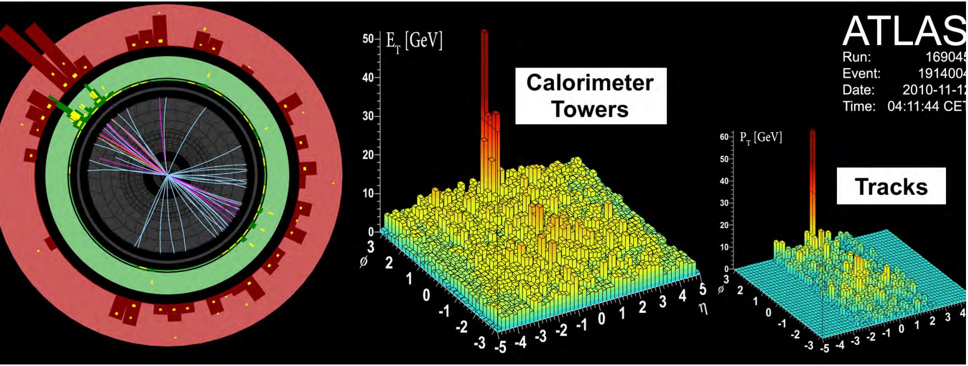
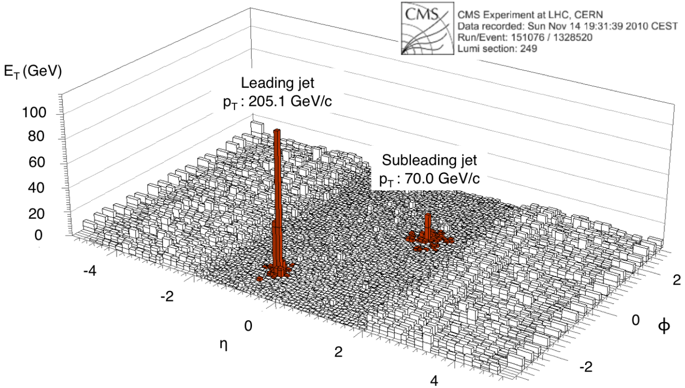
Φ

Di–jet asymmetry *(ATLAS)*



- Central Pb+Pb: 'mono–jet' events
- The secondary jet cannot be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV

Di–jet asymmetry *(CMS)*

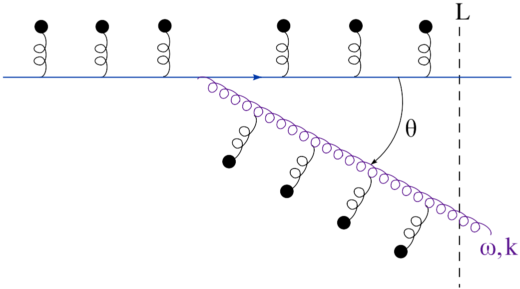


- Additional energy imbalance as compared to p+p : 20 to 30 GeV
- Compare to the typical scale in the medium: $T \sim 1$ GeV (average $p_\perp$)
- Detailed studies show that the 'missing energy' is carried by many soft ($p_\perp < 2$ GeV) hadrons propagating at large angles

pQCD : the BDMPSZ mechanism

Baier, Dokshitzer, Mueller, Peigné, and Schiff; Zakharov (96–97)
Wiedemann (2000); Arnold, Moore, and Yaffe (2002–03); ...

- Additional gluon radiation triggered by interactions in the medium



- Originally developed for a single gluon emission (energy loss by the LP)
- The LHC data call for a global understanding of the jet evolution
- Recent extension of the theory to multiple medium–induced emissions

  Blaizot, Dominguez, E.I., Mehtar-Tani (2012–13)

Transverse momentum broadening

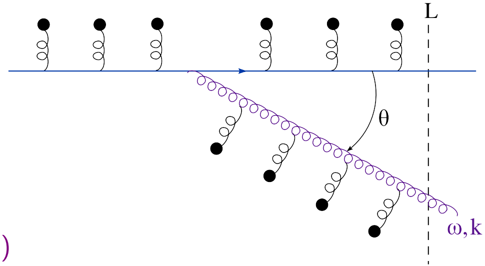
- Gluon emission is linked to transverse momentum broadening
  - transverse kicks provide acceleration and thus allow for radiation
  - they increase the emission angle θ
  - they occur randomly $\Longrightarrow$ Brownian motion in $k_\perp$

$$\langle k_\perp^2 \rangle \simeq \hat{q}\,\Delta t$$

$$\hat{q} \simeq \frac{m_D^2}{\lambda} = \frac{(\text{Debye mass})^2}{\text{mean free path}}$$

'jet quenching parameter'

(quasi–local transport coefficient)



- Gluon emissions require a formation time $\tau_f \simeq \omega/k_\perp^2$
- During formation, the gluon acquires a momentum $k_\perp^2 \sim \hat{q}\tau_f$

Formation time & emission angle

$$\tau_f \simeq \frac{\omega}{k_\perp^2} \quad \& \quad k_\perp^2 \simeq \hat{q}\tau_f \quad \Longrightarrow \quad \tau_f \simeq \sqrt{\frac{\omega}{\hat{q}}}$$

- Maximal ω for this mechanism: $\tau_f \le L \Rightarrow \omega \le \omega_c \equiv \hat{q}L^2$

  ▷ soft gluons ($\omega \ll \omega_c$) have small formation times: $\tau_f \ll L$

- The emission angle keeps increasing with time, via rescattering

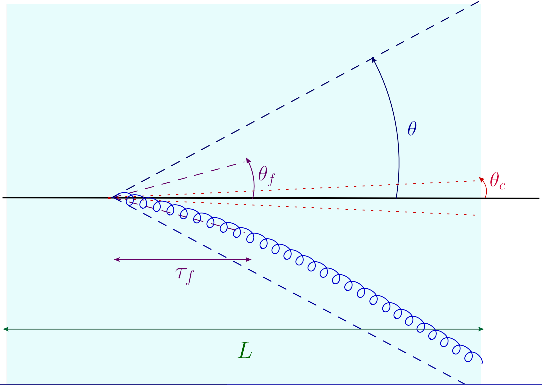
$$\theta(\omega) \simeq \frac{\sqrt{\hat{q}L}}{\omega} > \theta_f(\omega)$$

maximal energy ⇔ minimal angle

$$\theta_c \equiv \theta(\omega_c)$$

soft gluons ⇔ large angles

$$\omega \ll \omega_c \Longrightarrow \theta \gg \theta_c$$



Emission probability

- Spectrum : Bremsstrahlung $\times$ average number of emissions

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \alpha \frac{L}{\tau_f(\omega)} \simeq \alpha \sqrt{\frac{\omega_c}{\omega}} \qquad (\omega_c = \hat{q}L^2)$$

- LPM effect : the emission rate decreases with increasing ω

  (from Landau, Pomeranchuk, Migdal, within QED)

  - coherence: many collisions contribute to a single, hard, emission
  - formation time $\tau_f(\omega) \gg$ mean free path λ

- Energy loss by the leading particle :

$$\Delta E = \int^{\omega_c} \mathrm{d}\omega \, \omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \sim \alpha\omega_c$$

  - integral dominated by its upper limit $\omega = \omega_c$ (hard emission)
  - rare event : probability of $\mathcal{O}(\alpha)$
  - small emission angle $\theta_c \Rightarrow$ the energy remains inside the jet

Soft emissions at large angles

- Spectrum : Bremsstrahlung $\times$ average number of emissions

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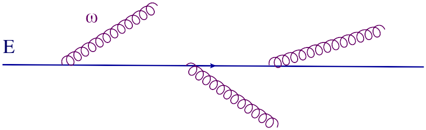
- Relatively soft emissions with $\omega \ll \omega_c$:
  - small formation times : $\tau_f \ll L$
  - quasi–deterministic : probability of $\mathcal{O}(1)$ for $\omega \lesssim \omega_s \equiv \alpha^2\omega_c$
  - a relatively smaller contribution to the energy loss : $\Delta E_s \sim \alpha^2\omega_c$
  - ... but this can be lost at very large angles : $\theta \gtrsim \theta_s \equiv \theta_c/\alpha^2$
- Potentially relevant for the di–jet asymmetry ☺
- When probability of $\mathcal{O}(1) \Longrightarrow$ multiple branchings become important

Multiple branchings

- Multiple 'primary' emissions with $\omega \lesssim \alpha^2\omega_c$ by the leading particle



$$\omega\,\frac{\mathrm{d}N}{\mathrm{d}\omega}\,\simeq\,\alpha\sqrt{\frac{\omega_c}{\omega}}$$

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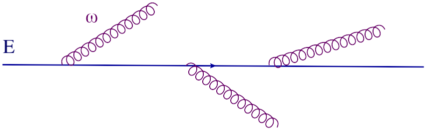
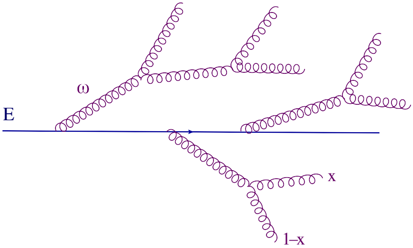


$$\omega\,\frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \alpha\sqrt{\frac{\omega_c}{\omega}}$$

- Each primary gluon develops its own gluon cascade



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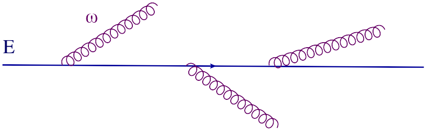

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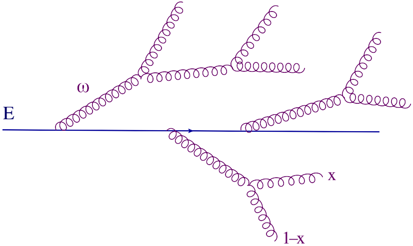
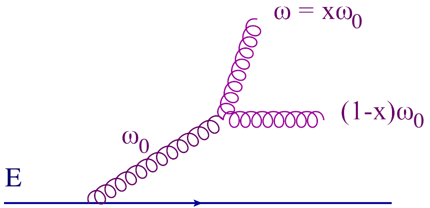


- Their subsequent branchings are quasi–democratic
  - the daughter gluons carry comparable energy fractions: $x \sim 1/2$

Quasi–democratic branchings

- Non–trivial ! Not true for bremsstrahlung in the vacuum !
- Bremsstrahlung in the vacuum : splittings are strongly asymmetric



$$\mathrm{d}N \sim \alpha \frac{\mathrm{d}\omega}{\omega} \sim \alpha \frac{\mathrm{d}x}{x}$$

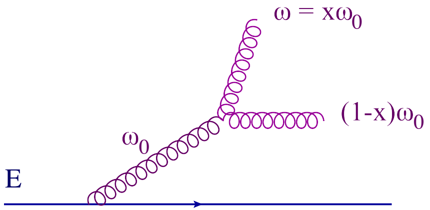
$$\Delta N \sim \alpha \int \frac{\mathrm{d}x}{x} \sim \alpha \ln \frac{1}{x}$$

- probability of $\mathcal{O}(1)$ when $\alpha \ln(1/x) \sim 1 \Longrightarrow$ favors $x \ll 1$
- argument independent of the parent energy ω_0

  ▷ all that matters is the splitting fraction x
- 'soft singularity' $(x \to 0)$ of bremsstrahlung

Quasi–democratic branchings

- In–medium radiation : a consequence of the LPM effect



$$\mathrm{d}N \sim \alpha \frac{\mathrm{d}\omega}{\omega}\sqrt{\frac{\omega_c}{\omega}}$$

$$\sim \alpha \frac{\mathrm{d}x}{x}\sqrt{\frac{\omega_c}{x\omega_0}}$$

- the rate also depends upon the parent gluon energy ω_0
- probability of $\mathcal{O}(1)$ when $\omega_0 \sim \alpha^2\omega_c$ for any value of x
- the phase space favors generic values of x: 'quasi–democratic'

Quasi–democratic branchings

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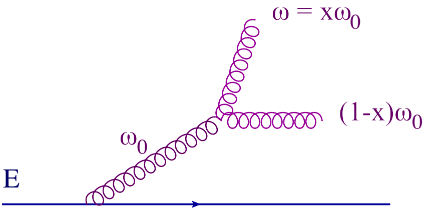

$$\mathrm{d}N \sim \alpha \frac{\mathrm{d}\omega}{\omega}\sqrt{\frac{\omega_c}{\omega}}$$

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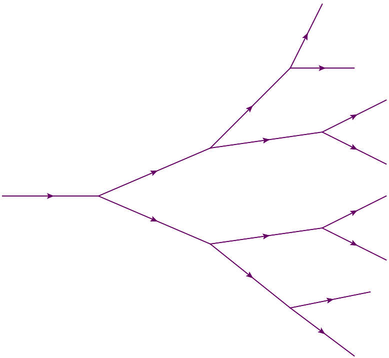
  - the phase space favors generic values of x: 'quasi–democratic'

- A similar scenario at strong coupling *(Y. Hatta, E.I., Al Mueller '08)*
- ... but no other known example in a weakly coupled gauge theory

Wave turbulence

- Democratic branchings imply an energy flux independent of ω (or x)
  - the energy flows from large x to small x w/o accumulating at any intermediate value of x



- the cascade stops when $\omega \sim T$
- gluons with $\omega \sim T$ 'thermalize' (lose their energy towards the medium)
- since very soft, such gluons propagate at very large angles w.r.t. jet axis

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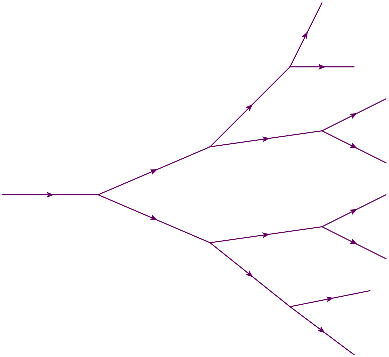
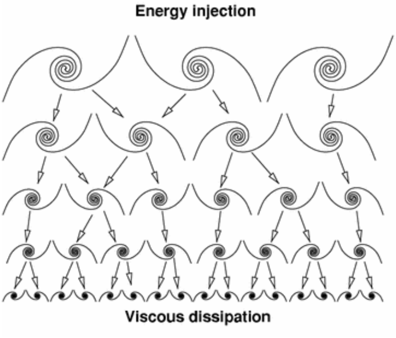
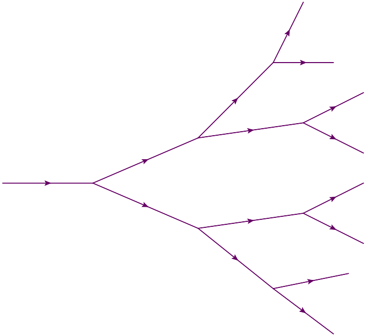
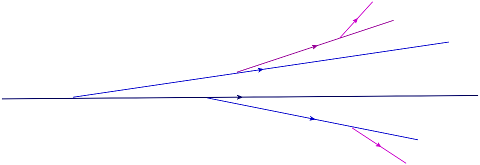




- Uniform flux $\Longleftrightarrow$ turbulent cascade *(Kolmogorov, '41; Zakharov, '92)*
  - the prototype: Richardson cascade for breaking-up vortices

Compare to DGLAP cascade (jet in the vacuum)





- The asymmetric splittings amplify the number of gluons at small x
- Yet, the energy remains in the few partons with larger values of x
- That is, the energy remains at small angles

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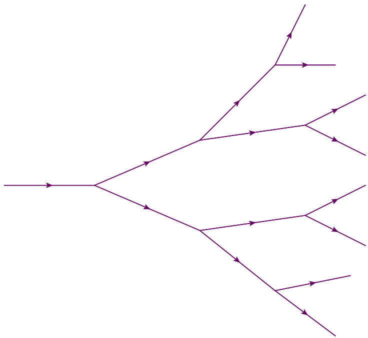
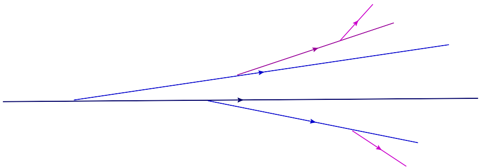


in–medium cascade



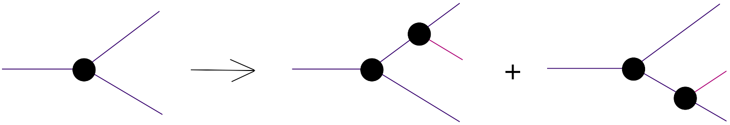
DGLAP cascade

$\tau = \ln Q^2$ ('virtuality')

- The asymmetric splittings amplify the number of gluons at small x
- Yet, the energy remains in the few partons with larger values of x
- That is, the energy remains at small angles
- Di-jet asymmetry strongly suggests a turbulent cascade

The rate equation

- Multiple branching $\approx$ a classical branching process (Markovian)
  - independent splittings with the rate given by BDMPSZ



  - interference effects are suppressed by scattering in the medium
    Mehtar-Tani, Salgado, Tywoniuk; Casalderrey-Solana, E. I. (10 –11)
    Blaizot, Dominguez, E.I., Mehtar-Tani (arXiv: 1209.4585)

- Evolution equation for the gluon spectrum ('rate equation')

$$D(x,t) \equiv x\frac{\mathrm{d}N}{\mathrm{d}x} \quad \text{where} \quad x = \frac{\omega}{E} \quad \text{and} \quad t \le L$$

- Previously conjectured and used for phenomenological studies
  Baier, Mueller, Schiff, Son '01 ('bottom–up thermalization');
  Arnold, Moore, Yaffe, '03; Jeon, Moore '05; MARTINI (McGill)

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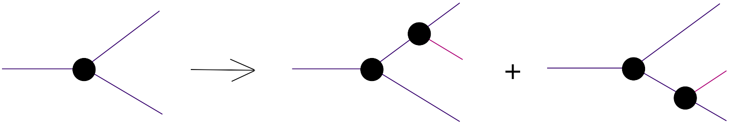

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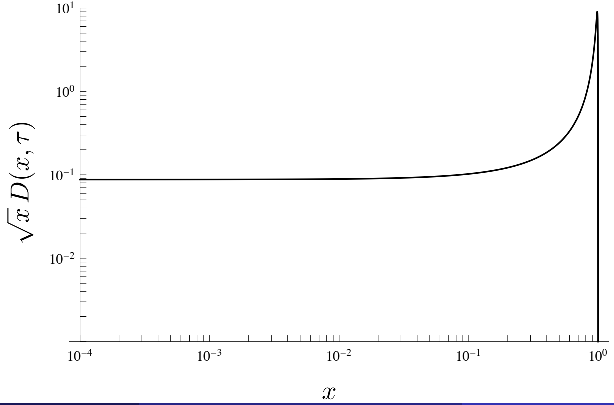
- Turbulence aspects only recently recognized (exact solutions)
  J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)

Small times : the small–x region

- Initial condition: $D(x, \tau = 0) = \delta(x - 1)$ (the leading particle)
- At small times: just one branching $\Longrightarrow$ BDMPSZ spectrum :

$$D^{(1)}(x,\tau) \simeq \alpha \sqrt{\frac{\hat{q}}{\omega}}\, t \equiv \frac{\tau}{\sqrt{x}} \qquad (\tau = \text{dimensionless 'time'})$$

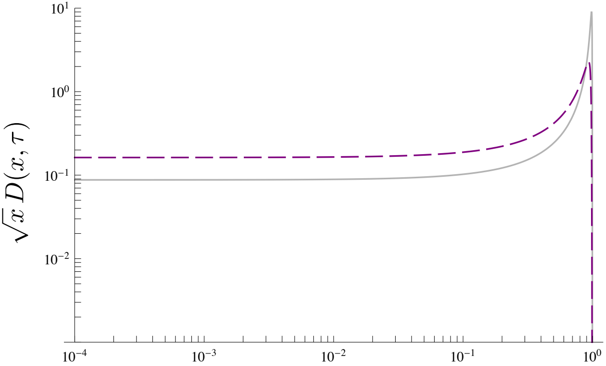


Small times: the leading particle

- Already for small times, multiple branchings are visible via the broadening of the leading particle

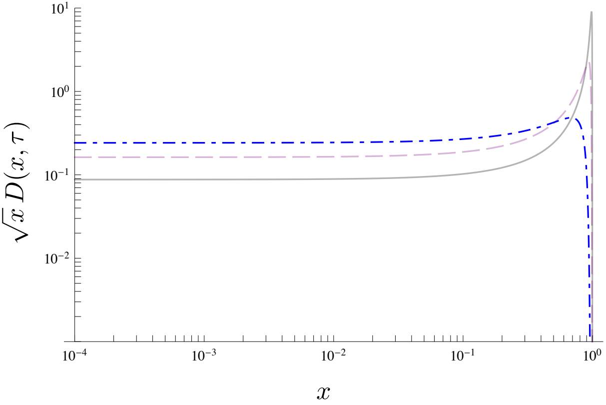
$$D(x,\tau) \,=\, \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\; \mathrm{e}^{-\pi\frac{\tau^2}{1-x}}$$

- Multiple emissions of non–perturbatively soft primary gluons



Larger times: suppression of the LP

- The leading particle peak disappears when $\tau \sim 1$
- Naively : "the energy moves from the LP into the bins at small x"



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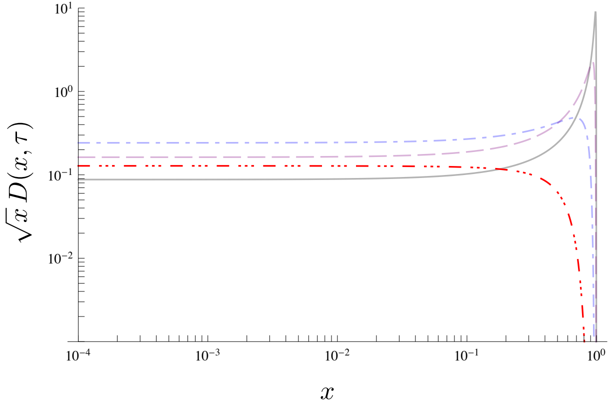

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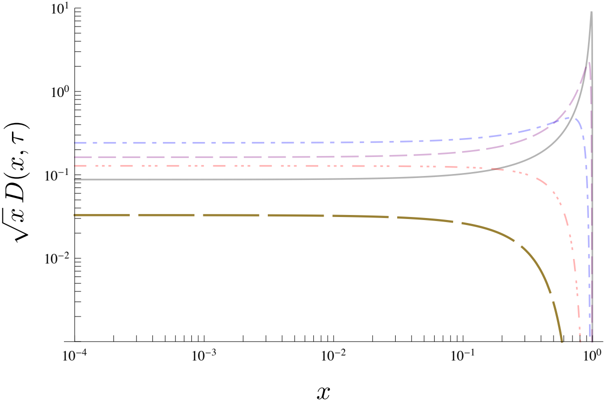
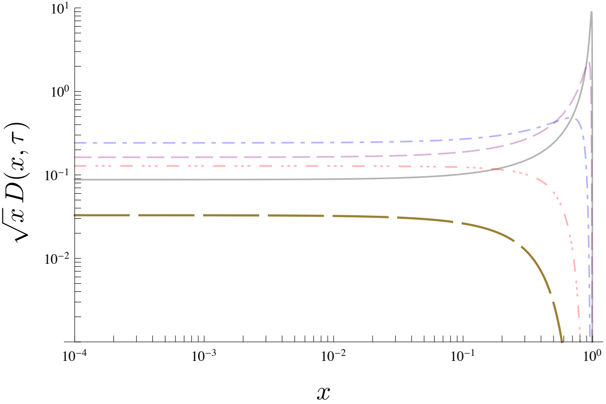


The Kolmogorov–Zakharov fixed point

- The BDMPSZ–like spectrum is preserved by multiple branchings

$$D(x,\tau) \simeq \frac{\tau}{\sqrt{x}}\, \mathrm{e}^{-\pi\tau^2}$$

- KZ fixed point $\Longrightarrow$ energy flux uniform in ω $\Longrightarrow$ turbulence

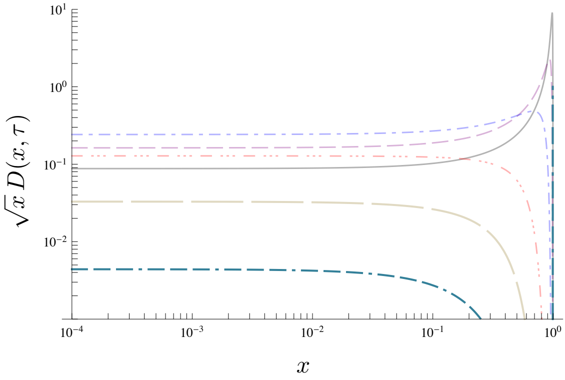


The turbulent flow

- The energy flows out from the spectrum ... exponentially fast :

$$\int_0^1 \mathrm{d}x\, D(x,\tau) = \mathrm{e}^{-\pi\tau^2} \implies \mathcal{E}_{\text{flow}}(\tau) = 1 - \mathrm{e}^{-\pi\tau^2}$$

- Formally, it accumulates into a condensate at $x = 0$



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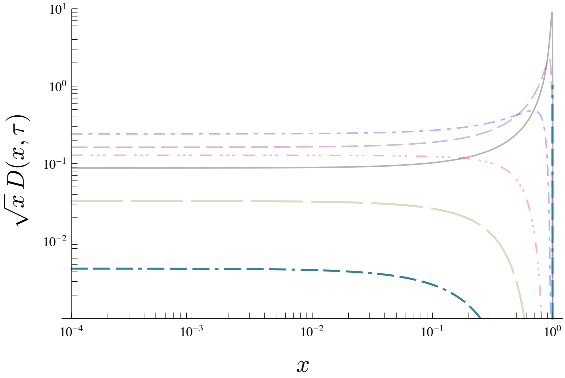
- Physically, it goes below $x_{\rm th} = T/E \ll 1$, meaning it thermalizes



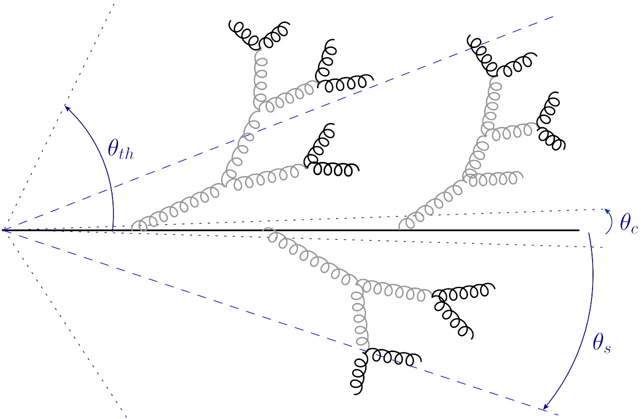
Di–jet asymmetry: energy loss

- Jets at the LHC : high energy kinematics ('small time τ')
  - $E = 100 \div 300$ GeV $\gg$ $\omega_c = \hat{q}L^2 = 10 \div 50$ GeV

$$\mathcal{E}_{\rm flow}(\tau) \simeq \pi\tau^2 \implies \Delta E \equiv E\,\mathcal{E}_{\rm flow} \simeq \upsilon\omega_s$$

  - $\omega_s = \alpha^2\omega_c$: characteristic scale for multiple branching
  - $\upsilon = 2\pi$: average number of primary gluons with energy ω_s
- Typical energy loss (event by event): $\Delta E \simeq 10 \div 20$ GeV ✓
- Universality : the energy lost via turbulent flow is
  - independent of the energy E of the leading particle
  - independent of the details of the thermalization mechanism
  - carried by soft quanta with energies $\omega \sim T \lesssim 1$ GeV
  - ... which propagate at large angles: $\theta \simeq k_\perp/\omega \sim \mathcal{O}(1)$

A typical gluon cascade



- The leading particle emits mostly soft gluons: $\omega \lesssim \omega_s \equiv \alpha^2 \omega_c$
- These primary gluons rapidly split into even softer ones.
- The primary gluons propagate along typical angles $\theta_s \simeq \theta_c/\alpha^2 \sim 0.5$
- The final gluons $(\omega \sim T)$ make even larger angles $\theta_{\rm th} > \theta_s \gtrsim 1$

Di–jet asymmetry: angular dependence

(L. Fister, E. I., september 2014, tomorrow on arXiv)

- $\mathcal{E}(\theta < \theta_0)$: the energy contained within a jet with opening angle θ

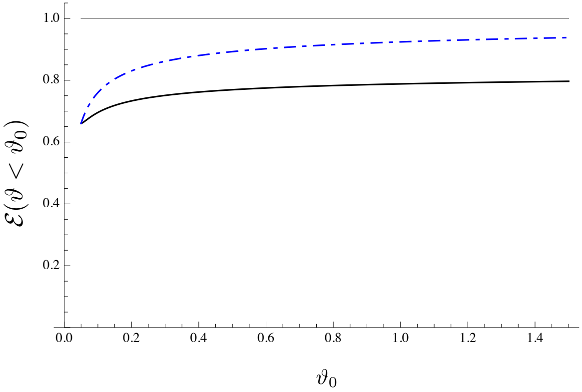
blue : BDMPSZ

black: multiple branchings

$\hat{q} \simeq 1$ GeV^2/fm

$L \simeq 4$ fm

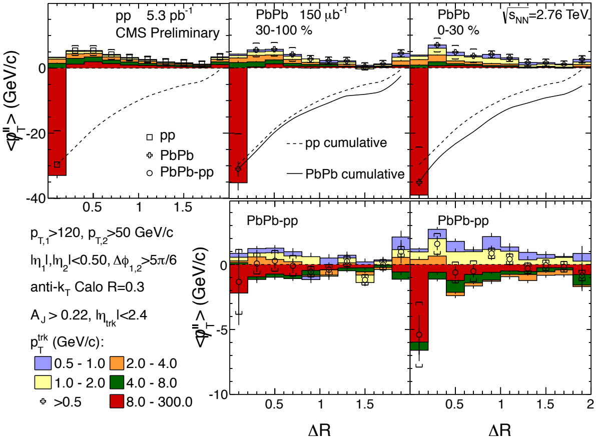
$\omega_c \simeq 40$ GeV, $\theta_s \simeq 0.5$



- offset at $\theta_0 \sim \pi/2$: the energy $\mathcal{E}_{\rm flow}$ taken away by the flow
- almost flat in θ_0 : energy is lost directly at large angles

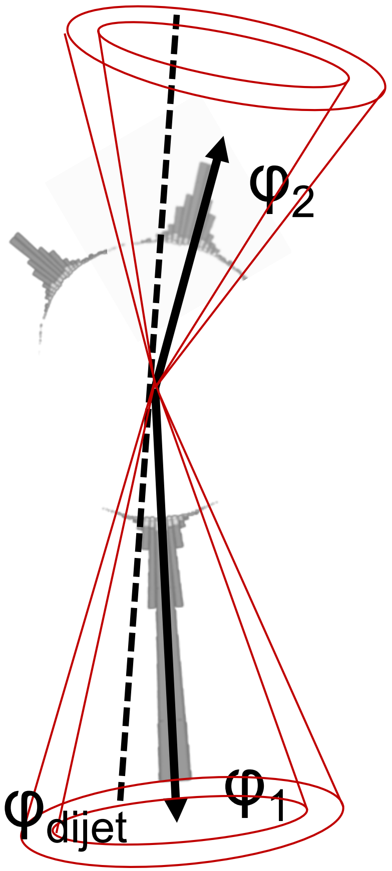
Angular distribution at the LHC *(CMS)*

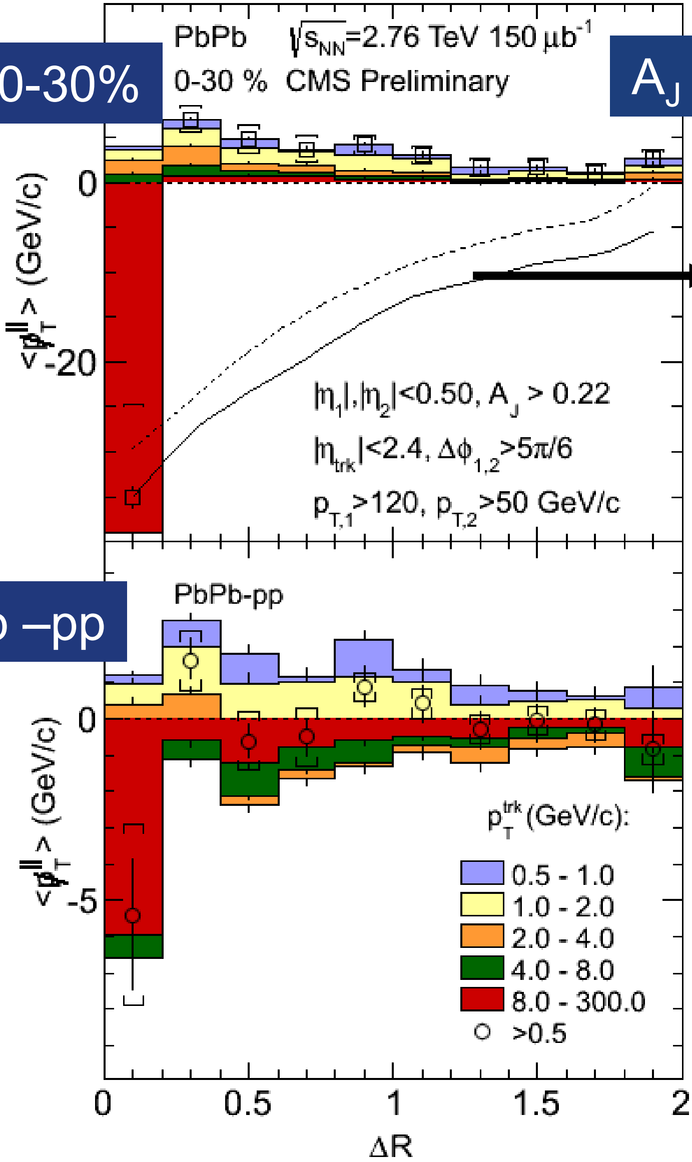
- For each bin in θ : energy difference between trigger jet and away jet



- the offset in Pb+Pb is clearly visible (larger than for p+p)
- the ΔR dependence looks stepper ... but is exactly the same in p+p

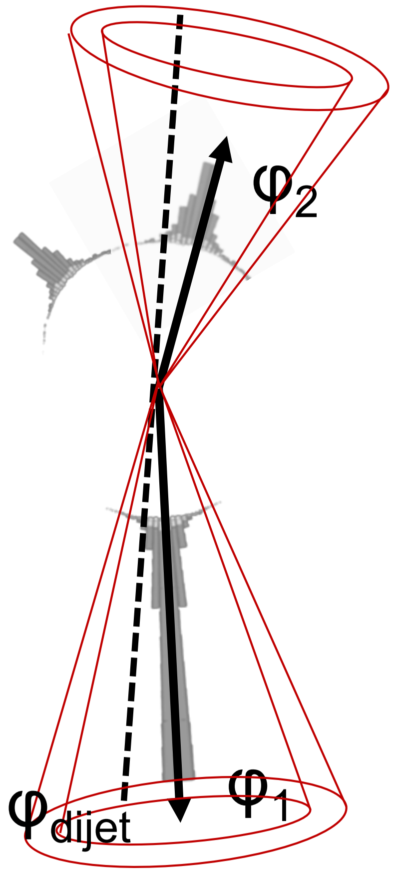
Results - Missing p_T vs. ΔR

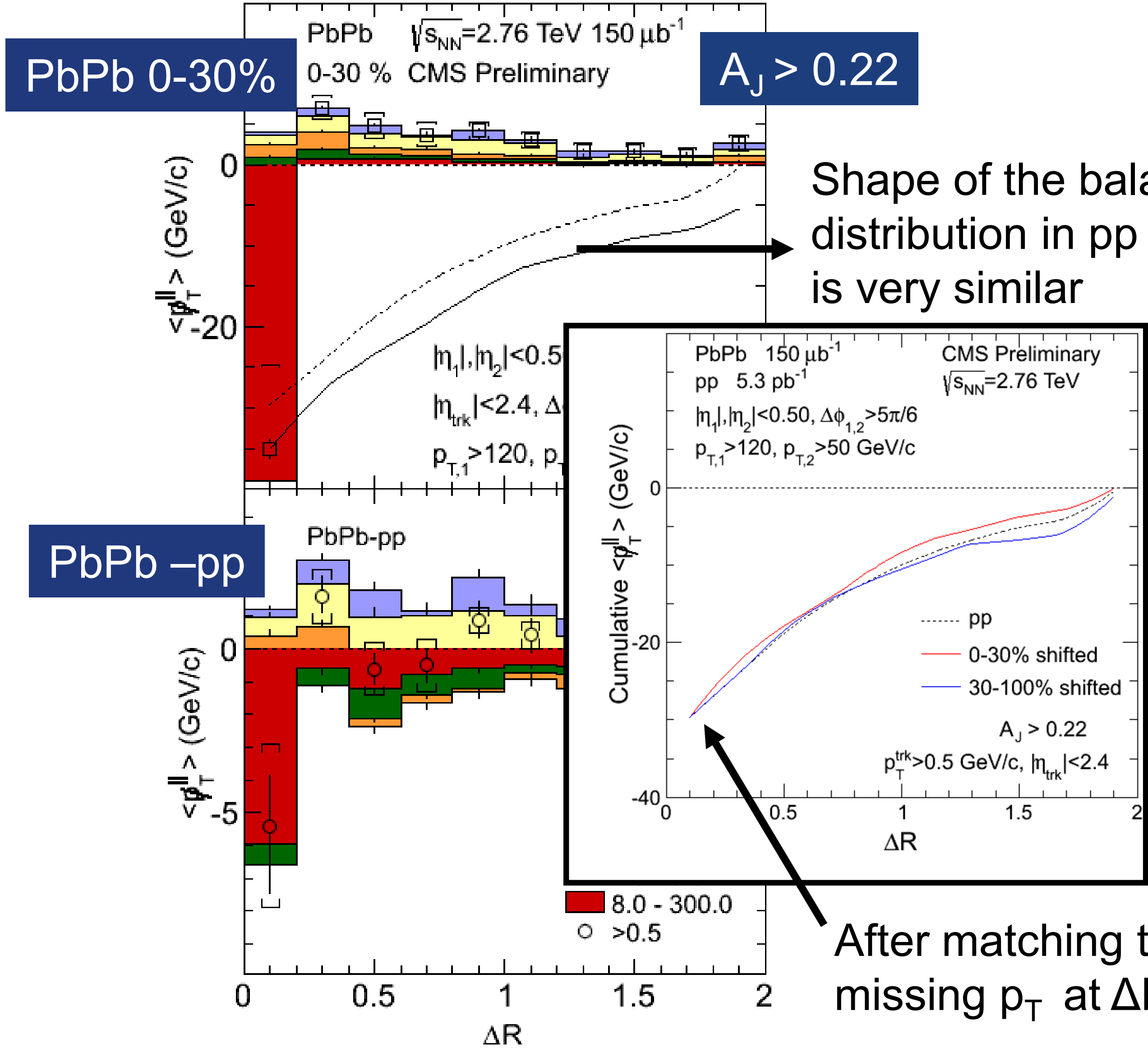




Shape of the balancing distribution in pp and PbPb is very similar

Results - Missing p_T vs. ΔR



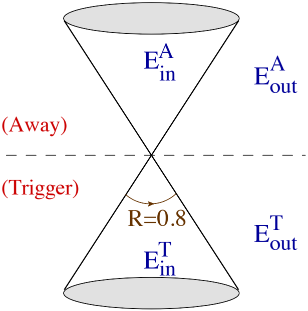


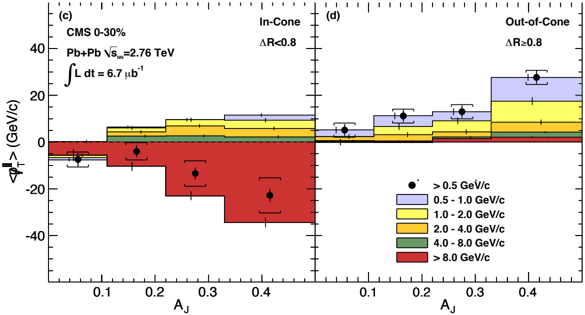
Shape of the balancing distribution in pp and PbPb is very similar

After matching the missing p_T at $\Delta R<0.2$

Soft hadrons at large angles

- The energy (im)balance for a jet with a wide opening : $R = 0.8$



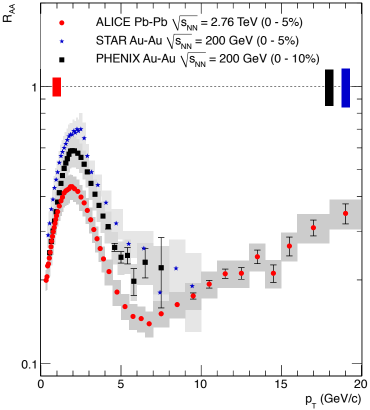


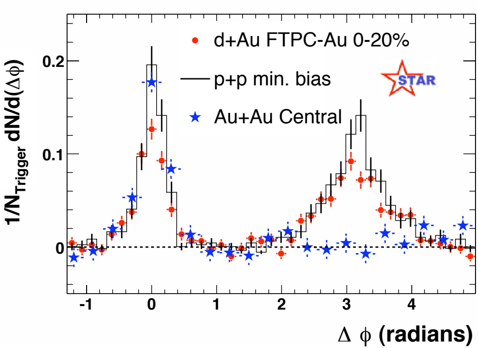
- Di–jet asymmetry : $E^{\rm T}_{\rm in} > E^{\rm A}_{\rm in}$
- No missing energy : $E^{\rm T}_{\rm in} + E^{\rm T}_{\rm out} = E^{\rm A}_{\rm in} + E^{\rm A}_{\rm out}$
- The energy lost at large angles, $E^{\rm A}_{\rm out} - E^{\rm T}_{\rm out}$ ...

  ... is carried mostly by soft hadrons with $p_T < 2$ GeV

Jet quenching

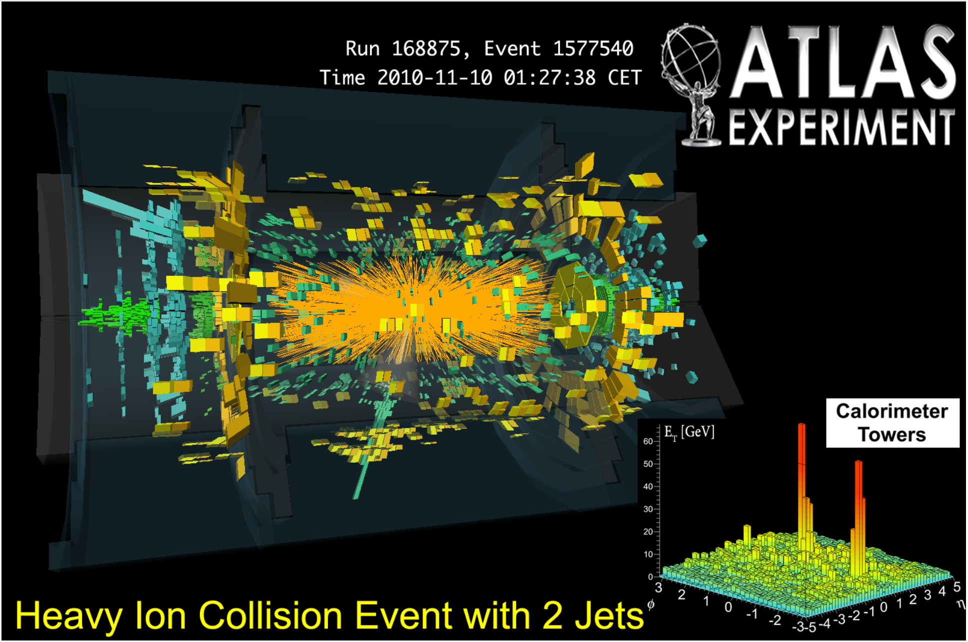
- Nuclear modification factor, di–hadron azimuthal correlations ...





- Energy loss & transverse momentum broadening by the leading particle

Jets in peripheral Pb+Pb collisions

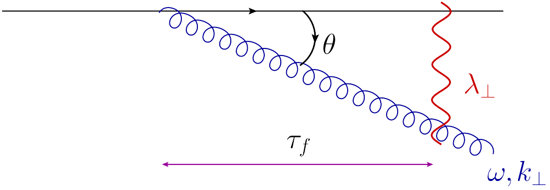


The formation time *(Ioffe's coherence time)*

- The gluon must lose quantum coherence with respect to its source

  ▷ the quark–gluon transverse separation $b_\perp$ at the formation time τ_f must be larger than the gluon transverse wavelength $\lambda_\perp \simeq 1/k_\perp$

- High energy kinematics: $\omega \gg k_\perp \implies$ small angle: $\theta \simeq k_\perp/\omega$



$$b_\perp \simeq \theta\,\tau_f \gtrsim \lambda_\perp \simeq 1/k_\perp \implies \tau_f \simeq \frac{1}{\omega\theta^2} \simeq \frac{\omega}{k_\perp^2}$$

- During formation, the gluon acquires a momentum $k_\perp^2 \sim \hat{q}\tau_f$