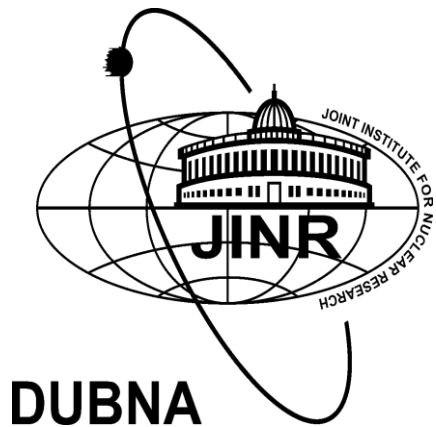


Supporting the existence of the QCD critical point by compact star observations



David Álvarez Castillo



XIth QUARK CONFINEMENT AND THE HADRON SPECTRUM

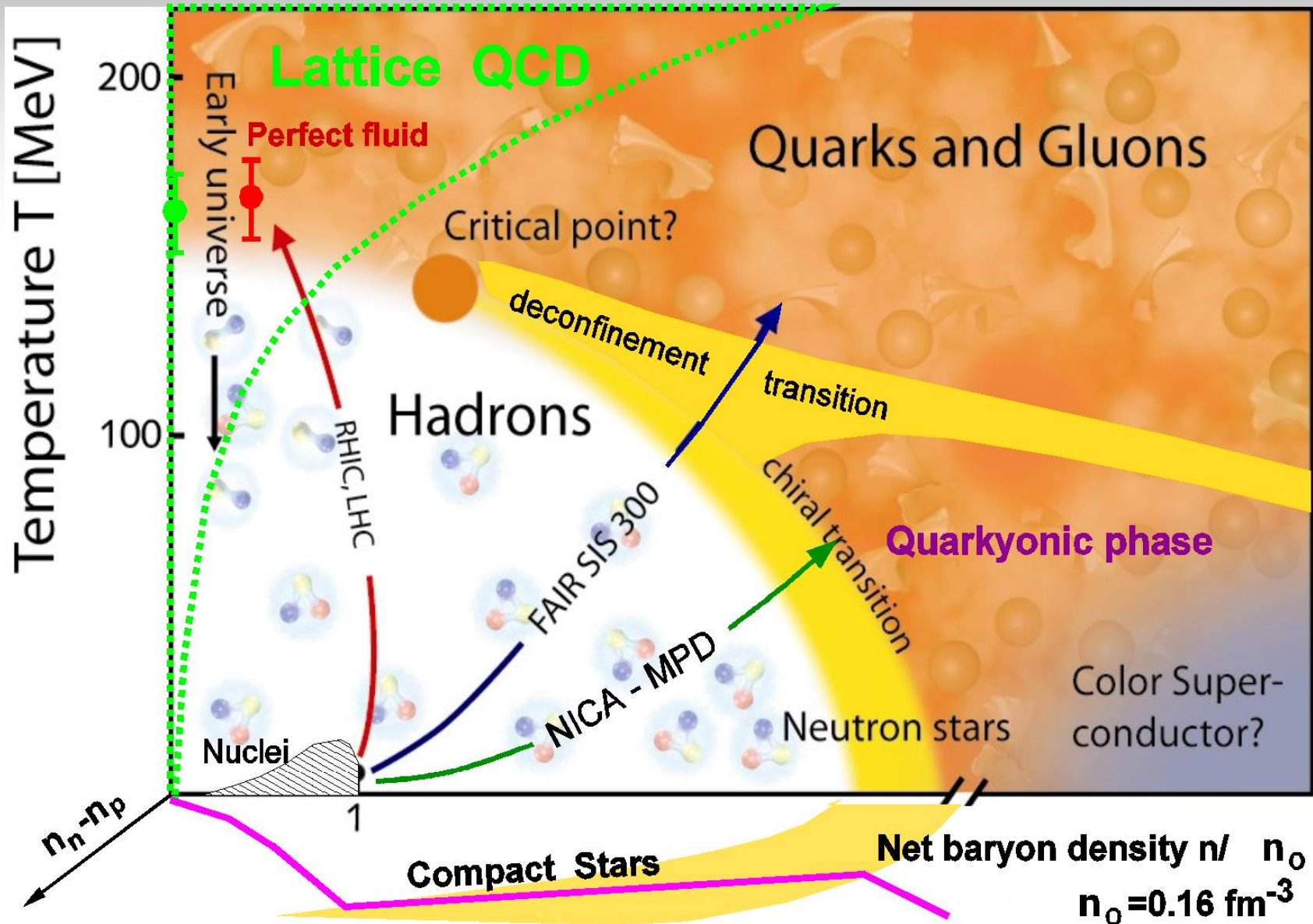
SAINT PETERSBURG STATE UNIVERSITY, RUSSIA

September 2014

Outline

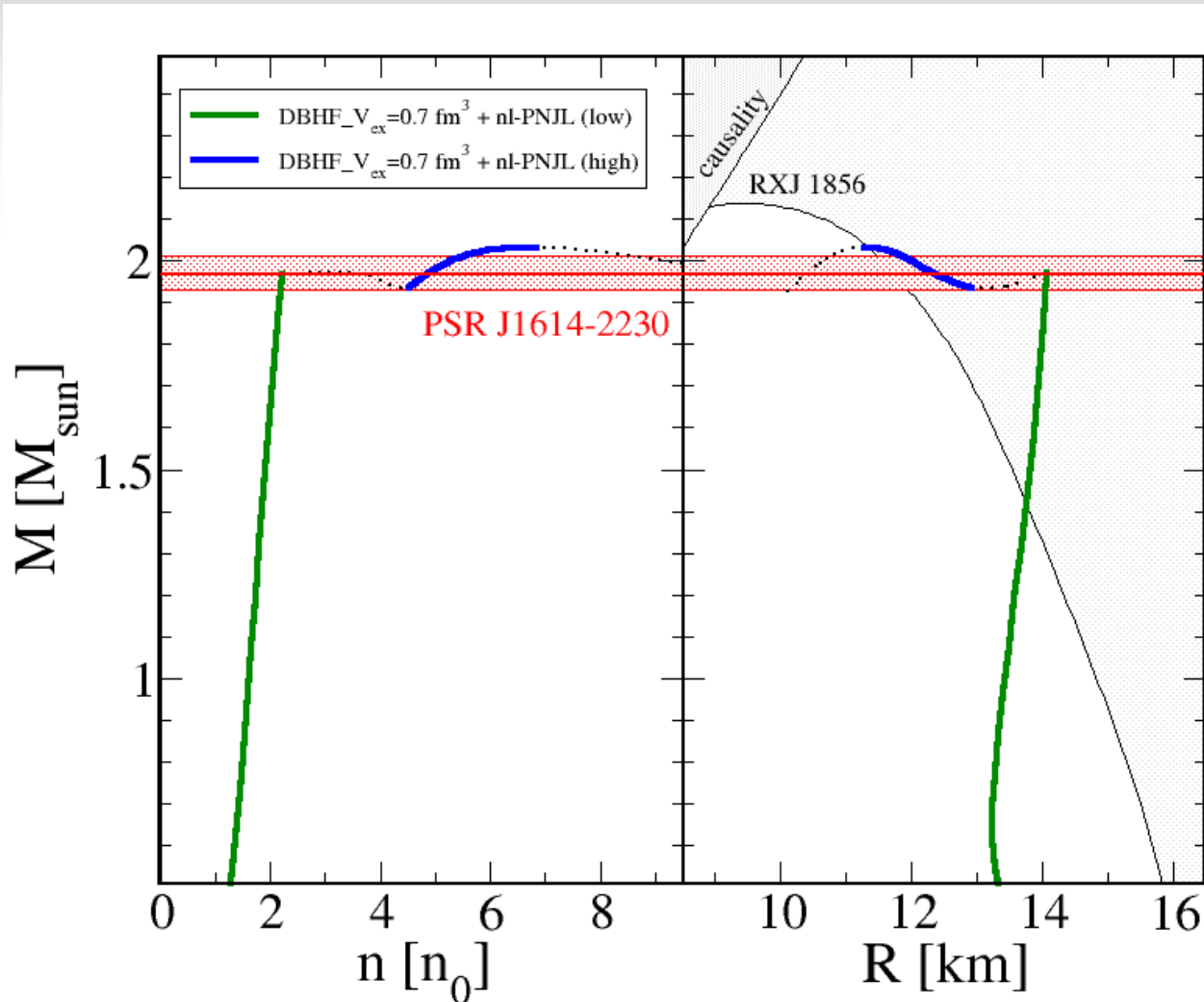
- Introduction to the QCD phase diagram and compact stars
- First order phase transition and deconfinement in compact stars: masquerade problem vs neutron star twins.
- Bayesian Analysis of hybrid star models.
- Astrophysical implications and perspectives

Critical End Point in QCD



Proving the CEP with Compact Stars

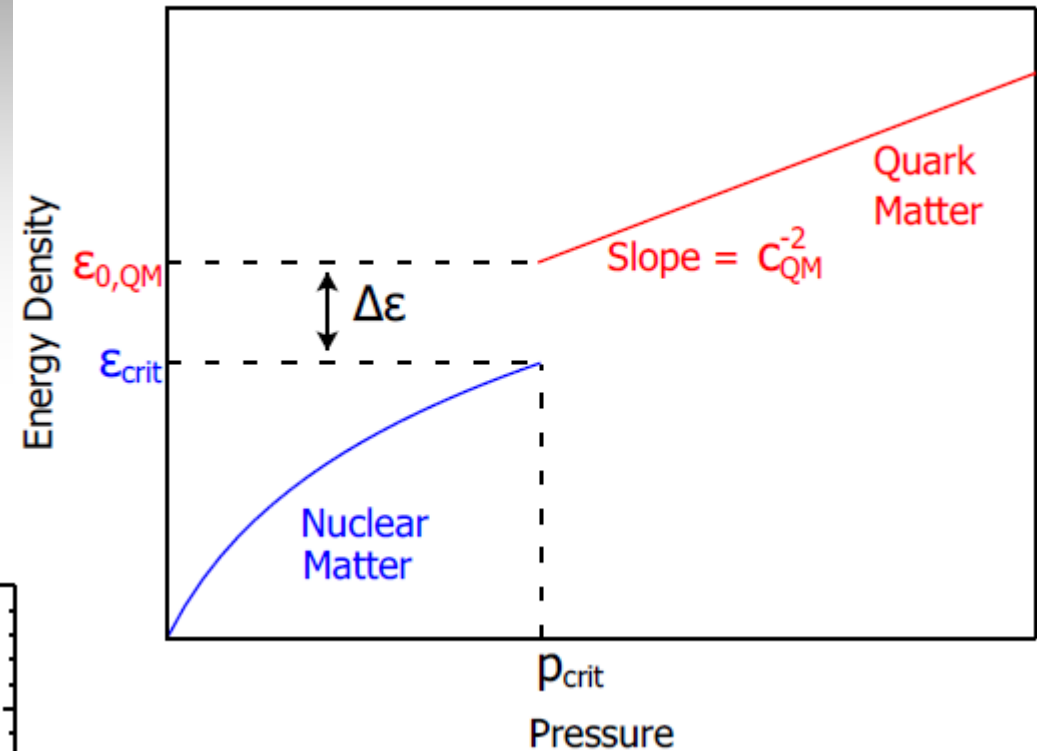
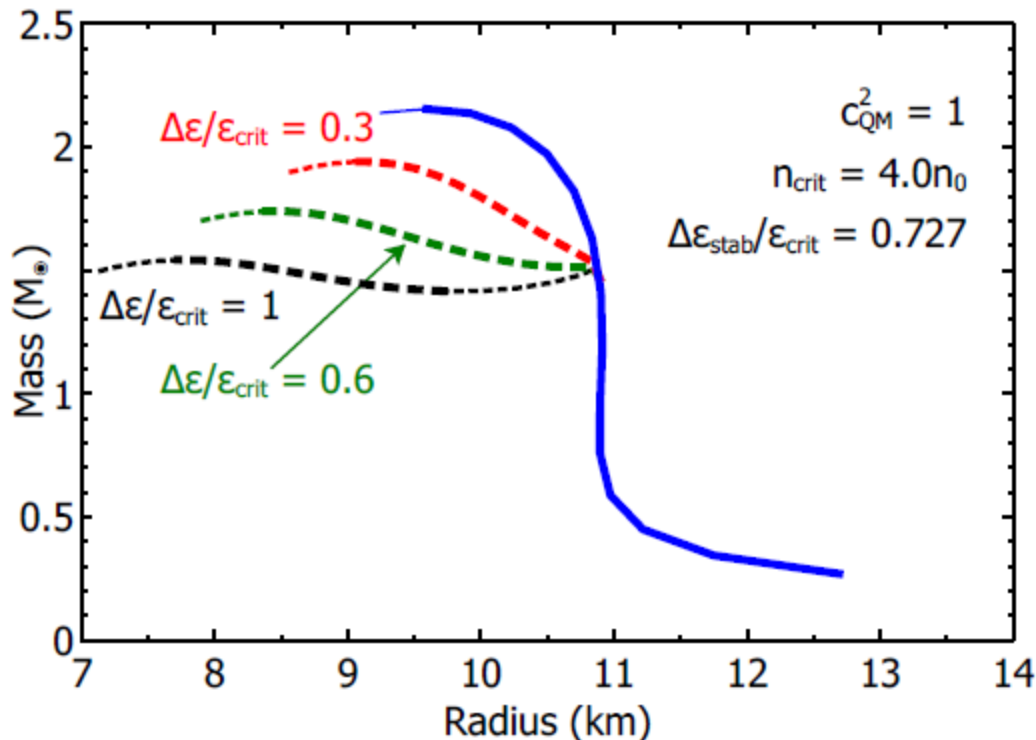
Third family (disconnected branch)



Neutron Star Twins and the AHP scheme

Alford, Han, Prakash, arxiv:1302.4732

First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the “latent heat” (jump in energy density), can even be disconnected from the hadronic one by an unstable branch → “**third family of CS**”.



Measuring two **disconnected populations** of compact stars in the M-R diagram would be the **detection of a first order phase transition** in compact star matter and thus the indirect proof for the existence of a **critical endpoint (CEP) in the QCD phase diagram!**

Baryon substructure effect (EVA)

Excluded volume approximation (EVA):

$$p_{\text{ex}}(\mu, T) = p(\tilde{\mu}, T), \quad \tilde{\mu} = \mu - v_0(\mu, T)p_{\text{ex}}(\mu, T)$$

$$n_{\text{ex}}(\mu, T) = \frac{\partial p_{\text{ex}}}{\partial \mu} = \frac{\partial \tilde{\mu}}{\partial \mu} \frac{\partial p(\tilde{\mu}, T)}{\partial \tilde{\mu}} = \left[1 - v_0 n_{\text{ex}}(\mu, T) - \frac{\partial v_0}{\partial \mu} p_{\text{ex}}(\mu, T) \right] n(\tilde{\mu}, T)$$

Thermodynamic consistency:

$$\epsilon_{\text{ex}}(\mu, T) = -p_{\text{ex}}(\mu, T) + \mu n_{\text{ex}}(\mu, T) + T s_{\text{ex}}(\mu, T)$$

Parametrization of excluded volume with nonlinear dependence on the chemical potential:

$$v_0(\mu, T) = (4\pi/3)r^3(\mu), \quad r^3(\mu) = r_0 + r_1(\mu/\mu_c)^2 + r_2(\mu/\mu_c)^4$$

NJL model with multiquark interactions

$$\mathcal{L} = \bar{q}(i\cancel{\partial} - m)q + \mu_q \bar{q}\gamma^0 q + \mathcal{L}_4 + \mathcal{L}_8, \quad \mathcal{L}_4 = \frac{g_{20}}{\Lambda^2} [(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2] - \frac{g_{02}}{\Lambda^2} (\bar{q}\gamma_\mu q)^2,$$

$$\mathcal{L}_8 = \frac{g_{40}}{\Lambda^8} [(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]^2 - \frac{g_{04}}{\Lambda^8} (\bar{q}\gamma_\mu q)^4 - \frac{g_{22}}{\Lambda^8} (\bar{q}\gamma_\mu q)^2 [(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]$$

Meanfield approximation: $\mathcal{L}_{\text{MF}} = \bar{q}(i\cancel{\partial} - M)q + \tilde{\mu}_q \bar{q}\gamma^0 q - U,$

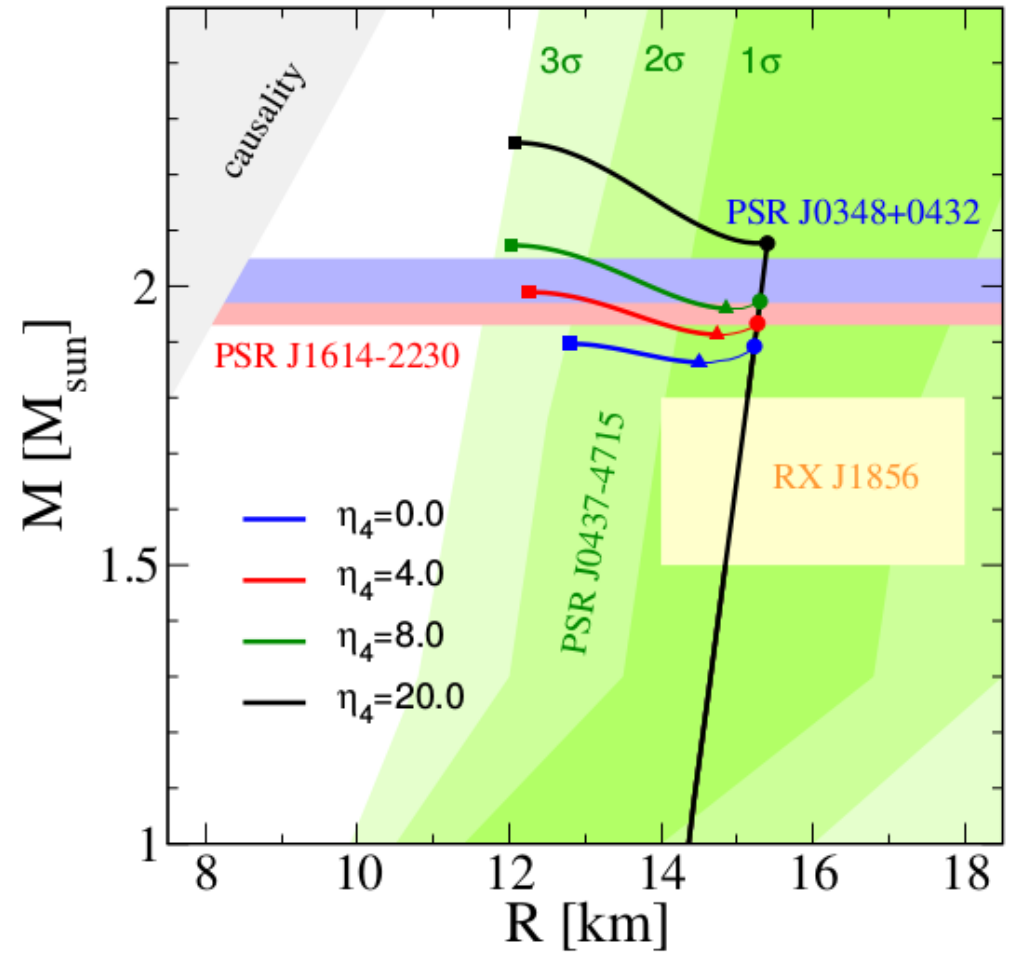
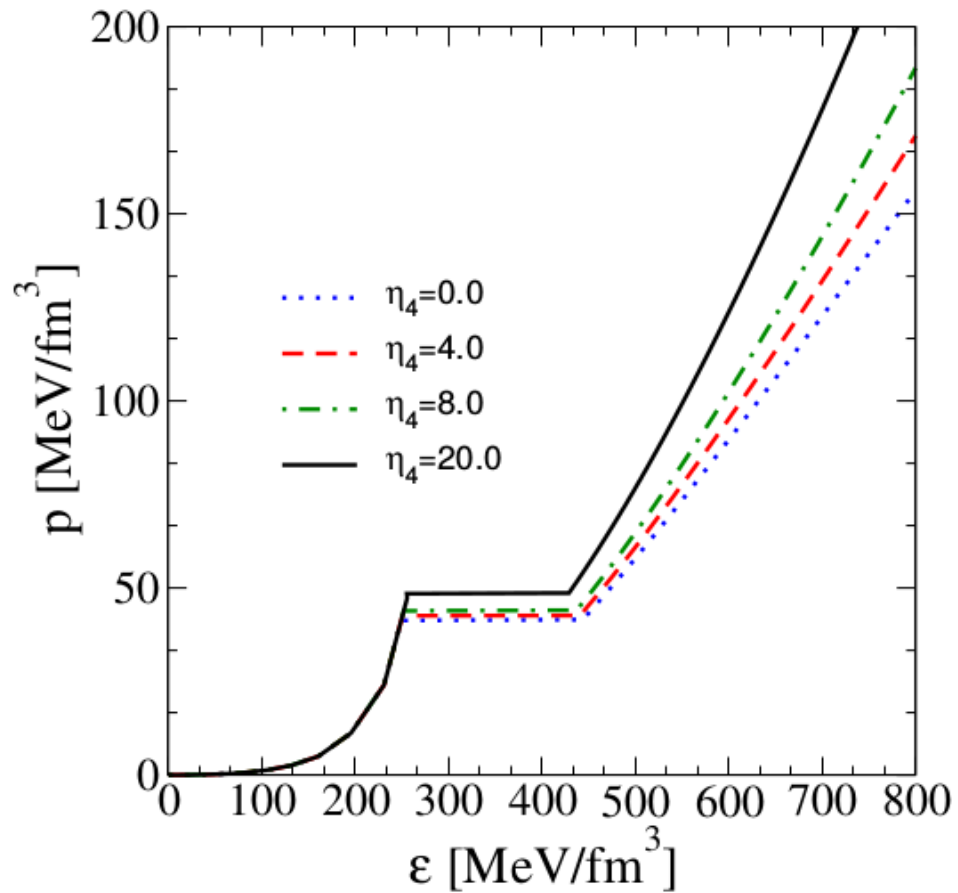
$$M = m + 2\frac{g_{20}}{\Lambda^2} \langle \bar{q}q \rangle + 4\frac{g_{40}}{\Lambda^8} \langle \bar{q}q \rangle^3 - 2\frac{g_{22}}{\Lambda^8} \langle \bar{q}q \rangle \langle q^\dagger q \rangle^2,$$

$$\tilde{\mu}_q = \mu_q - 2\frac{g_{02}}{\Lambda^2} \langle q^\dagger q \rangle - 4\frac{g_{04}}{\Lambda^8} \langle q^\dagger q \rangle^3 - 2\frac{g_{22}}{\Lambda^8} \langle \bar{q}q \rangle^2 \langle q^\dagger q \rangle,$$

$$U = \frac{g_{20}}{\Lambda^2} \langle \bar{q}q \rangle^2 + 3\frac{g_{40}}{\Lambda^8} \langle \bar{q}q \rangle^4 - 3\frac{g_{22}}{\Lambda^8} \langle \bar{q}q \rangle^2 \langle q^\dagger q \rangle^2 - \frac{g_{02}}{\Lambda^2} \langle q^\dagger q \rangle^2 - 3\frac{g_{04}}{\Lambda^8} \langle q^\dagger q \rangle^4.$$

Thermodynamic Potential:

$$\Omega = U - 2N_f N_c \int \frac{d^3 p}{(2\pi)^3} \left\{ E + T \log[1 + e^{-\beta(E - \tilde{\mu}_q)}] + T \log[1 + e^{-\beta(E + \tilde{\mu}_q)}] \right\} + \Omega_0$$

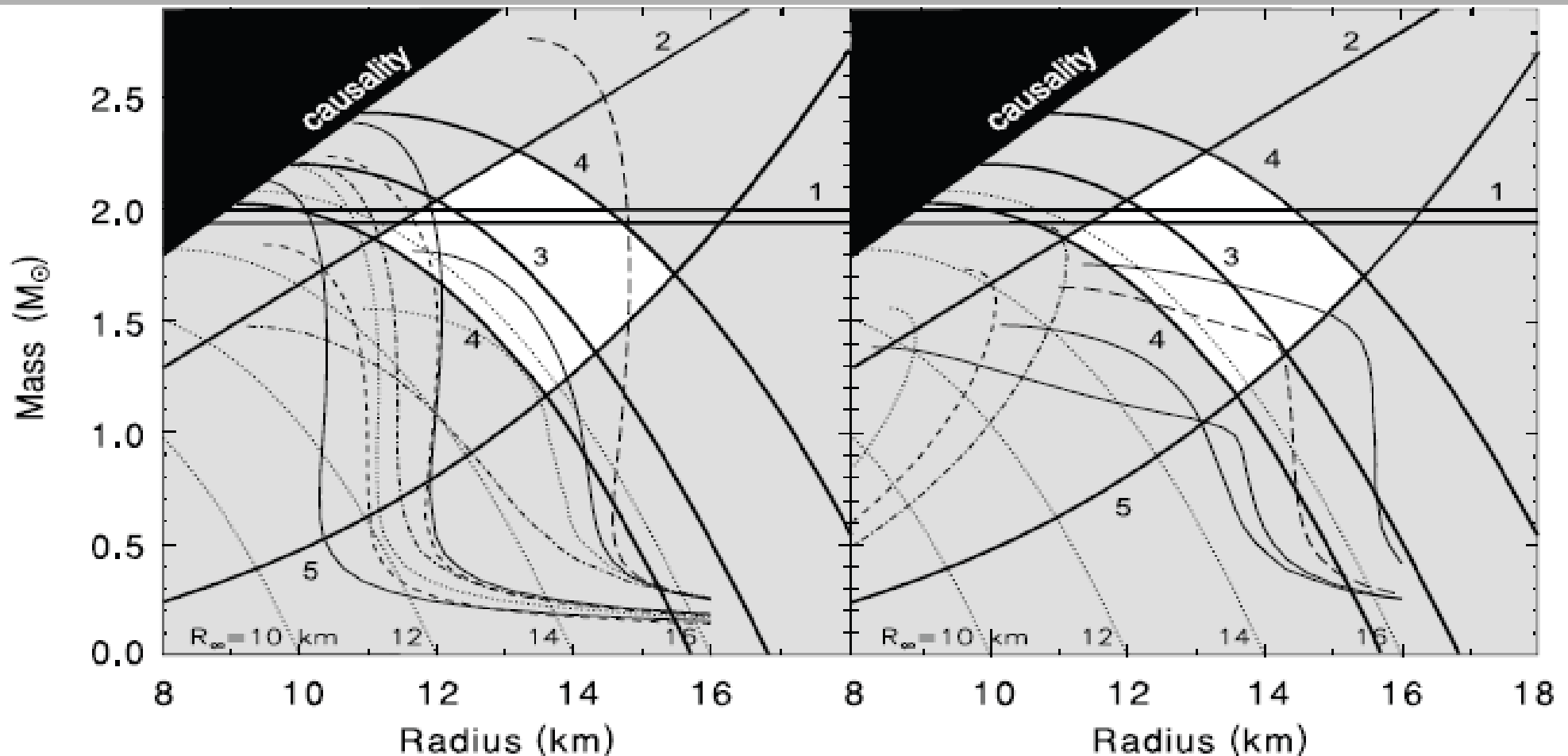


Benic, Blaschke, Alvarez-Castillo, Fischer, in progress (2014)

Compact Star Measurements

The background of the slide features several thick, light gray, wavy lines that flow from the bottom right towards the center, creating a sense of movement and depth.

Which constraints can be trusted ?



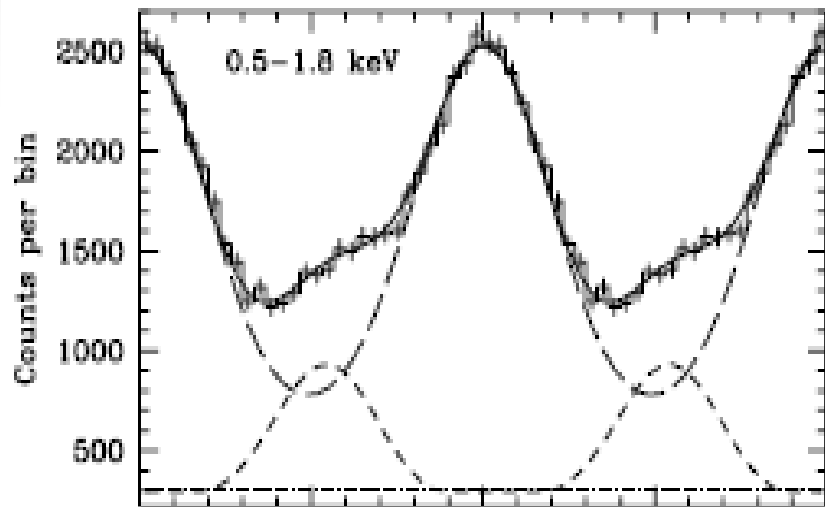
- 1 – Largest mass J1614 – 2230 (Demorest et al. 2010)
- 2 – Maximum gravity XTE 1814 – 338 (Bhattacharyya et al. (2005)
- 3 – Maximum radius RXJ 1856 – 3754 (Trumper et al. 2004)
- 4 – Radius, 90% confidence limits LMXB X7 in 47 Tuc (Heinke et al. 2006)
- 5 – Largest spin frequency J1748 – 2446 (Hessels et al. 2006)

Which constraints can be trusted ?

Nearest millisecond pulsar PSR J0437 – 4715 revisited by XMM Newton

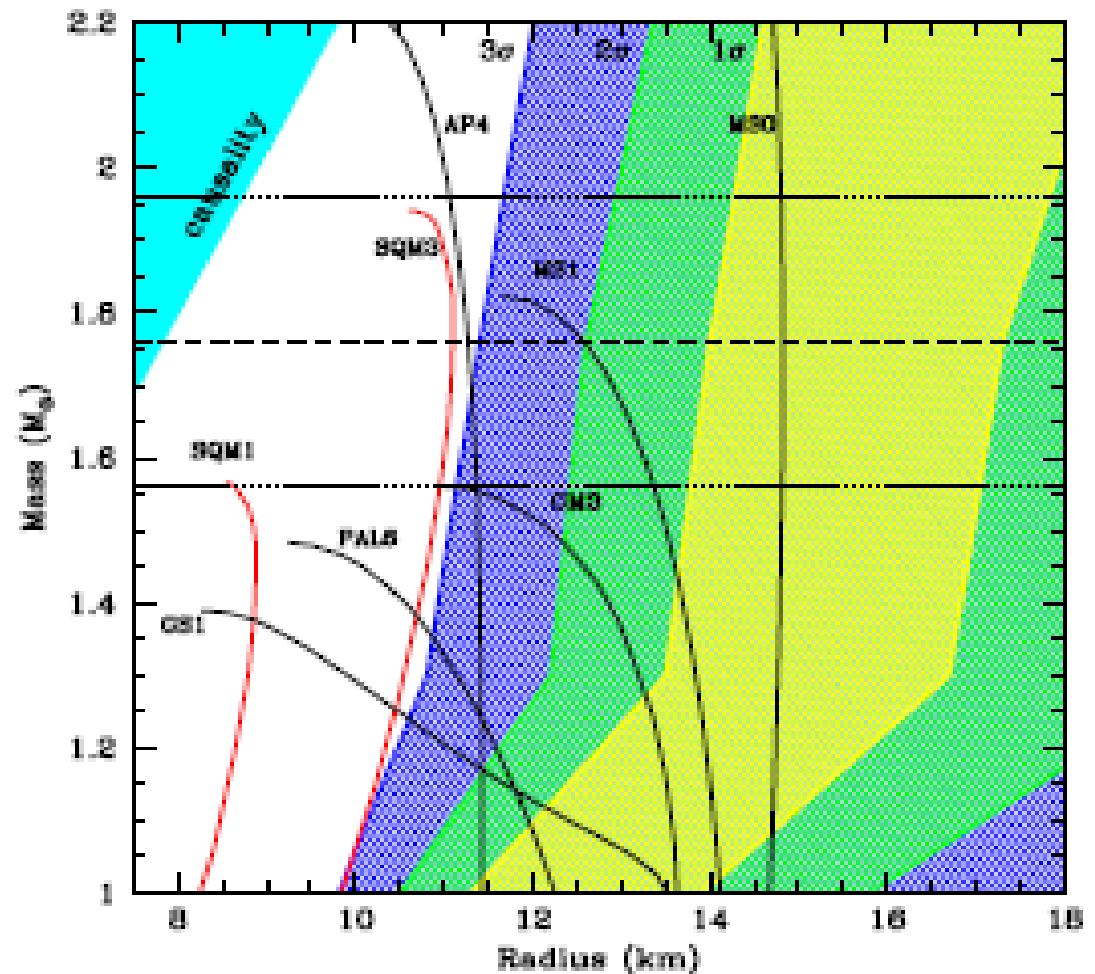
Distance: $d = 156.3 \pm 1.3$ pc

Period: $P = 5.76$ ms, $\dot{P} = 10^{-20}$ s/s, field strength $B = 3 \times 10^8$ G



Three thermal component fit
 $R > 11.1$ km (at 3 sigma level)
 $M = 1.76 M_{\text{sun}}$

S. Bogdanov, arxiv:1211.6113 (2012)



Bayesian Analysis for the EoS

Bayesian TOV analysis:

Steiner, Lattimer, Brown, ApJ 722 (2010) 33

Most Probable Values for Masses and Radii for Neutron Stars Constrained to Lie on One Mass Versus Radius Curve

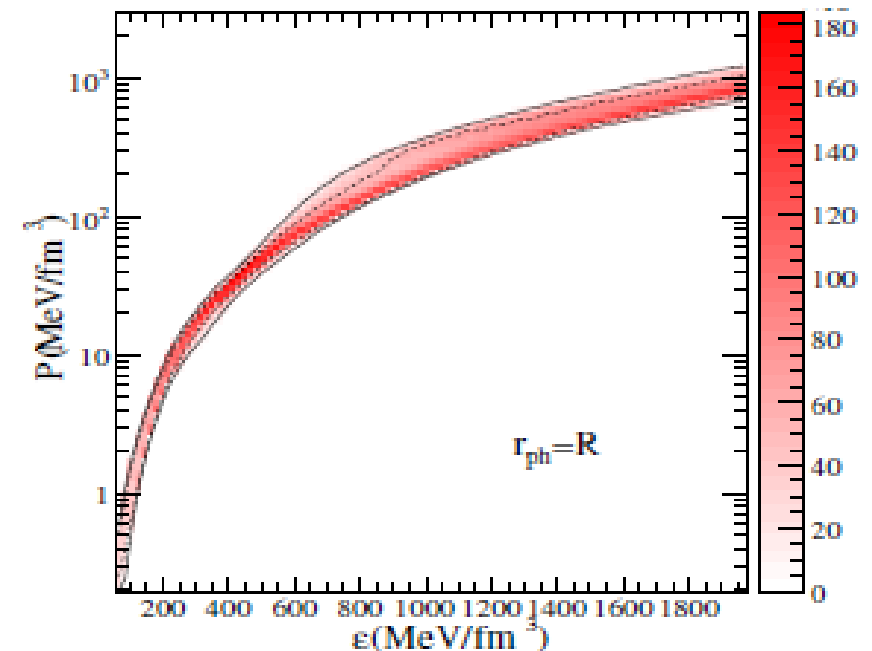
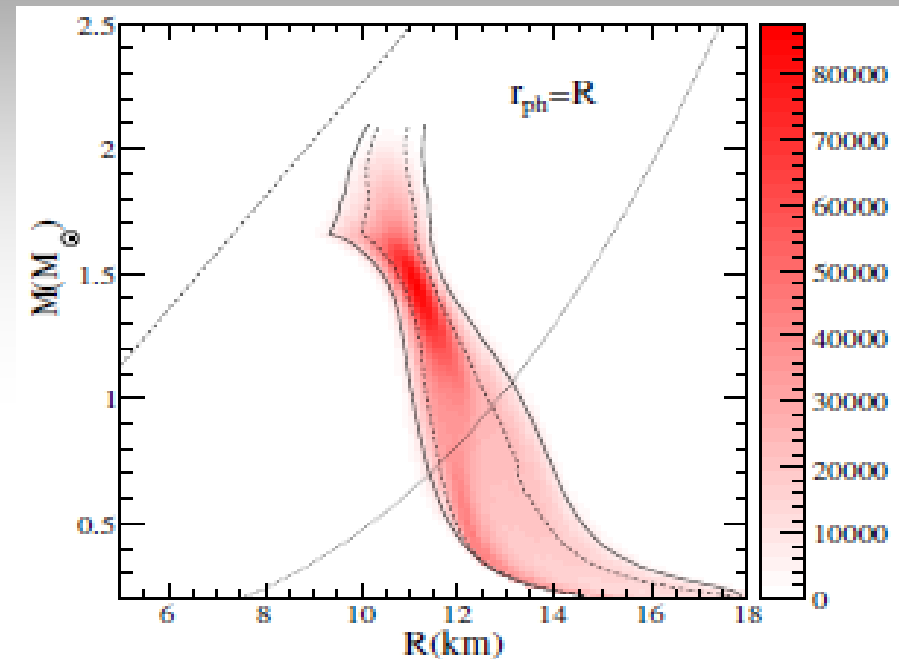
Object	$r_{\text{ph}} = R$		$r_{\text{ph}} \gg R$	
	$M (M_{\odot})$	$R \text{ (km)}$	$M (M_{\odot})$	$R \text{ (km)}$
4U 1608–522	$1.52^{+0.22}_{-0.18}$	$11.04^{+0.53}_{-1.50}$	$1.64^{+0.34}_{-0.41}$	$11.82^{+0.42}_{-0.89}$
EXO 1745–248	$1.55^{+0.12}_{-0.36}$	$10.91^{+0.86}_{-0.65}$	$1.34^{+0.450}_{-0.28}$	$11.82^{+0.47}_{-0.72}$
4U 1820–30	$1.57^{+0.13}_{-0.15}$	$10.91^{+0.39}_{-0.92}$	$1.57^{+0.37}_{-0.31}$	$11.82^{+0.42}_{-0.82}$
M13	$1.48^{+0.21}_{-0.64}$	$11.04^{+1.00}_{-1.28}$	$0.901^{+0.28}_{-0.12}$	$12.21^{+0.18}_{-0.62}$
ω Cen	$1.43^{+0.26}_{-0.61}$	$11.18^{+1.14}_{-1.27}$	$0.994^{+0.51}_{-0.21}$	$12.09^{+0.27}_{-0.66}$
X7	$0.832^{+1.19}_{-0.051}$	$13.25^{+1.37}_{-3.50}$	$1.98^{+0.10}_{-0.36}$	$11.3^{+0.95}_{-1.03}$

Caution:

If optical spectra are not measured, the observed X-ray spectrum may not come from the entire surface
But from a hot spot at the magnetic pole!

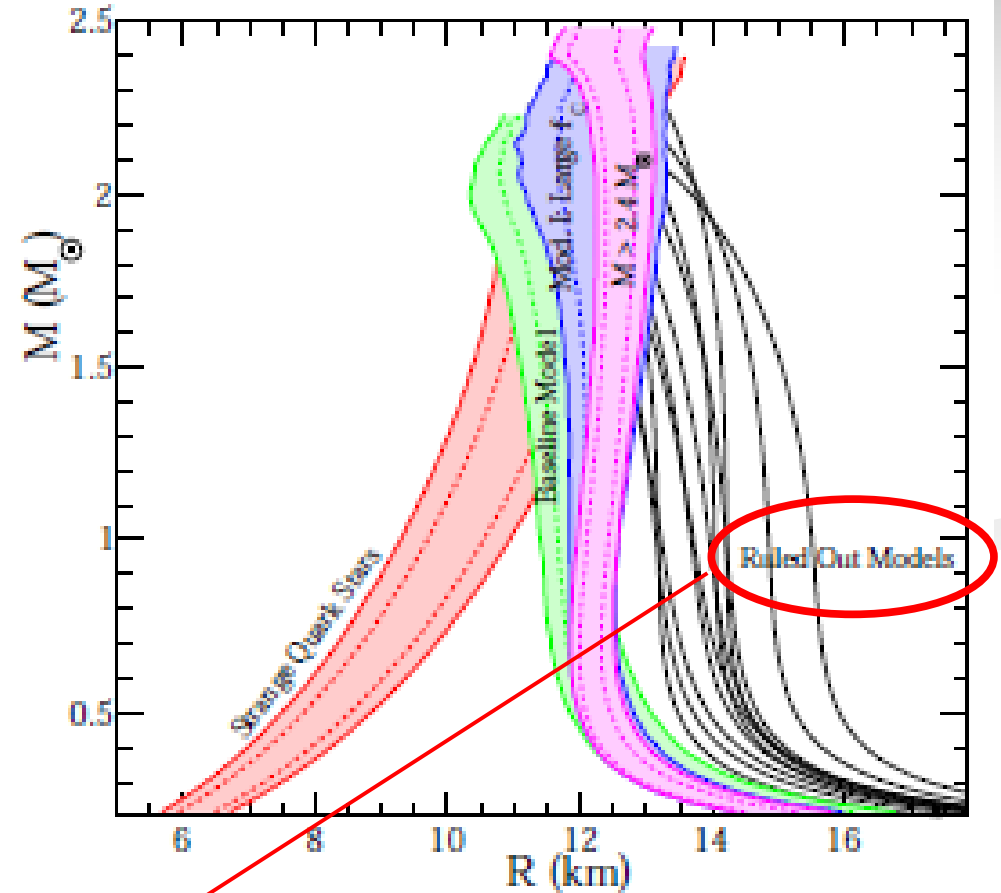
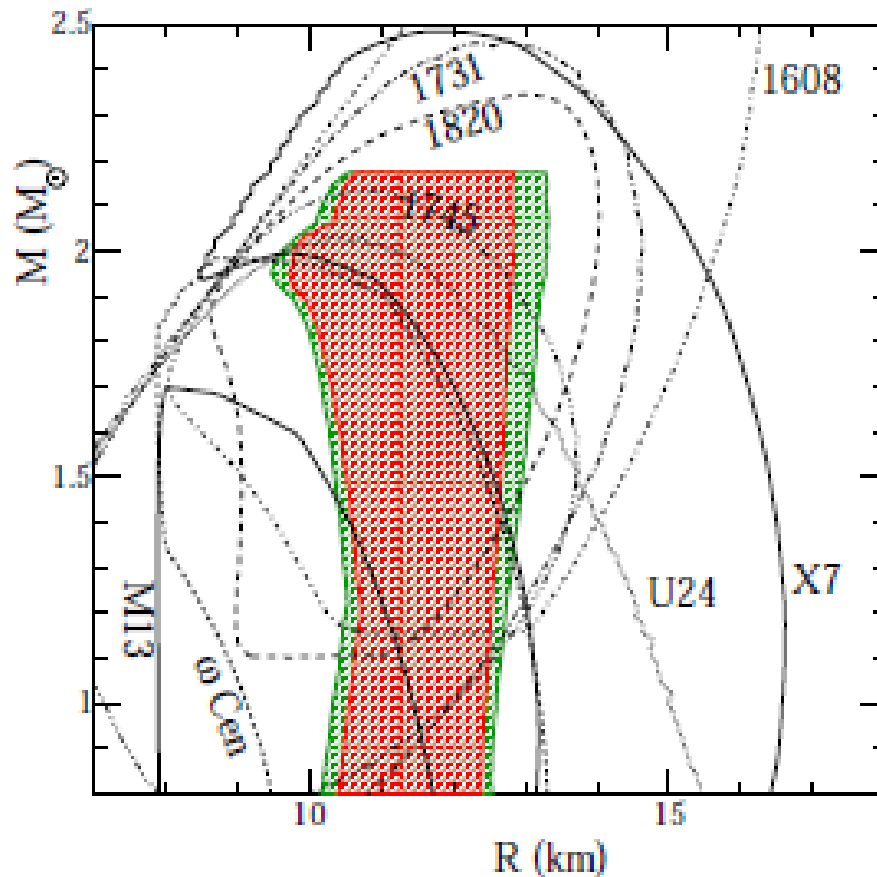
J. Trumper, Prog. Part. Nucl. Phys. 66 (2011) 674

Such systematic errors are not accounted for in Steiner et al. $\rightarrow M(R)$ is a lower limit \rightarrow softer EoS



Which constraints require caution ?


A. Steiner, J. Lattimer, E. Brown, ApJ Lett. 765 (2013) L5



“Ruled out models” - too strong a conclusion!

$M(R)$ constraint is a lower limit, which is itself included in that from RX J1856, which is one of the best known sources.

Bayesian Analysis

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Bayesian Analysis

Calculation of *a posteriori* Probabilities

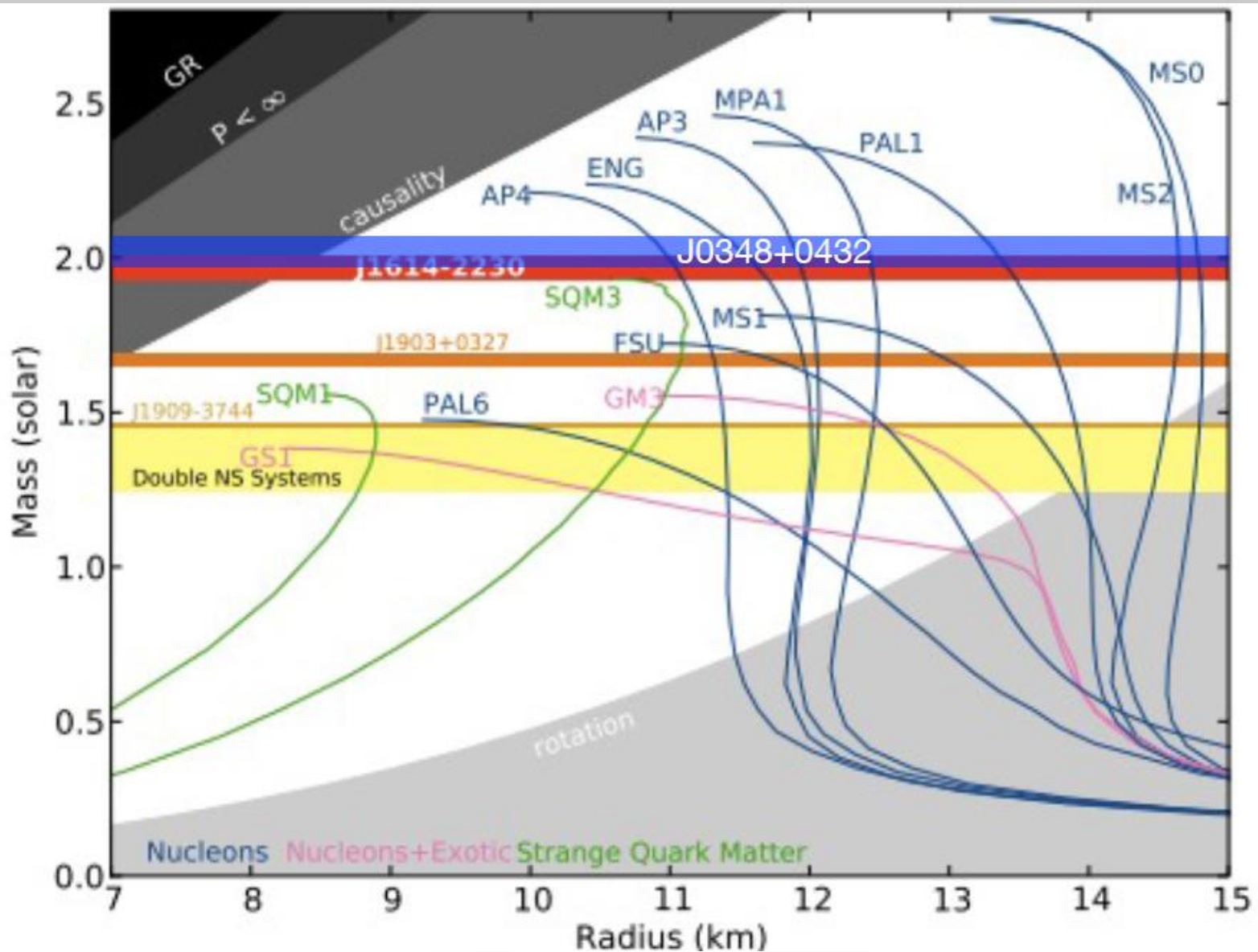
Note, that these measurements are independent on each other. That means that we can calculate complete conditional probability of event that contracted by π_i object corresponds to all measurements:

$$P(E|\pi_i) = P(E_A|\pi_i) \times P(E_B|\pi_i) \times P(E_K|\pi_i). \quad (6)$$

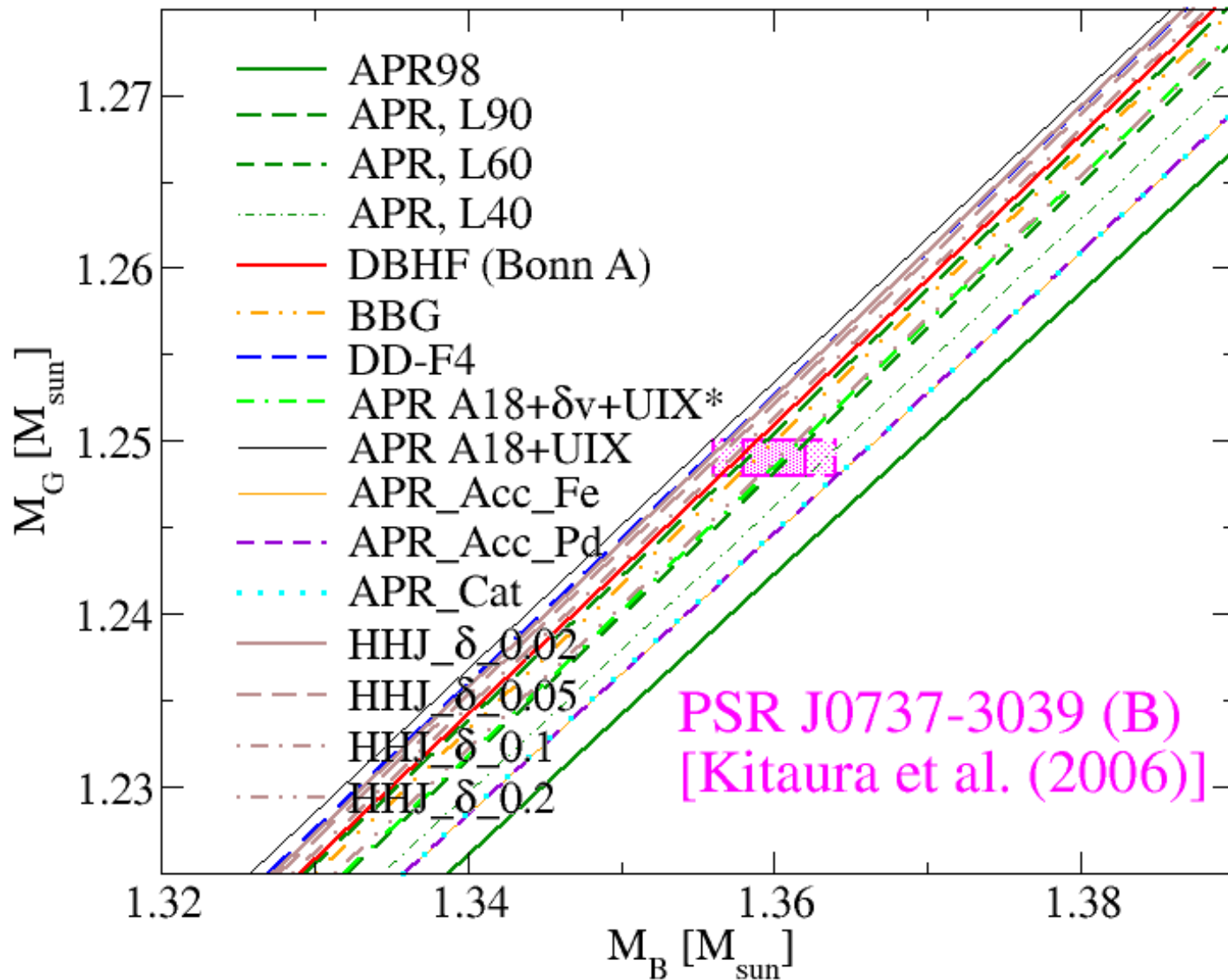
Now, we can calculate probability of π_i using Bayes' theorem:

$$P(\pi_i|E) = \frac{P(E|\pi_i) P(\pi_i)}{\sum_{j=0}^{N-1} P(E|\pi_j) P(\pi_j)}. \quad (7)$$

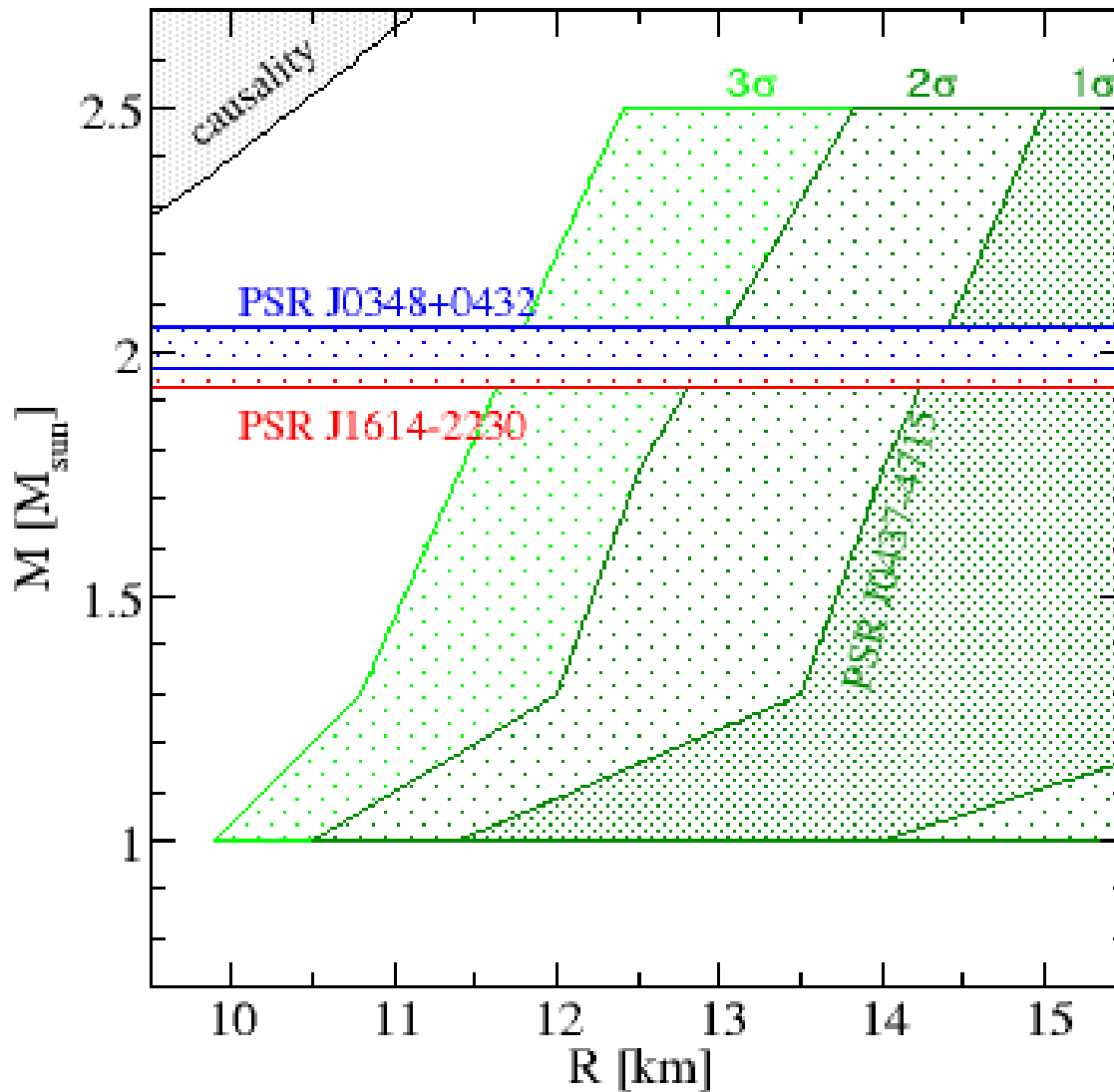
Mass vs. Radius Relation



Baryonic Mass



Mass-radius Constraints



EoS parameterization

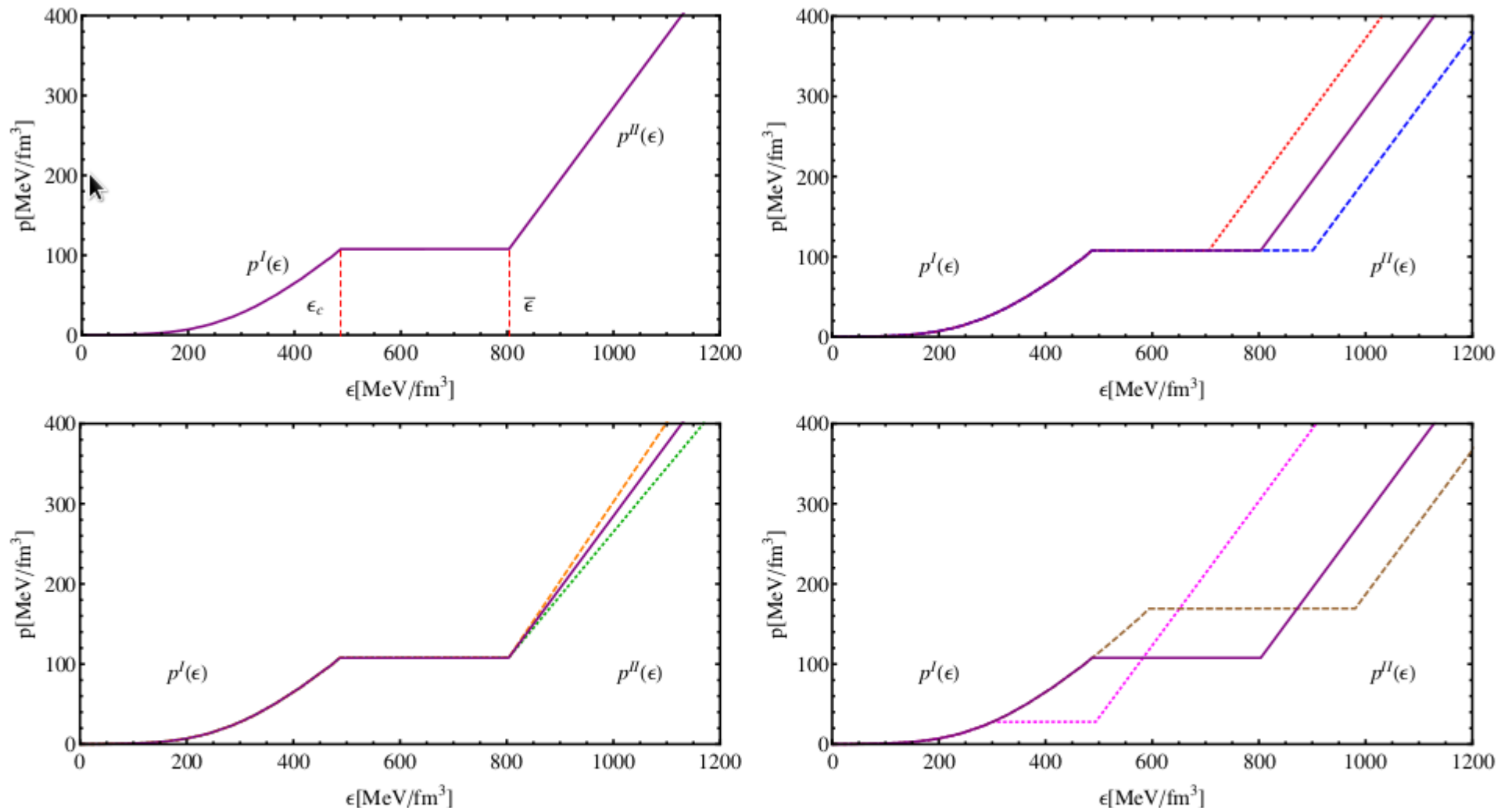
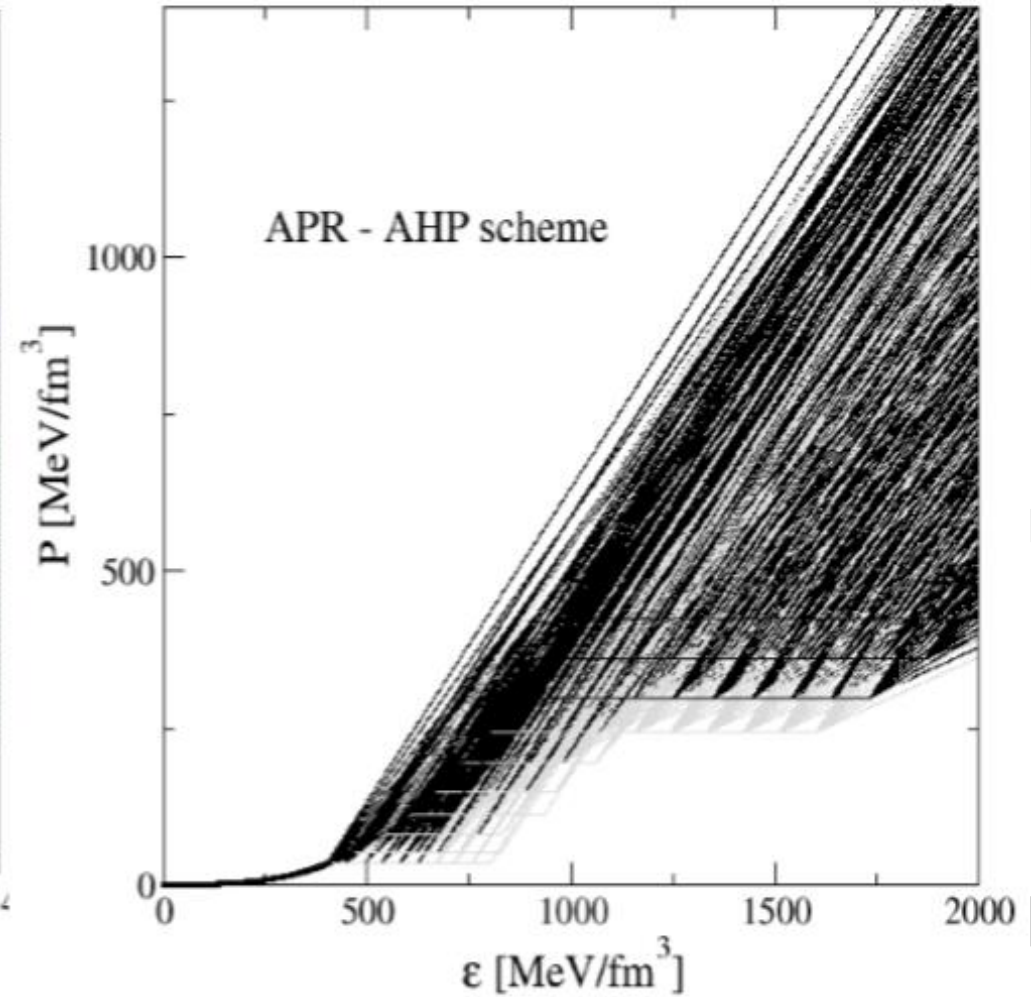
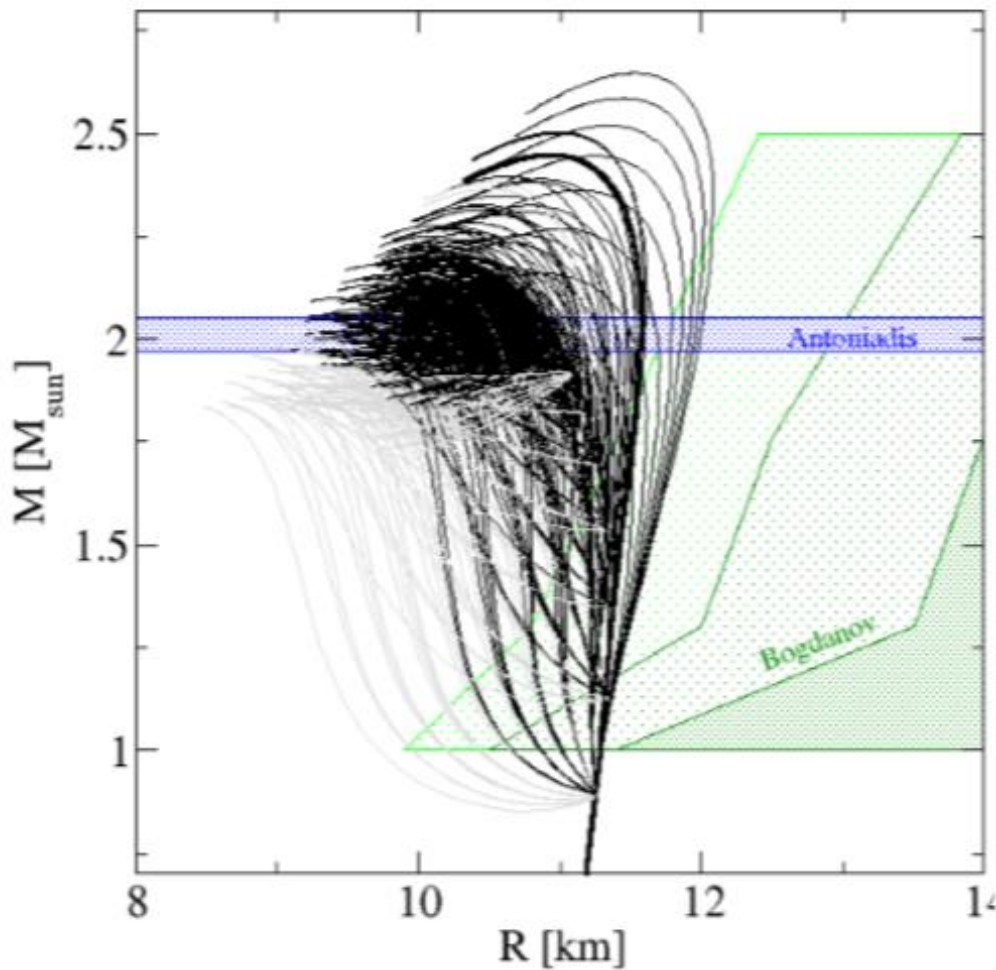


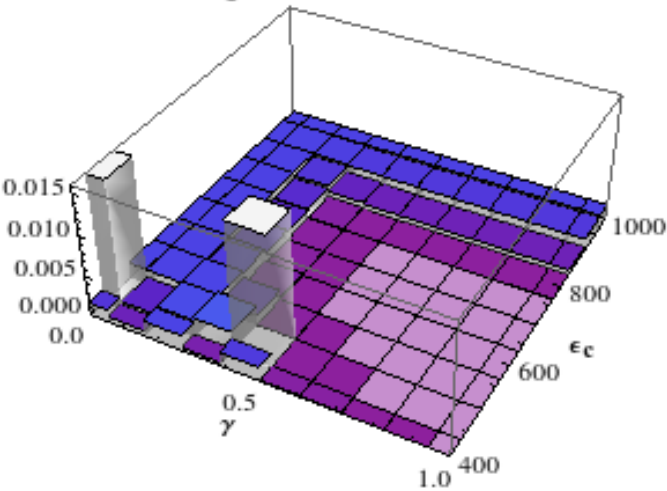
FIG. 2: Hybrid EoS scheme for two sets of parameters. *Upper left corner:* EoS with parameters $\epsilon_c = 446.966$ MeV/fm³, $c_{QM}^2 = 0.9$ and $\Delta\epsilon/\epsilon_c = 0.65$ used as a reference for the rest of the figures. *Upper right corner:* $\Delta\epsilon/\epsilon_c = 0.45$ (dotted), $\Delta\epsilon/\epsilon_c = 0.85$ (dashed) modified parameters in these EoS. *Lower left corner:* $c_{QM}^2 = 0.8$ (dotted) $c_{QM}^2 = 0.99$ (dashed) modifications. *Lower right corner:* $\epsilon_c = 299.359$ MeV/fm³ (dotted) and $\epsilon_c = 594.276$ MeV/fm³ (dashed) changes in these EoS with respect to the original.

Disjunct M-R constraints for Bayesian analysis !

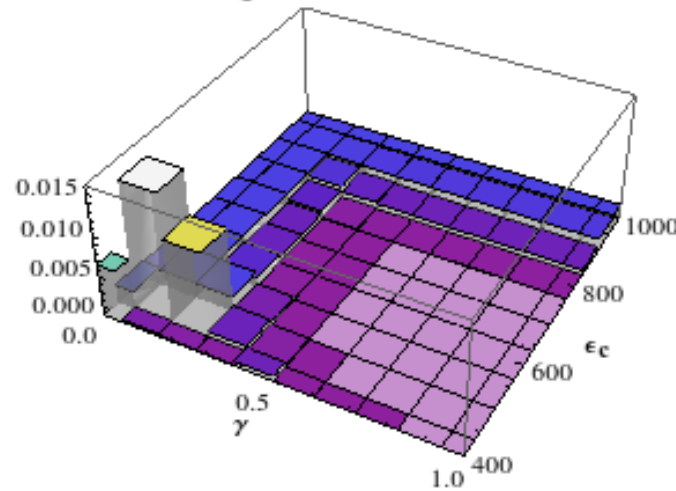


Posterior probabilities (highest values)

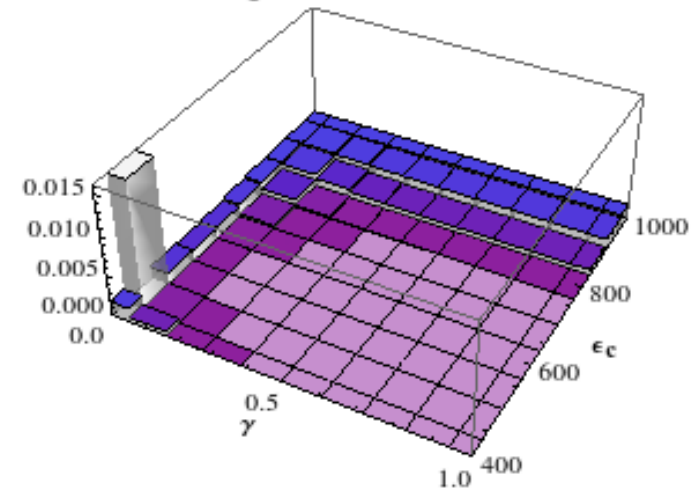
$$c^2_{QM}=0.922222$$



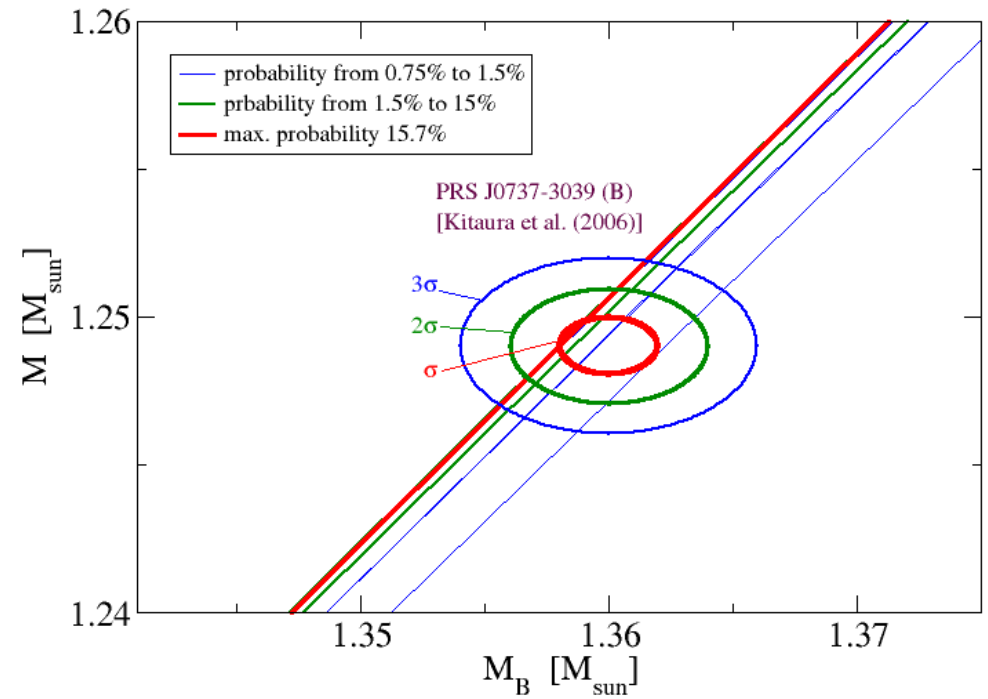
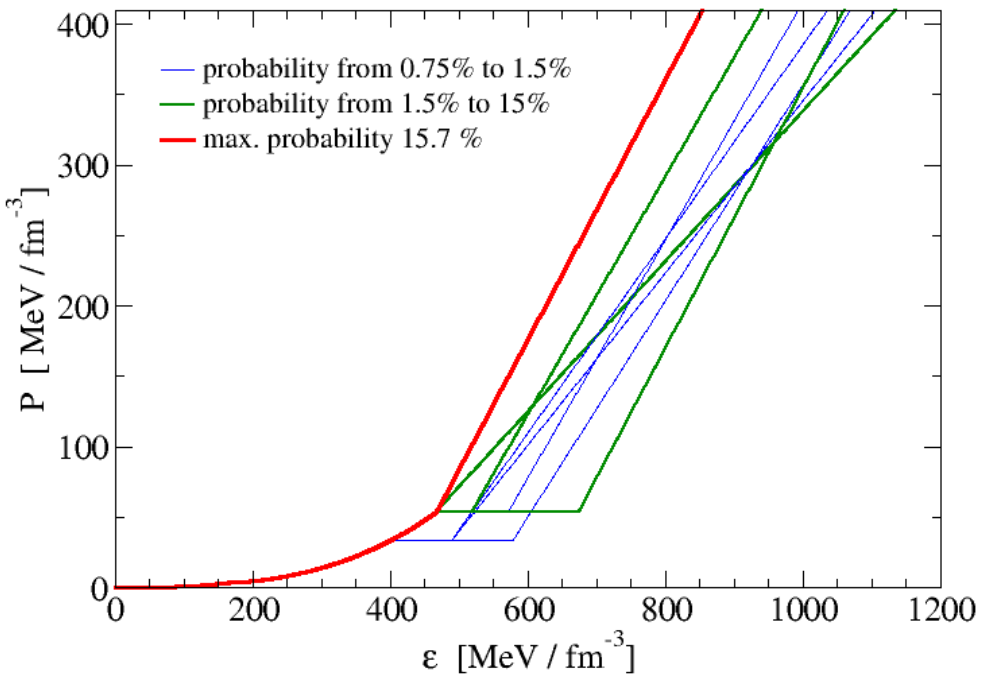
$$c^2_{QM}=0.844444$$



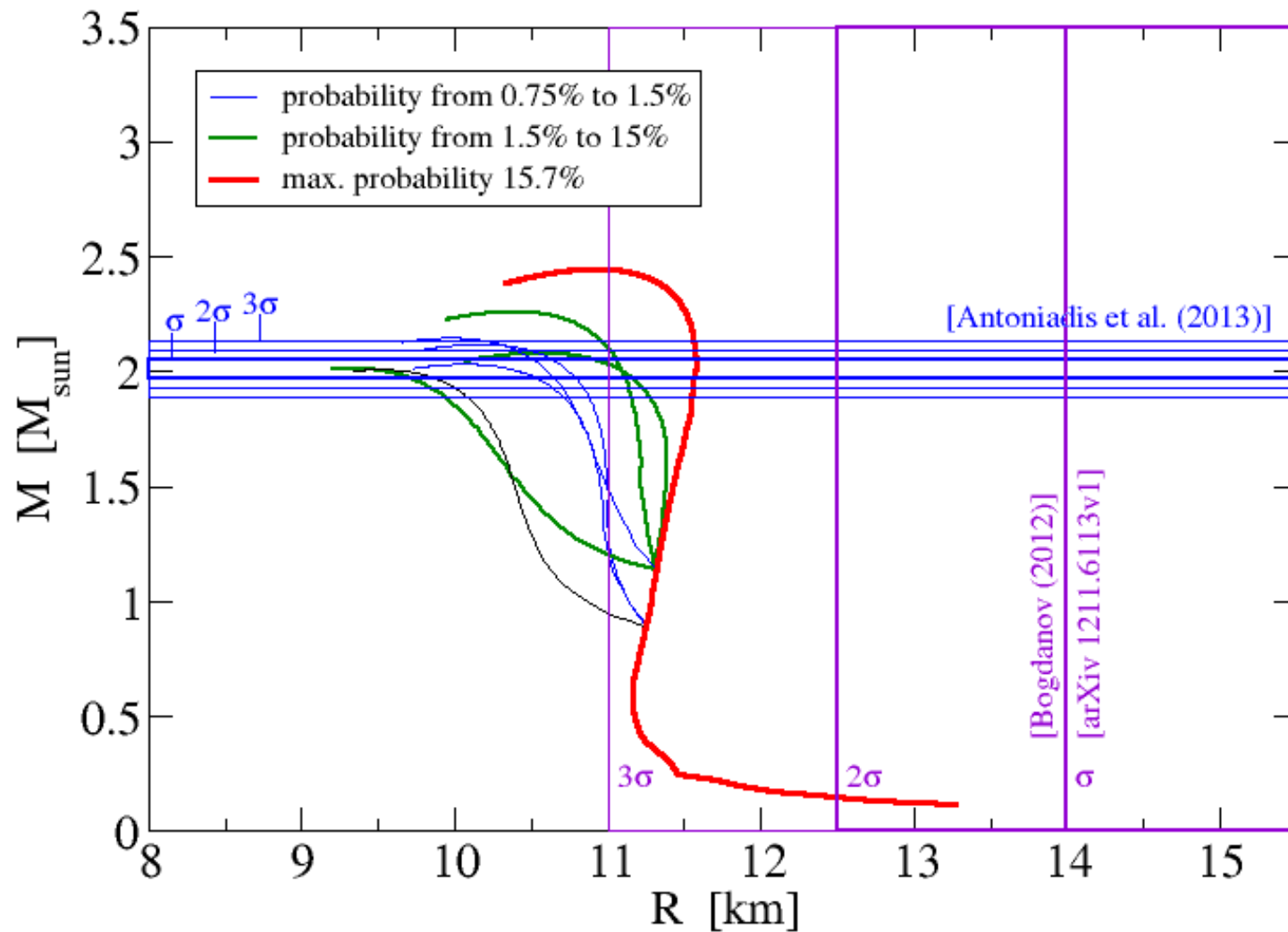
$$c^2_{QM}=0.533333$$



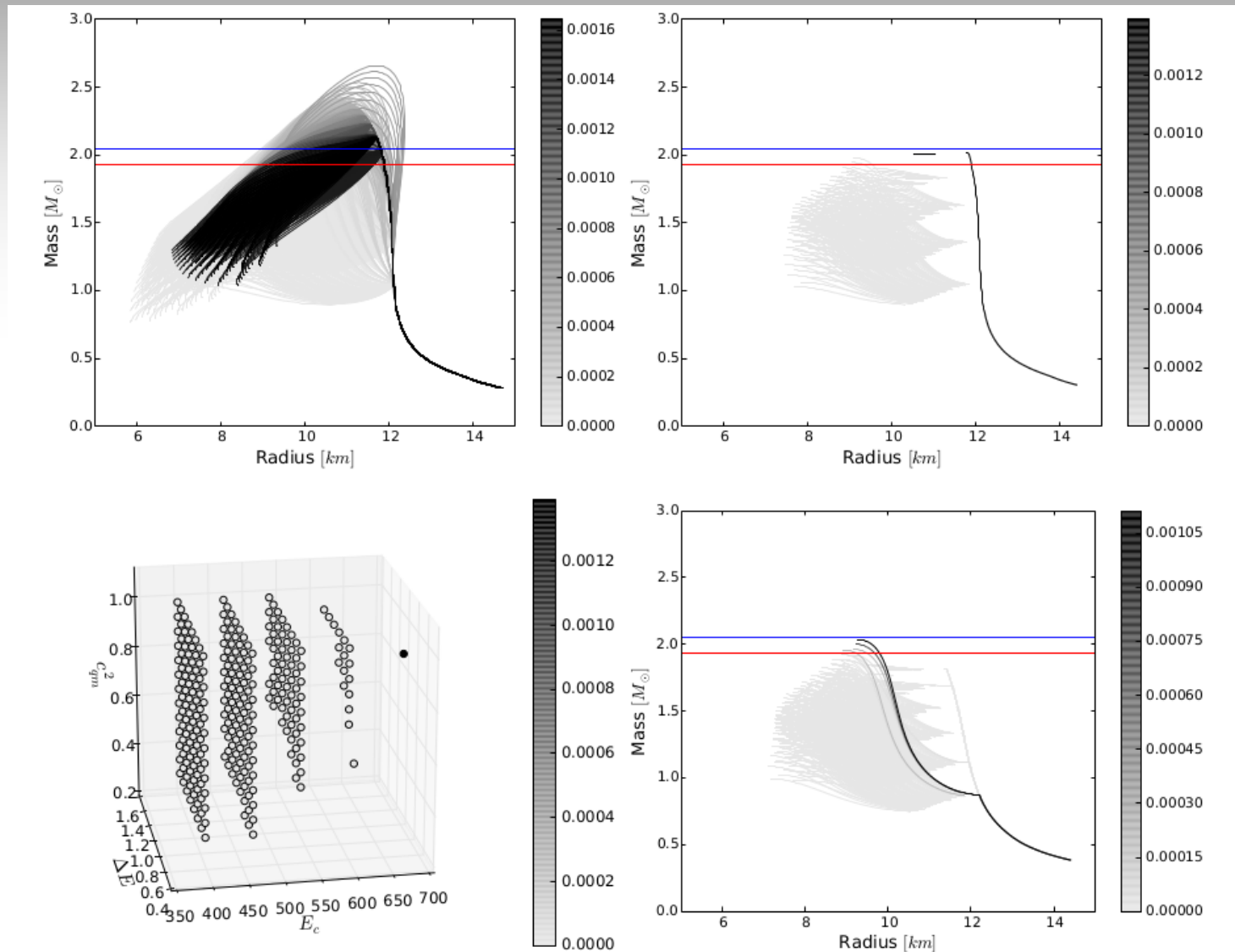
The most probable EoS set



The most probable EoS set



Phase transition measuring radii



Alvarez-Castillo, Ayriyan, Blaschke, Grigorian, Sokolowski (in progress, 2014)

Conclusions

- Given the knowledge from lattice QCD that at zero baryon density the QCD phase transition proceeds as a crossover, twins would then support the existence of a CEP in the QCD phase diagram.
- Modelling compact star twins is possible via realistic models based on a QCD motivated hPNJL models fulfilling observations.
- Bayesian Analysis on hybrid compact star EoS has the power to provide the corresponding probabilities for radius measurements in order to identify mass twins.

GRACIAS