Approach to equilibrium in weakly coupled nonabelian plasmas

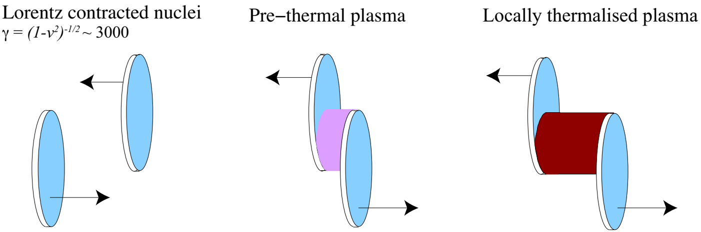
Aleksi Kurkela,

1401.3751 with M. Abraao York, E. Lu, and G. Moore (McGill)
1405.6318 with E. Lu

1107.5050, 1108.4684, 1209.4091, 1207.1663 with Moore

- What: Thermalization in $\alpha \ll 1$ nonabelian gauge theory
- How: Using combination of classical field theory and kinetic theory
- New: Smooth shift of d.o.f from fields to particles, first numerical estimates of bottom-up thermalization

Motivation:



- Rapid *thermalization* $\tau_0 \lesssim .5\text{fm/c}$

Questions:

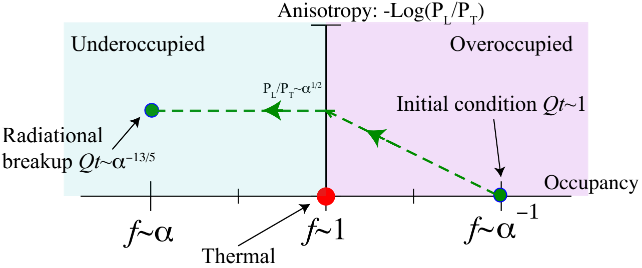
- How does the thermalization time arise?
- What happens before?
- What can be observed in experiment?

Theoretical challenge:

- Strong coupling, $\mathcal{N} = 4$: colliding shock waves — Chesler, Yaffe 1011.3562
- Weak coupling, QCD: "Bottom-up" scenario

Baier, Mueller, Schiff, Son hep-ph/0009237

Motivation: Bottom-up thermalization Baier et al. hep-ph/0009237



- Color Glass Condensate: Initial condition overoccupied

Iancu et. al hep-ph/0202270, Gelis et. al 0708.0047, 1002.0333

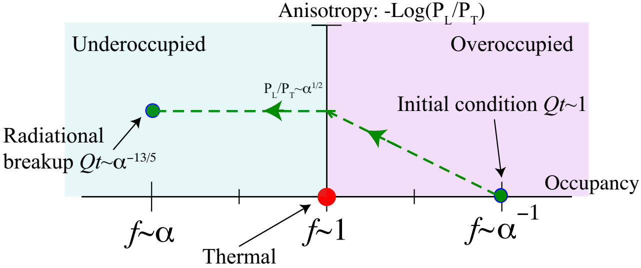
$$f(Q) \sim 1/\alpha$$

- Expansion makes system underoccupied before thermalizing

Baier et. al hep-ph/0009237, AK, Moore 1108.4684

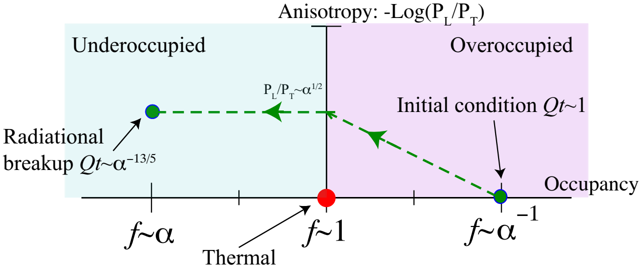
$$f(Q) \ll 1$$

Motivation: Bottom-up thermalization Baier et al. hep-ph/0009237



- Degrees of freedom:
  - Overoccupied: Classical field theory, $f \gg 1$
  - Underoccupied: Classical particles, eff. kinetic theory, $f \ll 1/\alpha$
- Full description: Need change of d.o.f. from fields to particles
- Overlapping domain of validity $1 \ll f \ll 1/\alpha$: Field-particle duality

Motivation: Bottom-up thermalization Baier et al. hep-ph/0009237



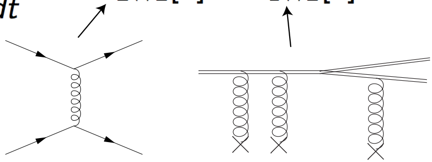
Recent works:

- Initial condition: Gelis, Epelbaum 1307.1765, 1307.2214, Lappi 1105.5511...
- Classical field evolution: Berges et al. 1303.5650, 1311.3005 ...
- Change of d.o.f. and radiational breakup, this talk.

Effective kinetic theory of Arnold, Moore and Yaffe

hep-ph/0209353

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$



- Soft and collinear divergences lead to nontrivial matrix elements

  soft: screening, Hard-loop; collinear: LPM, ladder resum

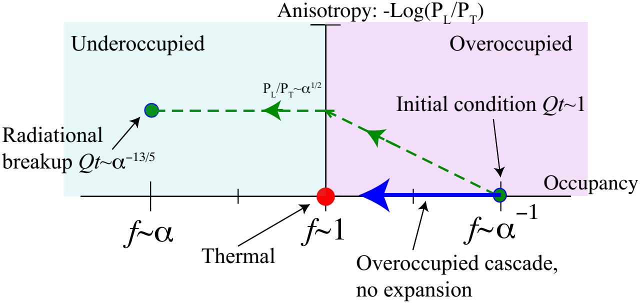
- No free parameters; LO accurate in the $\alpha \to 0$, $\alpha f \to 0$ limit.

- Used (in linearized form) *e.g.* for LO transport coefficients in QCD

  Arnold, Moore, Yaffe hep-ph/0302165

Outline:

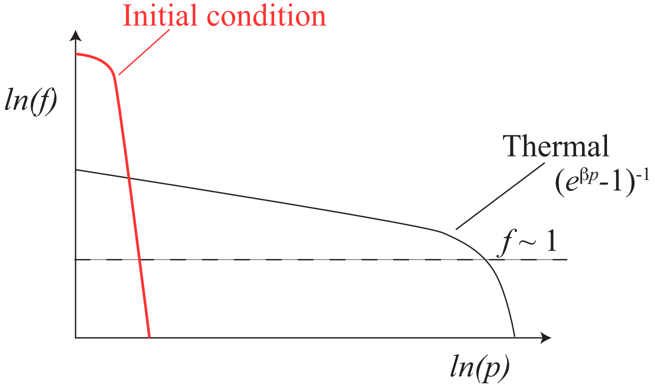
- Isotropic overoccupied system, field-particle duality
- Isotropic underoccupied system, radiational breakup
- Application to heavy-ion collisions a la BMSS



Overoccupied cascade

Abraao York, AK, Lu, Moore 1401.3751

What happens if you have too many soft gluons, $f \sim 1/\alpha$.
No longitudinal expansion.



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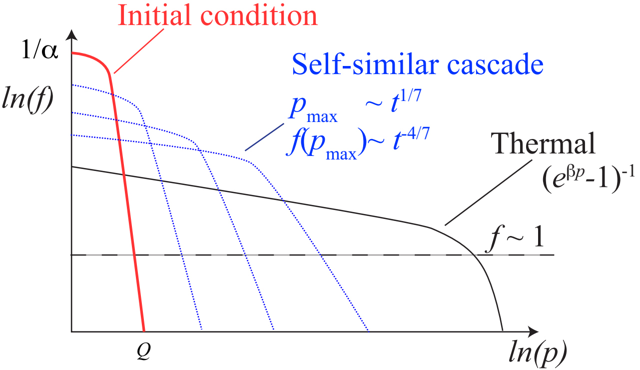
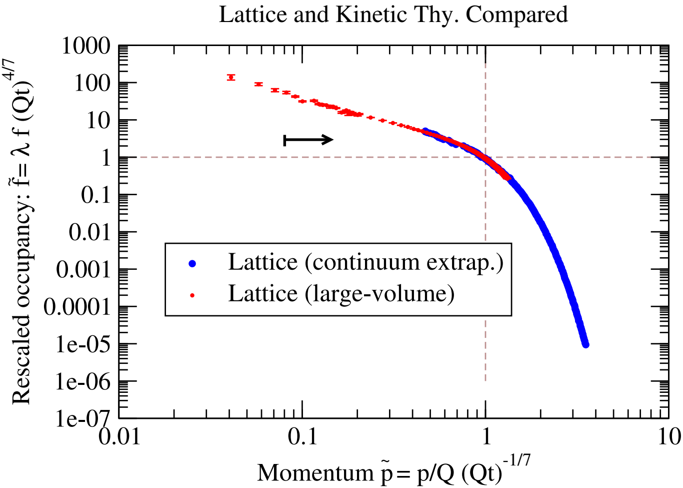


$$\tau_{\text{init}} \sim \left(\frac{Q}{T}\right)^7 \frac{1}{\alpha^2 T} \ll \frac{1}{\alpha^2 T} \sim \tau_{\text{them.}}$$

Overoccupied cascade

Abraao York, AK, Lu, Moore 1401.3751



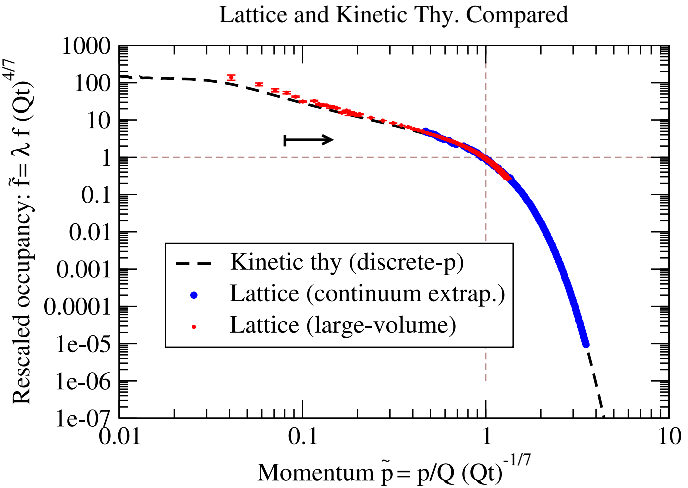
Form of cascade from classical lattice simulation,

$$1 \ll f \lesssim 1/\alpha$$

Large-volume: (Qa)=0.2, (QL)=51.2, Cont. extr.: down to (Qa)=0.1, (QL)=25.6, Qt=2000, $\tilde{m} = 0.08$

Overoccupied cascade

Abraao York, AK, Lu, Moore 1401.3751



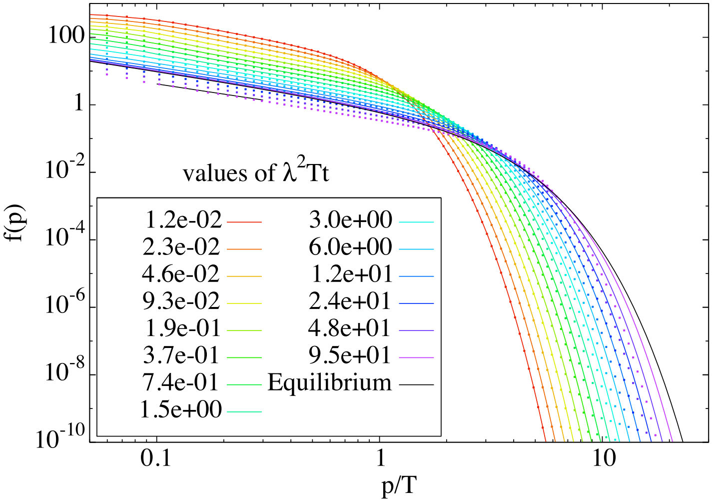
Same system, very different degrees of freedom

$$1 \lesssim f \ll 1/\alpha$$

Sensitive to the details of the collision terms

Ending of the overoccupied cascade

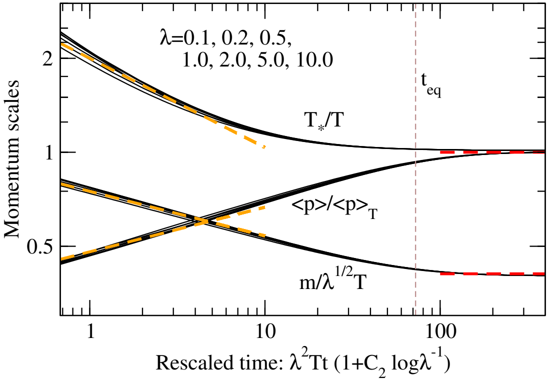
AK, Lu 1405.6318



Thermal equilibrium reached once $f \sim 1$ (or $t \sim \frac{1}{\alpha^2 T}$).

Ending of the overoccupied cascade

AK, Lu 1405.6318



$$m^2 = \lambda \int_{\mathbf{p}} \frac{f(p)}{p}$$

$$T_* = \frac{\lambda}{2} \int_{\mathbf{p}} f(p)[1 + f(p)]/m^2$$

$$\langle p \rangle = \frac{1}{n} \int_{\mathbf{p}} p\, f(p)$$

Therm. time through the approach of $\langle p \rangle - \langle p \rangle_T \sim \exp(-t/t_{\mathrm{eq}})$

$$t_{\mathrm{eq}} \approx \frac{72.}{1 + 0.12 \log \lambda^{-1}} \frac{1}{\lambda^2 T}, \qquad \lambda = 4\pi N_c \alpha.$$

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- Isotropic underoccupied system, radiational breakup
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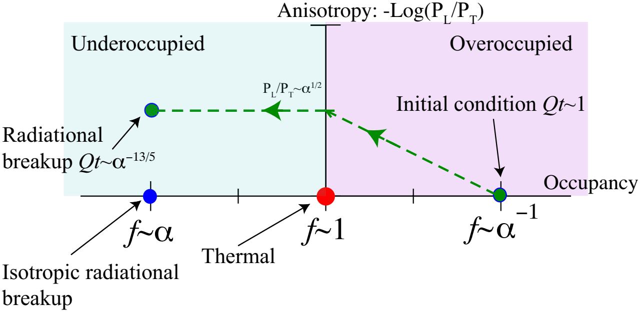
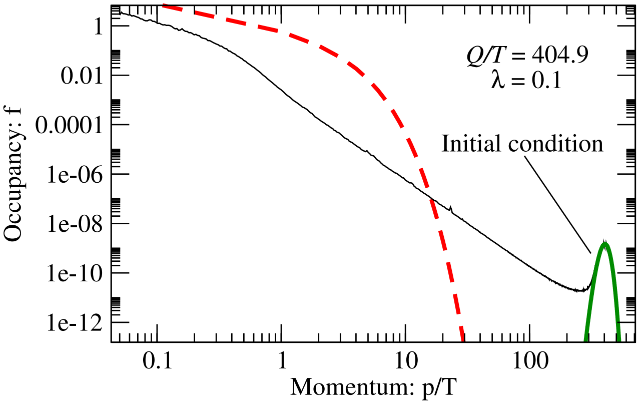


Underoccupied cascade

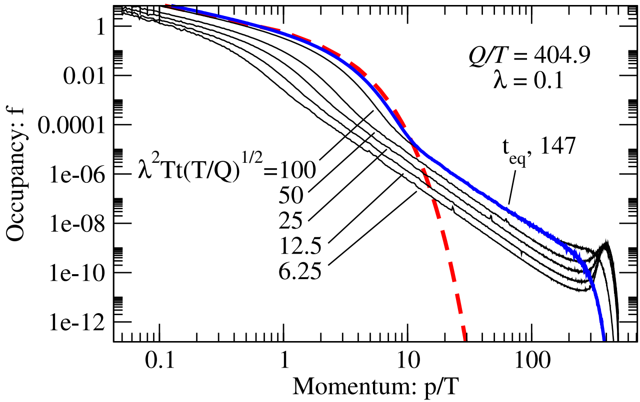
AK, Lu, 1405.6318



- Start with an underoccupied initial conditon
- after a very short time, an IR bath is created

Underoccupied cascade

AK, Lu, 1405.6318



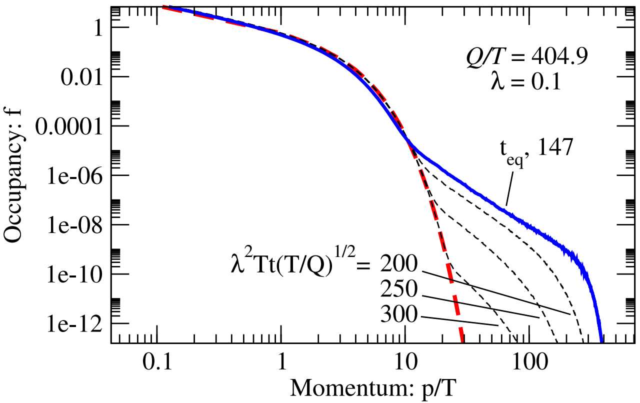
- More energy flows to the IR, temperature increases, "Bottom-up"
- Thermalization time given by the time to quench a jet of a momentum Q

AK, Moore 1107.5050

$$t_{\rm eq} \sim (Q/T)^{1/2}/\lambda^2 T$$

Underoccupied cascade

AK, Lu, 1405.6318



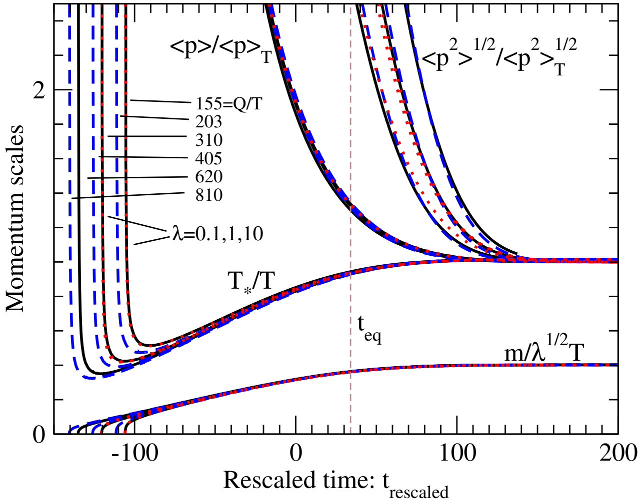
- Hardest scales reach equilibrium last.

Close resemblance to Blaizot, Iancu, Mehtar-tani 1301.6102

Underoccupied cascade, scaling analysis

AK, Lu 1405.6318

Scaling analysis with gaussian and step-cutoff initial conditions



$$t_{\text{eq}} \approx \frac{34. + 21.\ln(Q/T)}{1 + 0.037\log\lambda^{-1}} \left(\frac{Q}{T}\right)^{1/2} \frac{1}{\lambda^2 T}$$

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Connection to heavy-ion physics

AK, Lu 1405.6318

Bottom-up thermalization a la BMSS:

Baier et. al hep-ph/0009237, AK, Moore 1108.4684

- Underoccupied cascade, but expansion reduces the target temperature

$$\tau_{\rm eq} \sim \frac{1}{\alpha^2 T}\left(\frac{Q}{T}\right)^{1/2}, \quad \epsilon \sim T^4 \sim \frac{Q^4}{\alpha(Qt)} \Rightarrow Qt \sim \alpha^{-13/5}$$

- Rough estimate: replace parametric estimate by the numerical
  - Estimate for energy density $\epsilon \approx 1.5 d_A Q^4/\pi\lambda(Qt)$ and $\alpha = 0.3$

Lappi 1105.5511

$$Qt_{\rm eq} \approx 1.5$$

Caveats: Angular dependence, no fermions, definition dependence of $t_{\rm eq}$, extrapolation to $\alpha = 0.3$

Connection to heavy-ion physics

AK, Lu 1405.6318

Quantifying uncertainties:

- For $\alpha = 0.2$:

$$Qt_{\mathrm{eq}} \approx 4.0$$

- Varying ϵ by a factor of 2:

$$Qt_{\mathrm{eq}} \approx 2.5$$

    - with $\alpha = 0.2$: $Qt_{\mathrm{eq}} \approx 8.0$

- Replacing free streaming $(Qt)^{-1}$ by redshifting $(Qt)^{-4/3}$:

$$Qt_{\mathrm{eq}} < 4$$

For $Q_s \approx 2\mathrm{GeV}$, corresponds to

$$t_{\mathrm{eq}} \approx 0.1 - 1\mathrm{fm/c}$$

Conclusions

- Combination of classical simulations and effective kinetic theory allows to follow the time evolution from highly occupied initial condition to thermal equilibrium
- Thermalization times for simple systems faster than naively expected
- Inserting the underoccupied thermalization time to bottom-up thermalization yields a rough estimate for heavy-ion collisions

$$t_{\mathrm{eq}} \lesssim 0.1 - 1\mathrm{fm/c}$$

Outlook

- Proper treatment of expansion and angular dependence
- Implementation of fermions to kinetic theory — In preparation, Lu
- Inclusions of plasma unstable modes — AK, Moore 1108.4684
- NLO not inconceivable
- Applications to jets — AK, Wiedemann 1407.0293

p.s. No sign of BEC AK, Moore 1207.1663

Equilibration Mechanisms in Weakly and Strongly Coupled Quantum Field Theory

INT Program INT-15-2c : August 3 - 28, 2015

Organizers : J. Casalderrey Solana, F. Gelis, A. Kurkela, A. Vuorinen

- Thermalization at weak coupling
- Thermalization at strong coupling
- How to bridge the gap between weak and strong coupling thermalization studies?
- Jets as a nonequilibrium system
- Thermalization and heavy ion phenomenology
- What can thermalization studies in heavy ion physics learn from or teach to other fields
- and much more…

Scaling analysis

$$f_{\mathrm{step}}(p) \propto \Theta(Q_s - p), \quad f_g(p) \propto \exp\left[-\frac{(Q_s - p)^2}{(Q_s/10)^2}\right]$$

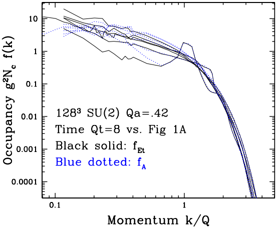
$$Q^2 \equiv \int_{\mathbf{p}} f(p)p^2 / \int_{\mathbf{p}} f(p)$$

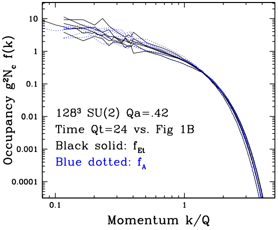
run	Q/T	n_H/n_T	λ	init	run	Q/T	n_H/n_T	λ	init
1	202.5	0.0134	0.1	g	4	155.1	0.01799	0.1	step
2	404.9	0.00668	0.1	g	5	310.0	0.00900	0.1	step
3	809.8	0.00334	0.1	g	6	620.0	0.00450	0.1	step
7	155.137	0.01799	1.0	step	9	310.0	0.00900	1.0	step
8	155.137	0.01799	10.0	step	10	310.0	0.00900	10.0	step

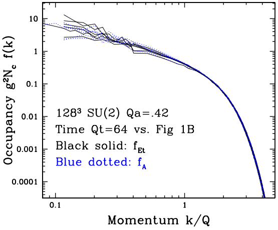
In reality many more simulations with varying parameters

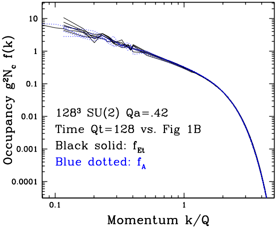
Information on the overoccupied initial condition lost in scattering time of the initial condition

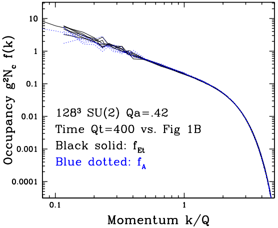
AK, Moore 1207.1663









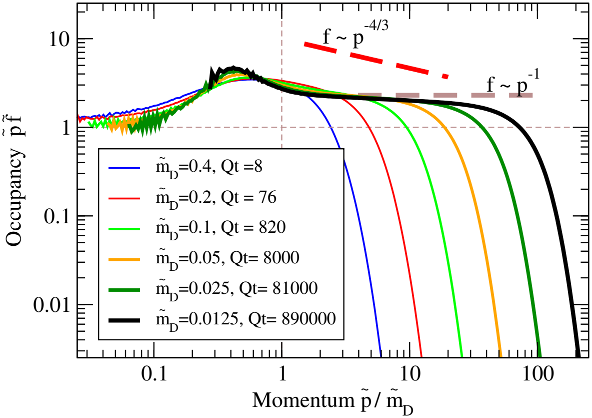


$$\tau_{\rm init} \sim \left(\frac{Q}{T}\right)^7 \frac{1}{\alpha^2 T} \ll \frac{1}{\alpha^2 T} \sim \tau_{\rm them}.$$

Power law from of the cascade

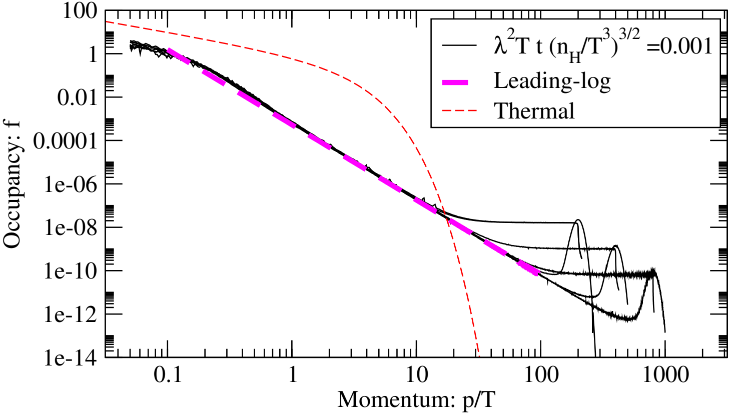
- Low scales have time to thermalize: $1/p$ — AK, Moore,1107.5050
- Turbulent kolmogorov cascade $1/p^{4/3}$, (BEC: $1/p^{3/2}$)?

Berges et al 0811.4293



Underoccupied cascade

Set of isotropic, underoccupied initial conditions, initial scale $\langle p^2 \rangle = Q^2$



At early times, connection to deep-LPM limit of $C_{1\leftrightarrow 2}$ by Arnold and Dogan

0804.3359