Primary and secondary production of heavy quarks in final-state jets

Vicent Mateu
University of Vienna

In collaboration with A. Hoang, I. Stewart, B. Dehnadi, M. Butenschön & P. Pietrulewicz

based on [1405.4860] and work in progress

XI Confinement (Saint Petersburg)

09-09-2014

Outline

- Motivation, aims & introduction
- Factorization theorem for massless quarks
- Secondary massive quark effects: scenarios
- Primary massive quark effects
- Conclusions & Outlook

Motivation, aims
&
Introduction

Motivations and aims

- Precision jet physics at c.o.m. energies of 14, 22 GeV needs full bottom mass dependence

  JADE, TASSO

  [See talk by VM on Friday, Parallel II: light quarks]

  e.g. α_s determinations or bottom mass determinations

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- Accurate top mass predictions at Tevatron and LHC, but unknown scheme

  what is m_t^{Pythia}? Does it correspond to a reach scheme?

  [See talk by J. Erler, this morning, plenary 3]

  Additional "conceptual" uncertainty of ~ $\mathcal{O}(1\,\mathrm{GeV})$ … respect to what?

$$m_t^{\mathrm{Pythia}} \sim m_t^{\mathrm{short-distance}} + \mathcal{O}(1\mathrm{GeV})$$

m_t^{Pythia}	$m_t^{\mathrm{short-distance}}$	$\mathcal{O}(1\mathrm{GeV})$
is this really a scheme??	what scheme exactly??	conservative enough??

We are able to do hadron level predictions with our formalism, allowing for a direct comparison to Pythia: fit α_s and a short distance top-mass from Pythia

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  We are able to do hadron level predictions with our formalism, allowing for a direct comparison to Pythia: fit α_s and a short distance top-mass from Pythia

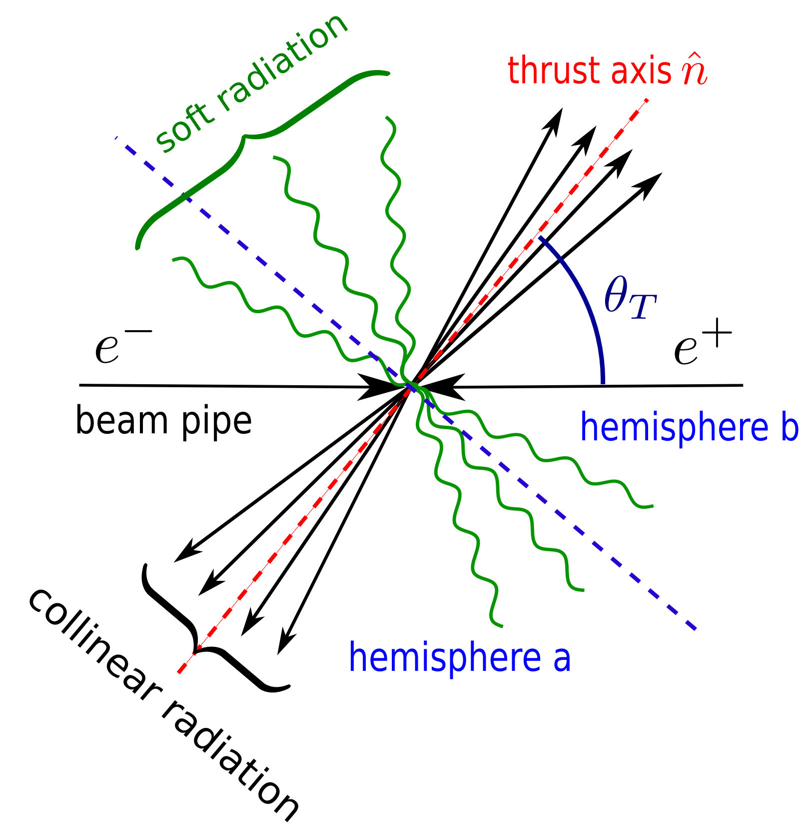
- Ultimate aim is to apply this technology to a hadron collider (boosted top production)

  accurate top mass determination

Event Shapes

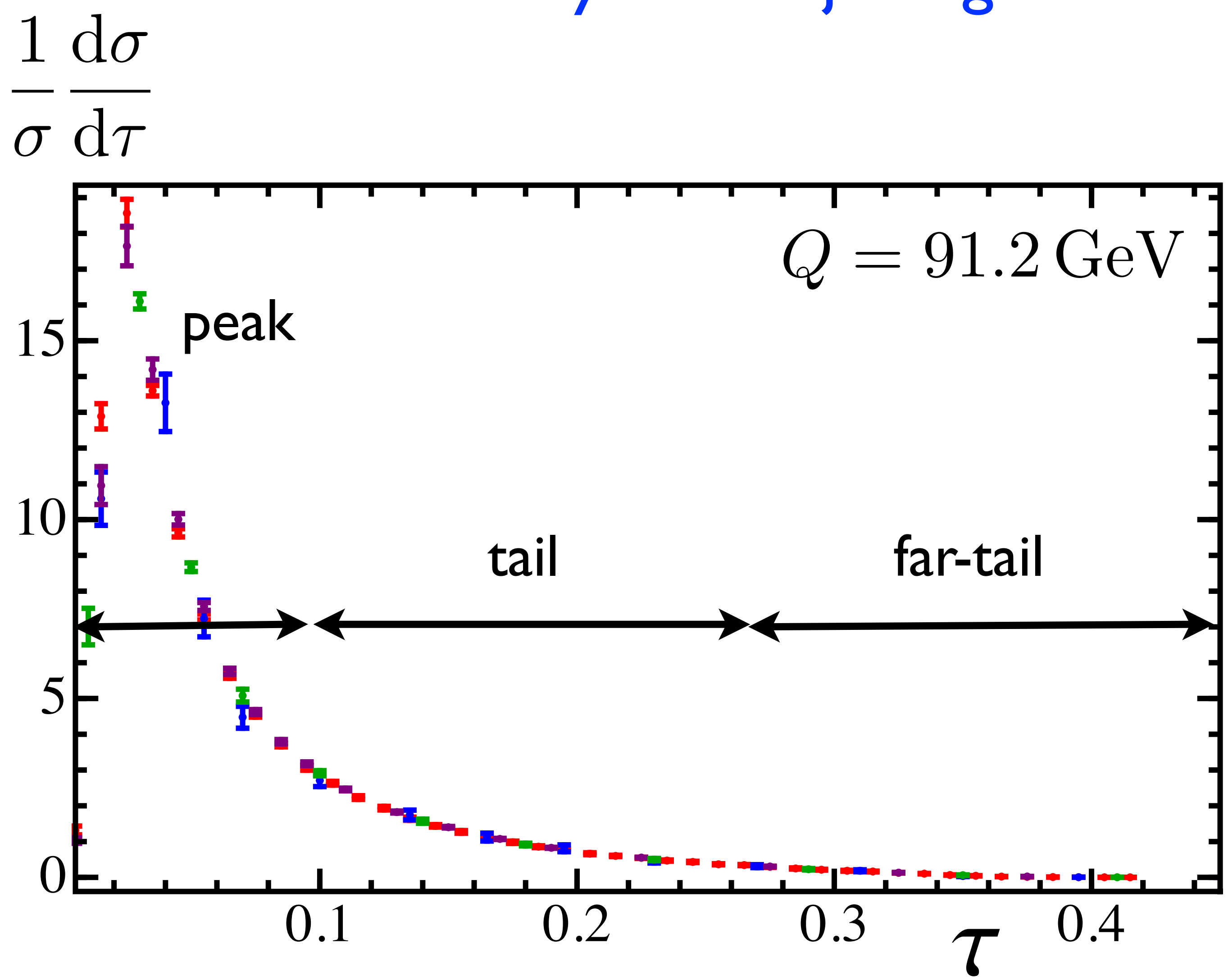
Event shapes characterize in a geometrical way the distribution of hadrons in the final state

$e^+ e^- \to \text{jets}$



Thrust $$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

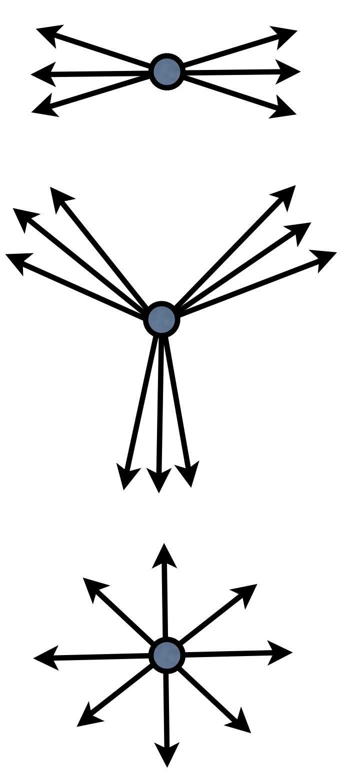
more friendly than a jet algorithm



dijet $\tau \sim 0$

three jets $\tau \sim 0.3$

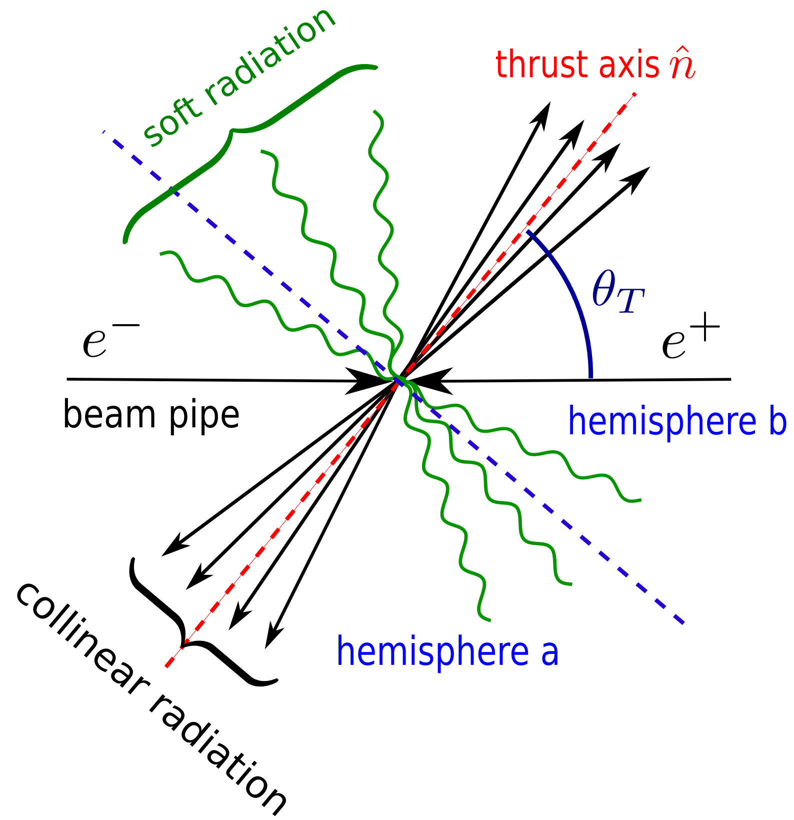
spherical $\tau \sim 0.5$



Continuous transition from 2-jet to 3-jet, ... multi-jet events

Event Shapes

$e^+ e^- \to \text{jets}$

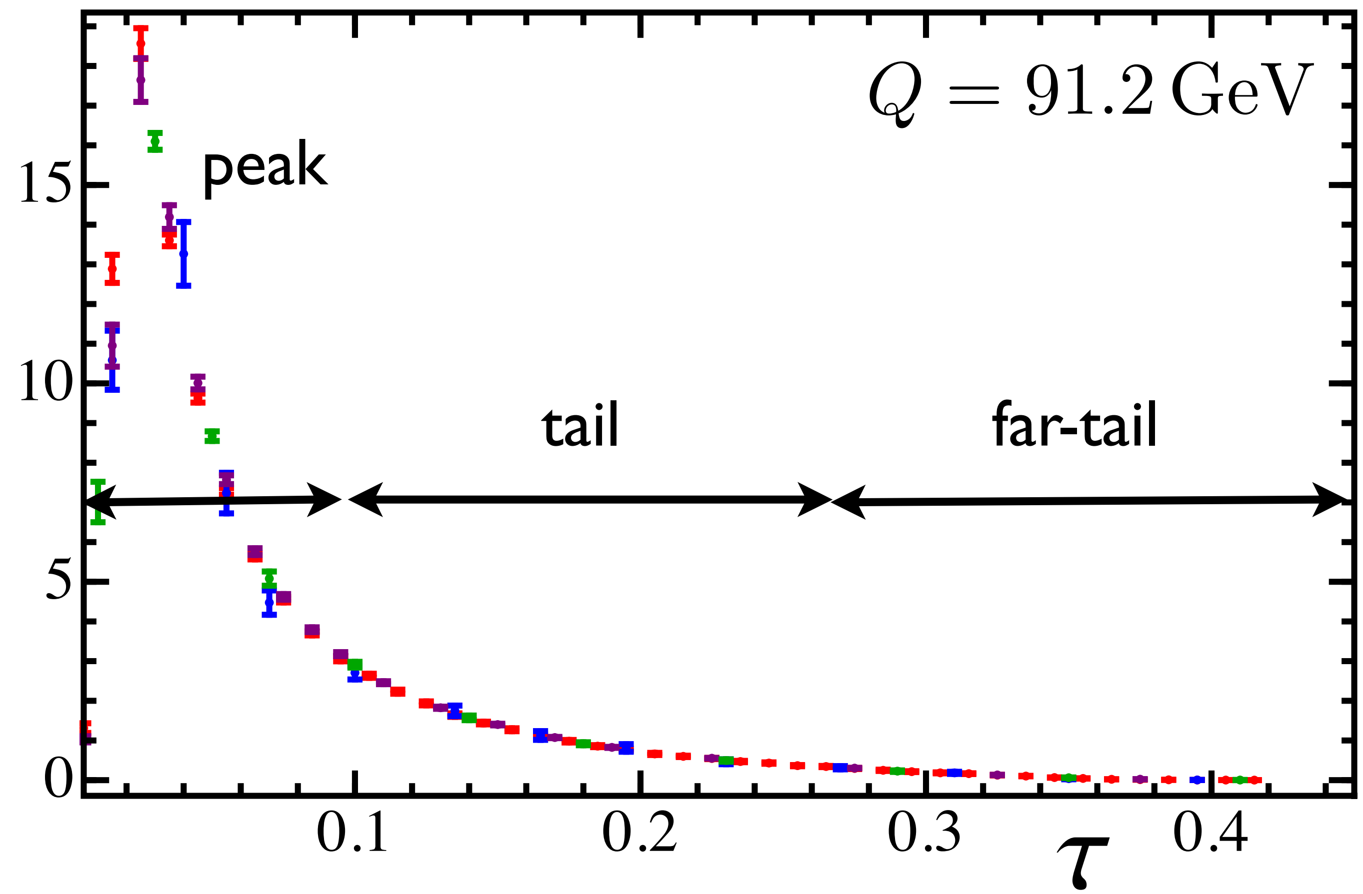


Event shapes characterize in a geometrical way the distribution of hadrons in the final state

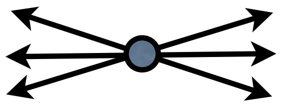
2-jettiness $\tau = 1 - \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{Q}$

modification that enhances quark mass dependence

$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}$



dijet $\tau \sim 0$



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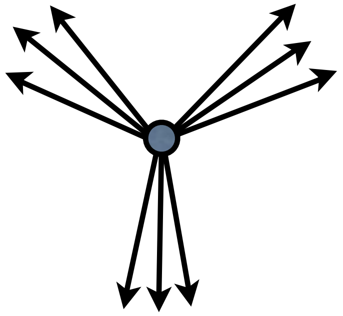
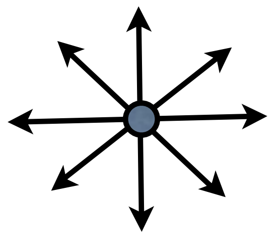


spherical $\tau \sim 0.5$



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Factorization for massless quarks

Resummation of large logarithms

Event shapes are not inclusive quantities

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = -\frac{2\alpha_s}{3\pi}\frac{1}{\tau}\Big(3 + 4\log\tau + \ldots\Big)$$

Large logs at small τ

Invalidates perturbative expression for small τ

One has to reorganize the expansion by considering $\alpha_s\,\lg(\tau) \sim \mathcal{O}(1)$

Counting more clear in the exponent of cumulant

$$\Sigma(\tau_c) \equiv \int_0^{\tau_c} \mathrm{d}\tau \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}$$

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$$\log\Sigma(\tau_c) = \alpha_s(\log^2\tau_c + \log\tau_c + 1)$$ LO

[Catani, Seymour] $$\alpha_s^2(\log^3\tau_c + \log^2\tau_c + \log\tau_c + 1)$$ NLO

State of the art $$\alpha_s^3(\log^4\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1)$$ NNLO

$$\alpha_s^4(\log^5\tau_c + \log^3\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1)$$

$\dots$

not known!

[Weinzierl]

[Gehrmann-De Rider, Gehrmann, Glover, Heinrich]

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$$\begin{aligned}
\log\Sigma(\tau_c) = {} & \alpha_s(\log^2\tau_c + \log\tau_c + 1) \\
& \alpha_s^2\,(\log^3\tau_c + \log^2\tau_c + \log\tau_c + 1) \\
& \alpha_s^3\,(\log^4\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1) \\
& \alpha_s^4\,(\log^5\tau_c + \log^3\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1) \\
& \quad\vdots
\end{aligned}$$

LL	NLL	N²LL	N³LL	not known!

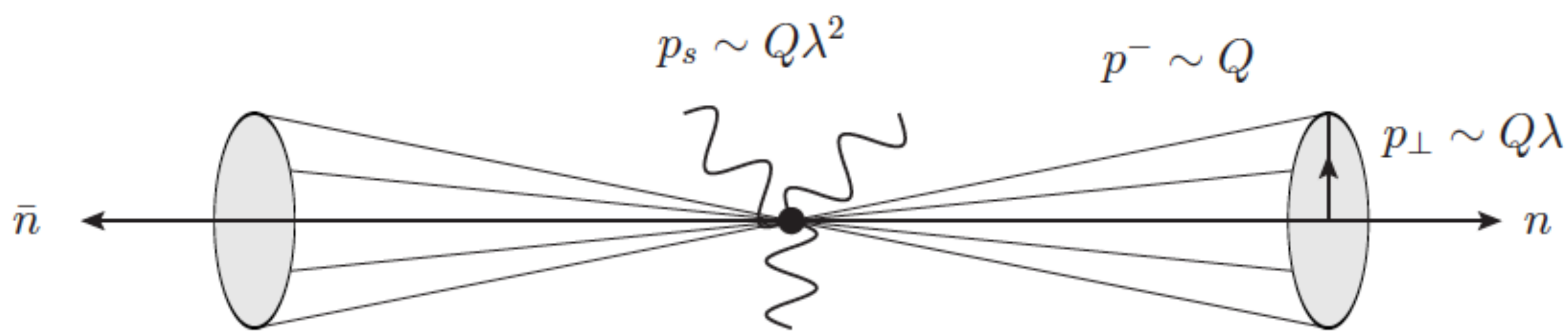
State of the art (N³LL)

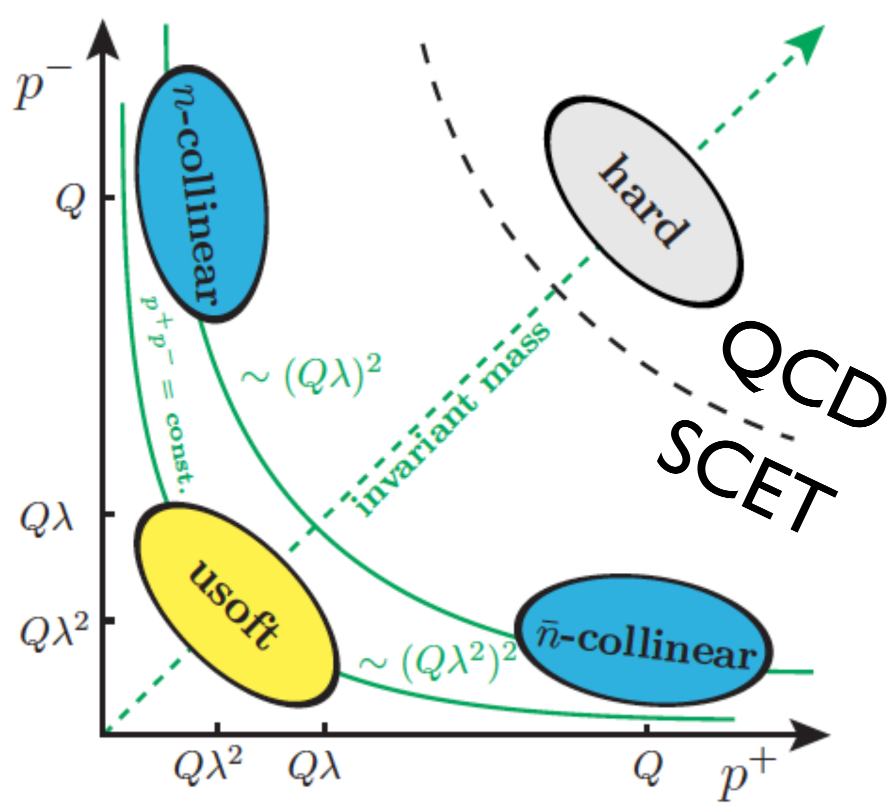
[Hoang, VM, Schwartz, Stewart]
[Becher, Schwartz]
[Chien, Schwartz]
[Abbate, Fickinger, Hoang, VM, Stewart]
[Hoang, Kolodrubetz, VM, Stewart]

SCET in a nutshell

[Bauer, Fleming, Luke, Pirjol, Stewart]

consider case of dijet production in e^+e^- annihilation (only light quarks)





$$n^\mu = (1,0,0,1) \qquad \bar{n}^\mu = (1,0,0,-1)$$

$$p^\mu = p^- \frac{n^\mu}{2} + p^+ \frac{\bar{n}^\mu}{2} + p_\perp^\mu \qquad p^- = E + |\vec{p}|$$

$$p^+ = E - |\vec{p}|$$

$$p^2 = p^- p^+ + p_\perp^2 \qquad p_\perp = |\vec{p}_T|$$

mode	$p^\mu = (+, -, \perp)$	p^2	fields
hard	$Q(1,1,1)$	Q^2	–
n-collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	ξ_n, A_n^μ
$\bar{n}$-collinear	$Q(1, \lambda^2, \lambda)$	$Q^2\lambda^2$	$\xi_{\bar{n}}$, $A_{\bar{n}}^\mu$
ultrasoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	q_{us}, A_{us}^μ

Factorization theorem for event shapes

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} = H_Q \times J_e \otimes S_e + \mathcal{O}\left(e^0, \frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

[Bauer, Lee, Fleming, Sterman]
[Berger, Kuks, Sterman]

Universal Wilson Coefficient

Jet function

Soft function

Nonsingular terms, power corrections

Calculable in perturbation theory

Perturbative and nonperturbative components

Factorization theorem for event shapes

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Leading power correction comes from soft function

$$S_e = \hat{S}_e \otimes F_e$$

[Korchemsky, Sterman, Tafat]

perturbative

nonperturbative & [Korchemsky & Sterman]

perturbative [VM, Thaler, Stewart]

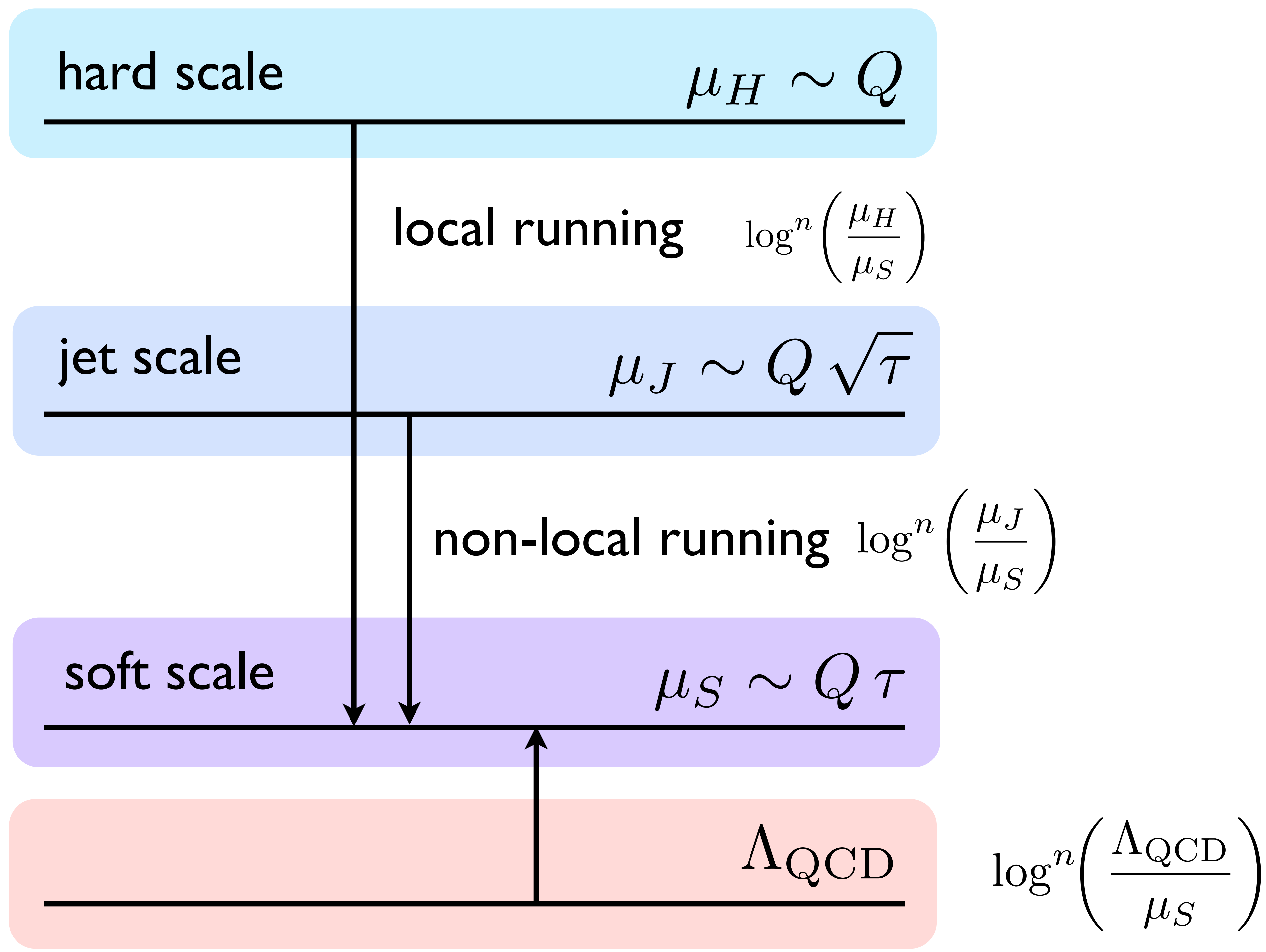
$$\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e} \otimes F_e$$

Renormalization group evolution

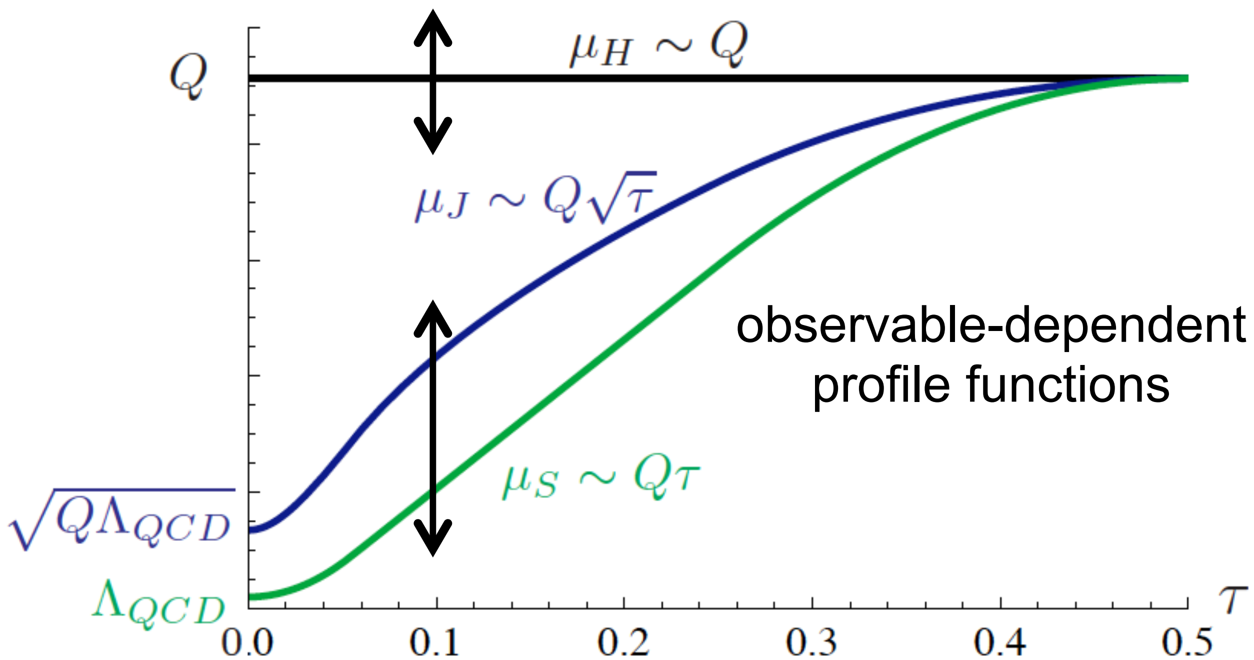
The hierarchy among the scales depends on the position on the spectrum

hard scale	$\mu_H \sim Q$	$\log^n\left(\frac{Q}{\mu}\right)$
jet scale	$\mu_J \sim Q\sqrt{\tau}$	$\log^n\left(\frac{Q^2\tau}{\mu^2}\right)$
soft scale	$\mu_S \sim Q\,\tau$	$\log^n\left(\frac{Q\tau}{\mu}\right)$
	Λ_{QCD}	large logs

Renormalization group evolution



Renormalization group evolution



$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} \sim H(\mu_Q)\, U_H(\mu_H,\mu_S) \times J(\mu_J) \otimes U_S(\mu_J,\mu_S) \otimes S(\mu_S)$$

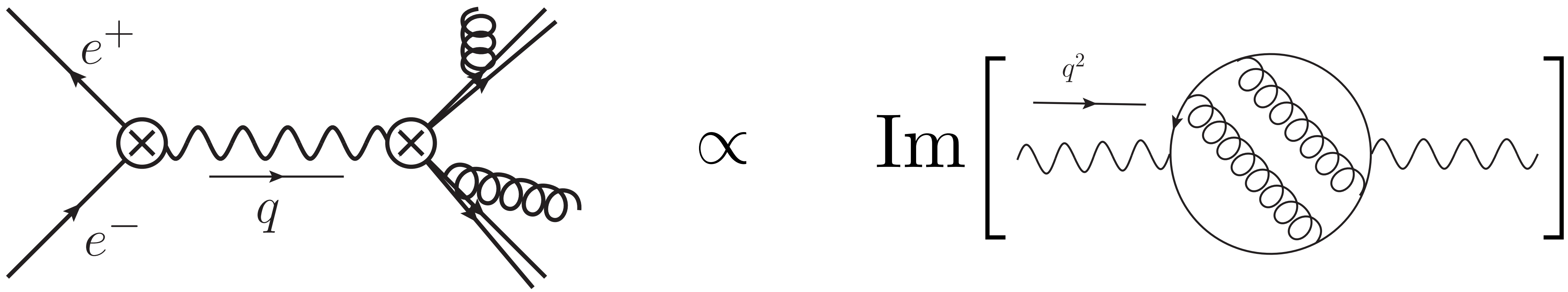
matrix elements

evolution factors

Secondary mass production

Case study: total hadronic cross section

$$R(Q^2) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \propto \text{Im}\Big[-i\int \mathrm{d}x\, e^{ix\cdot q}\langle 0|\mathrm{T}j_\mu(x)j^\mu(0)|0\rangle\Big]$$



$$= \; N_c\sum_q e_q^2\left\{1+\frac{\alpha_s^{(n_l)}(\mu)}{\pi}+\left(\frac{\alpha_s^{(n_l)}(\mu)}{\pi}\right)^2\left[r^2-\frac{\beta_0^{(n_l)}}{4}\ln\frac{Q^2}{\mu^2}\right]+\ldots\right\}$$

if only light quarks involved, only one characteristic scale, Q

no large logs if $\mu \sim Q$

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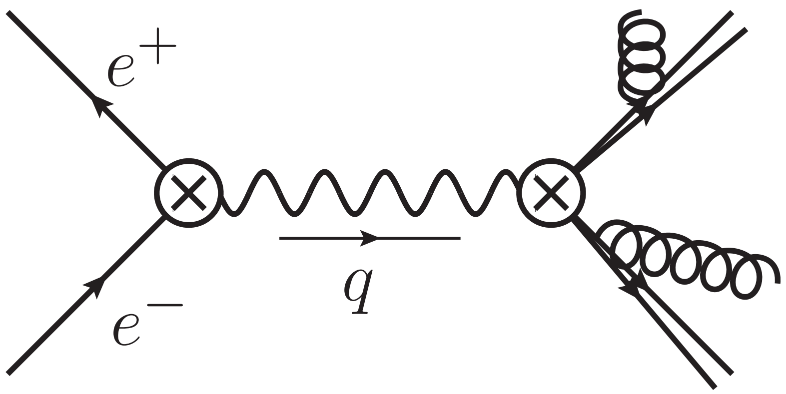
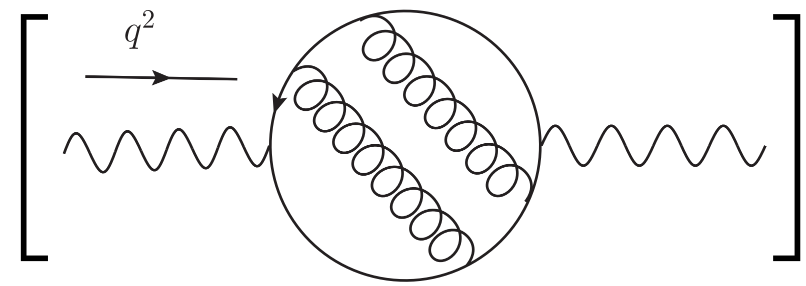


$\propto$ Im



$$= N_c \sum_q e_q^2 \left\{1 + \frac{\alpha_s^{(n_l)}(\mu)}{\pi} + \left(\frac{\alpha_s^{(n_l)}(\mu)}{\pi}\right)^2 \left[r^2 - \frac{\beta_0^{(n_l)}}{4}\ln\frac{Q^2}{\mu^2}\right] + \ldots\right\}$$

if only light quarks involved, only one characteristic scale, Q

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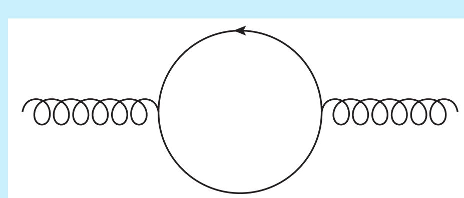
$$\beta_0 = 11 - \frac{2}{3}n_l$$



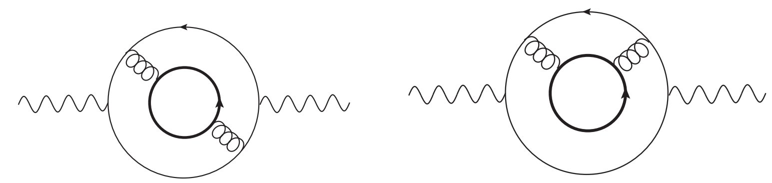
$\Pi(q^2)$

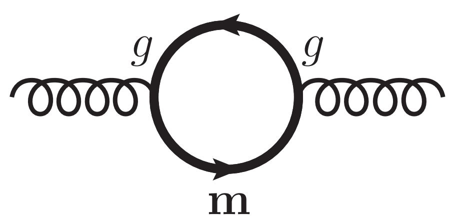
n_l dependence generated by vacuum polarization diagrams with massless quarks

$\overline{\text{MS}}$ renormalization ("only" possibility if $m_q = 0$) produces n_l term in β_0

Case study: total hadronic cross section

if heavy quarks are produced, another scale enters the game: m_h

$\mathrm{Im}\Big[$  $\Big]$

$\Pi(q^2) =$ 

$1/\epsilon$ can be subtracted: $\overline{\mathrm{MS}}$ scheme

or $\Pi(0)$ can be subtracted: OS scheme

well defined for massive quarks

$\Pi(q^2) - \Pi(0)$ is μ-independent, therefore does not contribute to β_0

$\Pi(q^2)$ in $\overline{\mathrm{MS}}$ scheme has same μ dependence as for $m_q = 0$

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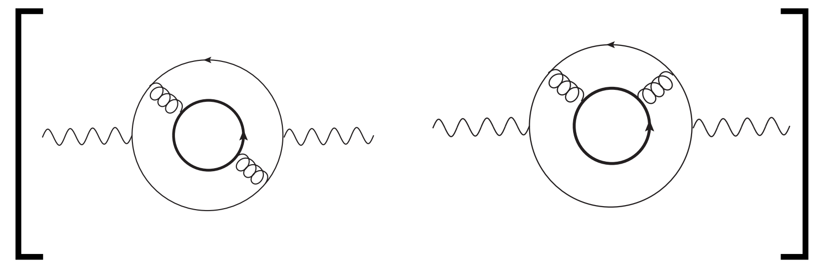
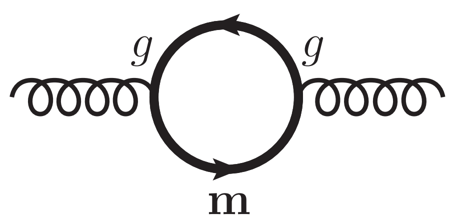
Im 

$\Pi(q^2) =$ 

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$\Pi(q^2)$ in $\overline{\mathrm{MS}}$ scheme has same μ dependence as for $m_q = 0$

$\overline{\mathrm{MS}}$ scheme: works for $m_h \sim Q$ and has smooth massless limit. Uses $\alpha_s^{(n_l+1)}$
large logs for $m_h \gg Q$ (no decoupling limit)

OS scheme: works for $m_h \sim Q$ and has smooth decoupling limit. Uses $\alpha_s^{(n_l)}$
large logs for $m_h \ll Q$ (no massless limit)

Both schemes related by the decoupling relation between $\alpha_s^{(n_l)}$ and $\alpha_s^{(n_l+1)}$

Case study: total hadronic cross section

$$\alpha_s^{(n_l)}(\mu) = \alpha_s^{(n_l+1)}(\mu)\Big(1 + \frac{T_f \alpha_s^{(n_l+1)}(\mu)}{3\pi} \ln\frac{m^2}{\mu^2} + \dots\Big)$$

Collins - Wilckek - Zee (CWZ) scheme

$\overline{\mathrm{MS}}$ scheme	$\mu \sim Q \geq m_h$
OS scheme	$\mu \sim Q \leq m_h$
matching at	$\mu \sim Q \sim m_h$

Exact mass dependence without approximations or large logs, massless and decoupling limit correctly reproduced. Introduces a matching scale $\mu_m \sim m$

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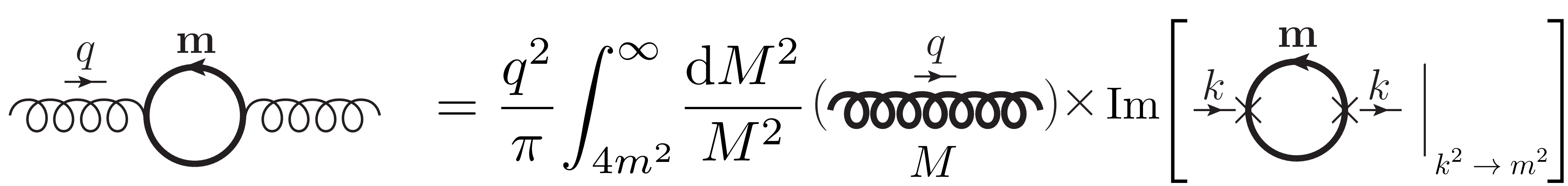
We will use exactly this ideas in our SCET factorization theorem
Situation more involved because matrix element have explicit μ dependence

dispersion relation



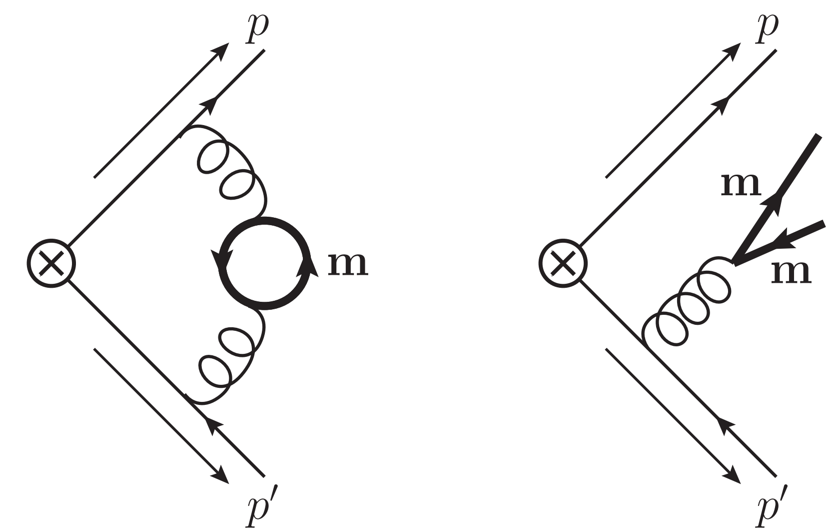
Secondary production of heavy quarks

[S.Gritschacher, A.Hoang, I.Jemos, P. Pietrulewicz (2013)]
[S.Gritschacher, A.Hoang, I.Jemos, VM, P. Pietrulewicz (2014)]

additional power counting parameter $\lambda_m \sim \frac{m}{Q}$

additional soft and collinear mass modes

mode	$p^\mu = (+, -, \perp)$	p^2
n-coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	m^2
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2



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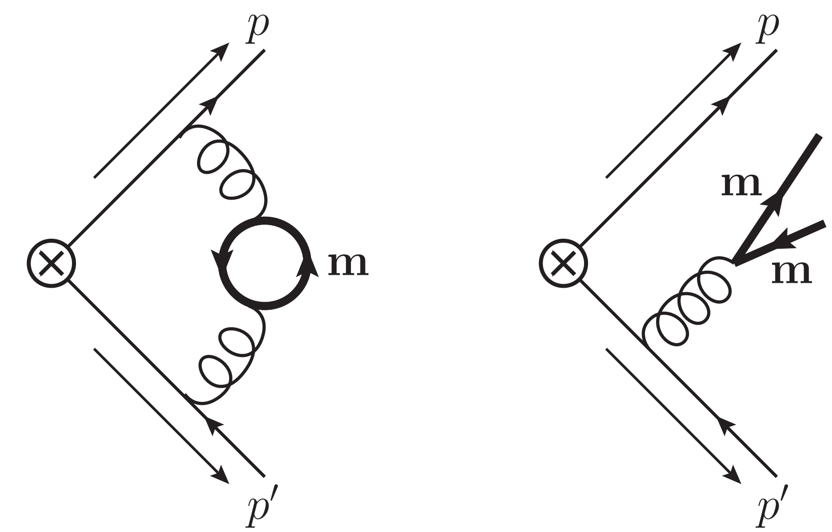

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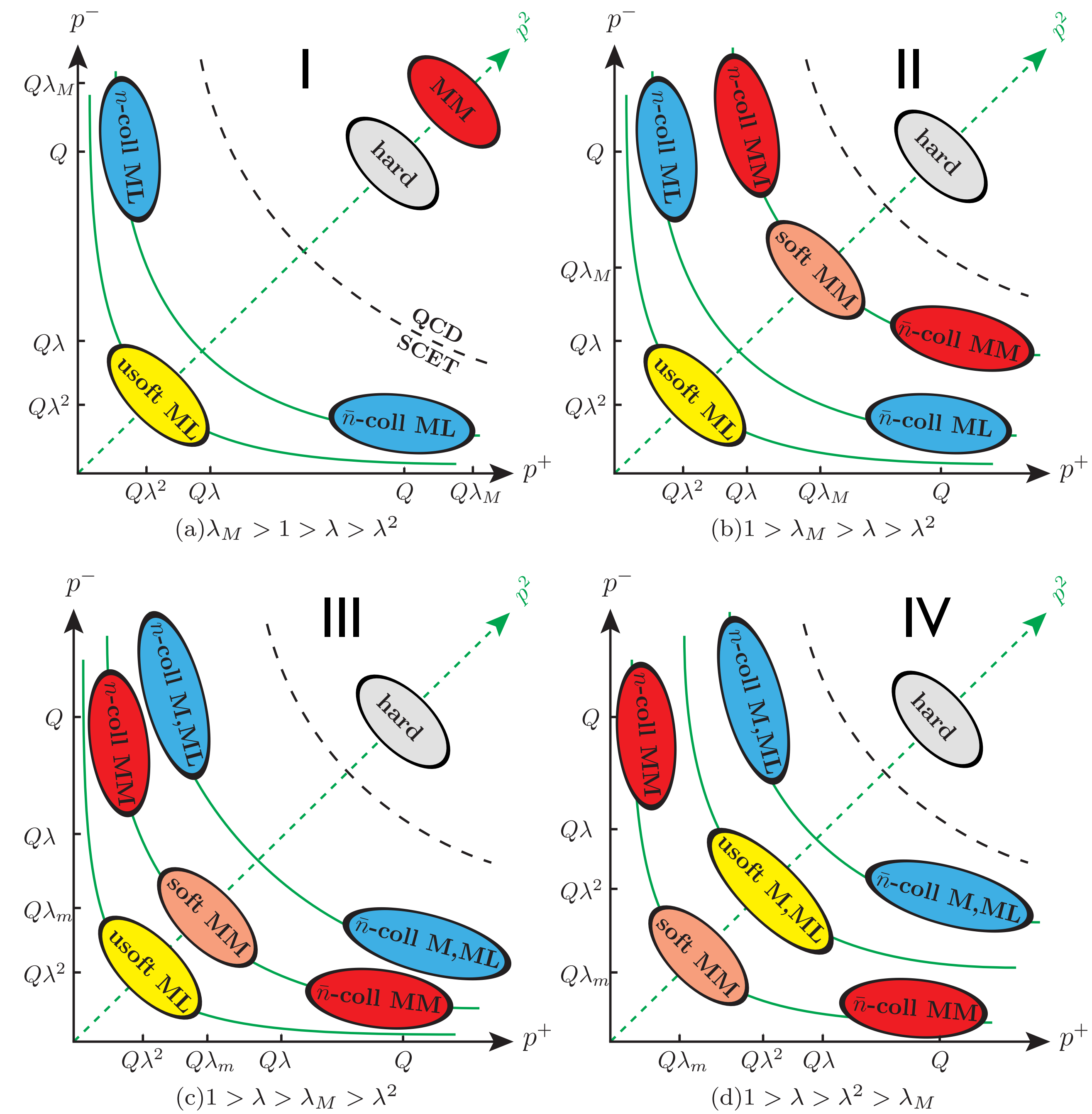
possible hierarchies:

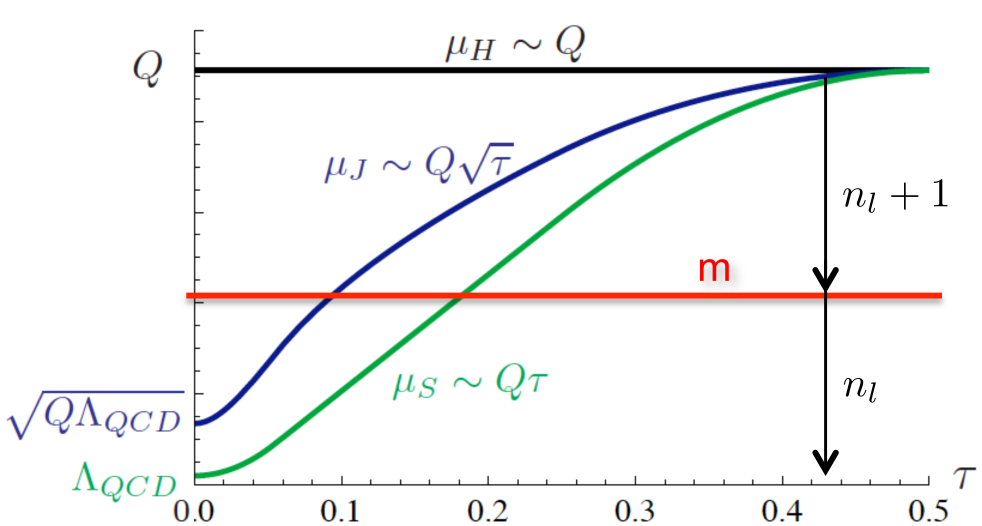
$\lambda_m > 1$ scenario I

$1 > \lambda_m > \lambda$ scenario II

$\lambda > \lambda_m > \lambda^2$ scenario III

$\lambda^2 > \lambda_m$ scenario IV





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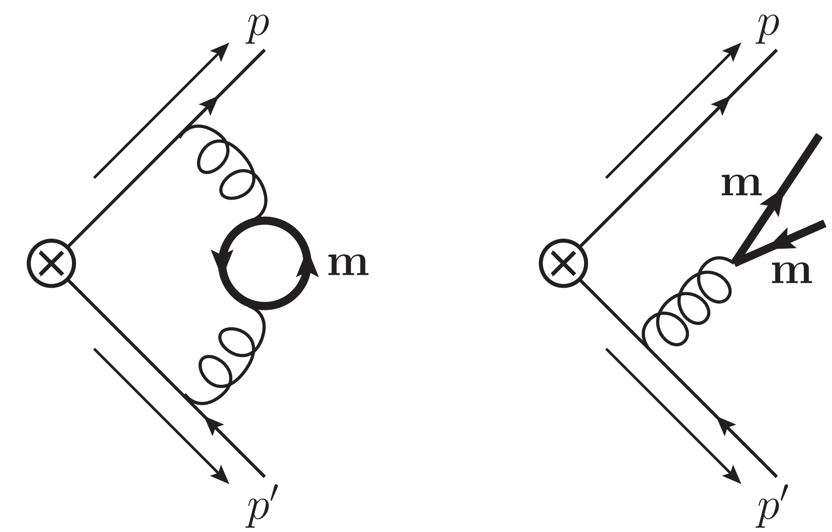
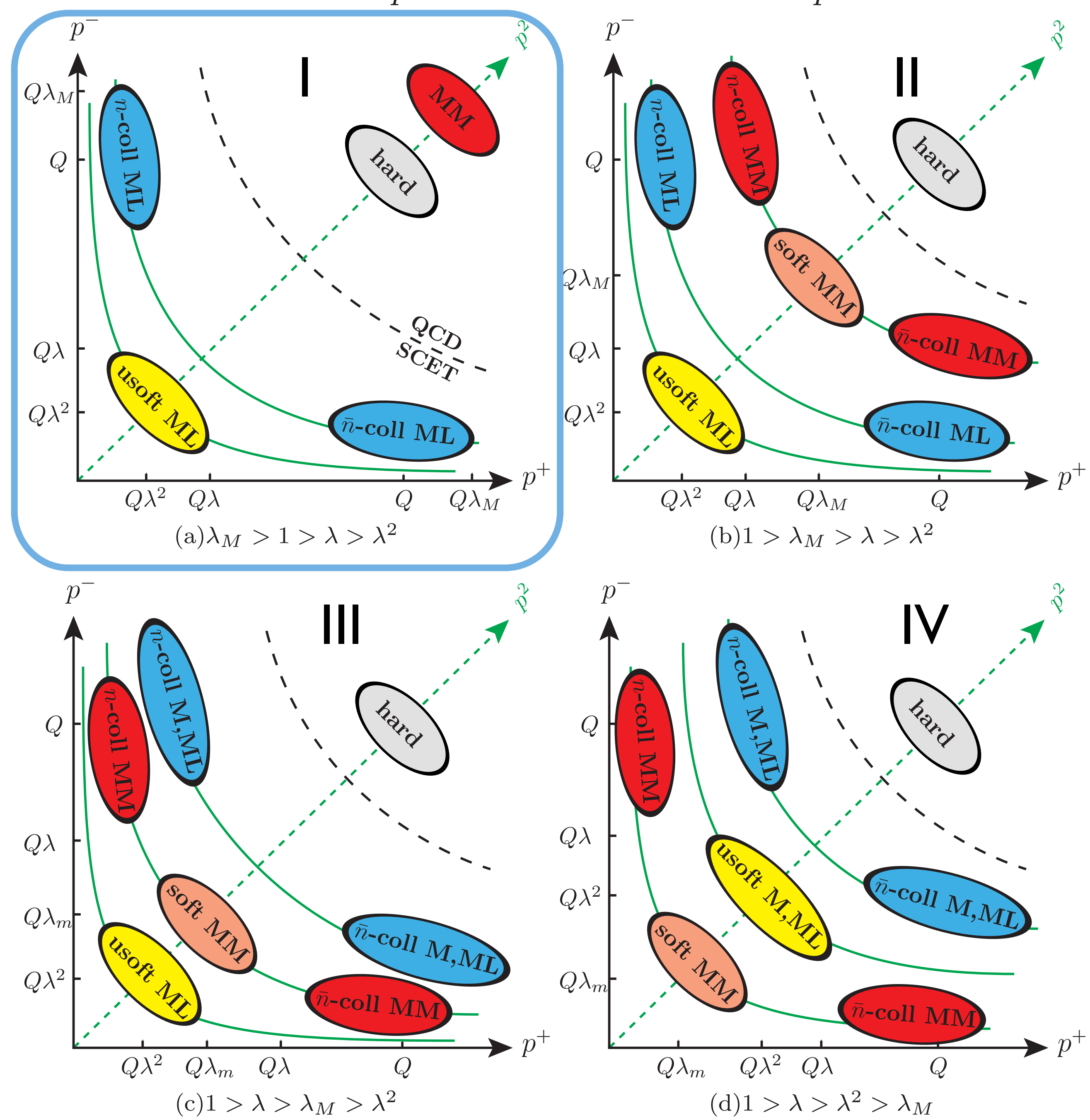
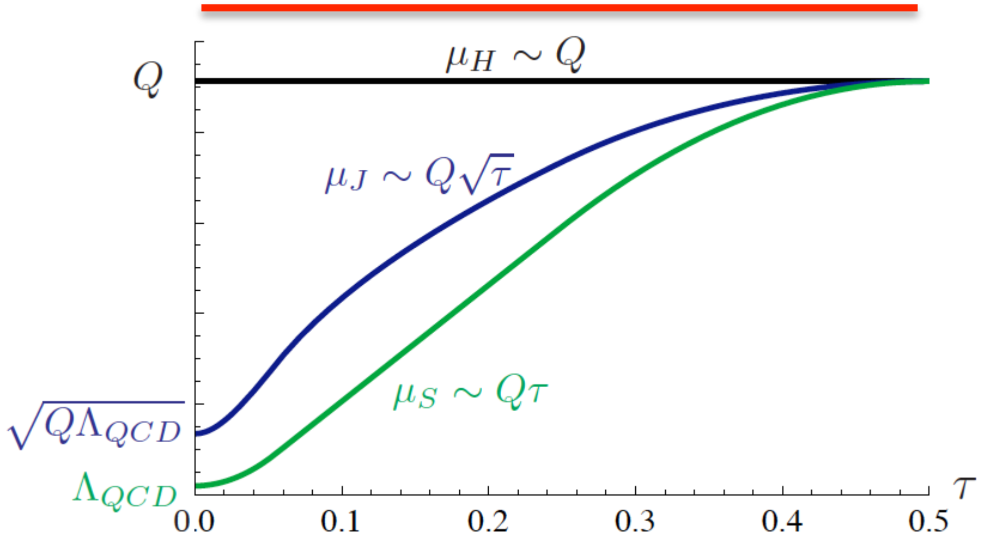
$\lambda > \lambda_m > \lambda^2$	scenario III
$\lambda^2 > \lambda_m$	scenario IV







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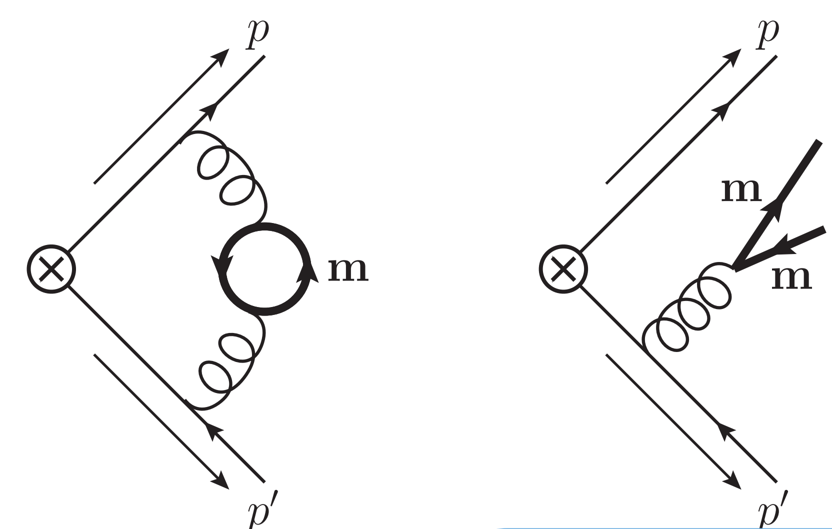
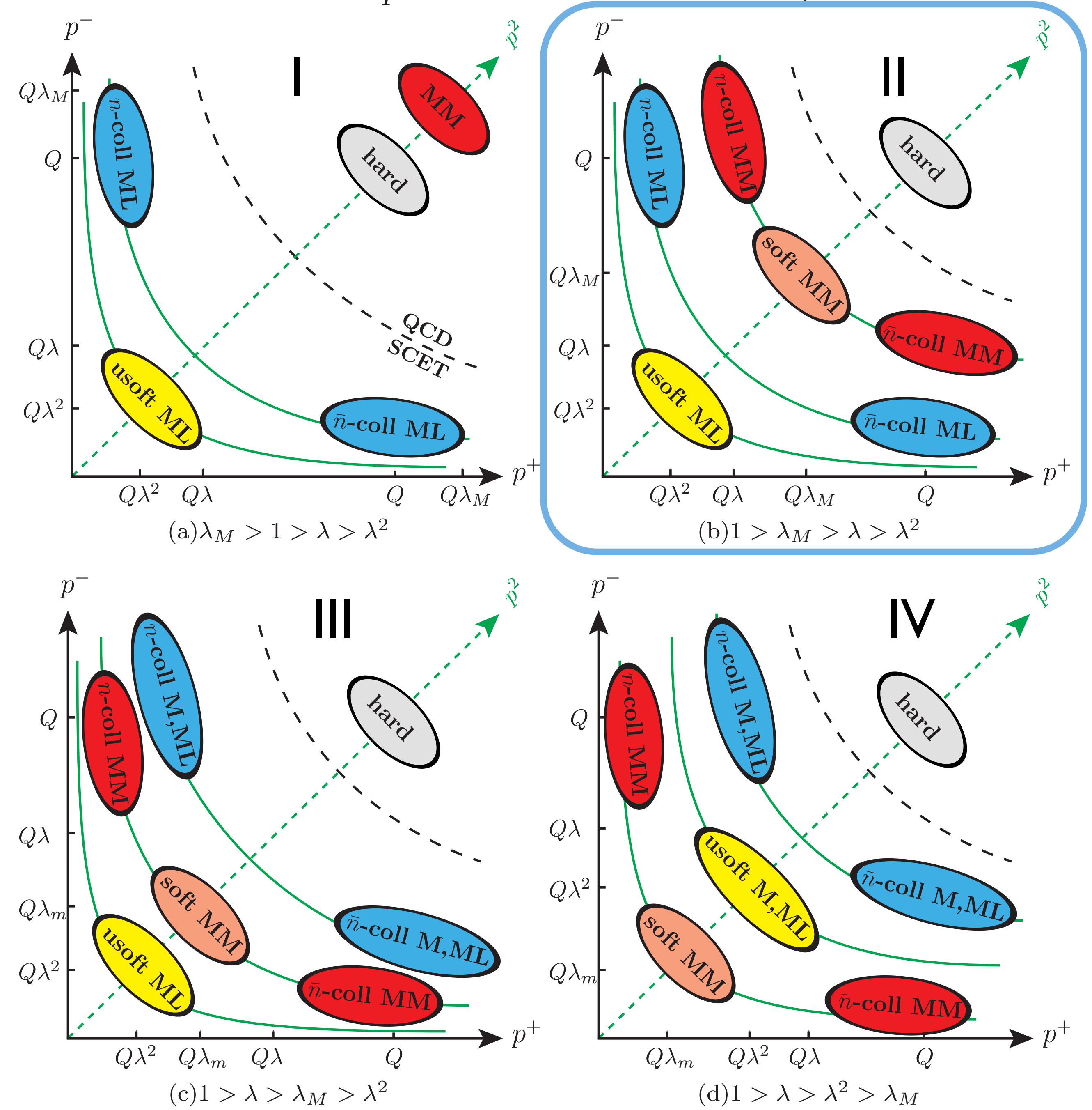
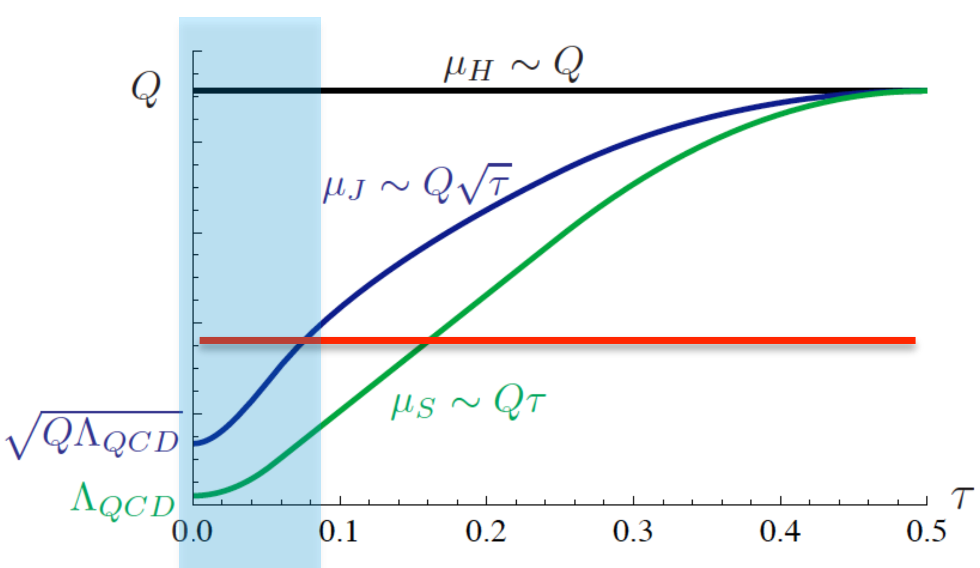
n-coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	m^2
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2



possible hierarchies:

$\lambda_m > 1$	scenario I
$1 > \lambda_m > \lambda$	scenario II
$\lambda > \lambda_m > \lambda^2$	scenario III
$\lambda^2 > \lambda_m$	scenario IV





Secondary production of heavy quarks

[S.Gritschacher, A.Hoang, I.Jemos, P. Pietrulewicz (2013)]
[S.Gritschacher, A.Hoang, I.Jemos, VM, P. Pietrulewicz (2014)]

additional power counting parameter $\lambda_m \sim \frac{m}{Q}$

additional soft and collinear mass modes

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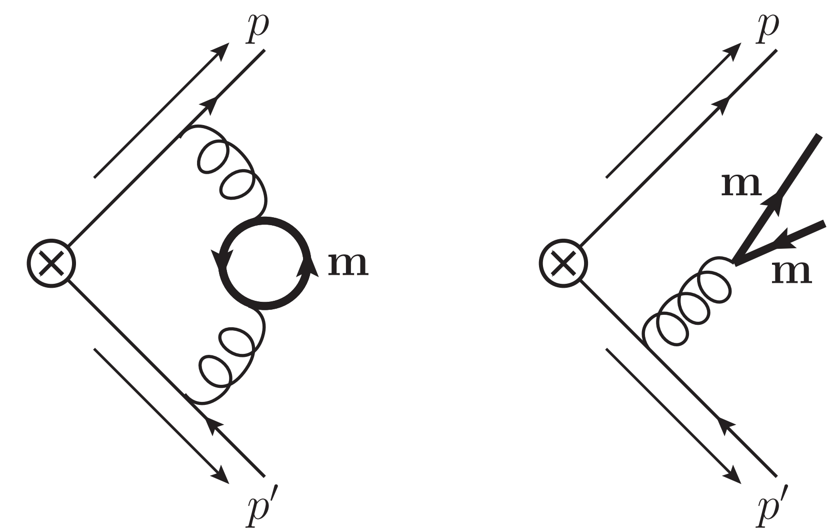

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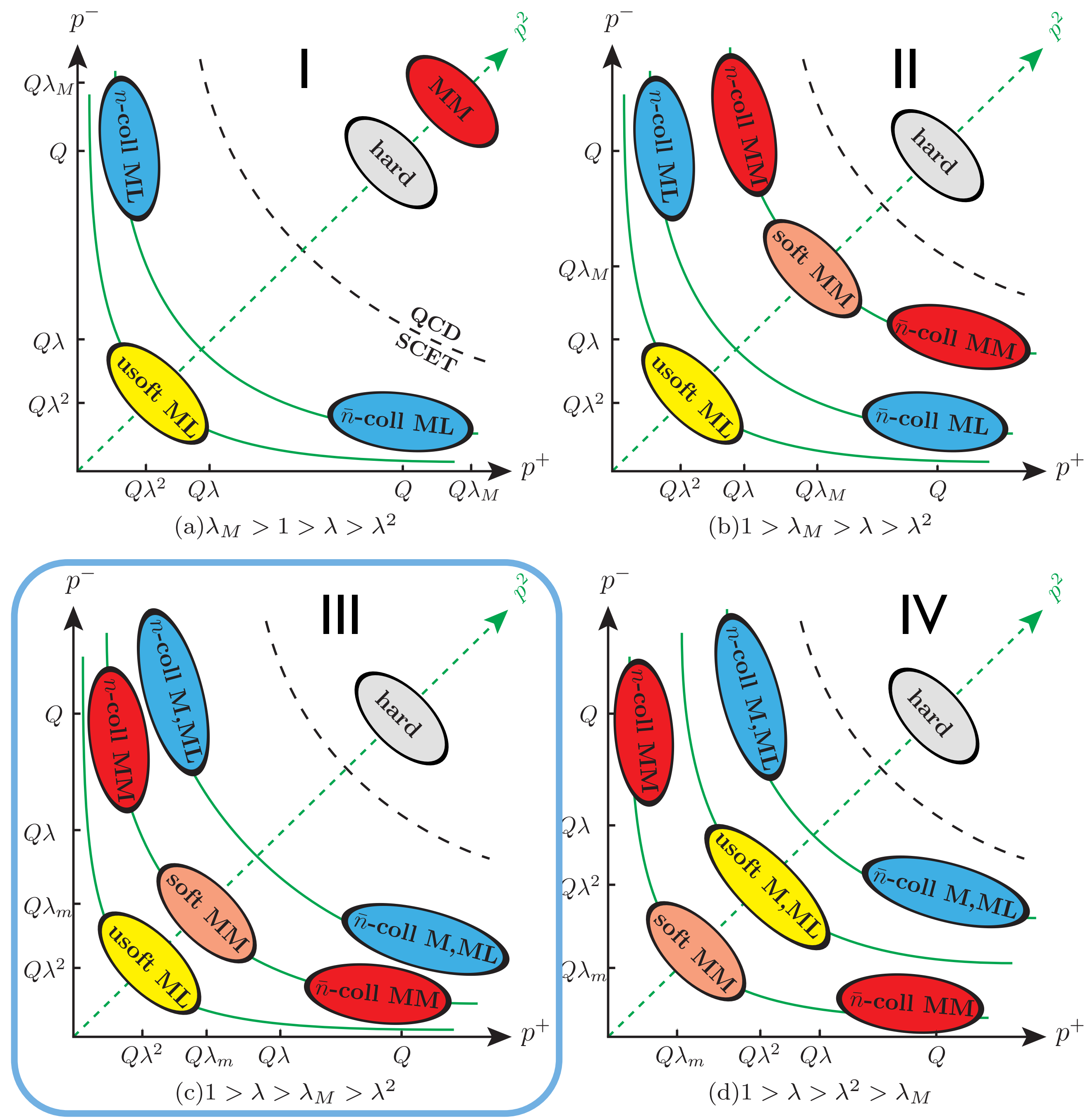

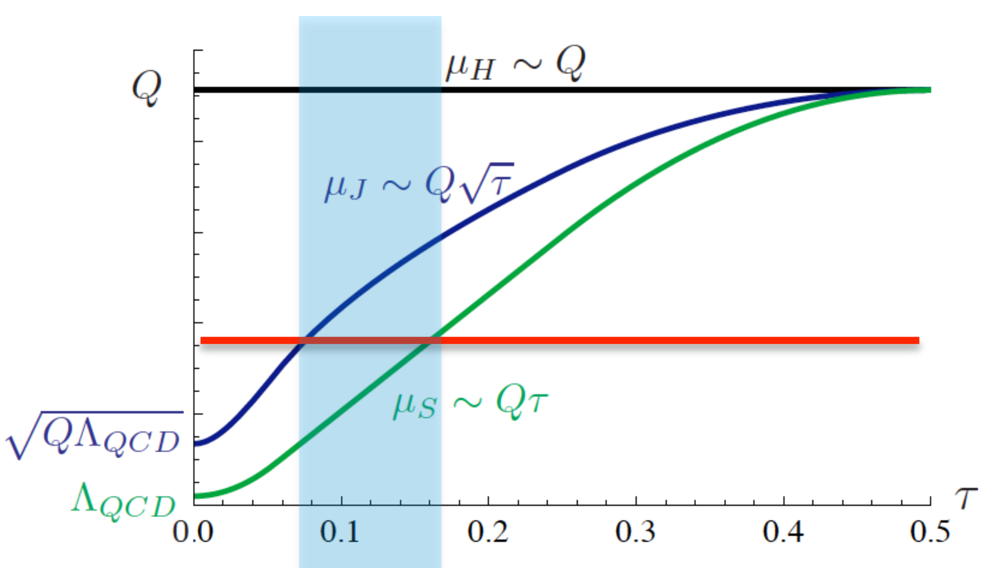

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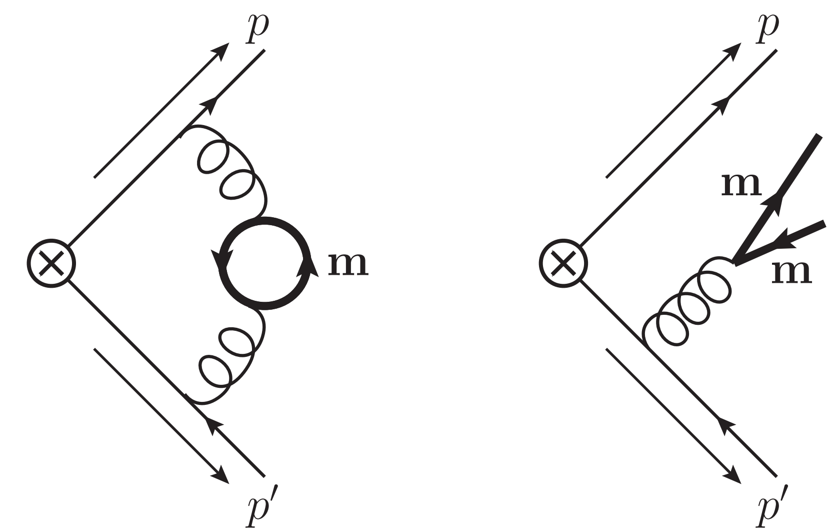

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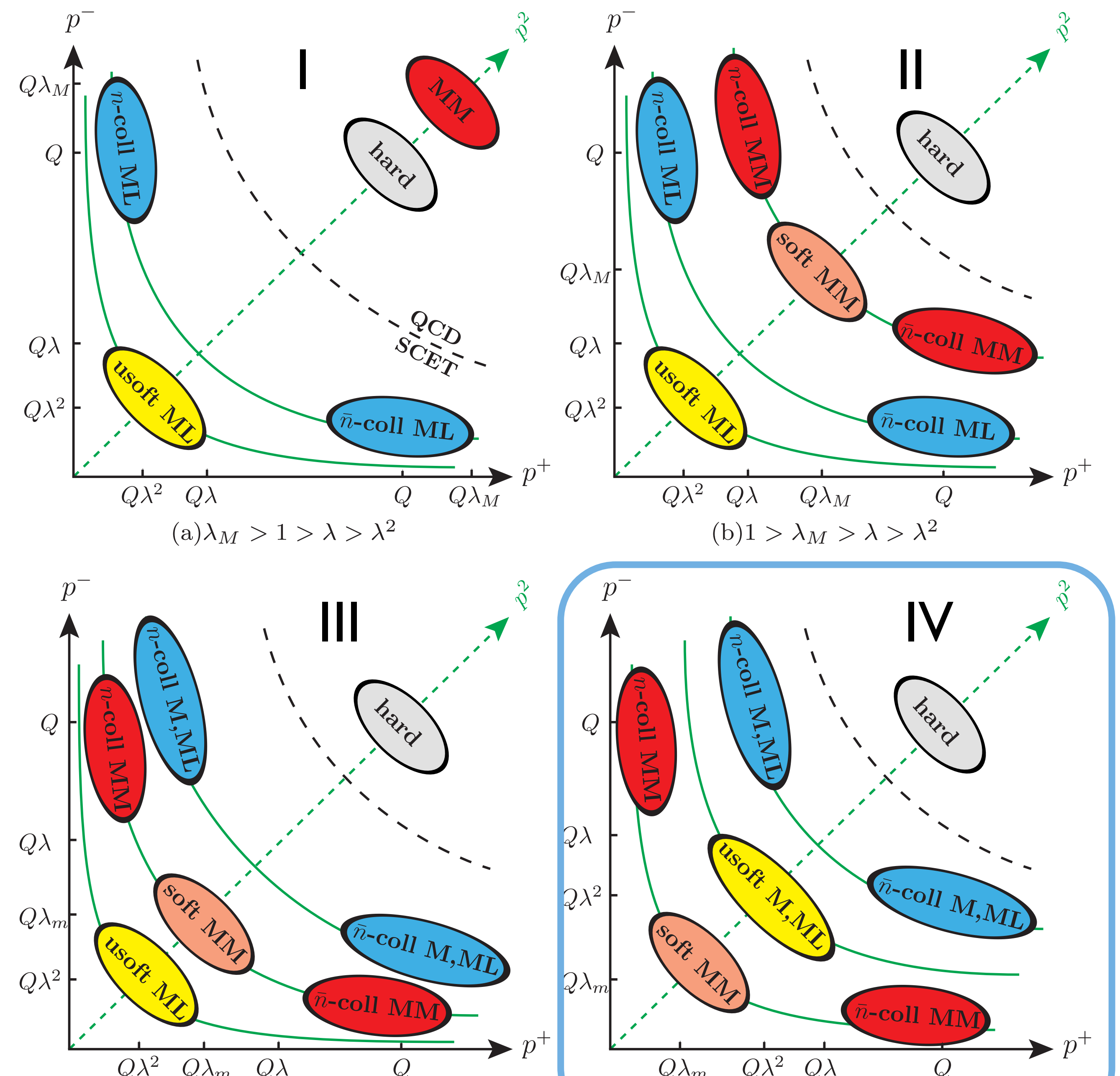
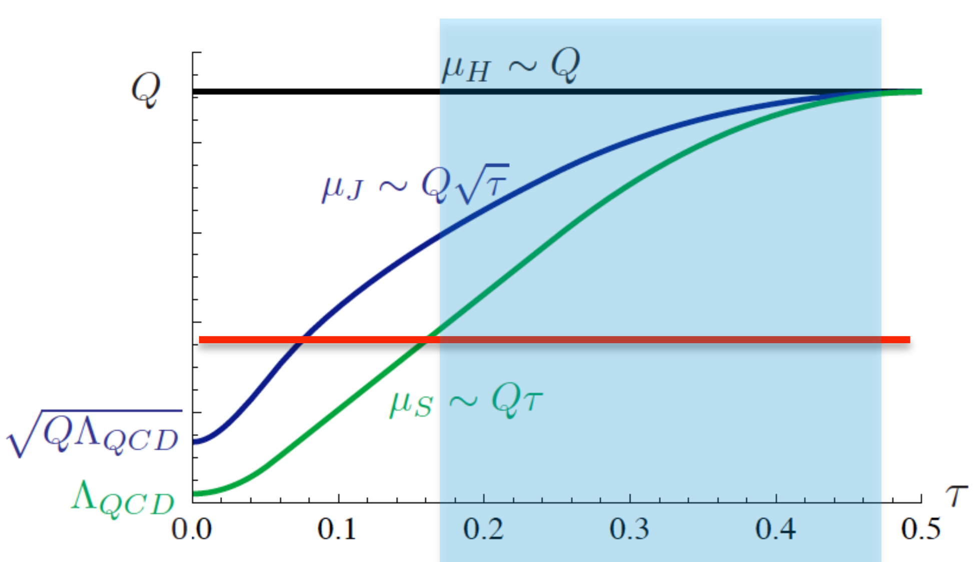


(a)$\lambda_M > 1 > \lambda > \lambda^2$

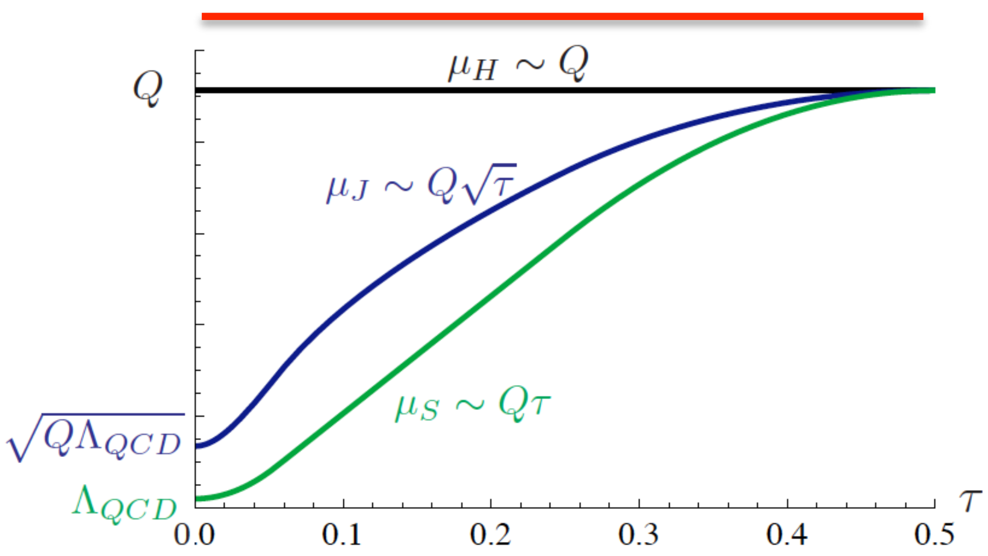
(b)$1 > \lambda_M > \lambda > \lambda^2$

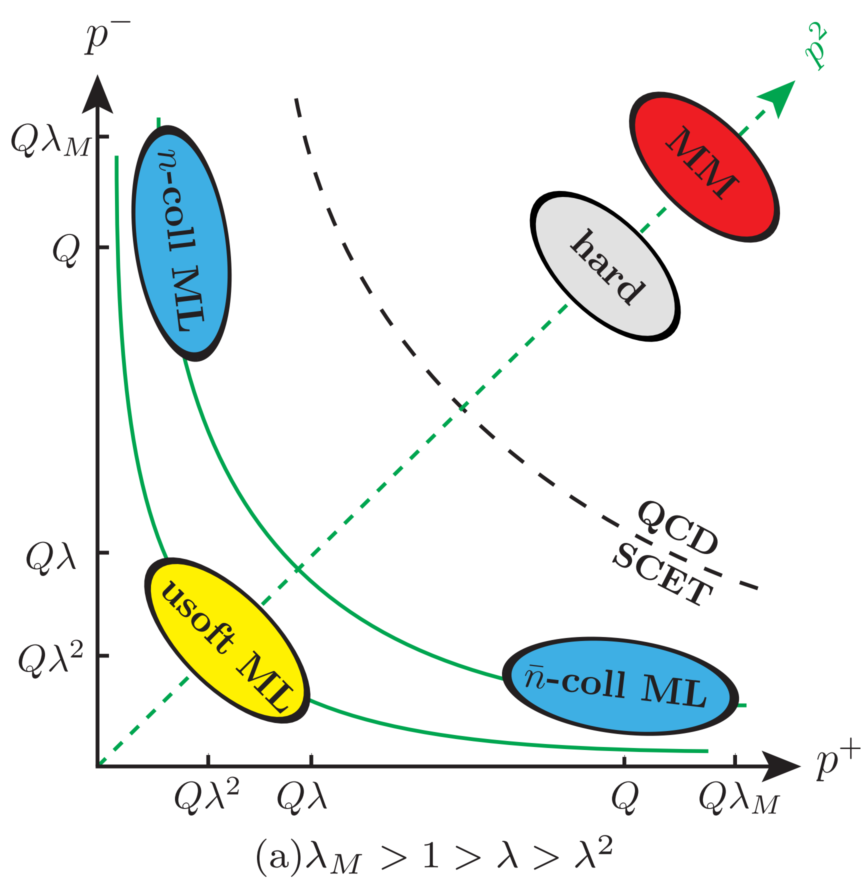
(c)$1 > \lambda > \lambda_M > \lambda^2$

(d)$1 > \lambda > \lambda^2 > \lambda_M$



Scenario I





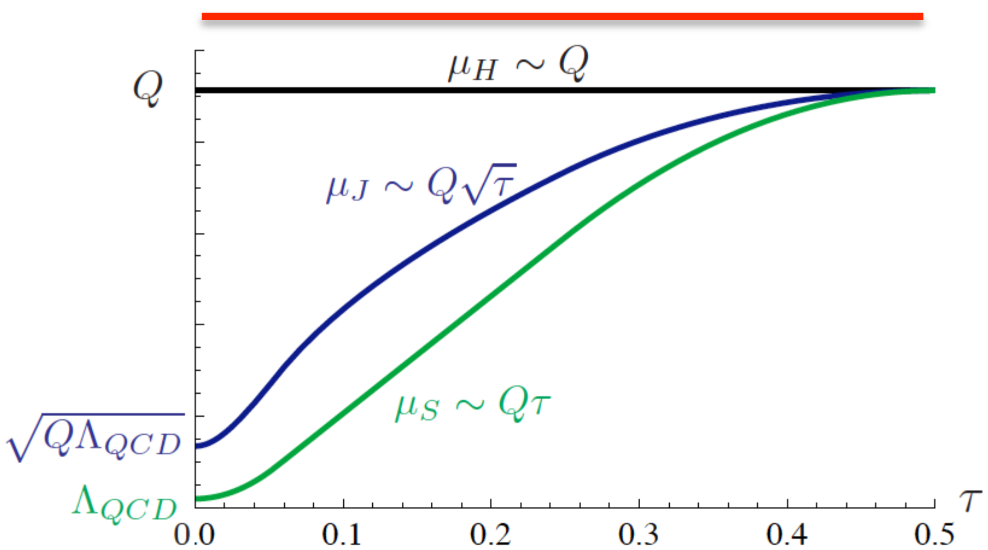
mass scale μ_m

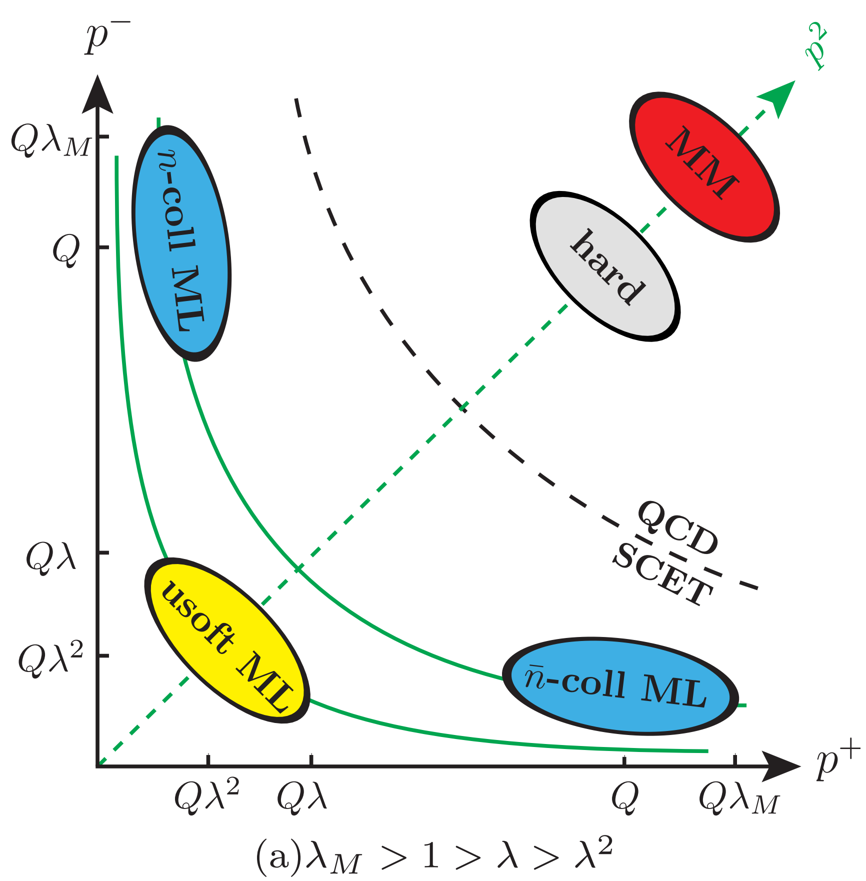
hard scale μ_H

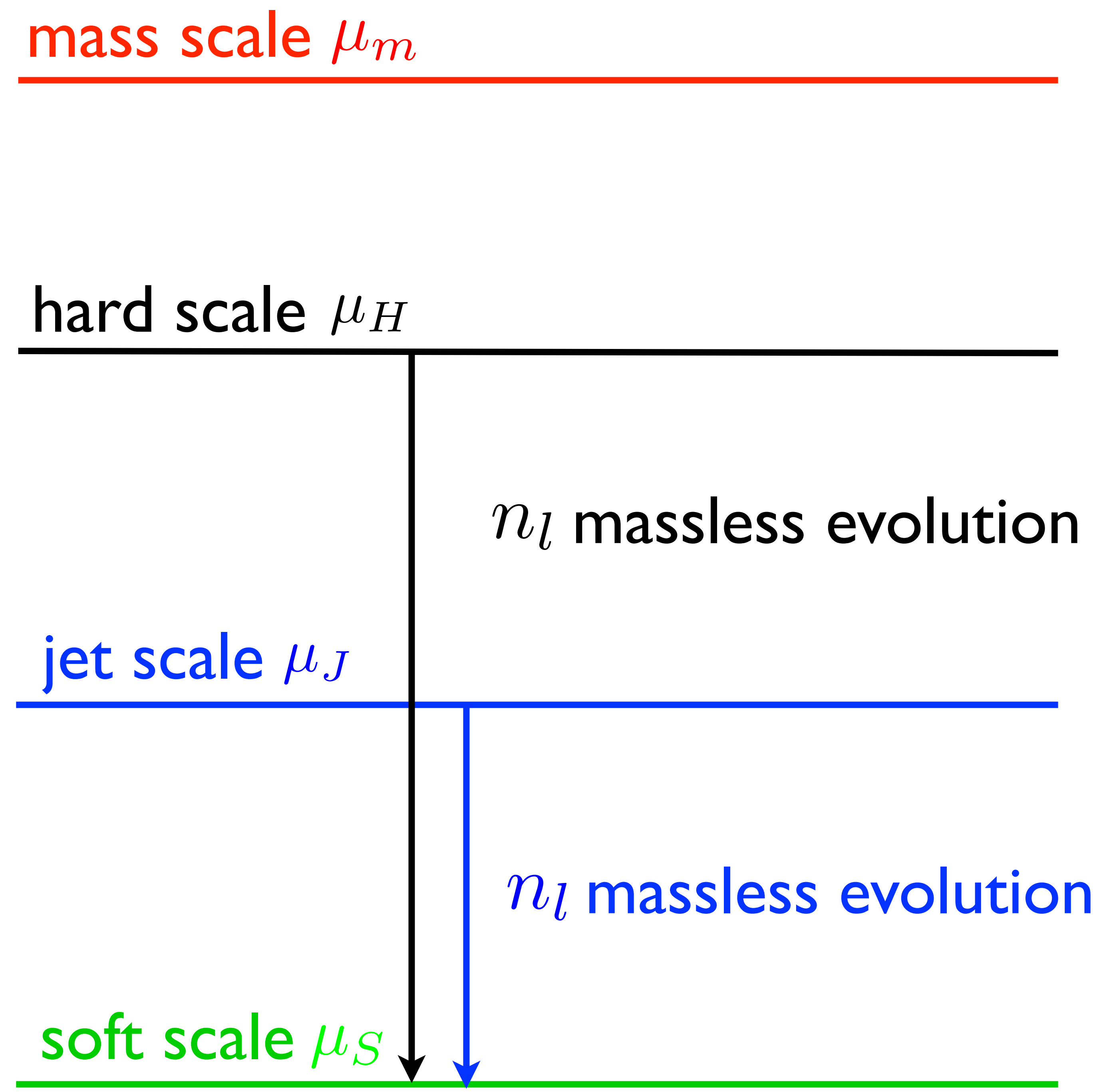
jet scale μ_J

soft scale μ_S

Scenario I


$\mu_H \sim Q$
$\mu_J \sim Q\sqrt{\tau}$
$\mu_S \sim Q\tau$
Q
$\sqrt{Q\Lambda_{QCD}}$
Λ_{QCD}
τ
0.0
0.1
0.2
0.3
0.4
0.5


p^-
p^+
p^2
$Q\lambda_M$
Q
$Q\lambda$
$Q\lambda^2$
n-coll ML
$\bar{n}$-coll ML
usoft ML
hard
MM
QCD
SCET
(a)$\lambda_M > 1 > \lambda > \lambda^2$


mass scale μ_m
hard scale μ_H
n_l massless evolution
jet scale μ_J
n_l massless evolution
soft scale μ_S

Scenario I

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim H^{(n_l)}(\mu_Q, m) U_H^{(n_l)}(\mu_H, \mu_S) \times J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_S) \otimes S^{(n_l)}(\mu_S)$$

SCET - QCD matching coefficient is mass-dependent

EFT matrix elements and running factors, same as in massless theory

all matching coefficients, matrix elements and running factors use $\alpha_s^{(n_l)}$

Scenario I

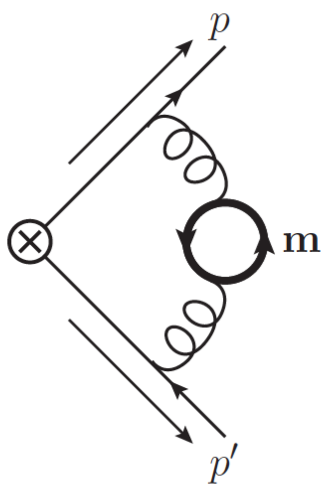
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim H^{(n_l)}(\mu_Q, m) U_H^{(n_l)}(\mu_H, \mu_S) \times J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_S) \otimes S^{(n_l)}(\mu_S)$$

SCET - QCD matching coefficient is mass-dependent

EFT matrix elements and running factors, same as in massless theory

all matching coefficients, matrix elements and running factors use $\alpha_s^{(n_l)}$

$H^{(n_l)}$ computed in the OS scheme (full QCD massive form factor)

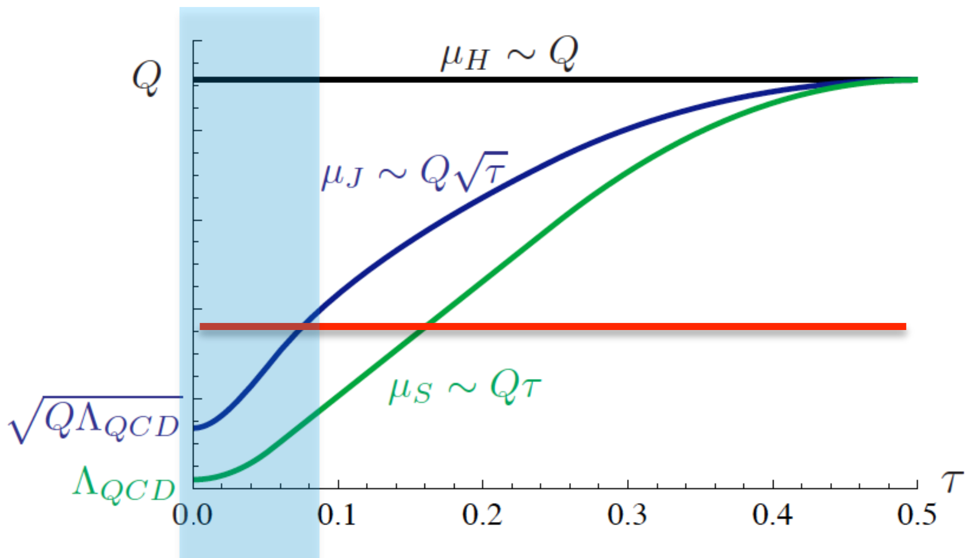


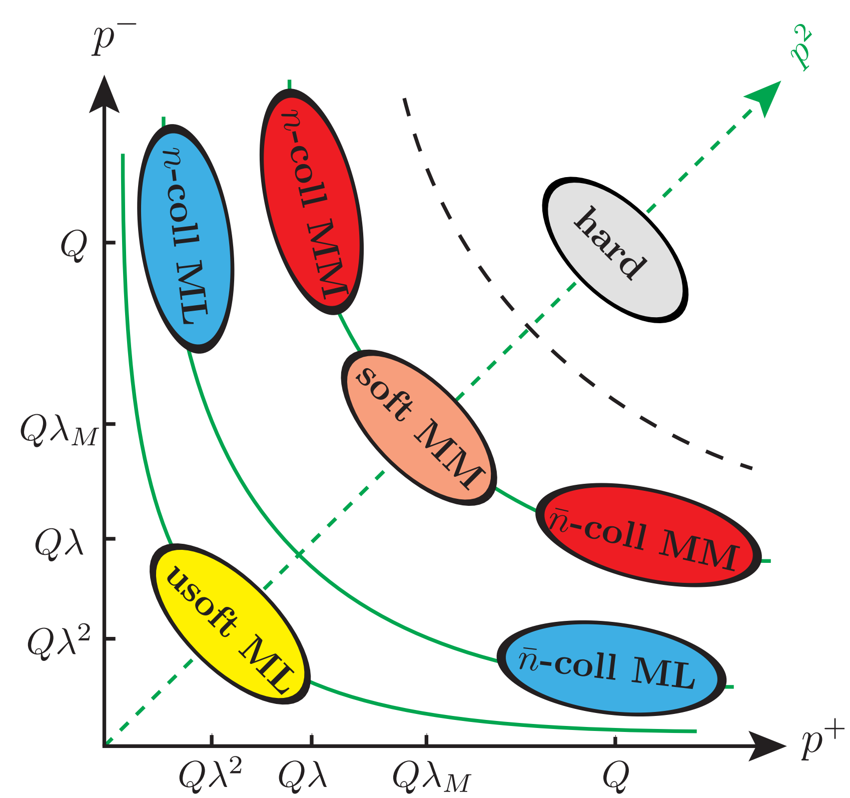
correct decoupling limit in $H^{(n_l)}$ for $m \gg Q$

but large log for $m \ll Q$

whole distribution has a smooth decoupling limit

Scenario II





(b)$1 > \lambda_M > \lambda > \lambda^2$

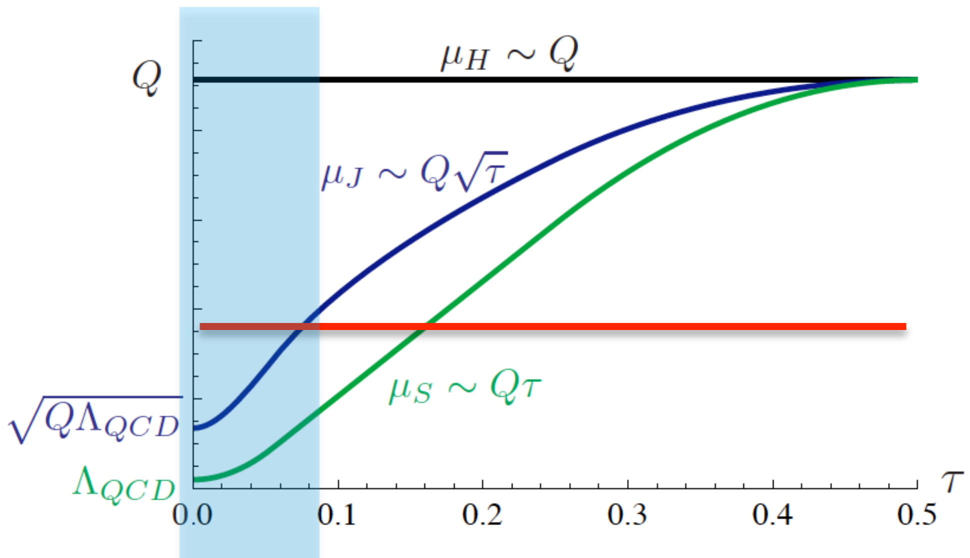
hard scale μ_H

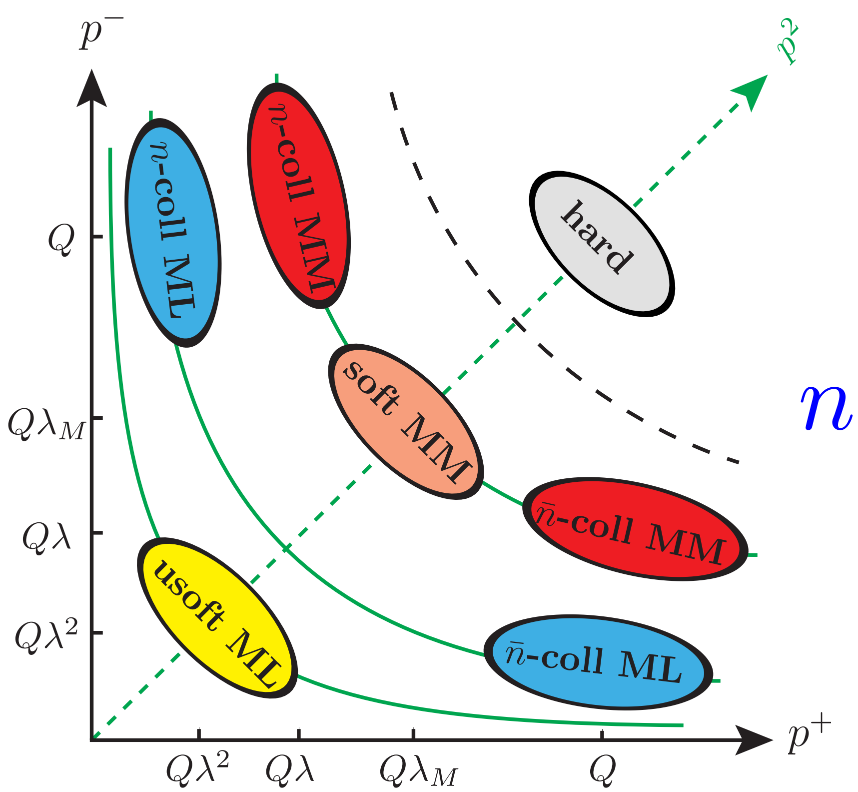
mass scale μ_m

jet scale μ_J

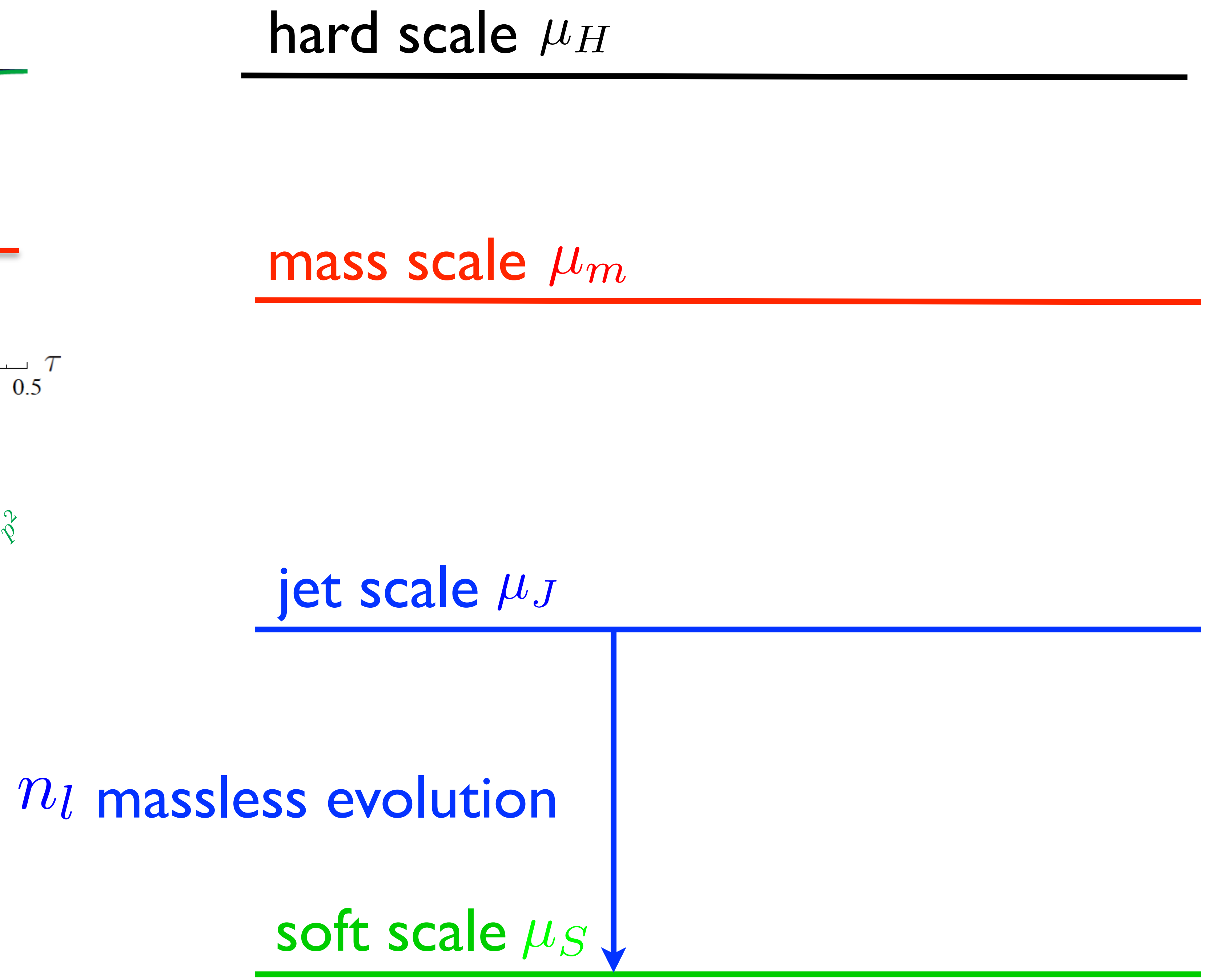
soft scale μ_S

Scenario II

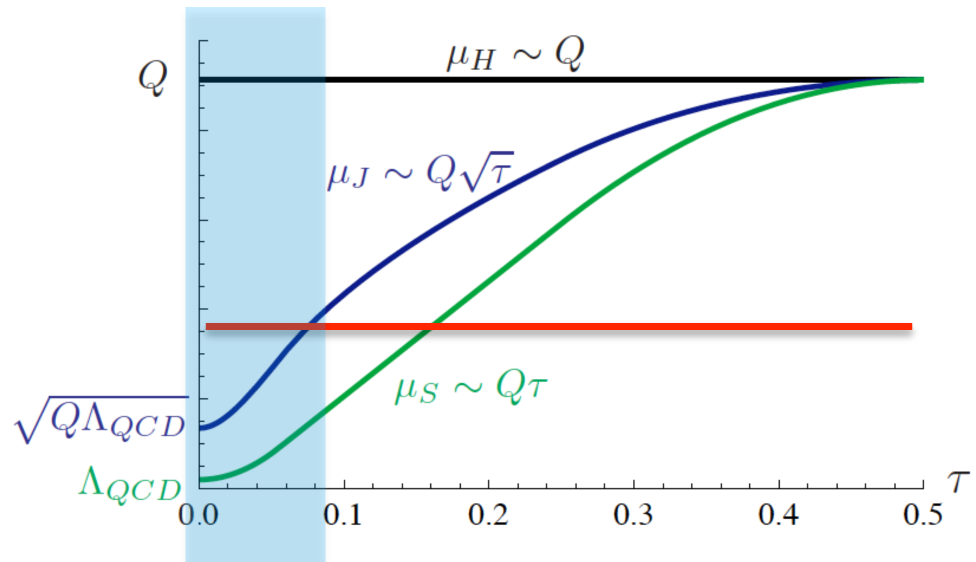


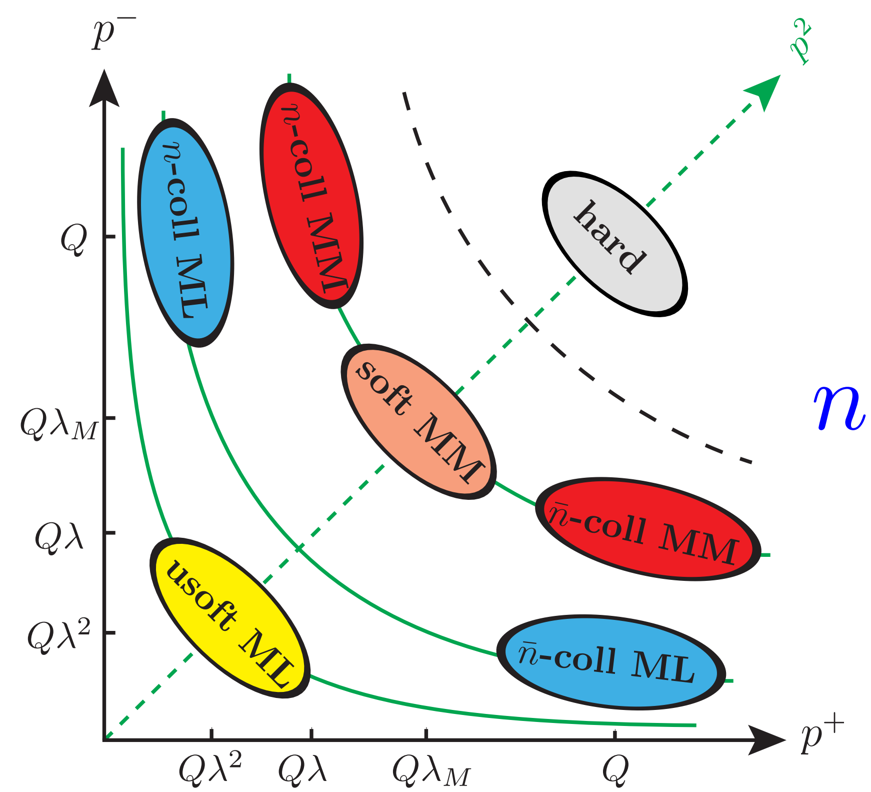


(b)$1 > \lambda_M > \lambda > \lambda^2$

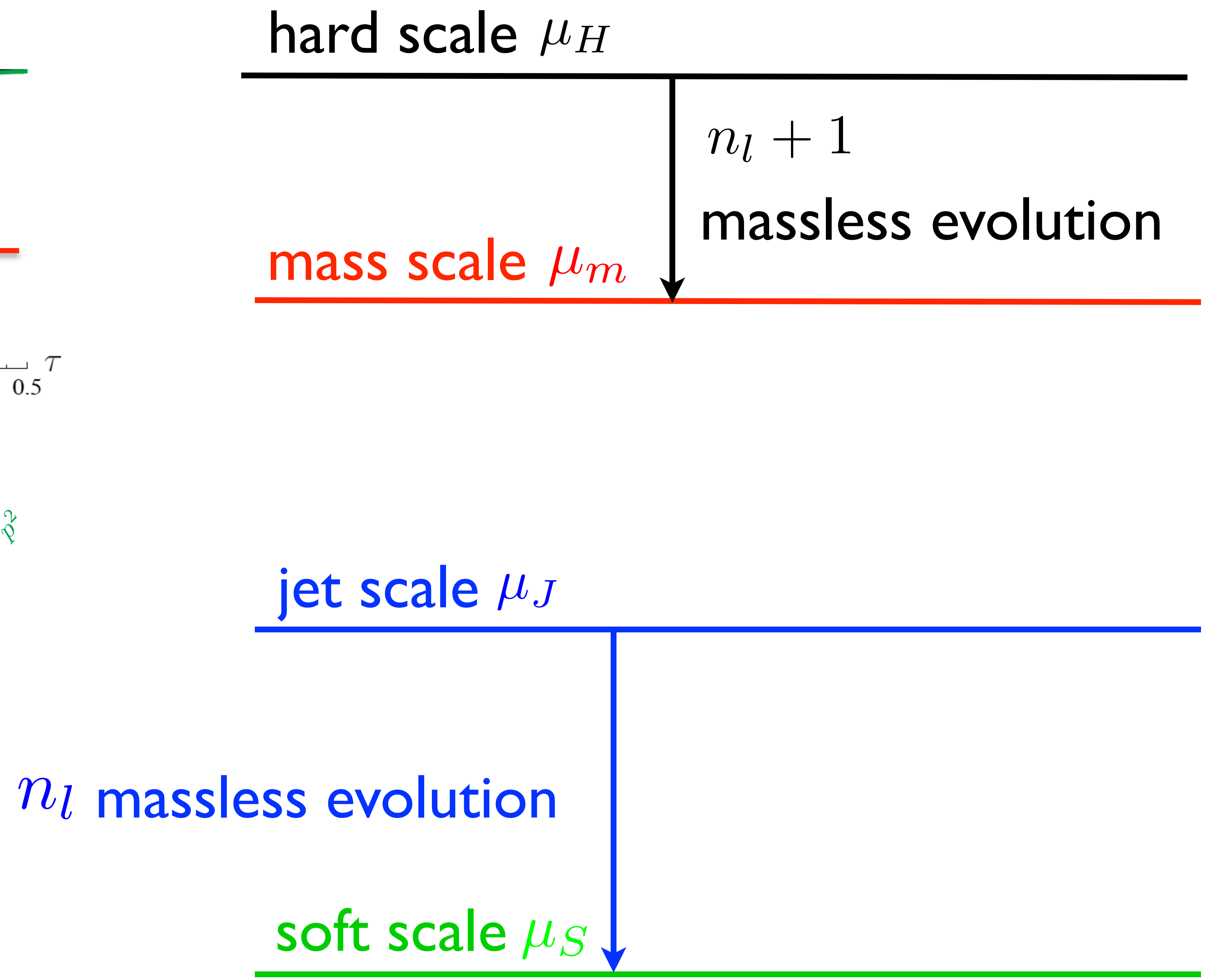


Scenario II

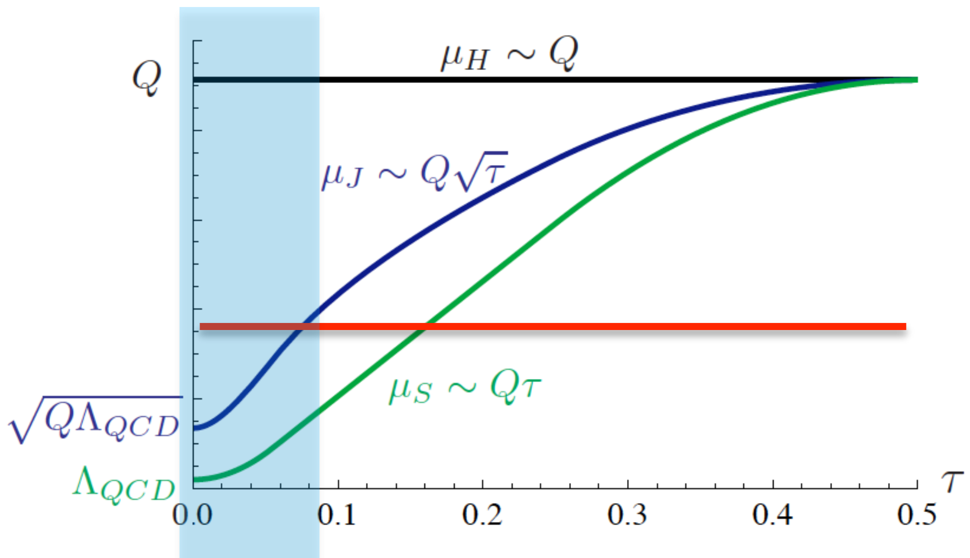


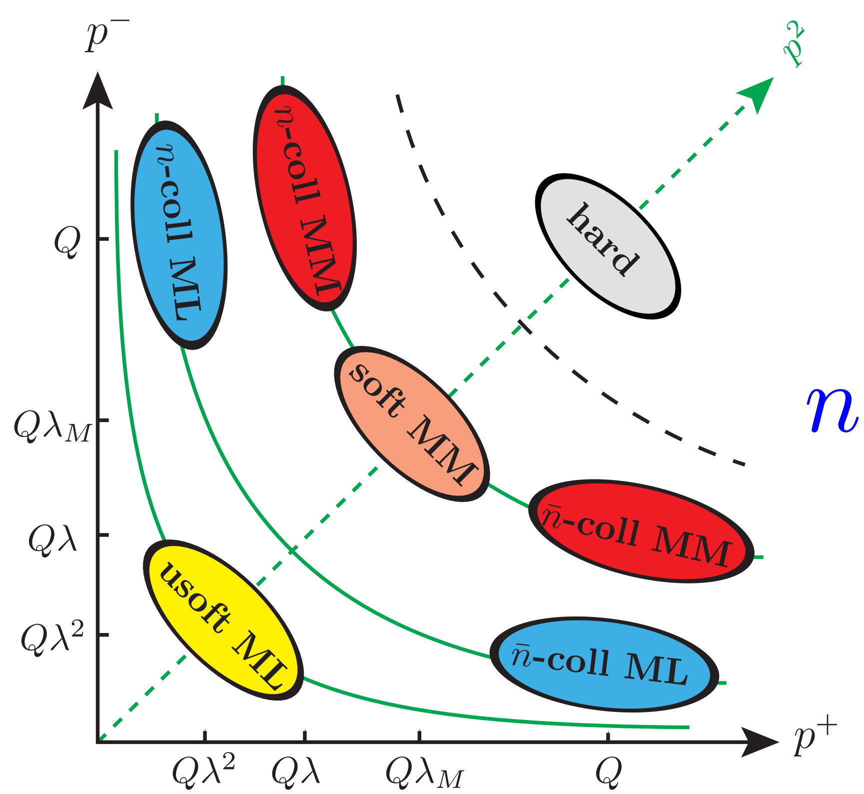


(b)$1 > \lambda_M > \lambda > \lambda^2$

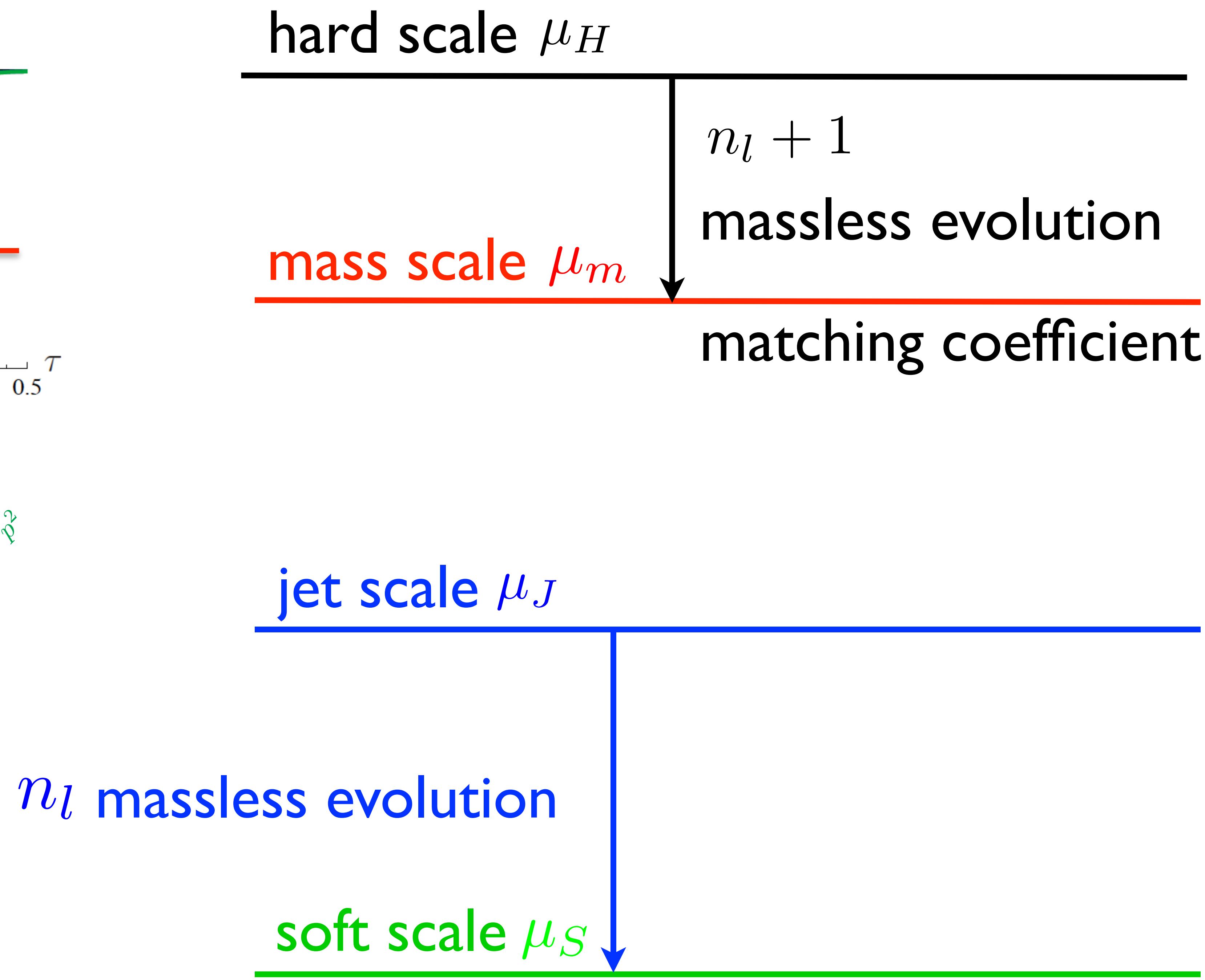


Scenario II

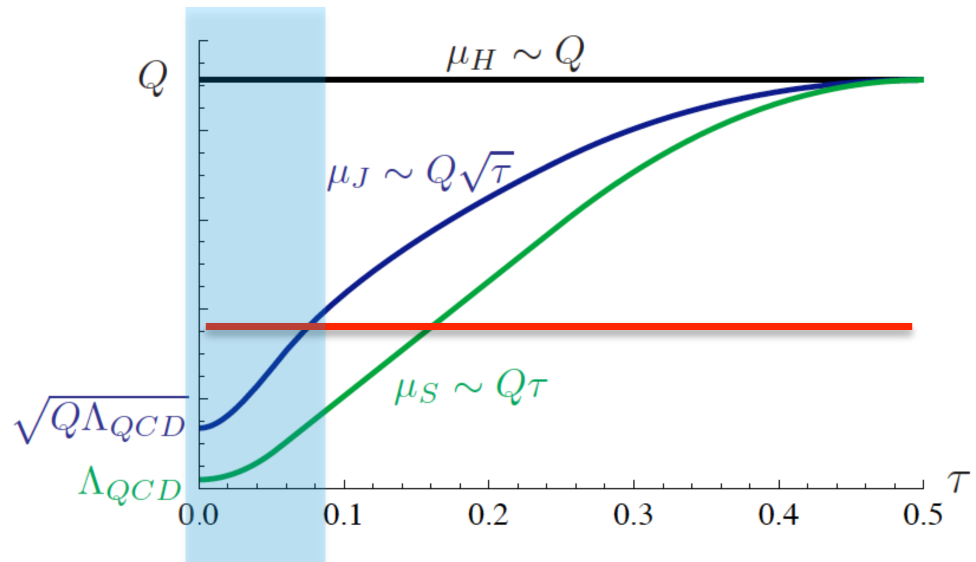


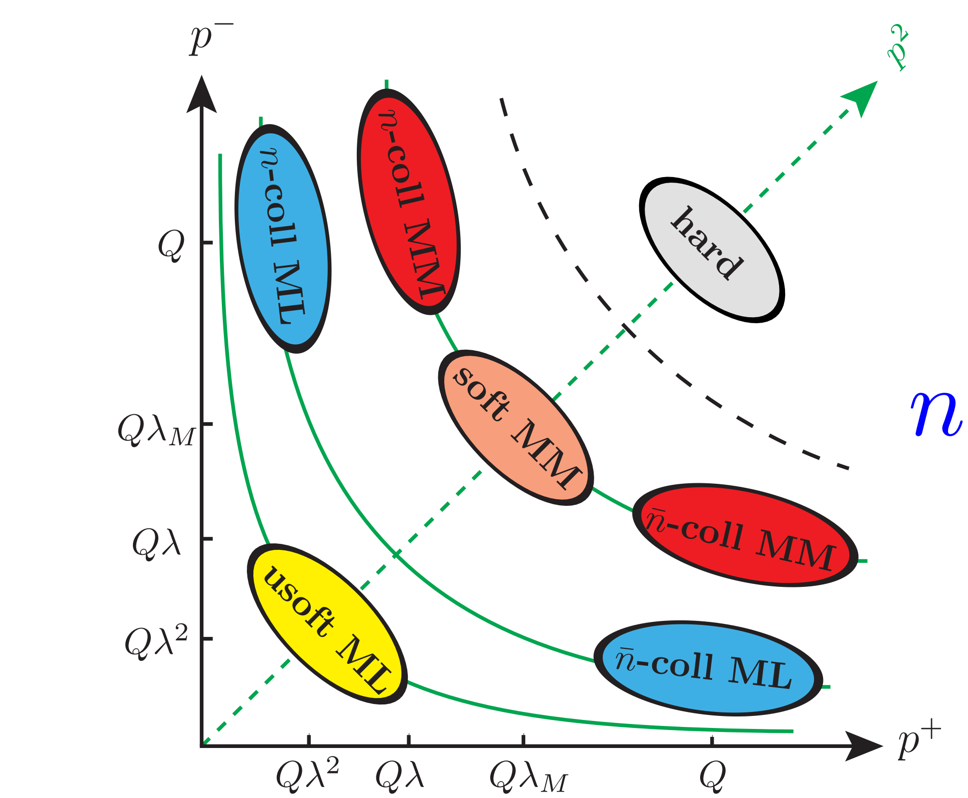


(b)$1 > \lambda_M > \lambda > \lambda^2$

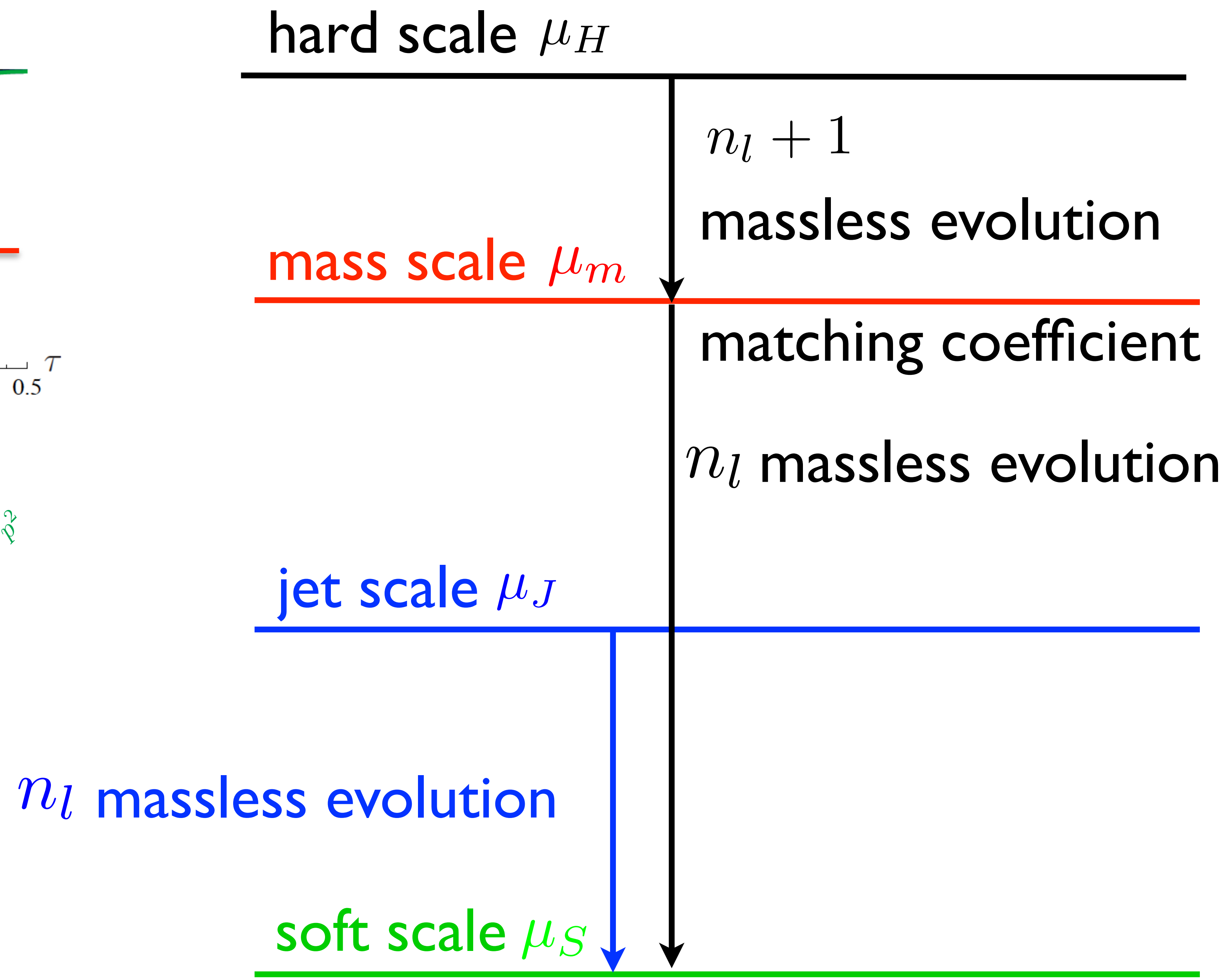


Scenario II





(b)$1 > \lambda_M > \lambda > \lambda^2$



Scenario II

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim H^{(n_l+1)}(\mu_Q, m) U_H^{(n_l+1)}(\mu_H, \mu_m) \mathcal{M}_H(\mu_m) U_H^{(n_l)}(\mu_m, \mu_S) \times J^{(n_l)}(\mu_J, m) \otimes U_J^{(n_l)}(\mu_J, \mu_S) \otimes S^{(n_l)}(\mu_S)$$

matching coefficient and jet function are mass-dependent

matching compensates difference between $H^{(n_l+1)}$ and $H^{(n_l)}$

soft matrix element and running factors, same as in massless theory

use $\alpha_s^{(n_l+1)}$

use $\alpha_s^{(n_l)}$

Scenario II

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim H^{(n_l+1)}(\mu_Q, m) U_H^{(n_l+1)}(\mu_H, \mu_m) \mathcal{M}_H(\mu_m) U_H^{(n_l)}(\mu_m, \mu_S) \times J^{(n_l)}(\mu_J, m) \otimes U_J^{(n_l)}(\mu_J, \mu_S) \otimes S^{(n_l)}(\mu_S)$$

matching coefficient and jet function are mass-dependent

matching compensates difference between $H^{(n_l+1)}$ and $H^{(n_l)}$

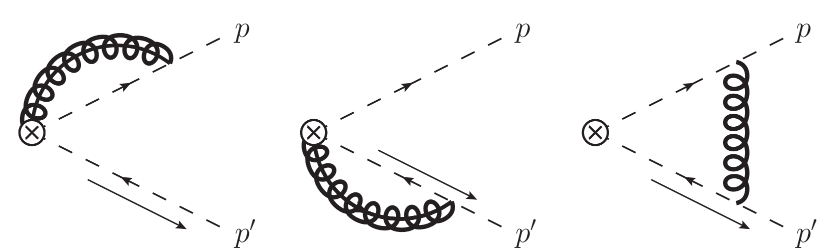
soft matrix element and running factors, same as in massless theory

use $\alpha_s^{(n_l+1)}$

use $\alpha_s^{(n_l)}$

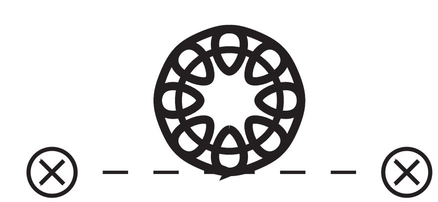
$H^{(n_l+1)}$ includes massive effects from SCET. Computation in $\overline{\mathrm{MS}}$ scheme

correct massless limit in $H^{(n_l+1)}$ for $m \ll Q$

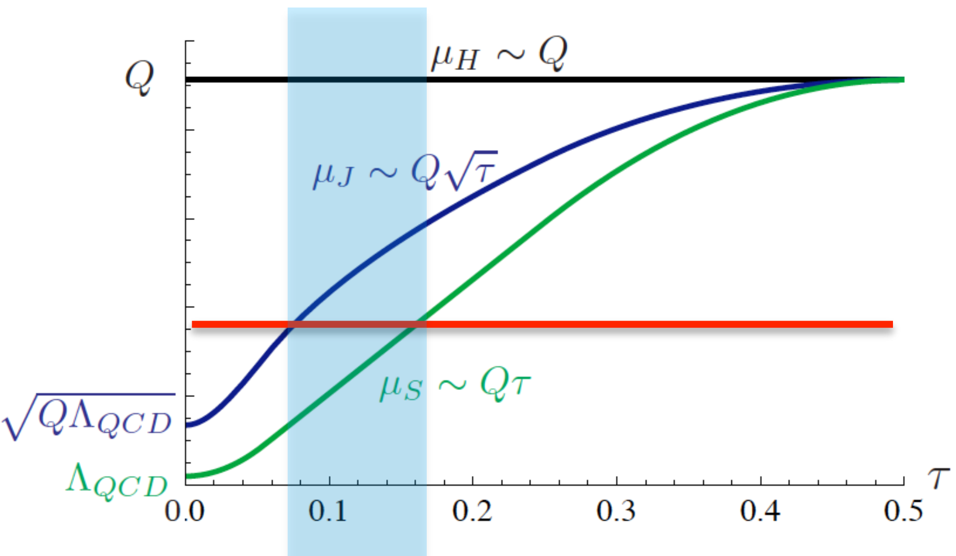


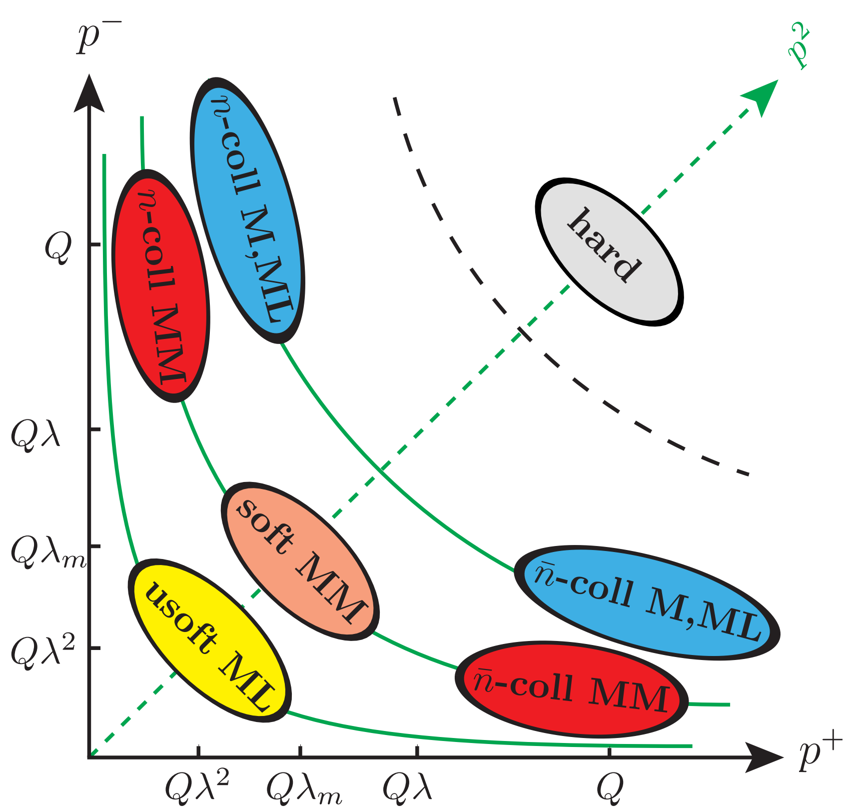
includes heavy quark virtual effects in OS scheme

correct decoupling limit but IR-divergent massless limit



Scenario III





(c) $1 > \lambda > \lambda_M > \lambda^2$

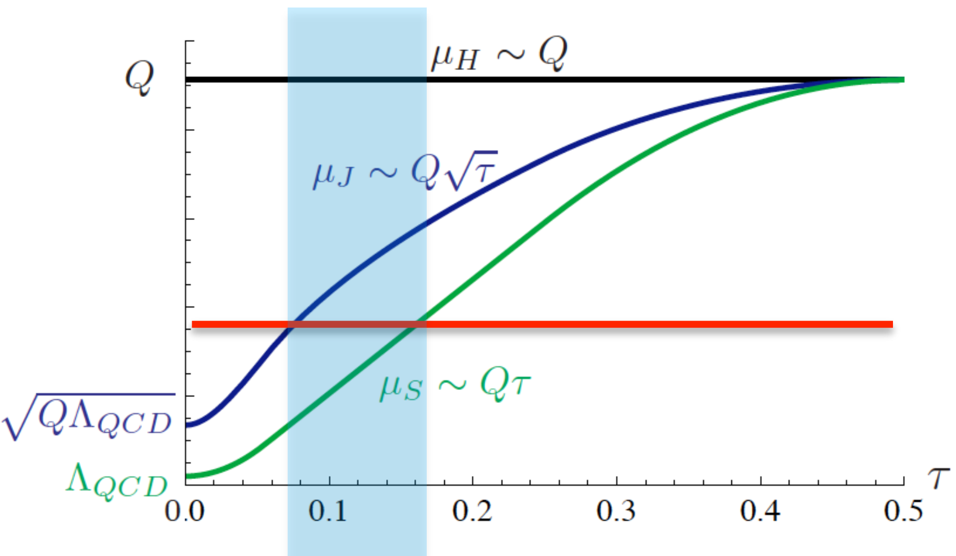
hard scale μ_H

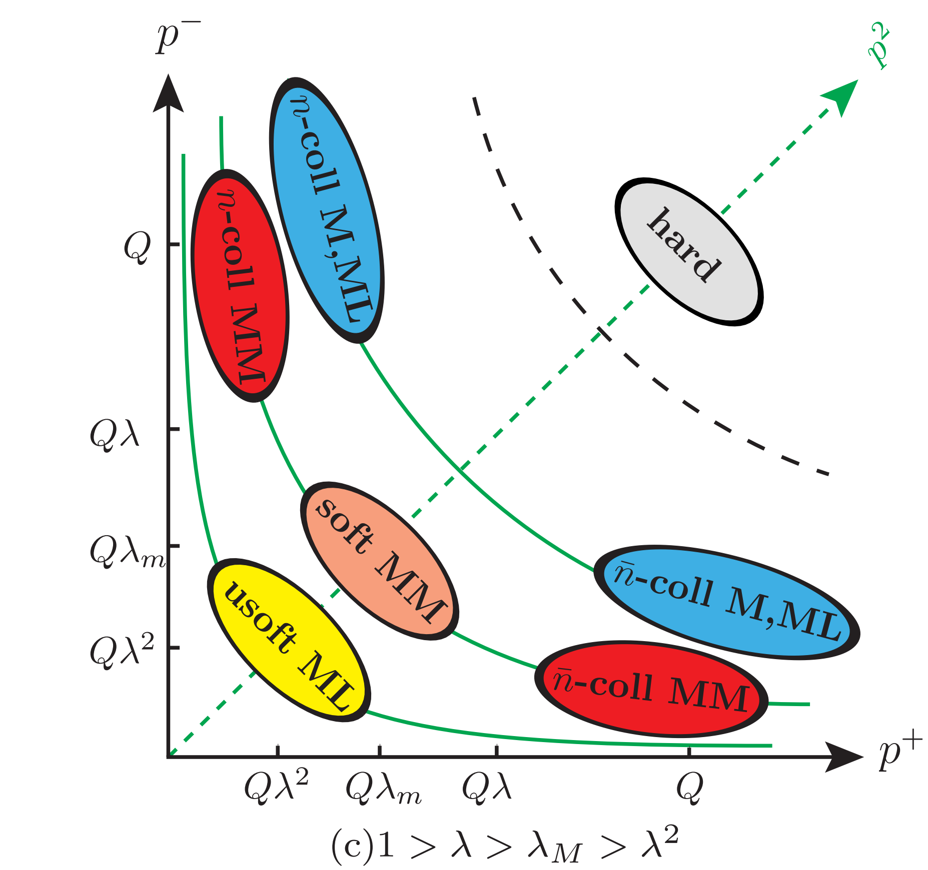
jet scale μ_J

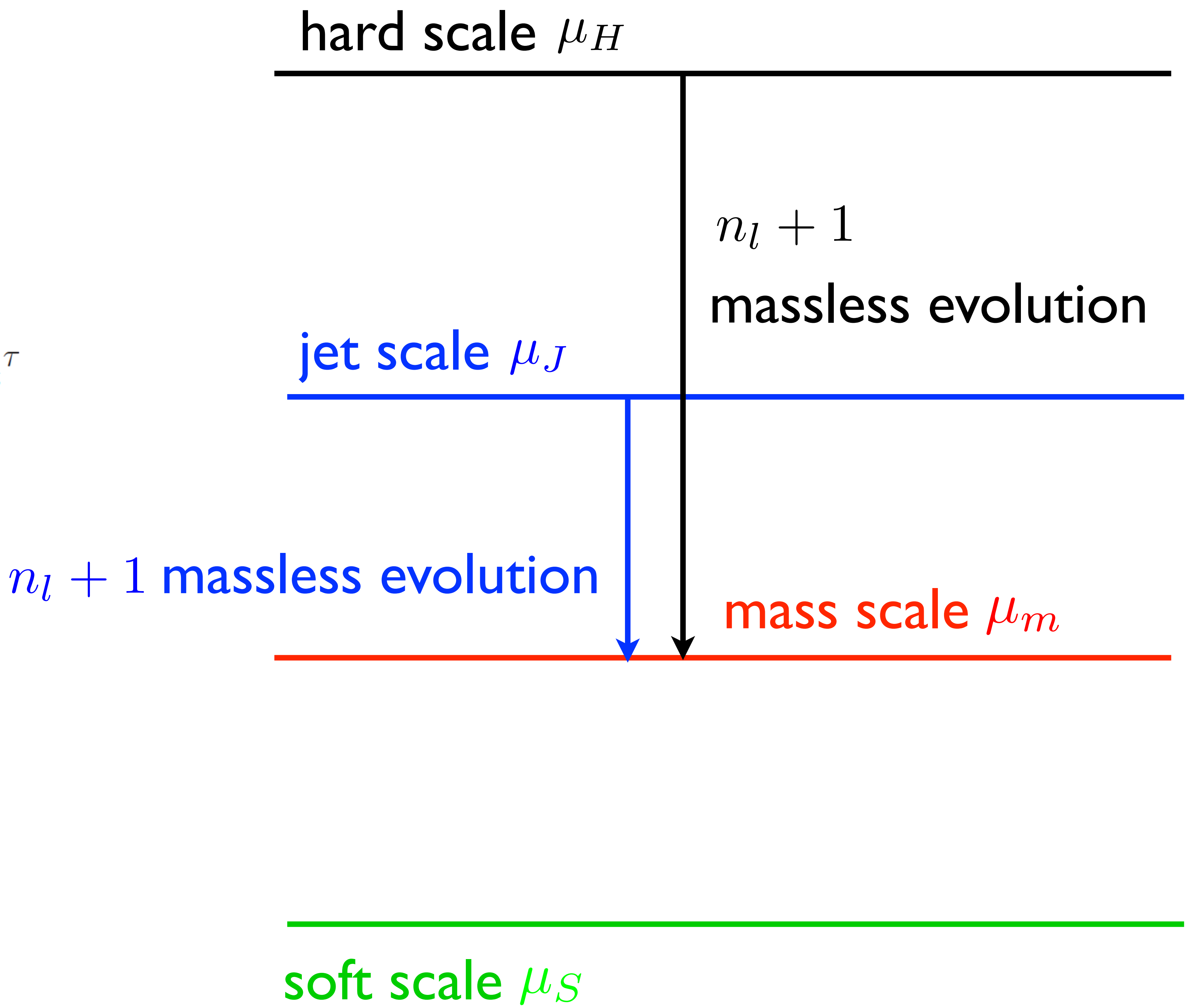
mass scale μ_m

soft scale μ_S

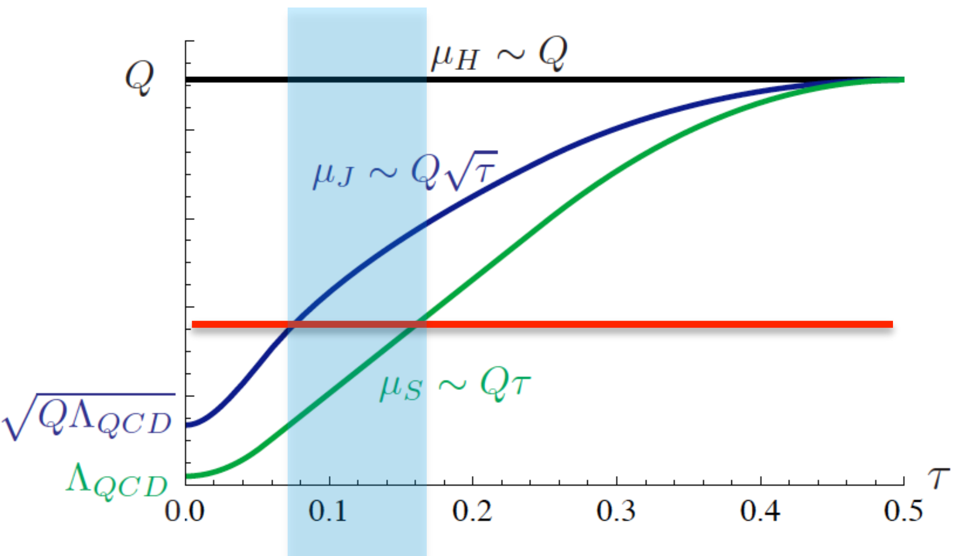
Scenario III

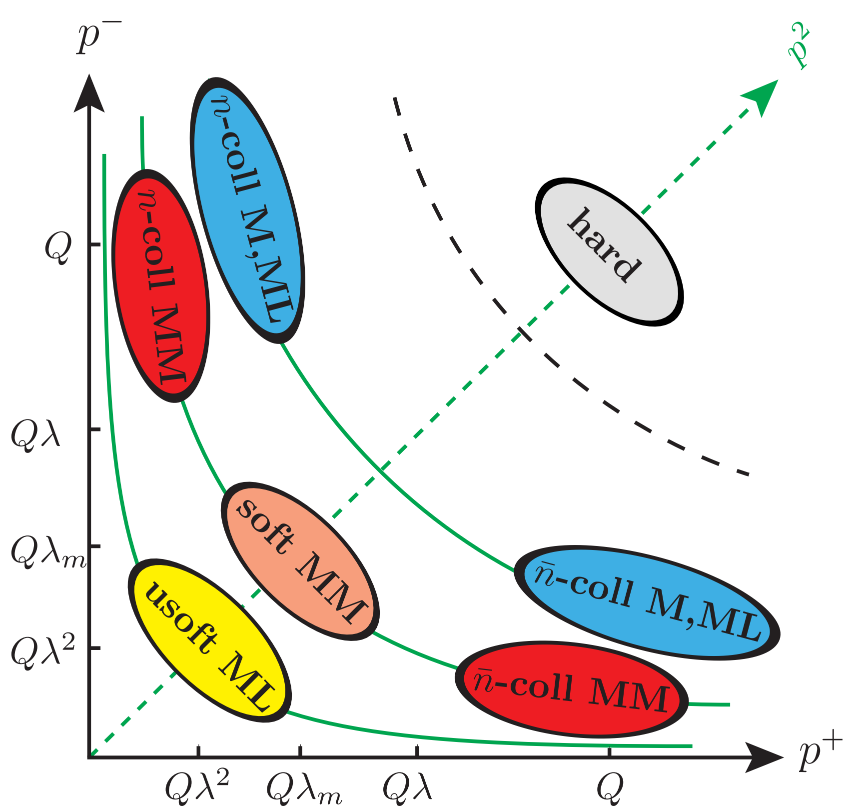




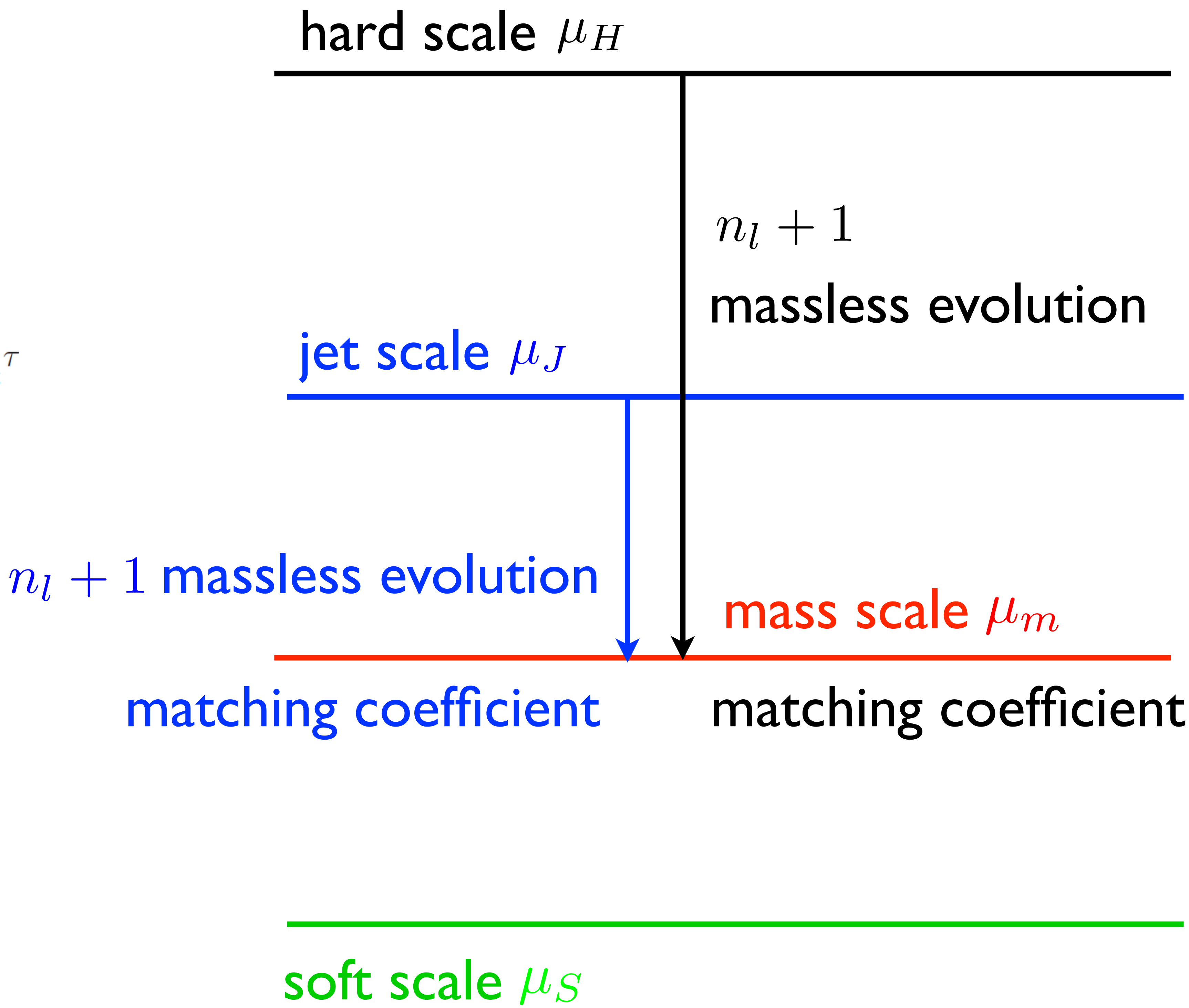


Scenario III

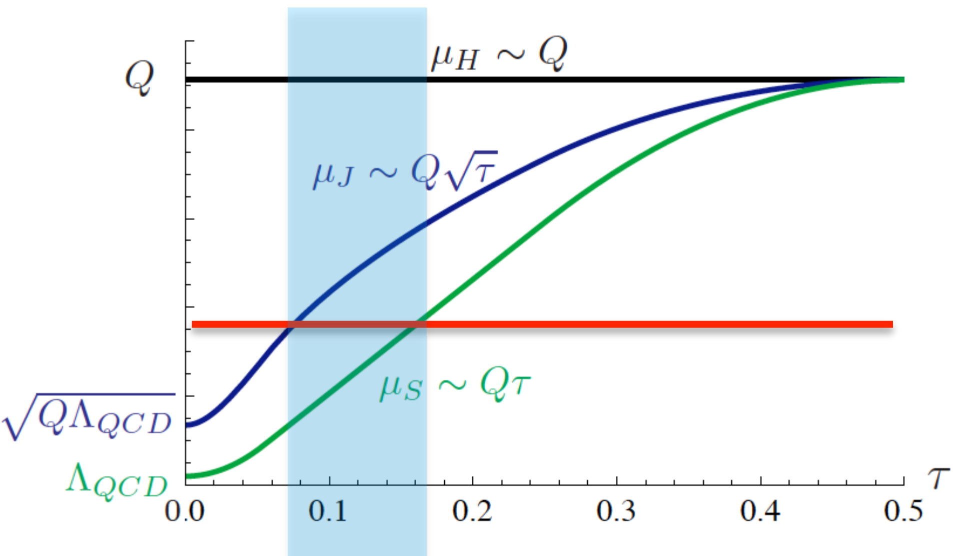


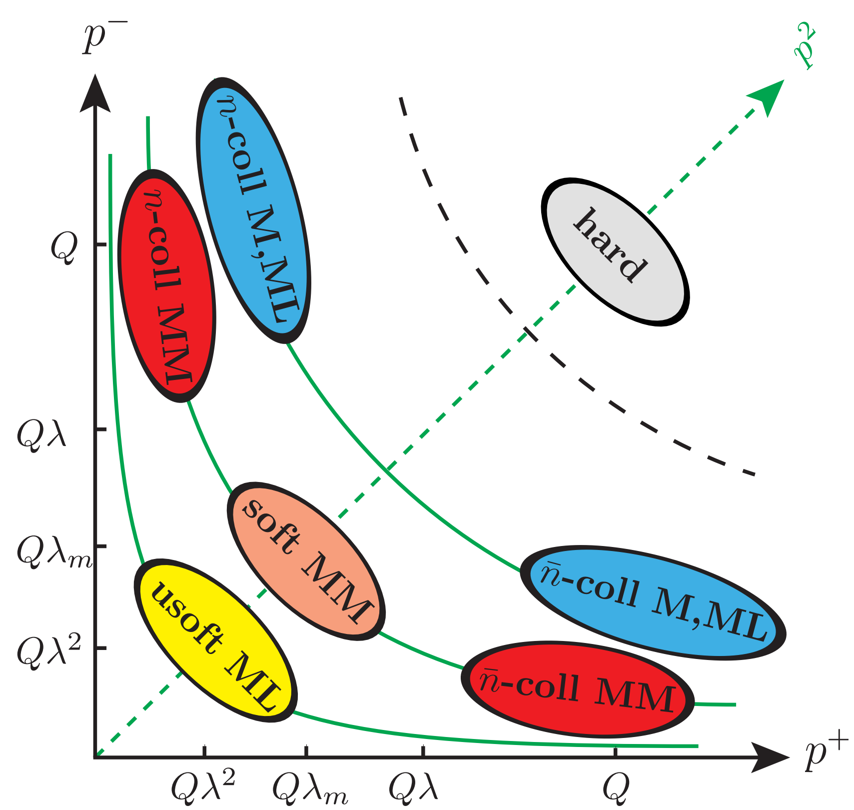


(c)$1 > \lambda > \lambda_M > \lambda^2$

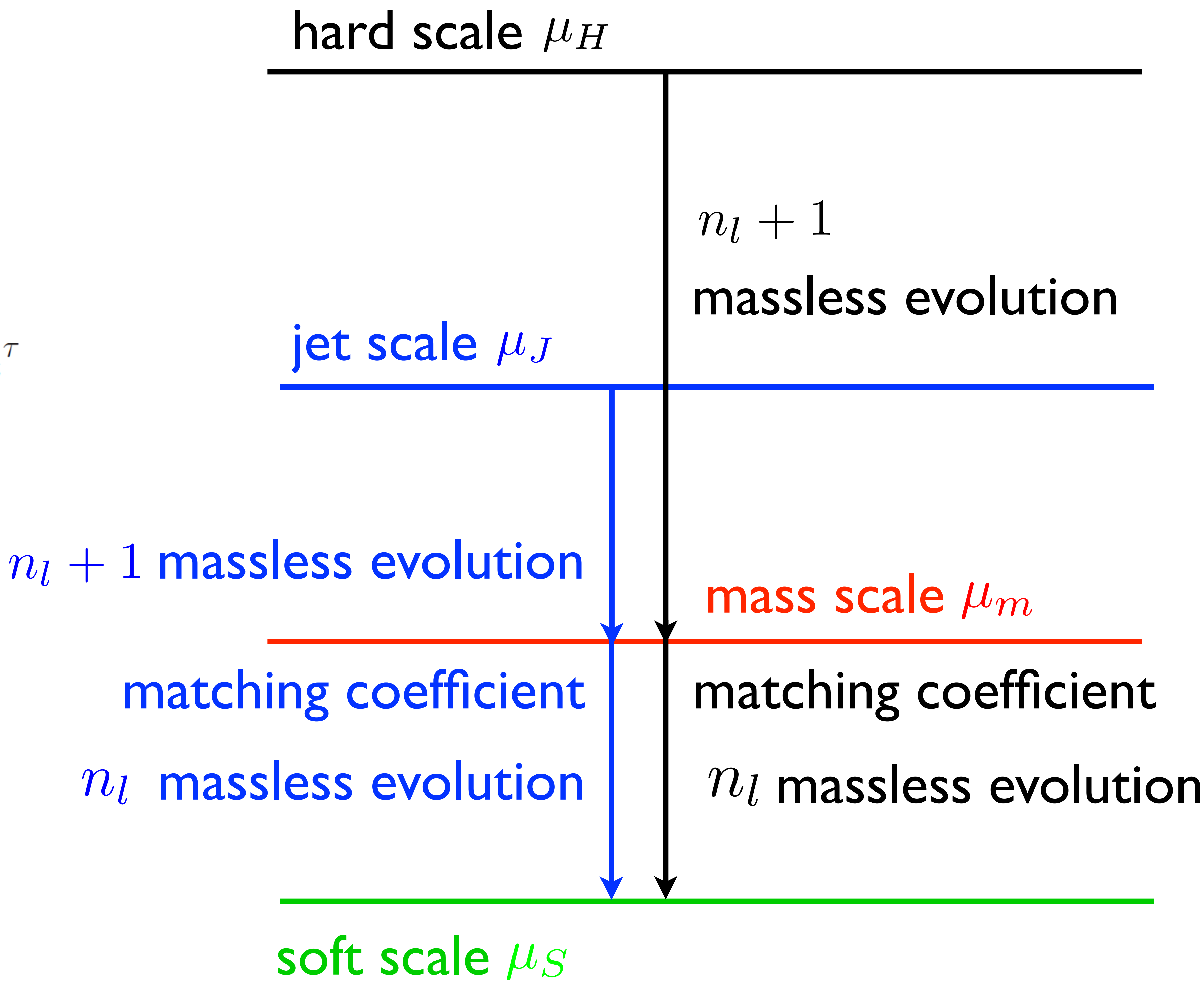


Scenario III





(c) $1 > \lambda > \lambda_M > \lambda^2$



Scenario III

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim H^{(n_l+1)}(\mu_Q, m) U_H^{(n_l+1)}(\mu_H, \mu_m) \mathcal{M}_H(\mu_m) U_H^{(n_l)}(\mu_m, \mu_S) \times$$

$$J^{(n_l+1)}(\mu_J, m) \otimes U_J^{(n_l+1)}(\mu_J, \mu_m) \otimes \mathcal{M}_J(\mu_m) \otimes U_J^{(n_l)}(\mu_m, \mu_S) \otimes S^{(n_l)}(\mu_S, m)$$

all matrix elements are mass-dependent

matching compensates difference between $H^{(n_l+1)}$ and $H^{(n_l)}$

$J^{(n_l)}$ $J^{(n_l+1)}$

running factors, same as in massless theory

Scenario III

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim H^{(n_l+1)}(\mu_Q, m)U_H^{(n_l+1)}(\mu_H, \mu_m)\mathcal{M}_H(\mu_m)U_H^{(n_l)}(\mu_m, \mu_S)\times$$

$$J^{(n_l+1)}(\mu_J, m) \otimes U_J^{(n_l+1)}(\mu_J, \mu_m) \otimes \mathcal{M}_J(\mu_m) \otimes U_J^{(n_l)}(\mu_m, \mu_S) \otimes S^{(n_l)}(\mu_S, m)$$

all matrix elements are mass-dependent

matching compensates difference between $H^{(n_l+1)}$ and $H^{(n_l)}$

running factors, same as in massless theory

use $\alpha_s^{(n_l+1)}$

use $\alpha_s^{(n_l)}$

Scenario III

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim H^{(n_l+1)}(\mu_Q, m) U_H^{(n_l+1)}(\mu_H, \mu_m) \mathcal{M}_H(\mu_m) U_H^{(n_l)}(\mu_m, \mu_S) \times$$

$$J^{(n_l+1)}(\mu_J, m) \otimes U_J^{(n_l+1)}(\mu_J, \mu_m) \otimes \mathcal{M}_J(\mu_m) \otimes U_J^{(n_l)}(\mu_m, \mu_S) \otimes S^{(n_l)}(\mu_S, m)$$

all matrix elements are mass-dependent

matching compensates difference between $H^{(n_l+1)}$ and $H^{(n_l)}$

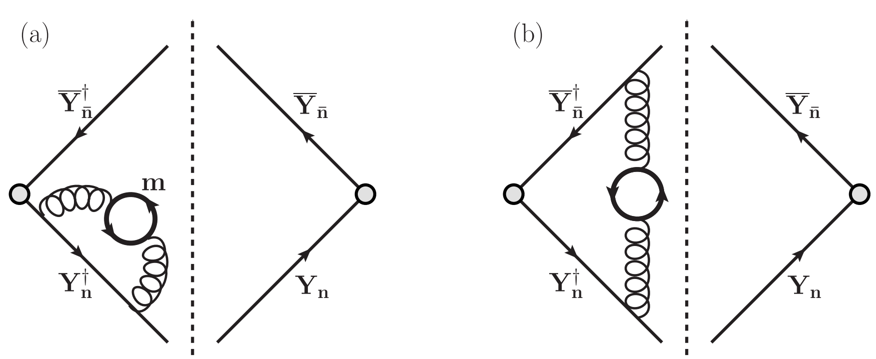
running factors, same as in massless theory

includes virtual heavy quark mass effects in $\overline{\mathrm{MS}}$ scheme and heavy quark real radiation

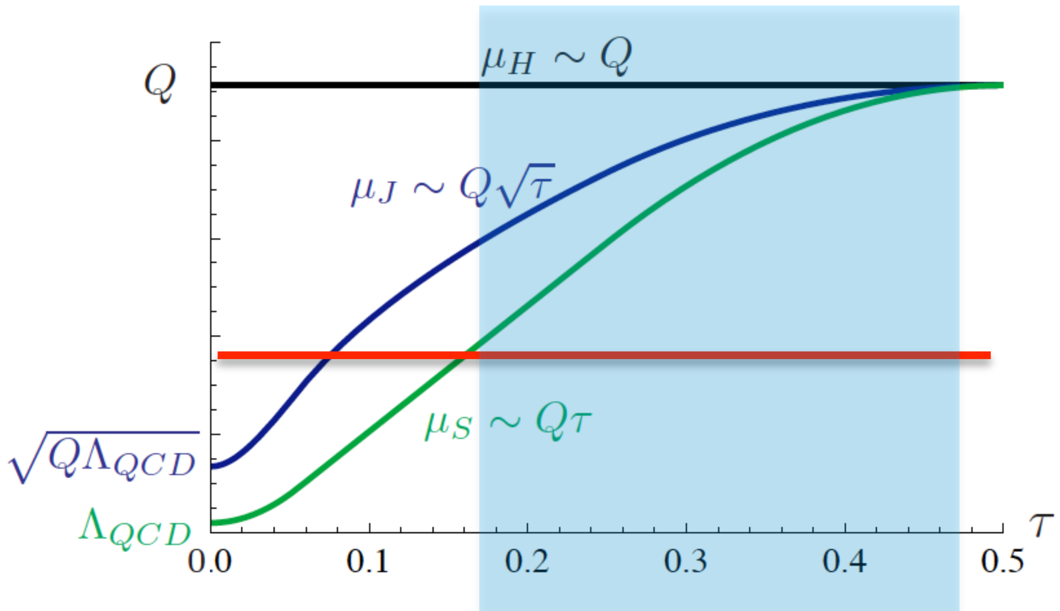
both contributions make for a smooth massless limit

includes virtual heavy quark mass effects in OS scheme

correct decoupling limit but IR-divergent massless limit

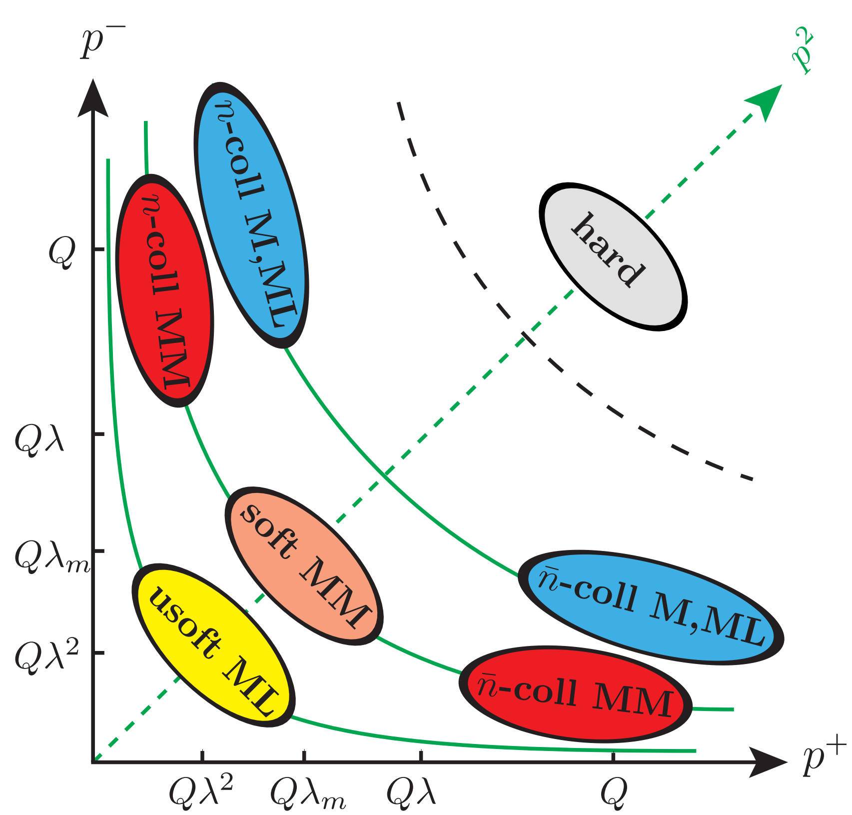


Scenario IV



hard scale μ_H

jet scale μ_J

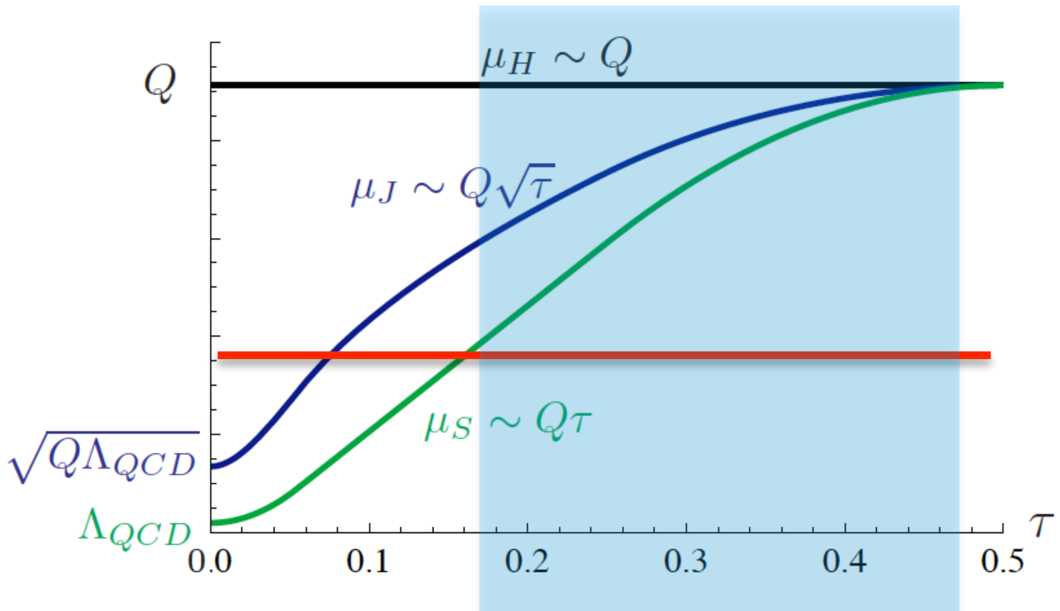


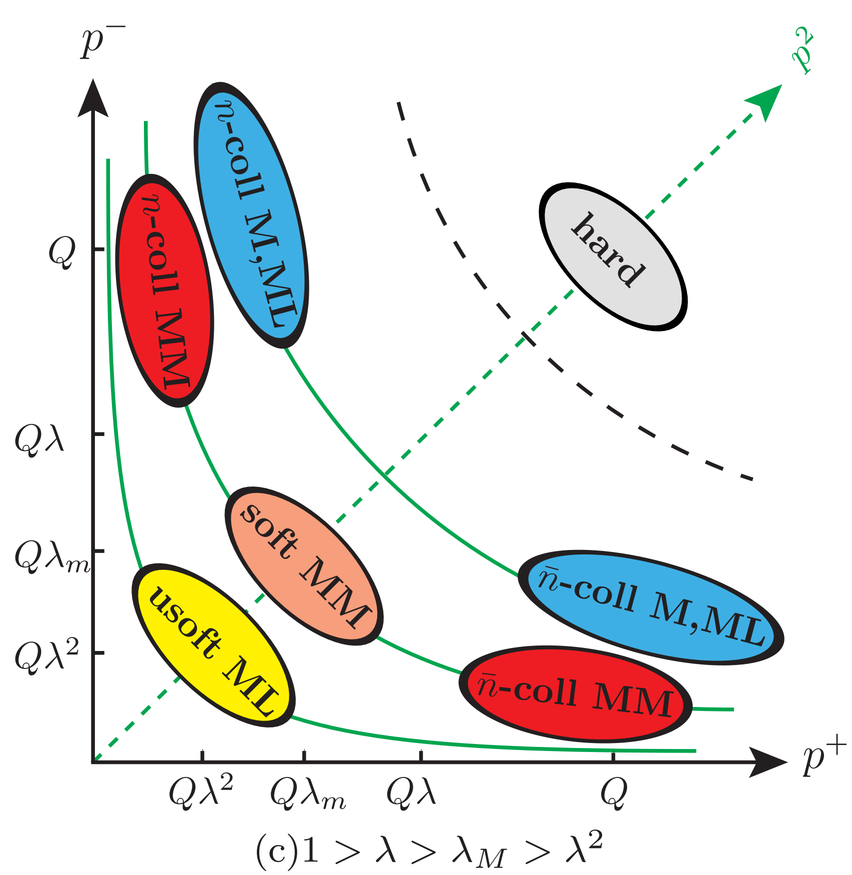
(c)$1 > \lambda > \lambda_M > \lambda^2$

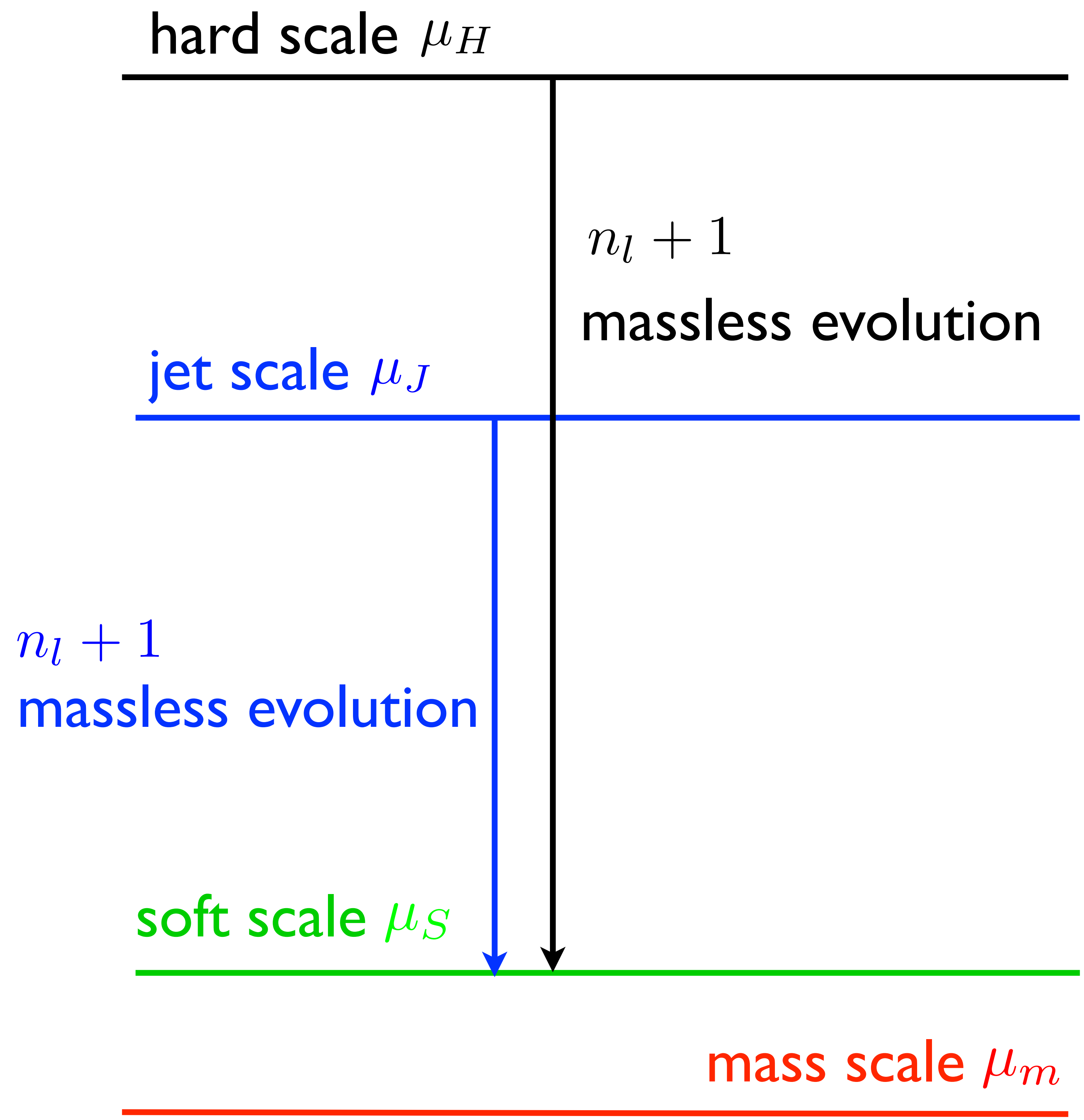
soft scale μ_S

mass scale μ_m

Scenario IV







no matching at the mass scale !

Scenario IV

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim H^{(n_l+1)}(\mu_Q, m) U_H^{(n_l+1)}(\mu_H, \mu_S) \times J^{(n_l+1)}(\mu_J, m) \otimes U_J^{(n_l+1)}(\mu_J, \mu_S) \otimes S^{(n_l+1)}(\mu_S, m)$$

all matrix elements are mass-dependent

running factors, same as in massless theory

all matching coefficients, matrix elements and running factors use $\alpha_s^{(n_l+1)}$

Scenario IV

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim H^{(n_l+1)}(\mu_Q, m) U_H^{(n_l+1)}(\mu_H, \mu_S) \times J^{(n_l+1)}(\mu_J, m) \otimes U_J^{(n_l+1)}(\mu_J, \mu_S) \otimes S^{(n_l+1)}(\mu_S, m)$$

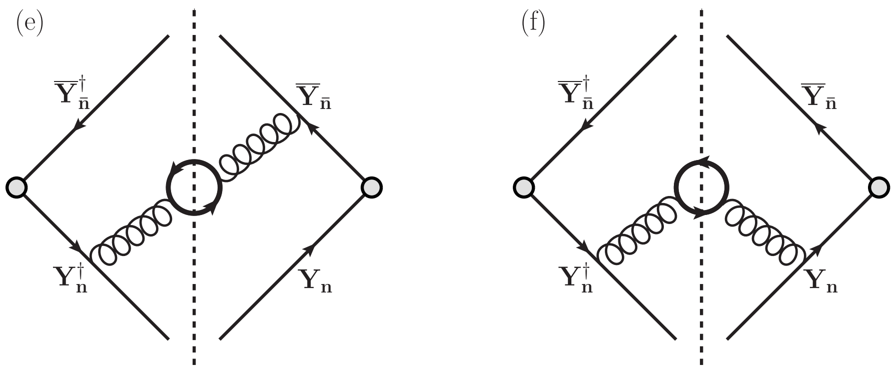
all matrix elements are mass-dependent

running factors, same as in massless theory

all matching coefficients, matrix elements and running factors use $\alpha_s^{(n_l+1)}$

includes virtual heavy quark mass effects in $\overline{\mathrm{MS}}$ scheme and heavy quark real radiation

both contributions make for a smooth massless limit



full distribution has a smooth massless limit

Theoretical remarks

- Secondary mass effects start at two loops
- However matching coefficients suffer from rapidity logs
- This logs exponentiate and can be summed up
- This makes their effect effectively one loop

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- This logs exponentiate and can be summed up
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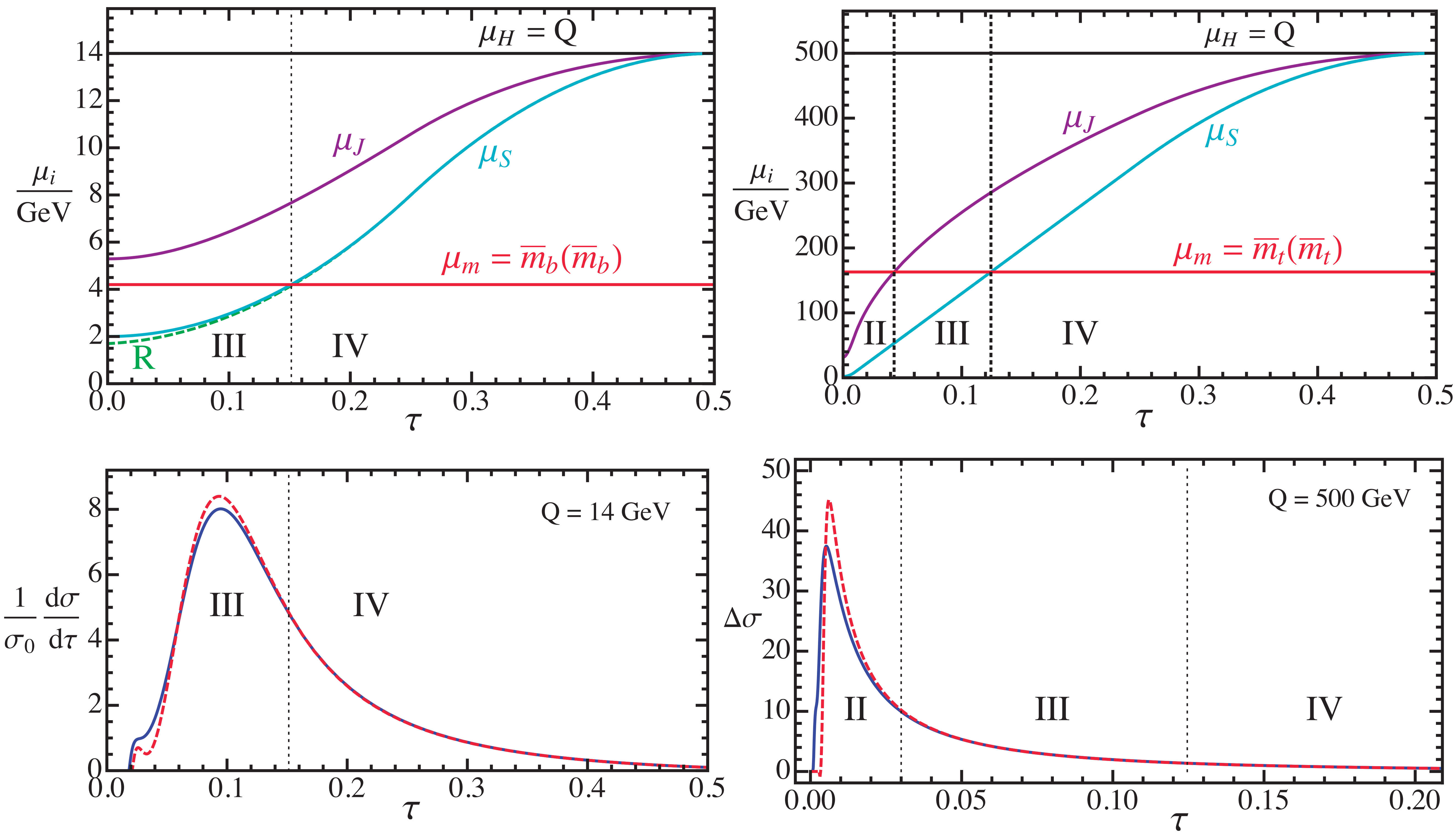
- The various scenarios join smoothly (by construction)
- Full mass dependence kept in every scenario
- All matrix elements for thrust computed at two loops
- All ingredients known for a $\mathrm{N^3LL}$ analysis

Numerical results

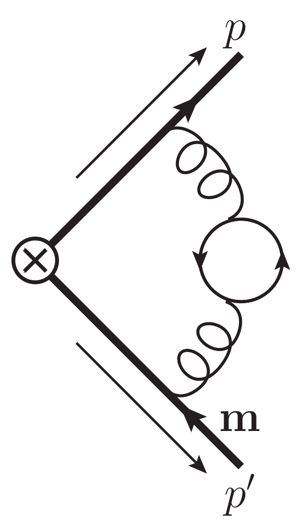
$\mathcal{O}(\alpha_s^2)$ matrix element and $\mathrm{N^3LL}$ resummation

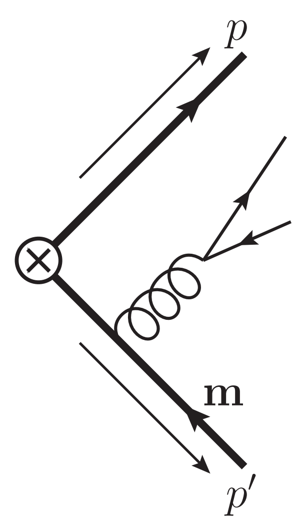
only secondary bottom and top mass effects (hadron level predictions)



Primary mass production

Primary production of heavy quarks





[Fleming, Mantry, Hoang Stewart]

in scenarios III and IV one can also produce, primary quarks, starting at tree level

This only modifies the jet function, which becomes mass dependent (same hard and soft function, same running factors)

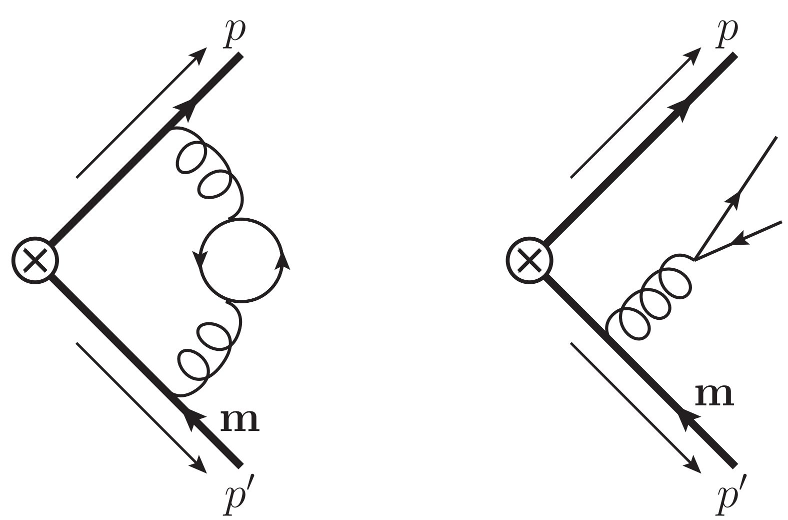
Jet function for thrust (and C-parameter) known at one loop: enables a $\mathrm{N^2LL}$ analysis

The primary massive jet function has a smooth massless limit

One needs two loop massive jet function for a $\mathrm{N^3LL}$ analysis

Short distance mass has to be used to avoid renormalon. $\overline{\mathrm{MS}}$ does the job

Primary production of heavy quarks



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Kinematically power suppressed contributions necessary for a precision analysis

Primary mass effects introduce distributive terms in non-singular terms

Thrust [Butenschön, Dehnadi, Hoang, VM, Stewart]

C-parameter [Hoang, VM, Preisser, w.i.p.]

b-HQET regime

[Fleming, Mantry, Hoang Stewart]

When $\mu_J - \mu_m \ll \mu_m$ a new hierarchy arises (together with new class of large logs)

One has to match SCET to a boosted HQET theory to sum them up

In this framework one can also treat finite width effects (mandatory for top !!!)

Effectively one has a bHQET jet function (and an additional matching coefficient)

bHQET jet function known at two loops: enables a $\mathrm{N^3LL}$ analysis *[Jain, Scimemi, Stewart]*

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bHQET jet function known at two loops: enables a $\mathrm{N^3LL}$ analysis *[Jain, Scimemi, Stewart]*

One needs to switch to a short-distance mass.
$\overline{\mathrm{MS}}$ does the job BUT breaks power counting

Jet mass: defined from the bHQET jet function *[Jain, Scimemi, Stewart]*

MSR mass: derived from $\overline{\mathrm{MS}}$ - pole relation *[Jain, Hoang, Scimemi, Stewart]*

Both remove the renormalon and respect the power counting.
They depend on an infrared scale R

Conclusions & Outlook

- Implemented Variable Flavor Number scheme for final-state jets
- Implemented primary massive quark effects
- Fast numerical fortran code already created
- 1^{st} step: fitting heavy quark masses to Pythia output
- 2^{nd} step: fitting bottom mass from low Q data
- Long range aim: top production at hadron collider