α_s determination from C-parameter

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In collaboration with A. Hoang (U. Vienna), D. Kolodrubetz and I. Stewart (MIT)

two papers to appear soon on the arXiv

XI Confinement (Saint Petersburg)

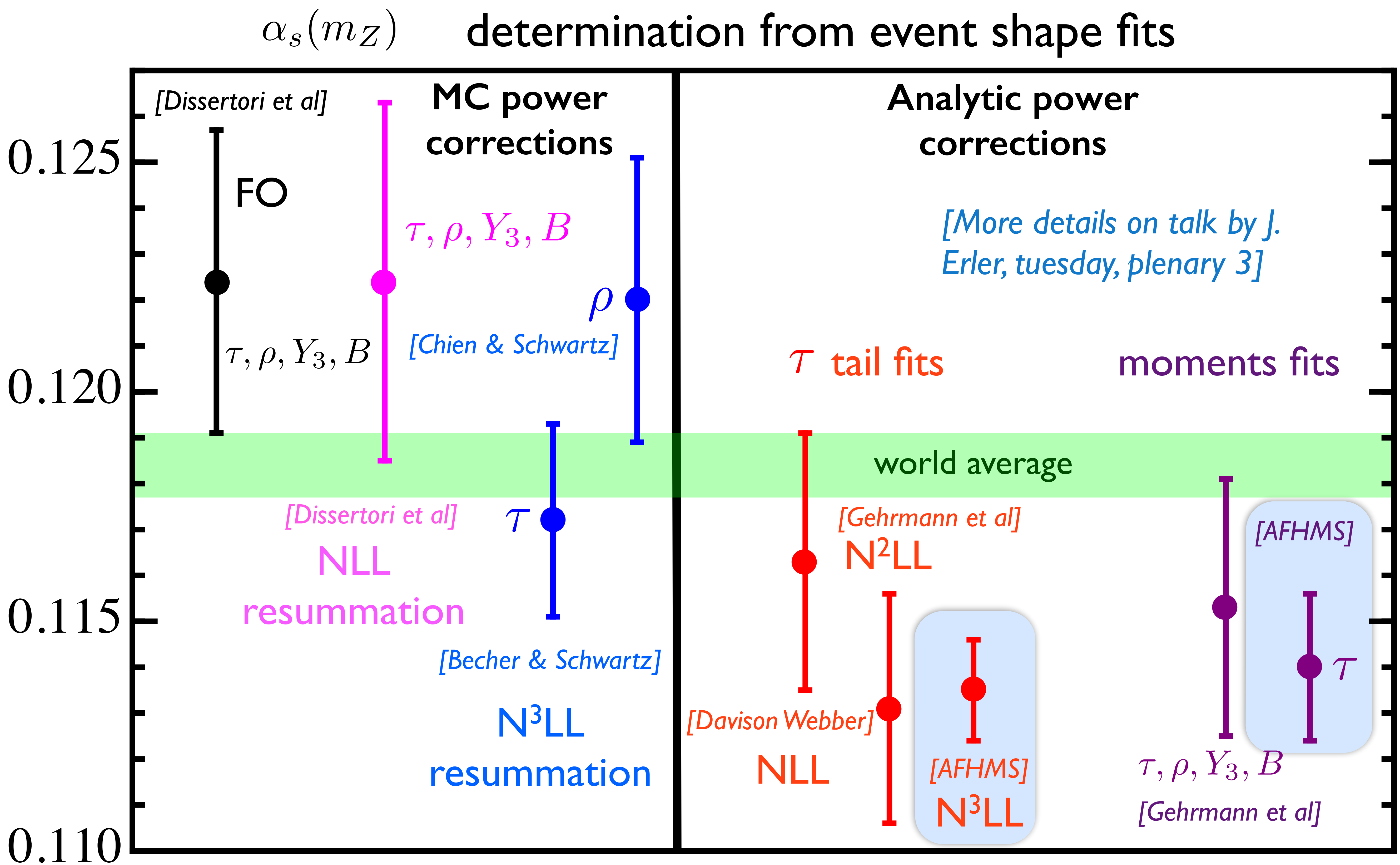
12-09-2014

Outline

- Motivation & Introduction
- Factorization & log resummation at N^3LL
- Singular & Non-singular terms
- Power corrections
- Fits for α_s
- Conclusions and Outlook

Motivation
&
Introduction

Event shape analyses with analytic power corrections get consistently low values for α_s



C-parameter definition $e^+ e^- \to \text{jets}$

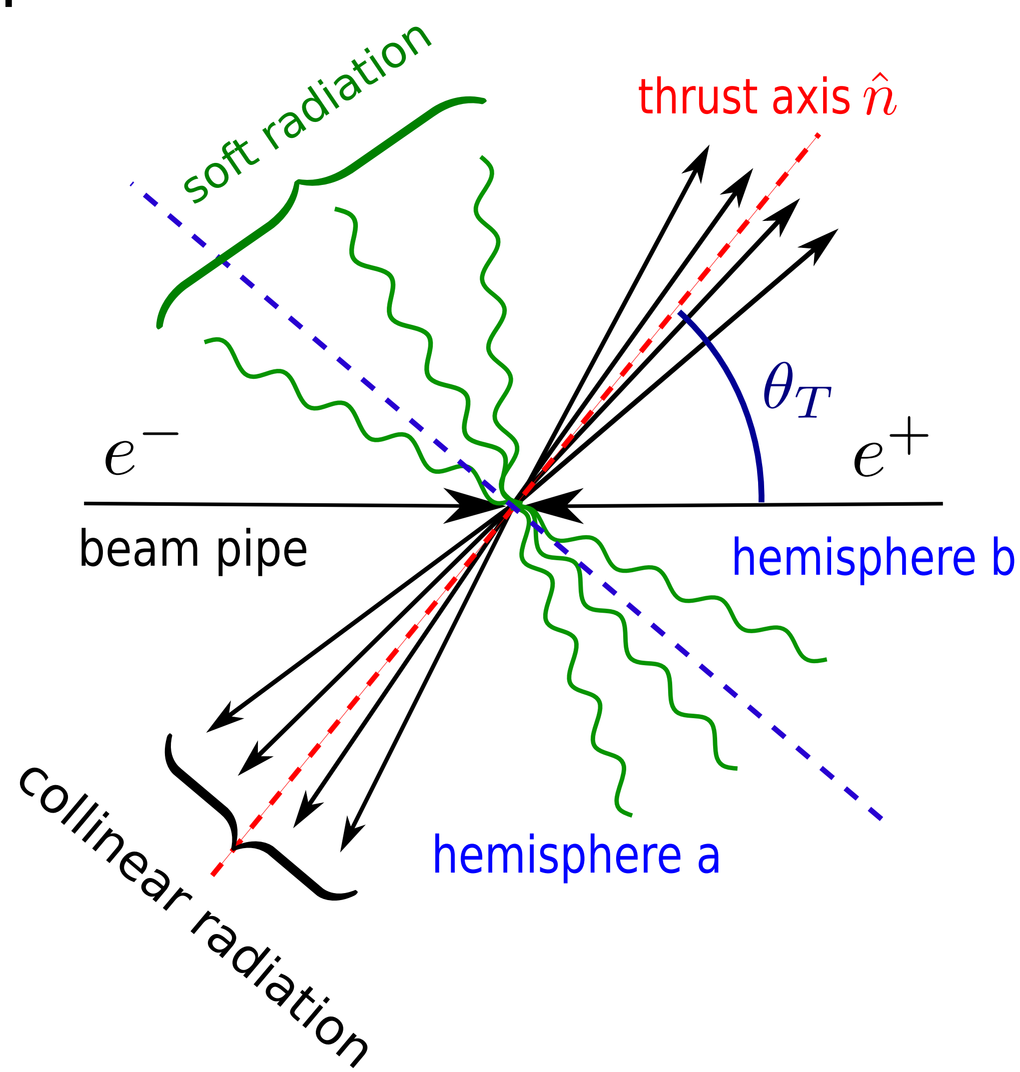
linearized momentum tensor $\Theta^{\alpha\beta} = \frac{1}{\sum_i |\vec{p}_i|} \sum_i \frac{p_i^\alpha p_i^\beta}{|\vec{p}_i|}$ with eigenvalues $\lambda_{1,2,3}$

$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$

$$C = 3\,(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3) = \frac{3}{2}\frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j|\sin^2\theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}$$

IR and collinear safe

- Double sum
- Does not require minimization



[For thrust see my talk on Tuesday, parallel III]

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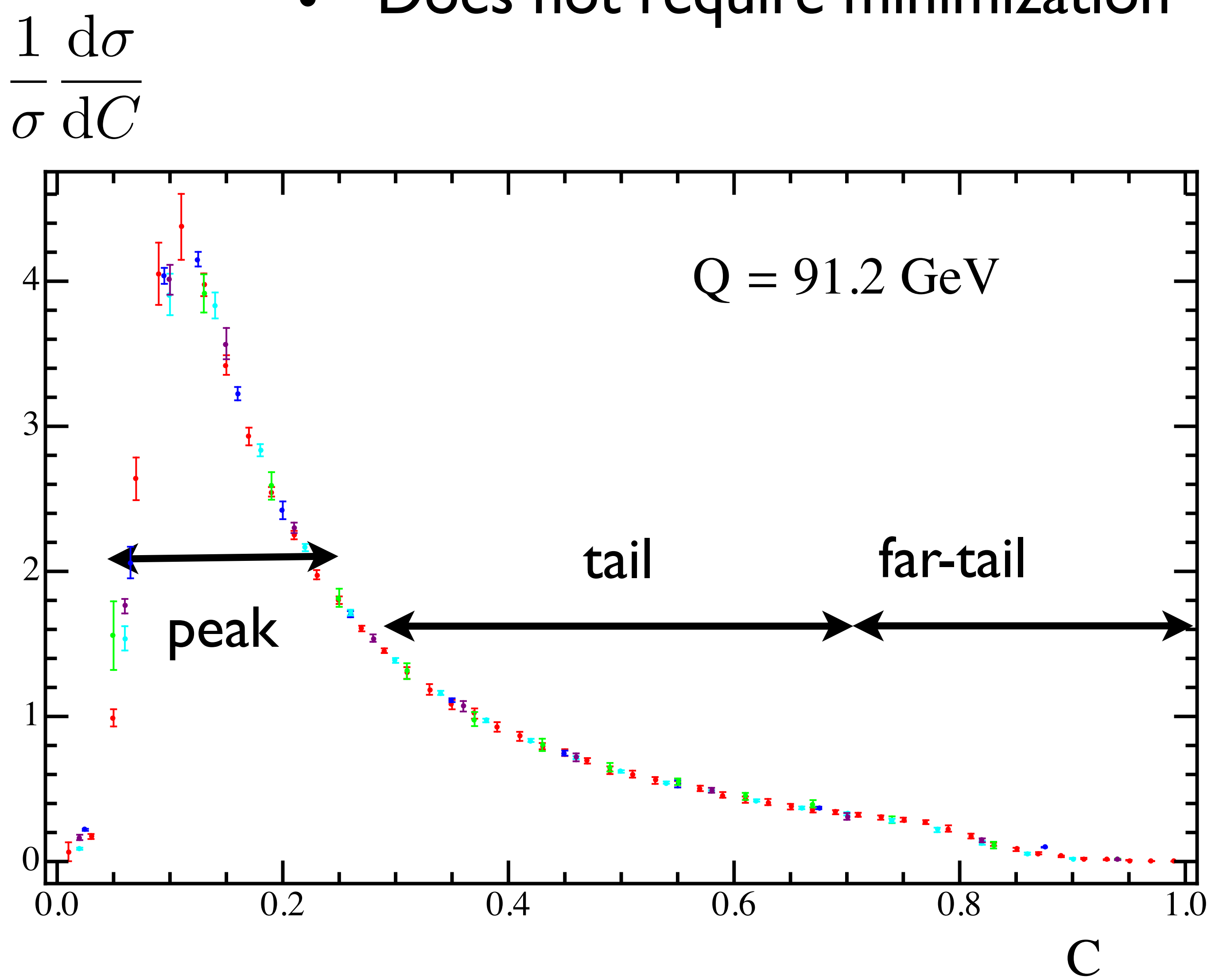
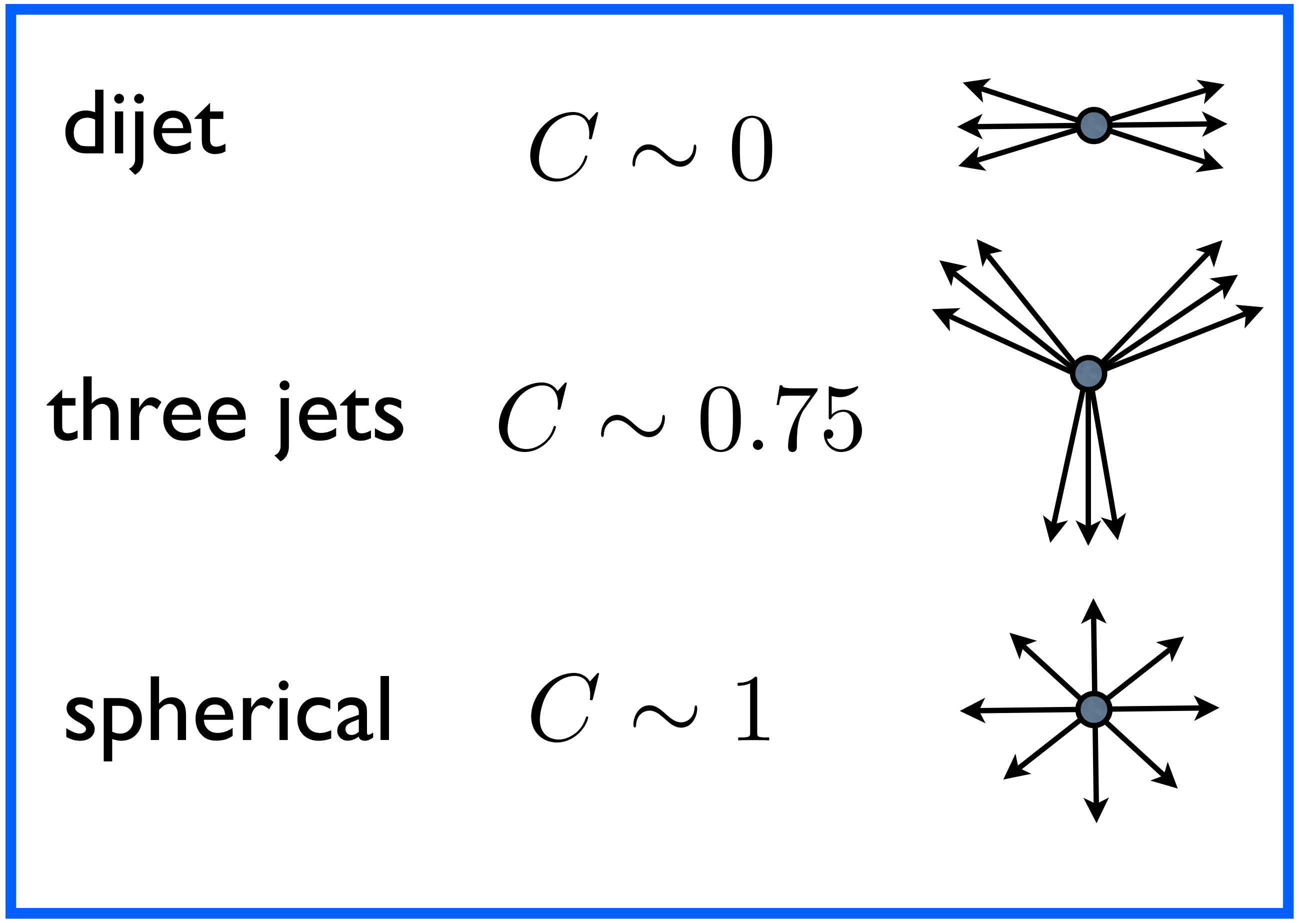
Continuous transition from 2-jet to 3-jet, ... multi-jet events





Factorization &
log resummation

Resummation of large logarithms

[For thrust see my talk on Tuesday, parallel III]

Event shapes are not inclusive quantities

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{dC}} = -\frac{2\alpha_s}{3\pi}\frac{1}{C}\left(3 + 4\log\frac{C}{6} + \dots\right)$$

Large logs at small C

Invalidates perturbative expression for small

One has to reorganize the expansion by considering $\alpha_s \log\frac{C}{6} \sim \mathcal{O}(1)$

Counting more clear in the exponent of cumulant

$$\Sigma(C_c) \equiv \int_0^{C_c} \mathrm{d}C \,\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{dC}}$$

$$\begin{aligned}
\log\Sigma(C_c) = {} & \alpha_s(\log^2 C_c + \log C_c + 1) \\
& \alpha_s^2\,(\log^3 C_c + \log^2 C_c + \log C_c + 1) \\
& \alpha_s^3\,(\log^4 C_c + \log^3 C_c + \log^2 C_c + \log C_c + 1) \\
& \alpha_s^4\,(\log^5 C_c + \log^4 C_c + \log^3 C_c + \log^2 C_c + \log C_c + 1) \\
& \dots \quad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots
\end{aligned}$$

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LL | NLL | N^{2}LL | N^{3}LL | not known!

State of the art

[Hoang, VM, Schwartz, Stewart]
[Becher, Schwartz]
[Chien, Schwartz]
[Abbate, Fickinger, Hoang, VM, Stewart]
[Hoang, Kolodrubetz, VM, Stewart]

Resumption of large logarithms

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Factorization theorem for event shapes

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \underbrace{H_Q \times J_e}_{\text{Calculable in perturbation theory}} \otimes \underbrace{S_e}_{\text{Perturbative and nonperturbative components}} + \mathcal{O}\left(e^0, \frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

[Bauer, Lee, Fleming, Sterman]

Universal Wilson Coefficient

Jet function

Soft function

Nonsingular terms, power corrections

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Universal Wilson Coefficient

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Nonsingular terms, power corrections

Calculable in perturbation theory

Perturbative and nonperturbative components

Leading power correction comes from soft function

$$S_e = \hat{S}_e \otimes F_e$$

perturbative

nonperturbative & <u>perturbative</u>

[Hoang & Stewart]

[VM, Thaler, Stewart]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e} \otimes F_e$$

Hadron mass effects taken into account, but no time to discuss them

Renormalization group evolution

large logs

hard scale $\mu_H \sim Q$ — $\log^n\left(\frac{Q}{\mu}\right)$

jet scale $\mu_J \sim Q\sqrt{C/6}$ — $\log^n\left(\frac{Q^2 C}{6\mu^2}\right)$

soft scale $\mu_S \sim Q\,C/6$ — $\log^n\left(\frac{QC}{6\mu}\right)$

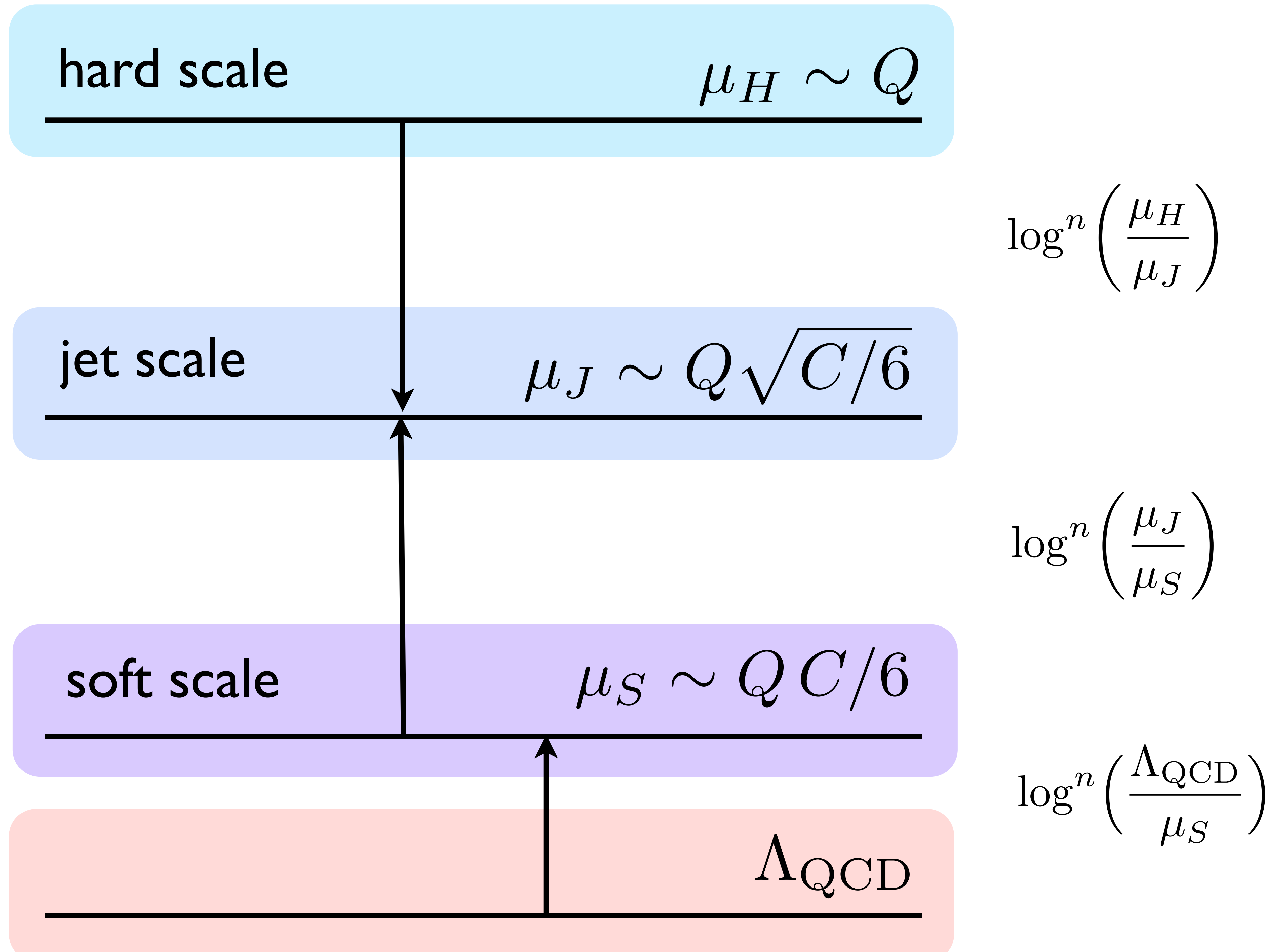
$\Lambda_{\rm QCD}$ — $\log^n\left(\frac{6\Lambda_{\rm QCD}}{QC}\right)$

The hierarchy among the scales depends on the position on the spectrum

Use profile function to describe the whole distribution

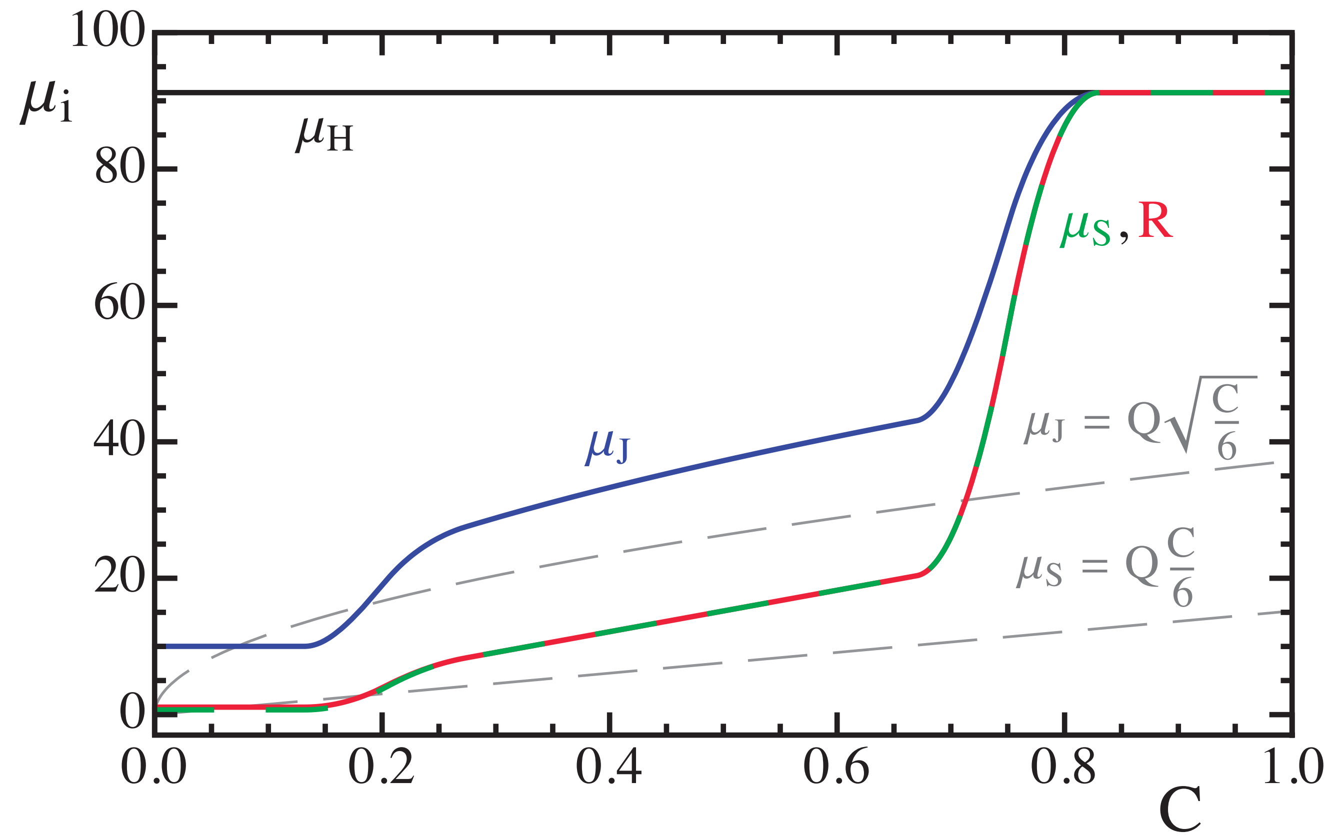
Renormalization group evolution

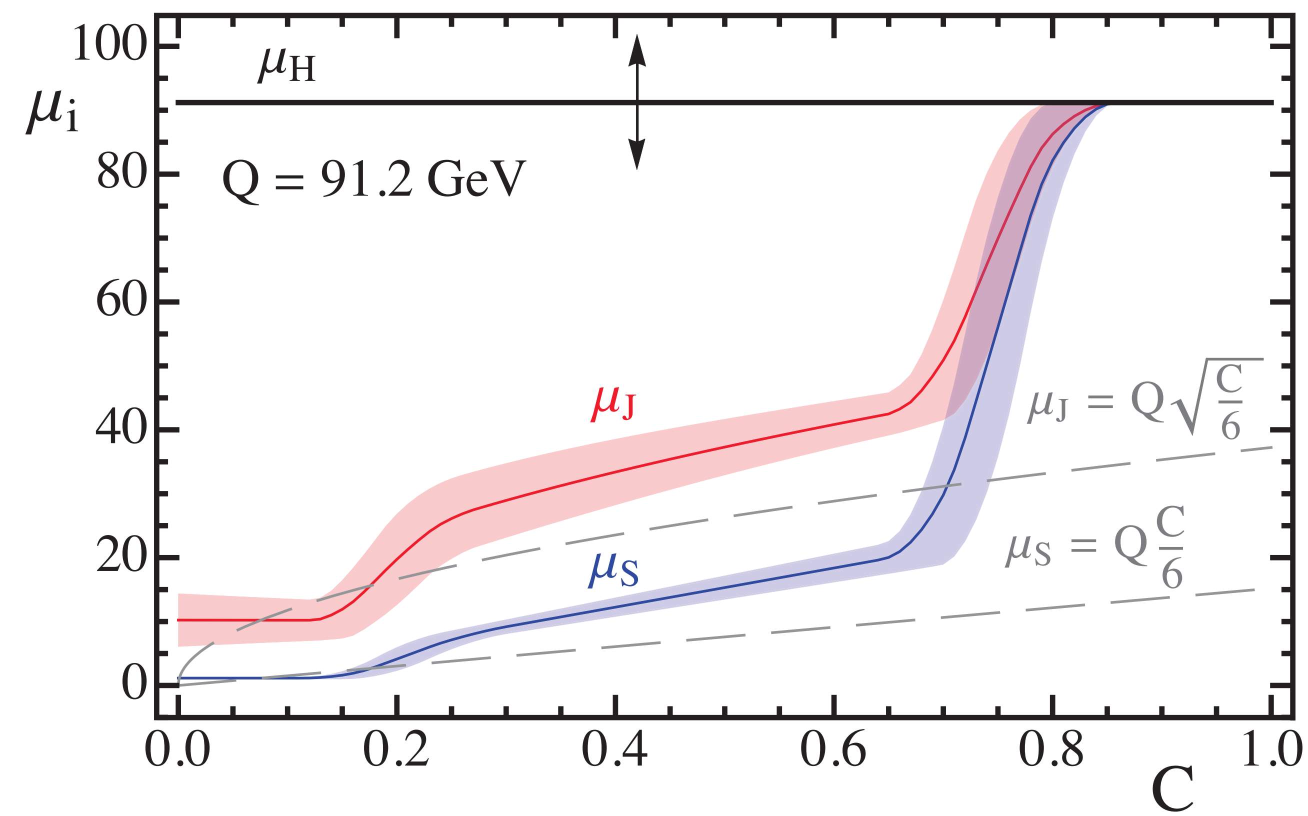
The hierarchy among the scales depends on the position on the spectrum



Renormalization scale setting

	parameter	default value	range of values
scale variation	μ_0	1.1 GeV	1 to 1.3 GeV
	R_0	0.7 GeV	0.6 to 0.9 GeV
	n_0	12	10 to 16
	n_1	25	22 to 28
	t_2	0.67	0.64 to 0.7
	t_s	0.83	0.8 to 0.86
	r	0.33	0.26 to 0.38
	e_J	0	-0.5 to 0.5
	e_H	1	0.5 to 2.0
	n_s	0	-1, 0, 1
unknowns	$\Gamma_3^{\rm cusp}$	1553.06	-1553.06 to $+4659.18$
	s_2	-43.2	-44.2 to -42.2
	j_3	0	-3000 to $+3000$
	s_3	0	-500 to $+500$
non-singular	$\epsilon_{2,\rm low}$	0	-1, 0, 1
	$\epsilon_{2,\rm high}$	0	-1, 0, 1
	$\epsilon_{3,\rm low}$	0	-1, 0, 1
	$\epsilon_{3,\rm high}$	0	-1, 0, 1





Singular
&
nonsingular terms

Theoretical knowledge

$H(Q, \mu)$ Hard function known at 3 loops

same as thrust

$J_n(s, \mu)$ Jet function known at two loops

Running known at three loops

$S_C(\ell, \mu)$ Soft function known analytically at one loop, numerically at two loops

Running known at three loops

Theoretical knowledge

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same as thrust

$S_C(\ell,\mu)$ Soft function known analytically at one loop, numerically at two loops
Running known at three loops

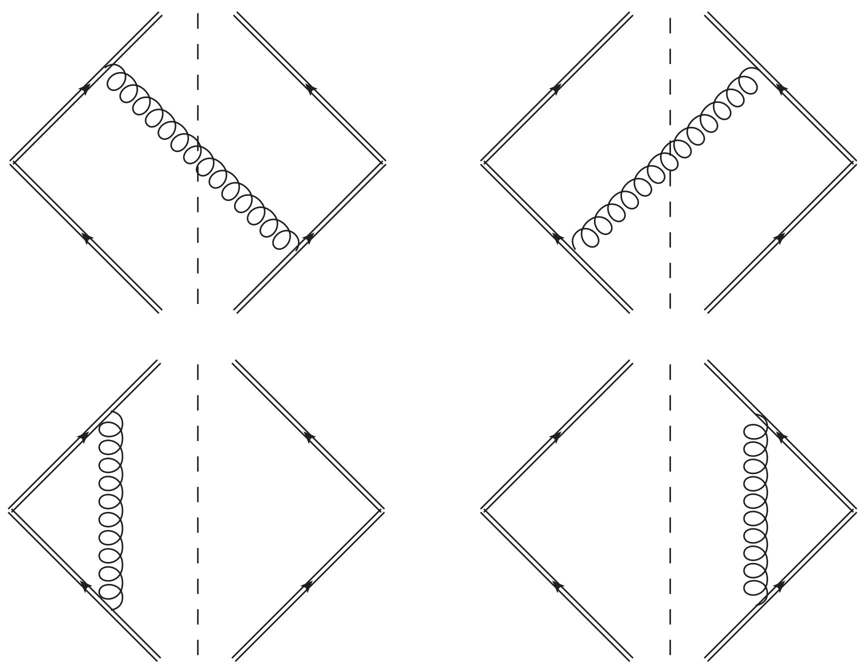
Fixed-order predictions known at three loops

Mass corrections known at N^2LL and two loops *[for more details see my talk on Tuesday, parallel III]*

FS QED corrections known at N^3LL

C-parameter soft function computation

[Kolodrubetz, Hoang, VM, Stewart]



Analytic computation of soft function at 1-loop

$$S_e^{1-\mathrm{loop}}(\ell) = \frac{2\,\alpha_s C_F e^{\epsilon\gamma_E}}{\mu\,\pi\,\Gamma(1-\epsilon)}\left(\frac{\ell}{\mu}\right)^{-1-2\epsilon} I_e(\epsilon)$$

universal formula for all event shapes

$$I_\tau(\epsilon) = \frac{1}{\epsilon} \qquad I_{\widetilde{C}}(\epsilon) = \frac{1}{2}\frac{\Gamma(\epsilon)^2}{\Gamma(2\epsilon)}$$

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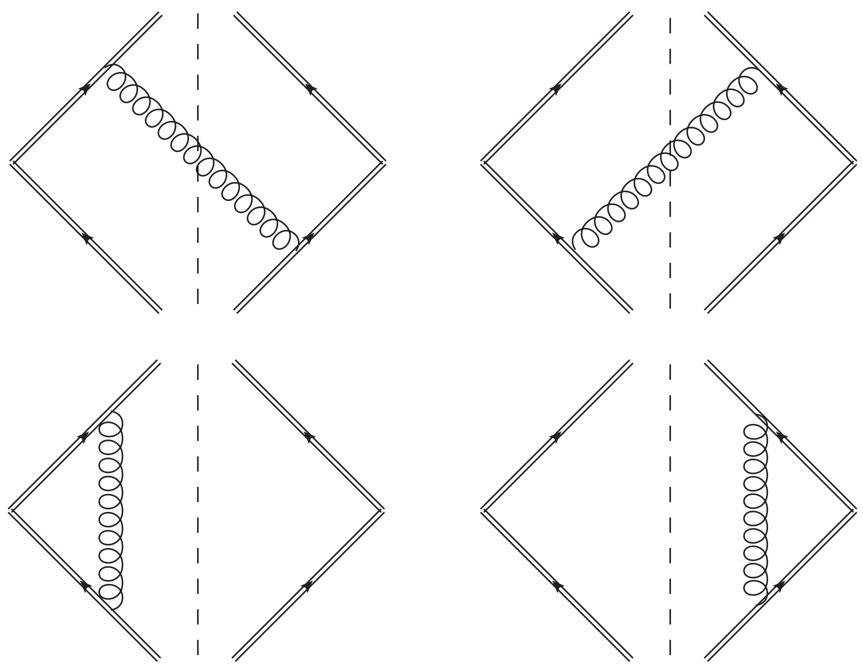

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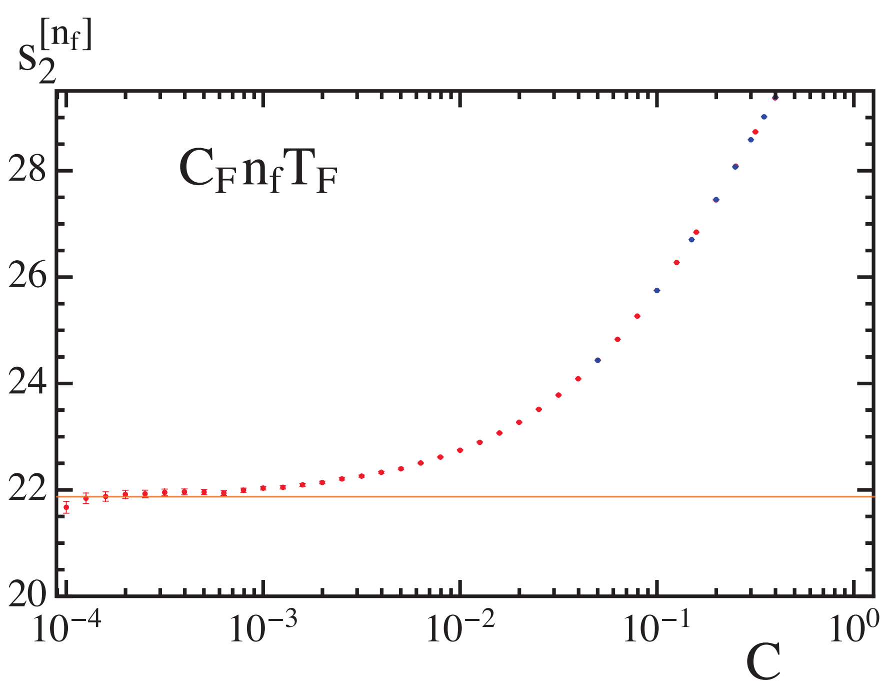

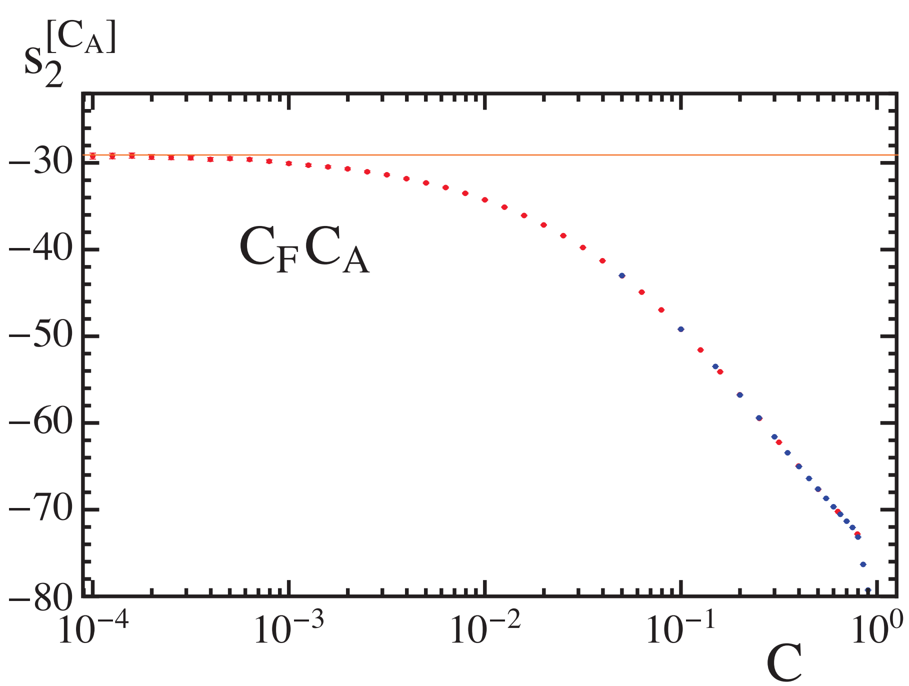

Numerical determination at 2-loops using Event2

Kinematic power corrections

a.k.a. nonsingular terms

$$\frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}C} = \frac{\mathrm{d}\hat{\sigma}^{\mathrm{FO}}_{\mathrm{full}}}{\mathrm{d}C} - \frac{\mathrm{d}\hat{\sigma}^{\mathrm{FO}}_{s}}{\mathrm{d}C}$$

full FO

SCET with fixed scales

$$\frac{\mathrm{d}\sigma_{\mathrm{ns}}}{\mathrm{d}C} = \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}C} \otimes F_C$$

same shape function as singular terms

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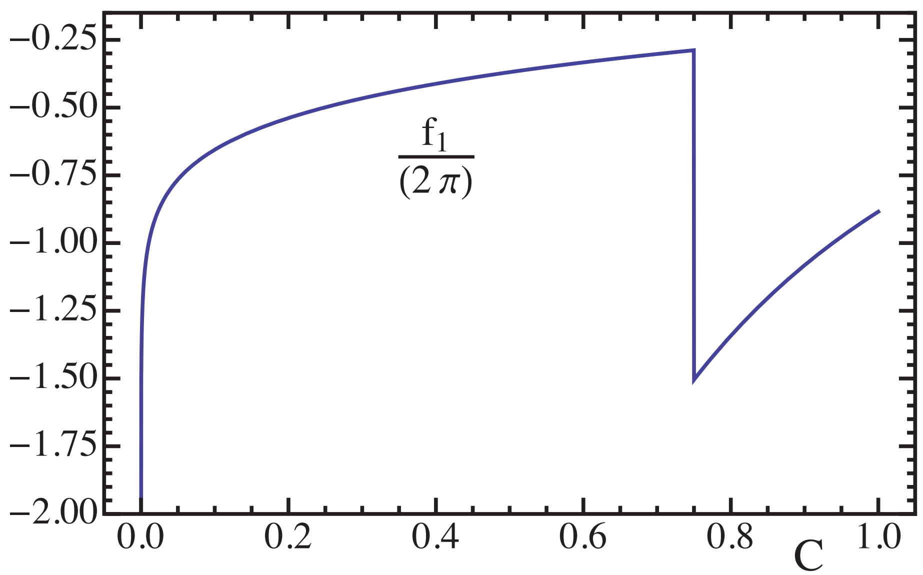
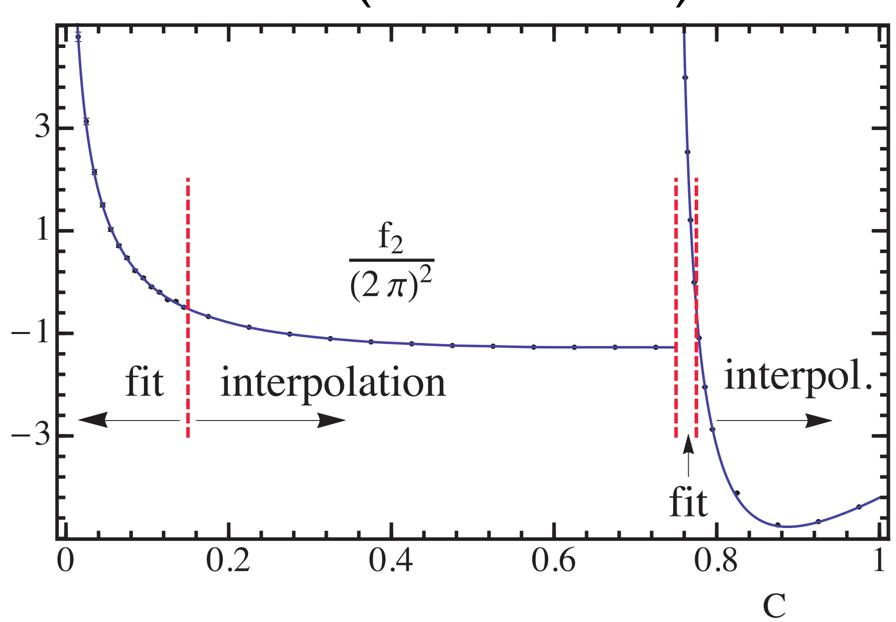
LO (analytically)

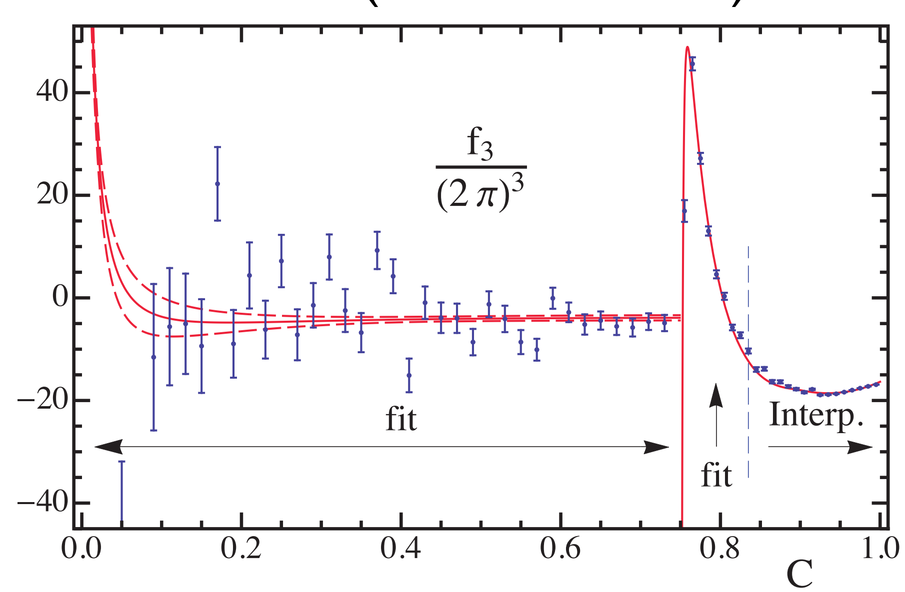


$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}C} = \frac{\alpha_s(Q)}{2\pi} f_1(C) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 f_2(C) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^3 f_3(C) + \ldots$$

NLO (from Event2)



NNLO (from EERAD3)



Power Corrections
&
full results

OPE for non-perturbative corrections

For $e \gg \frac{\Lambda_{\rm QCD}}{Q}$ Shape function can be expanded in the tail

$$F_e(\ell) \simeq \delta(\ell) - \Omega_1^{\rm e}\delta'(\ell)$$

$$\frac{d\sigma}{de} = \frac{d\hat{\sigma}}{de} - \frac{\Omega_1}{Q}\frac{d}{de}\frac{d\hat{\sigma}}{de} \simeq \frac{d\hat{\sigma}}{de}\left(e - \frac{\Omega_1}{Q}\right) + \mathcal{O}\left[\left(\frac{\Lambda_{\rm QCD}}{Q\,e}\right)^2\right]$$

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Universality: [Lee & Sterman] $\Omega_1^e = c_e\,\Omega_1^\rho$

Leading power corrections proportional to each other, calculable coefficient

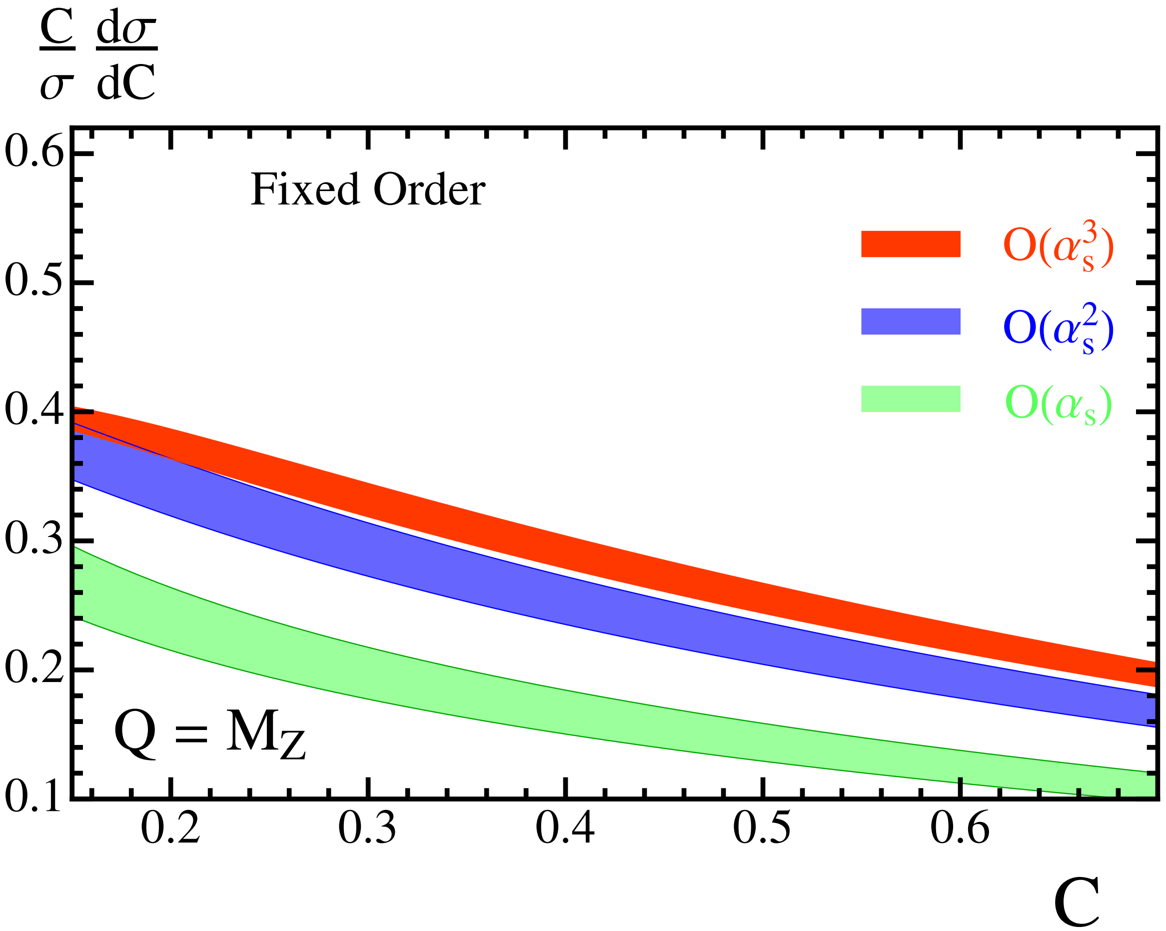
Hadron mass effect break this relation [VM, Stewart, Thaler]

No time to discuss this in detail

We define the gap scheme for $\Omega_1^{\rm e}$ in which it is renormalon-free

No time to discuss this in detail

Cross section convergence



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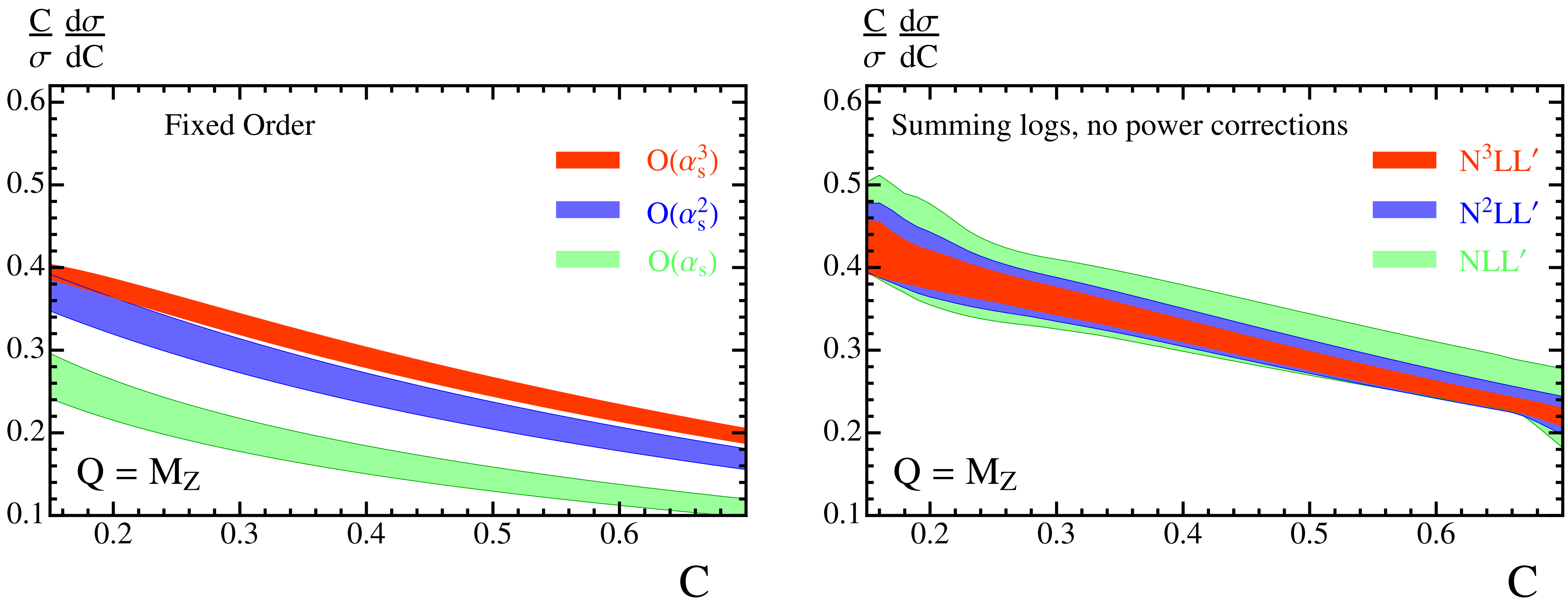

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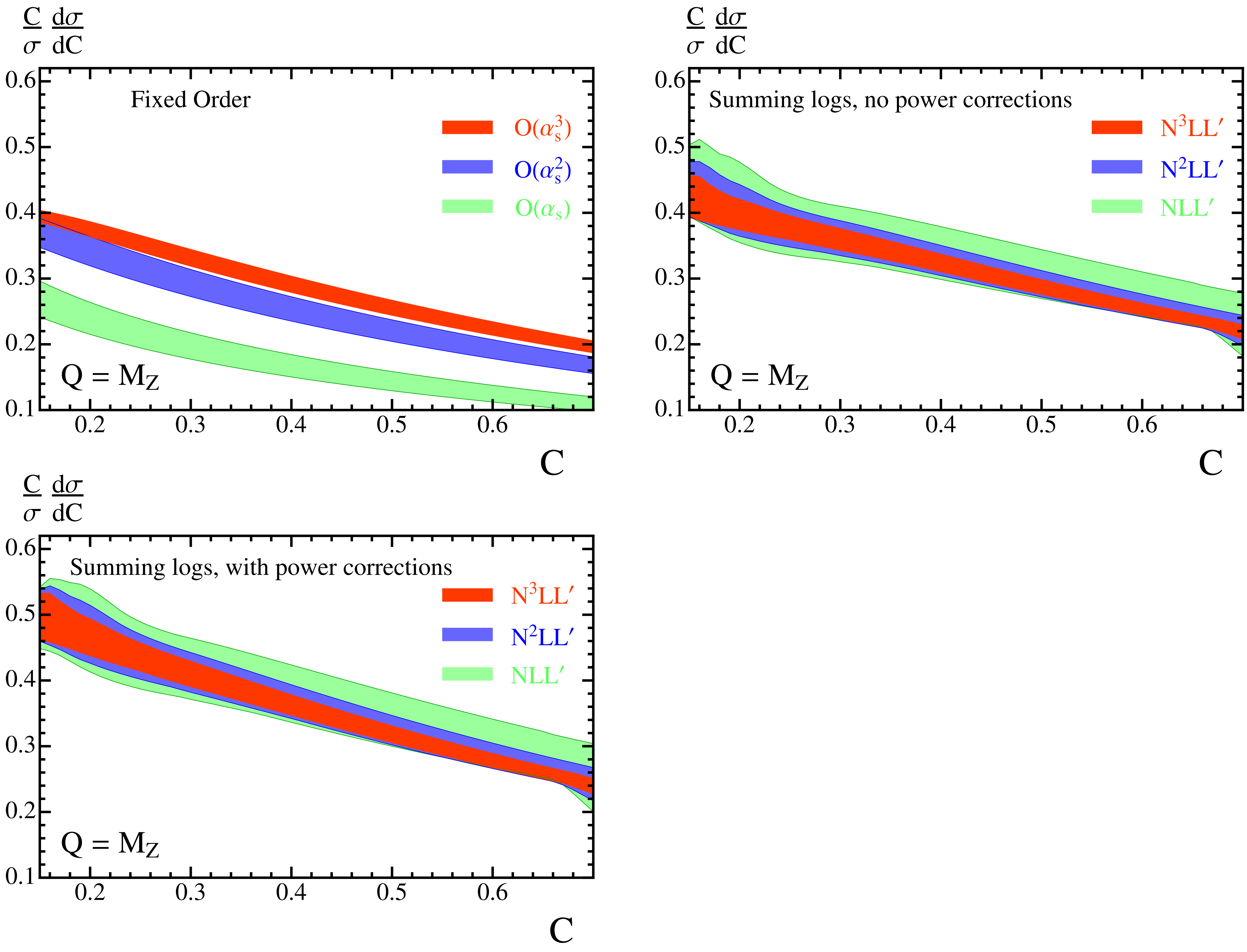

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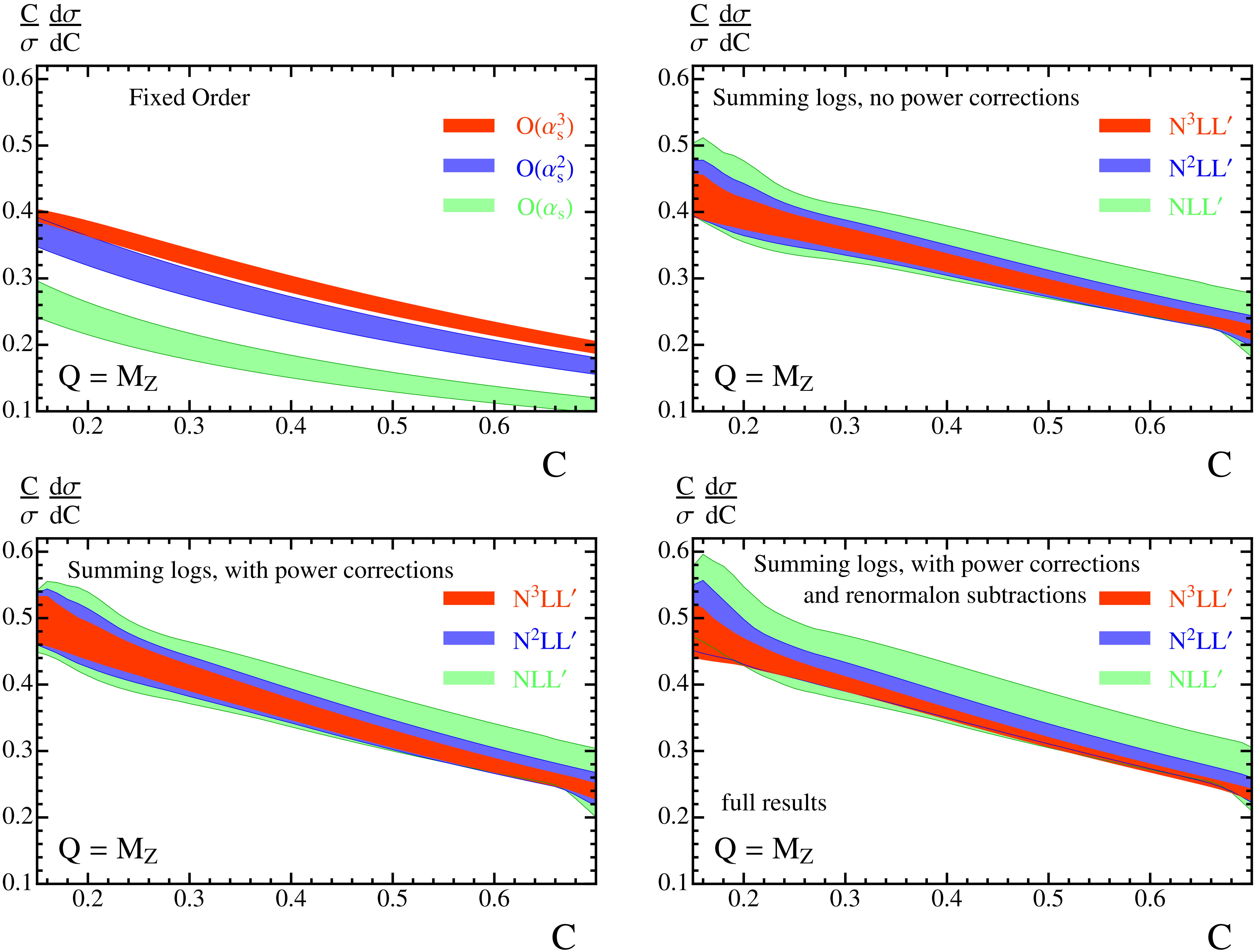
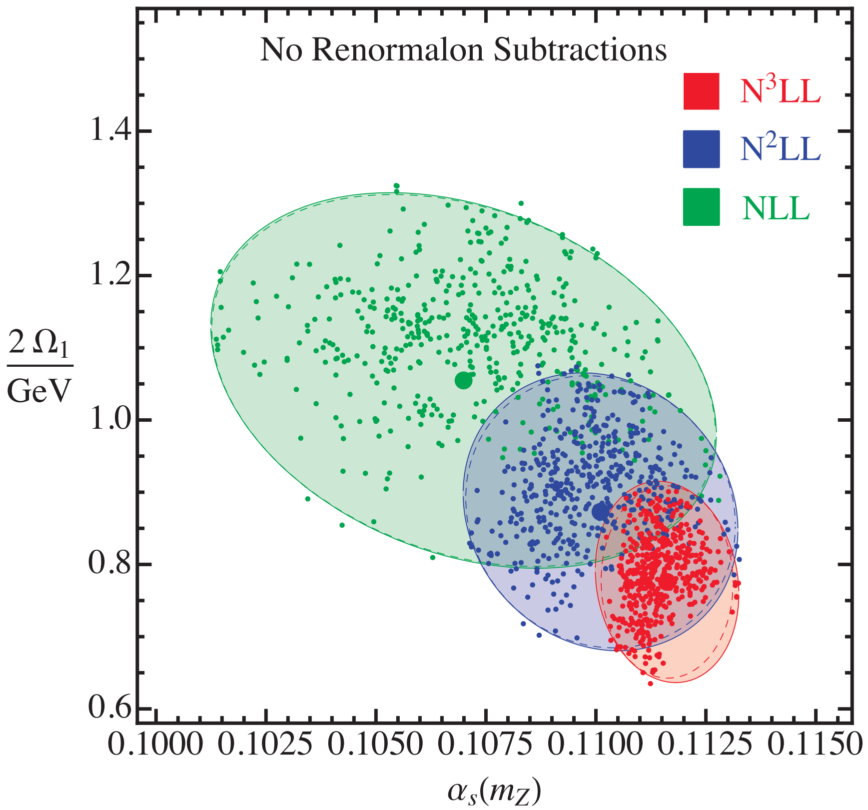
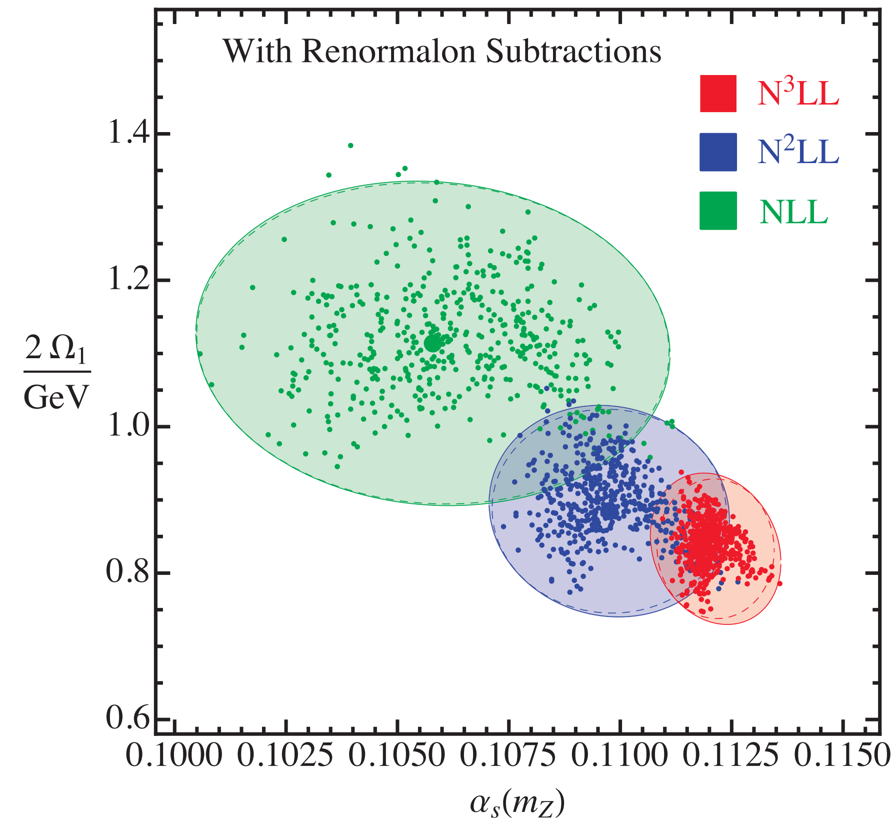


Fits for α_s

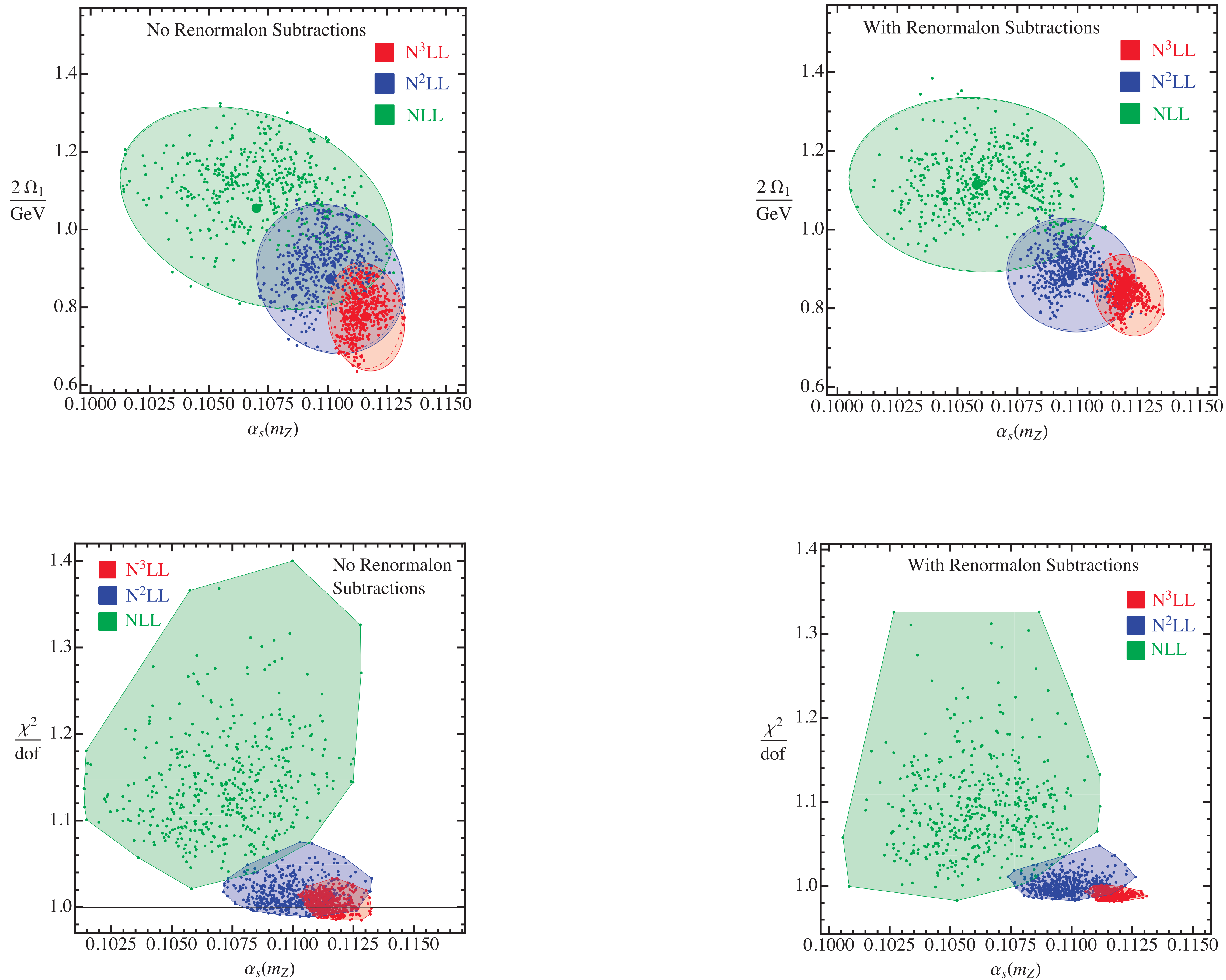
α_s determination: C-parameter tail fits





We perform global fits for energies between 35 and 206 GeV. We restrict ourselves to the tail of the distribution

α_s determination: C-parameter tail fits



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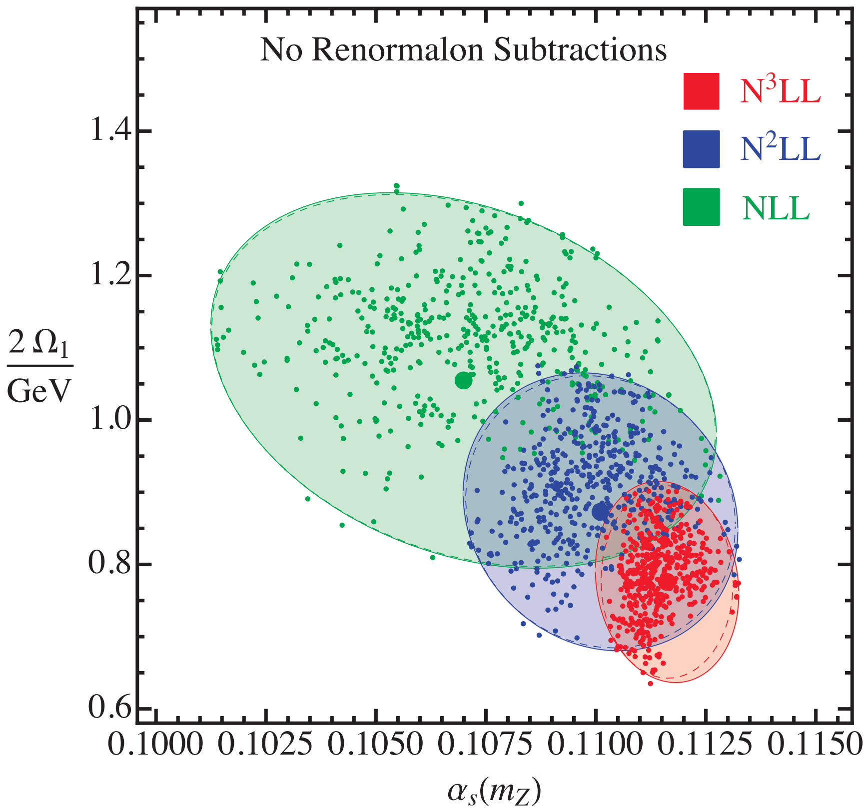
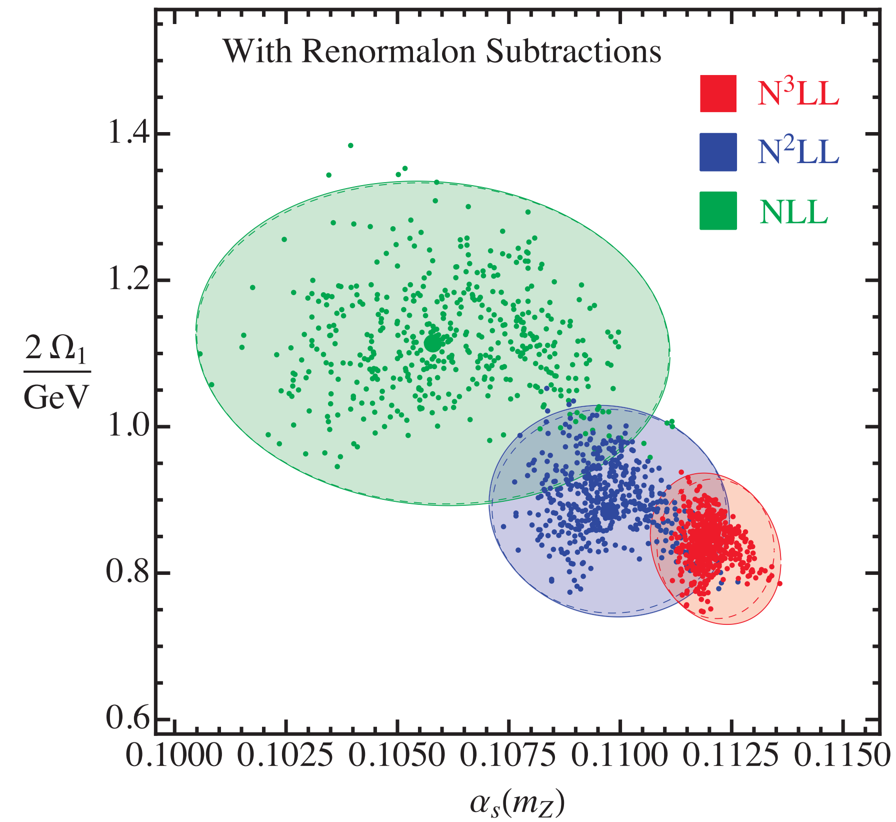
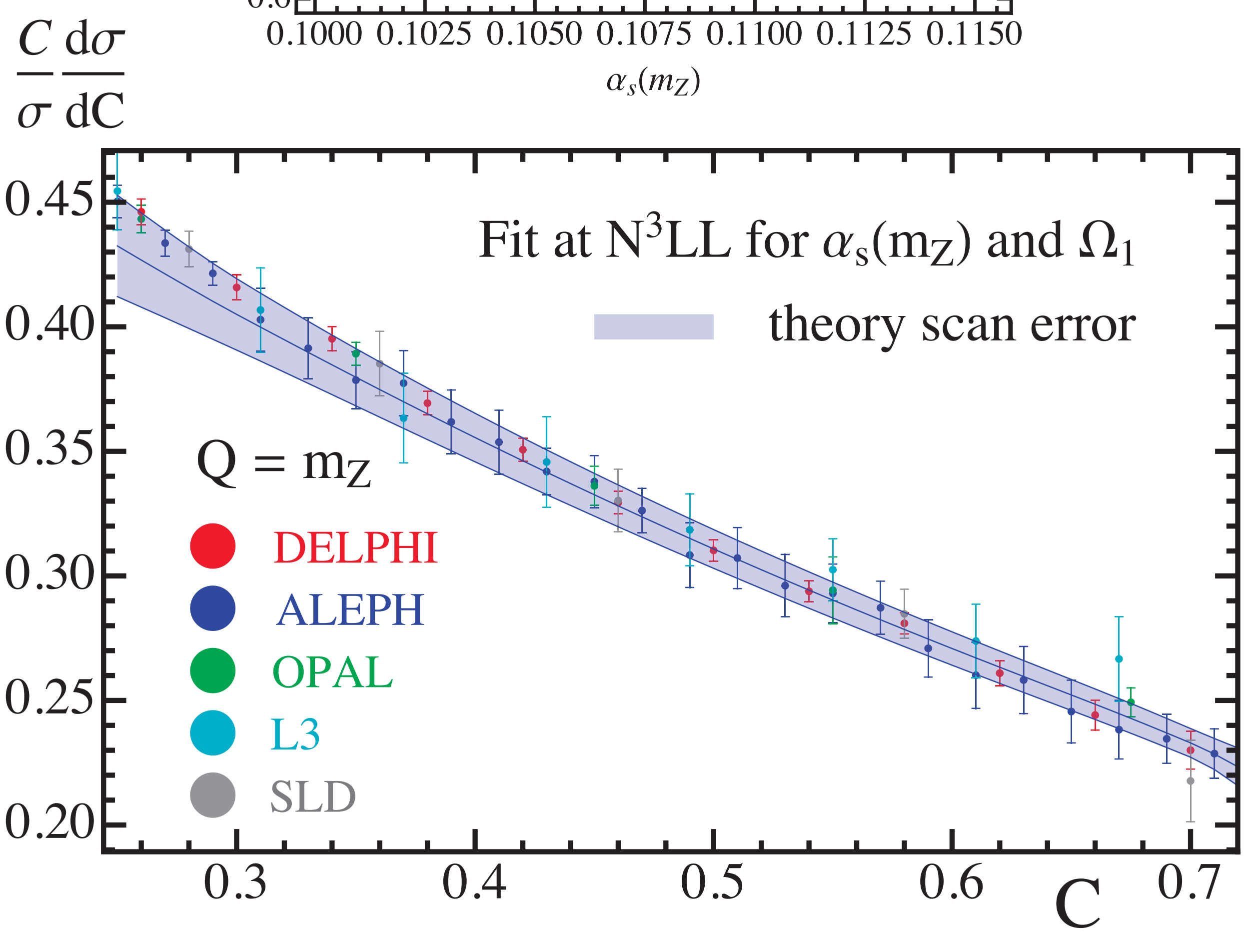






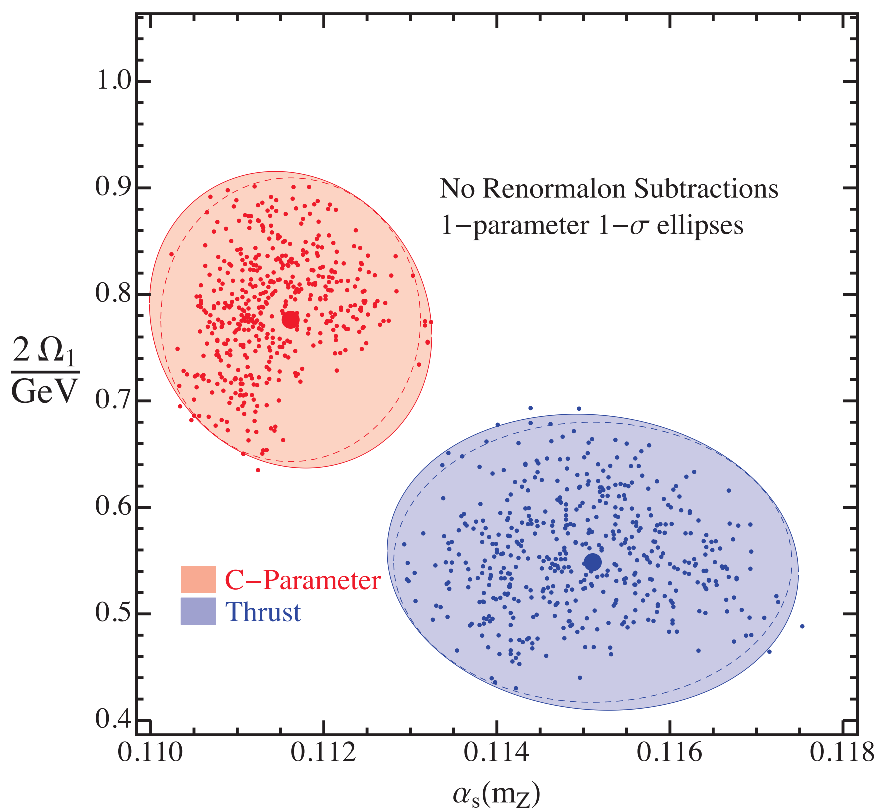
$$\alpha_s(m_Z) = 0.1121 \pm 0.0015$$

all errors combined

$$\alpha_s(m_Z) = 0.1121 \pm 0.0013_{\rm th} \pm 0.0006_{\rm exp} \pm 0.0002_{\rm had}$$

Universality: thrust vs C-parameter

thrust fits in [Abbate, Fickinger, Hoang, VM, Stewart]



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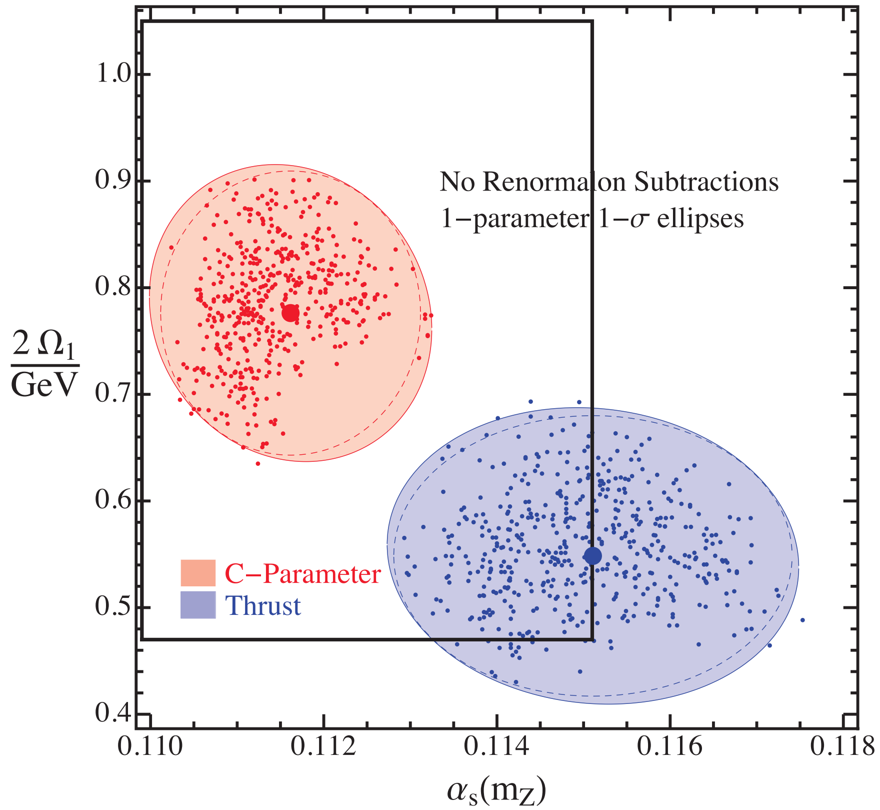
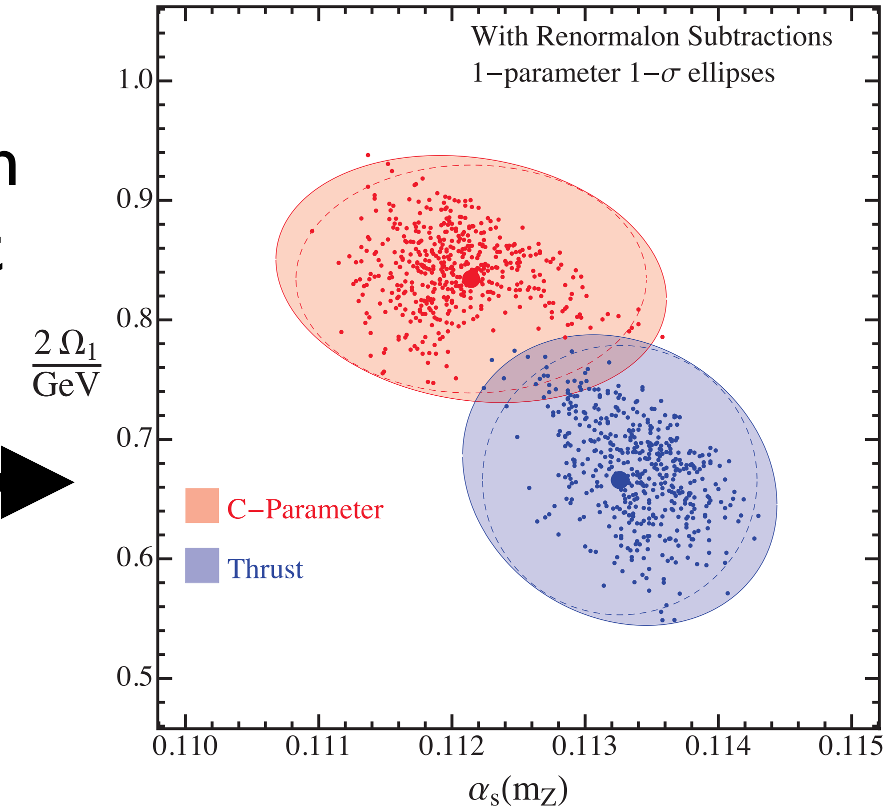


Renormalon subtraction improves α_s agreement



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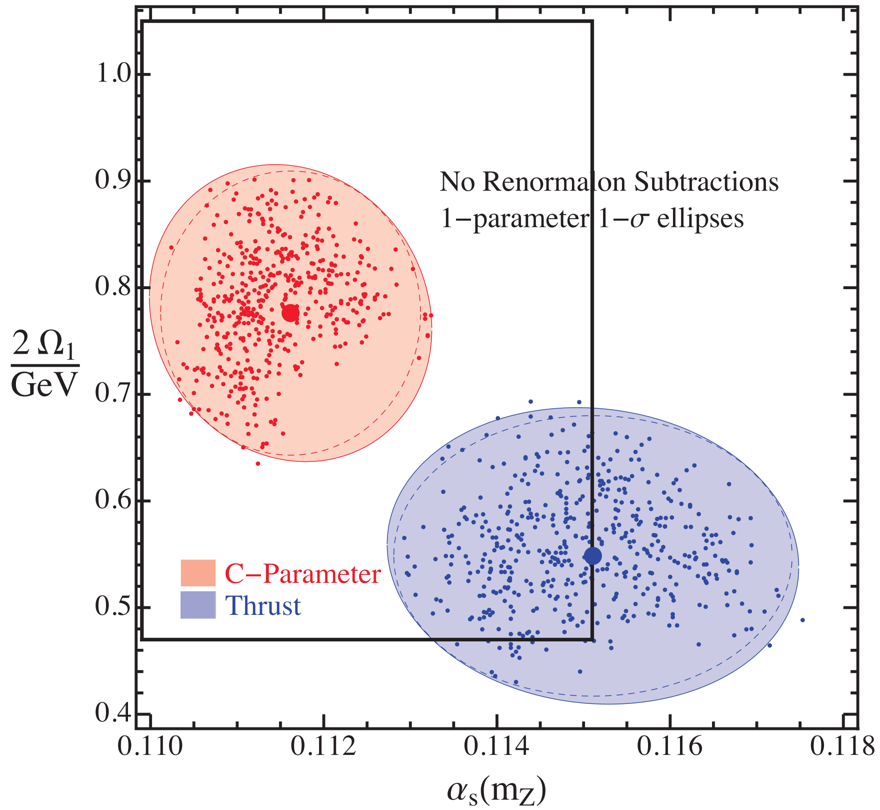

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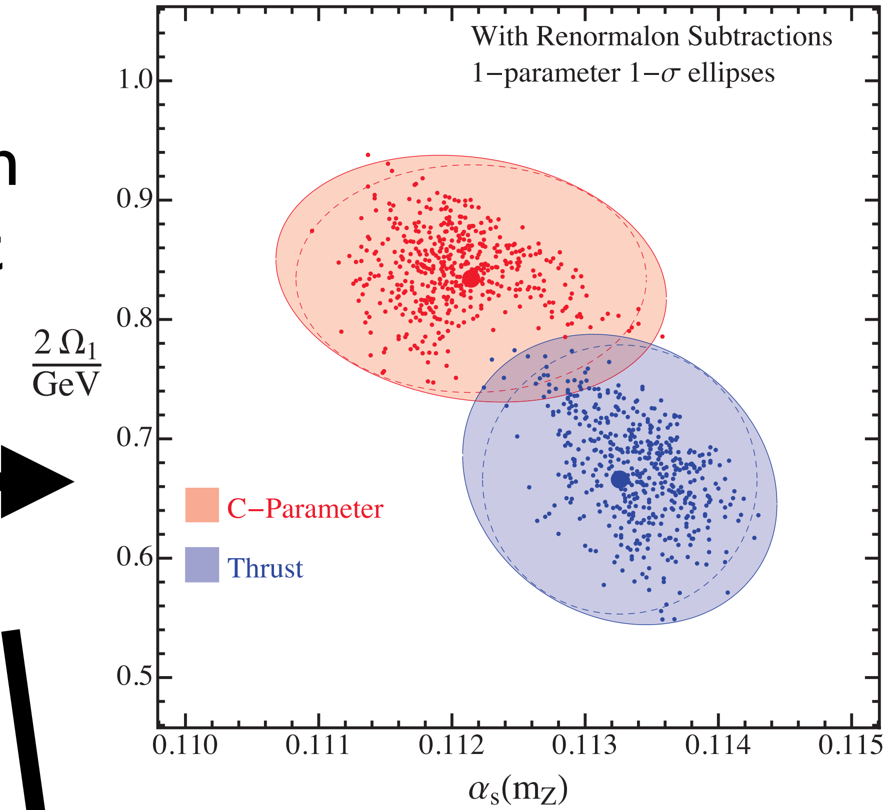
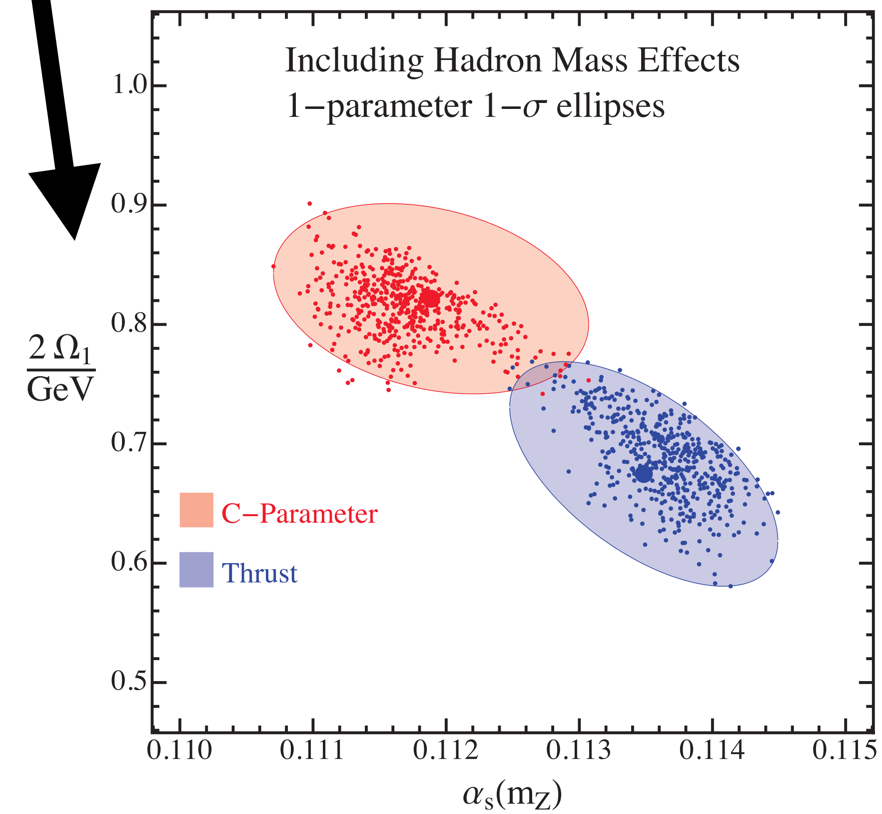


hadron-mass effects have small effect



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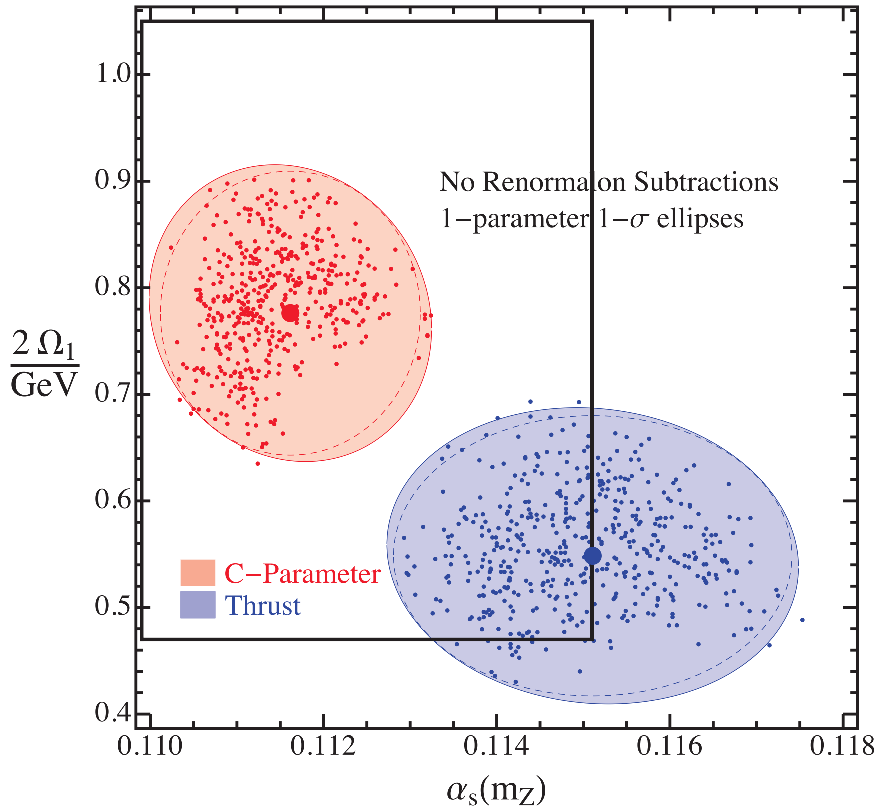

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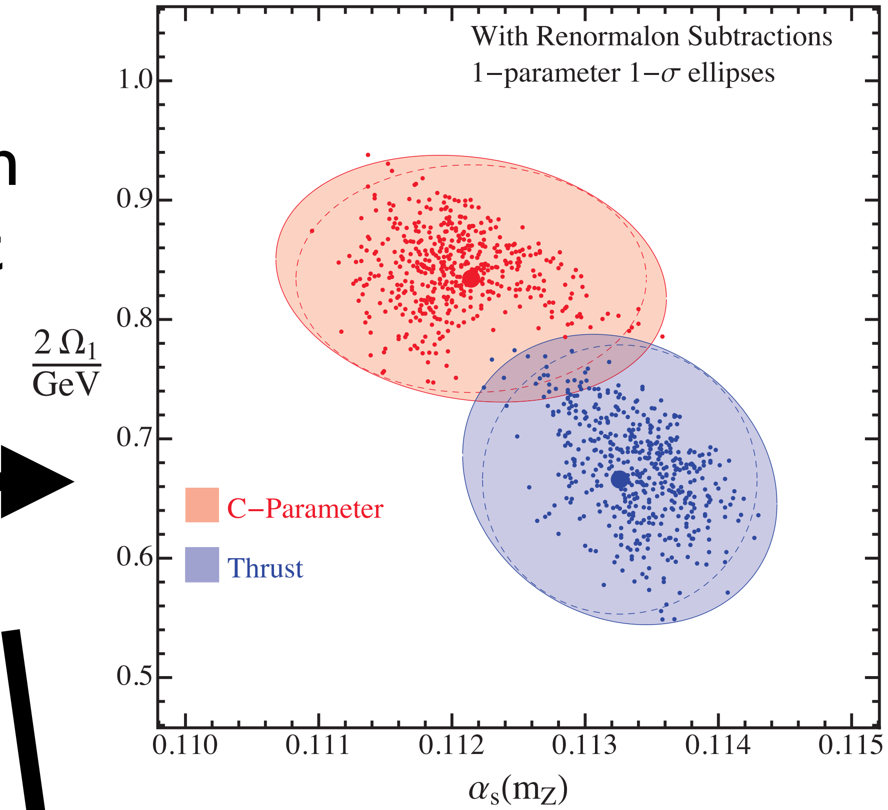

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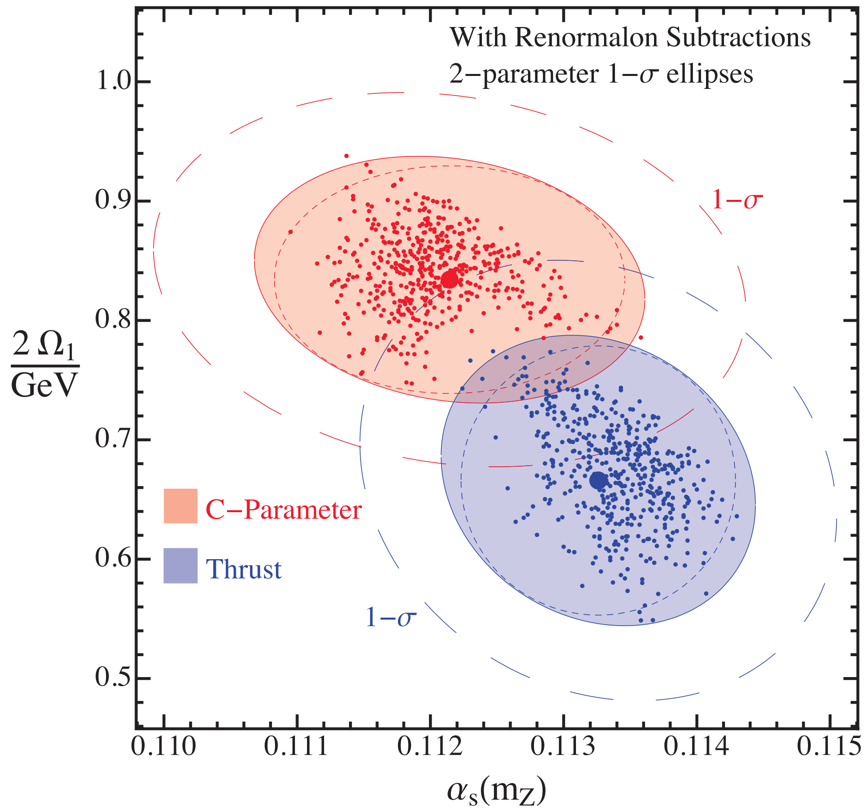
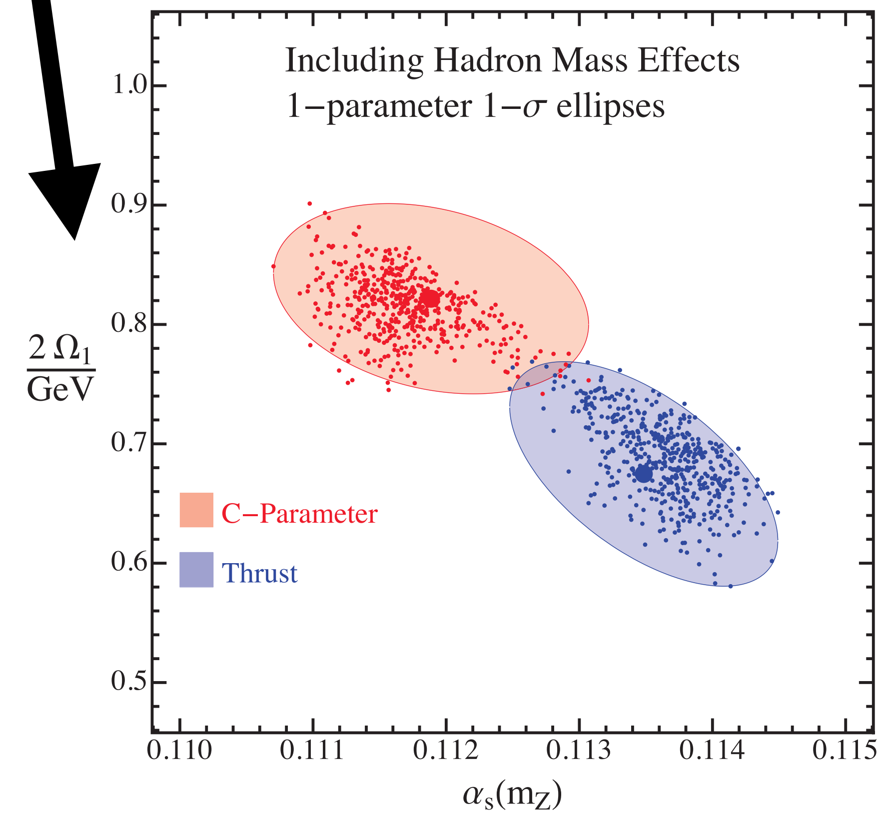


fair comparison with 2-parameter 1-σ ellipses

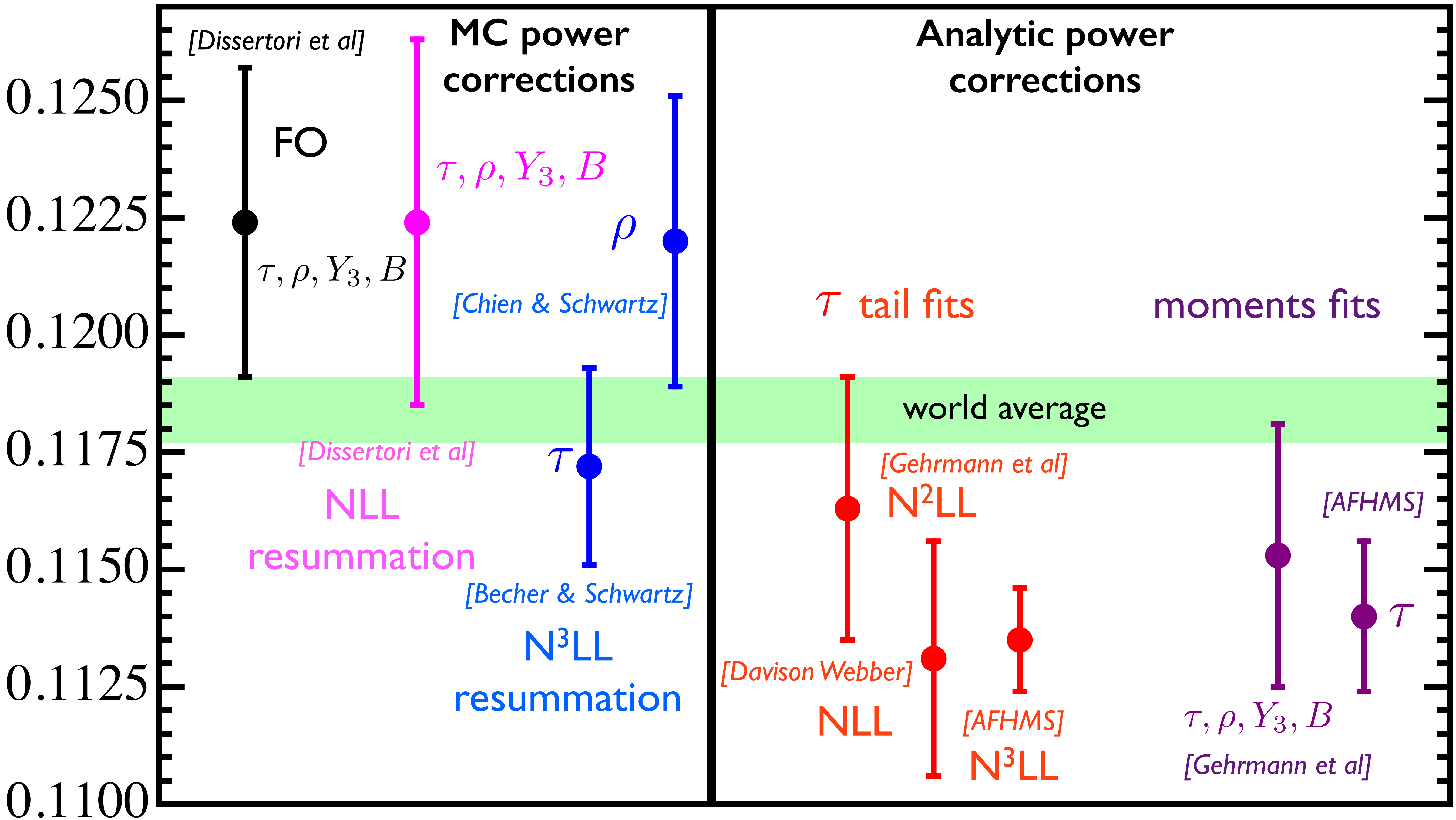


Conclusions & Outlook

Result

$$\alpha_s(m_Z) = 0.1121 \pm 0.0013_{\rm th} \pm 0.0006_{\rm exp} \pm 0.0002_{\rm had}$$

$\alpha_s(m_Z)$ determination from event shape fits



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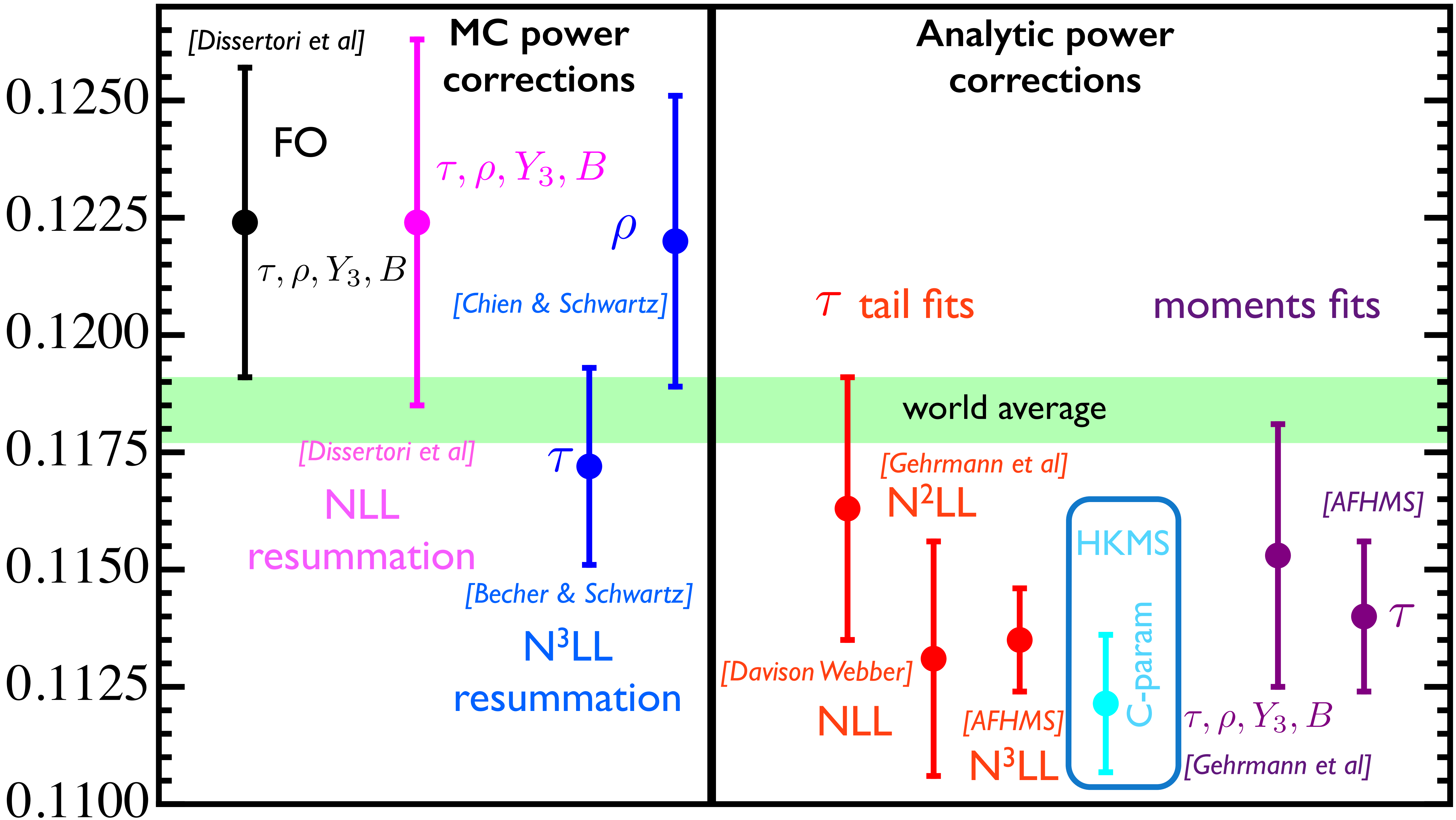


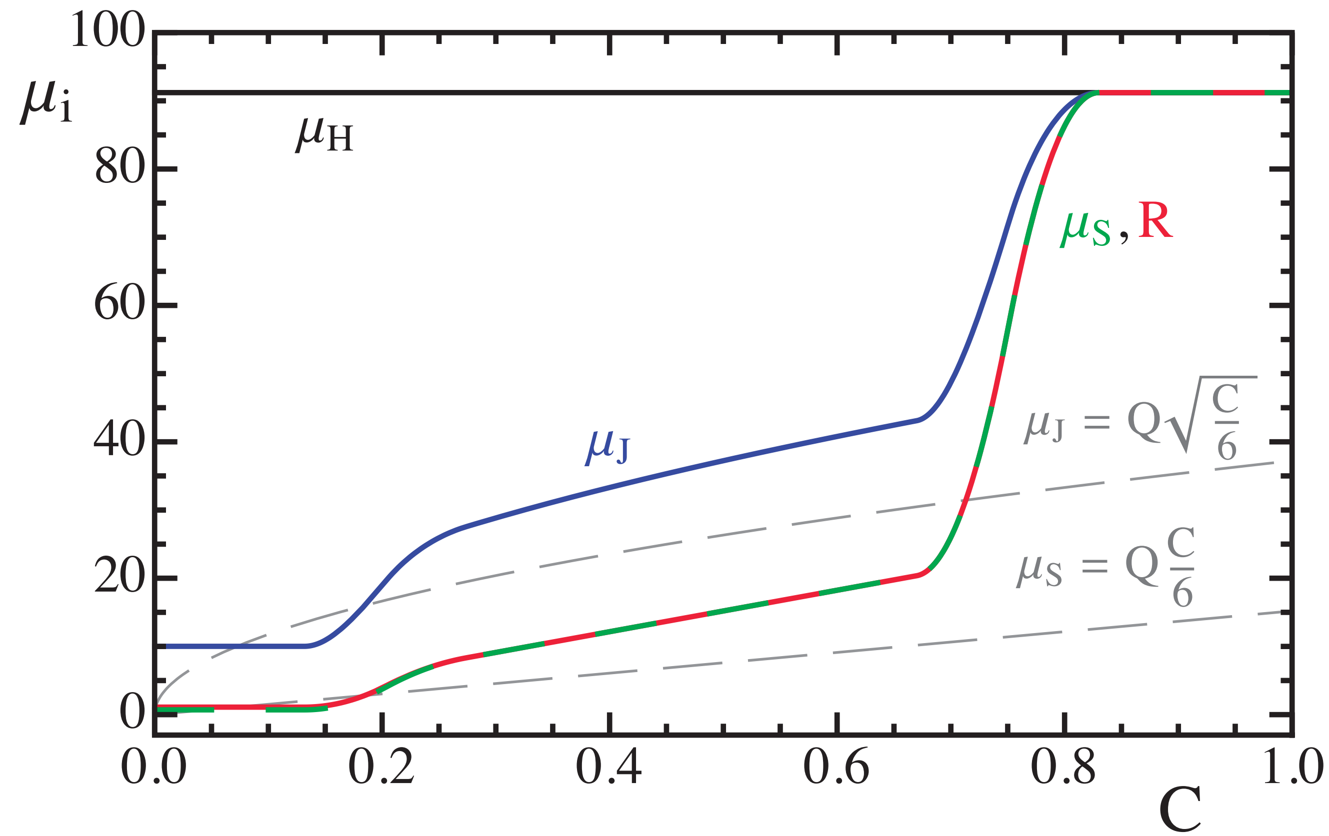
Conclusions & Outlook

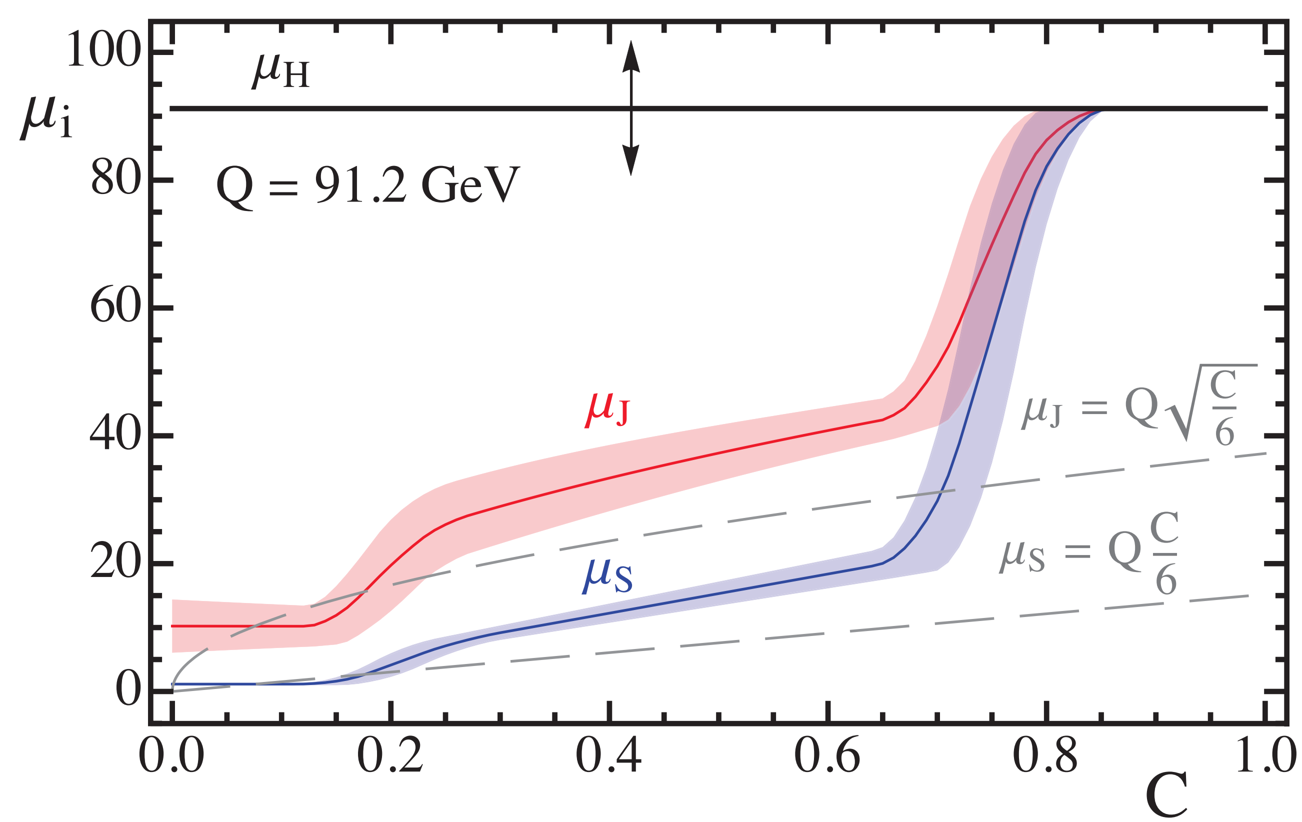
- Slightly less precision than thrust determination, but good consistency check of method + universality.
- First fits ever including hadron mass effects.
- Primary massive production computation (w.i.p.).
- QED effects can be easily added (w.i.p.).
- Fits to the first moment of C-parameter (w.i.p.).
- Close the picture with fits to HJM distribution (w.i.p.).

Backup slides

Renormalization scale setting

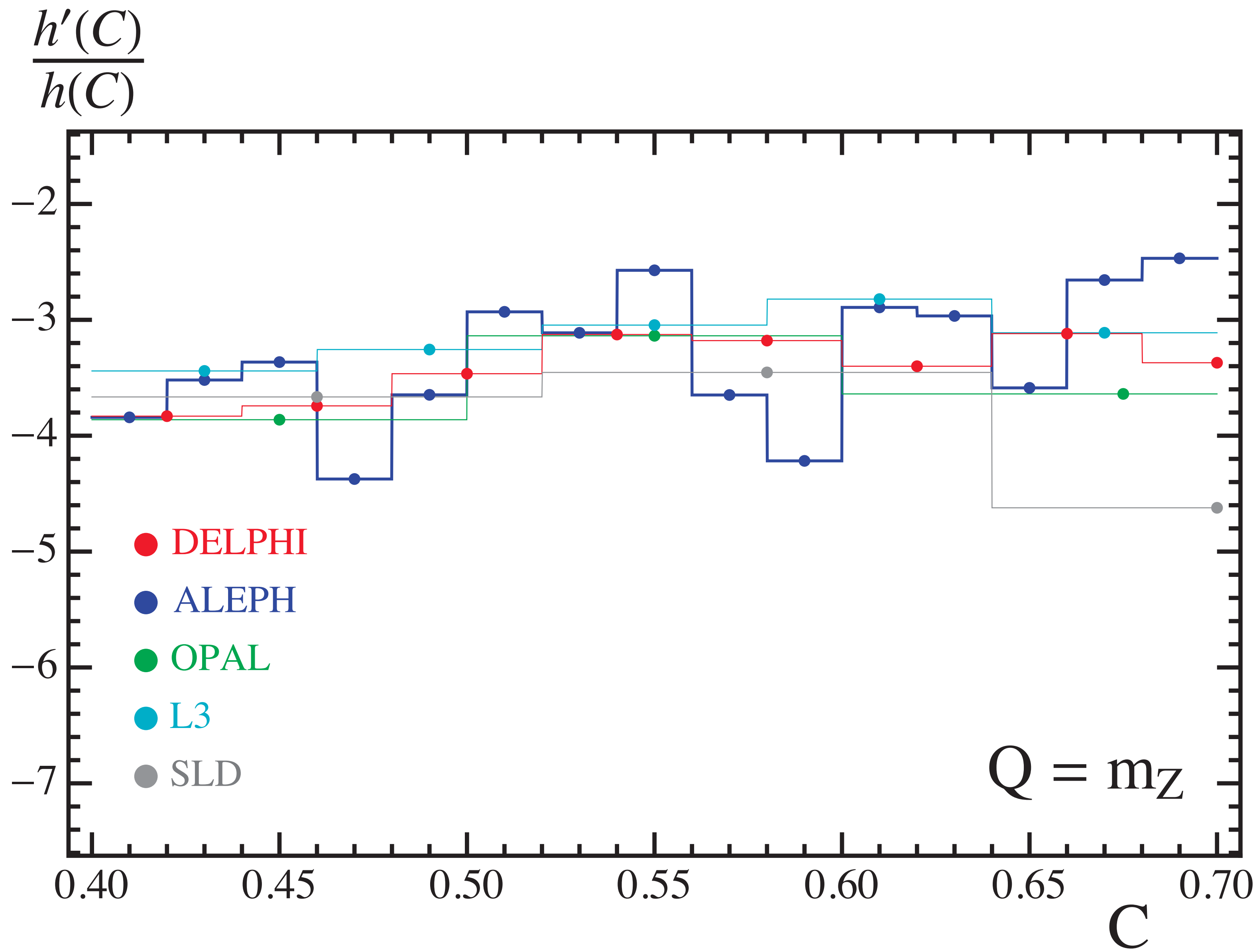
	parameter	default value	range of values
scale variation	μ_0	1.1 GeV	1 to 1.3 GeV
	R_0	0.7 GeV	0.6 to 0.9 GeV
	n_0	12	10 to 16
	n_1	25	22 to 28
	t_2	0.67	0.64 to 0.7
	t_s	0.83	0.8 to 0.86
	r	0.33	0.26 to 0.38
	e_J	0	-0.5 to 0.5
	e_H	1	0.5 to 2.0
	n_s	0	-1, 0, 1
unknowns	$\Gamma_3^{\rm cusp}$	1553.06	-1553.06 to $+4659.18$
	s_2	-43.2	-44.2 to -42.2
	j_3	0	-3000 to $+3000$
	s_3	0	-500 to $+500$
non-singular	$\epsilon_{2,\rm low}$	0	-1, 0, 1
	$\epsilon_{2,\rm high}$	0	-1, 0, 1
	$\epsilon_{3,\rm low}$	0	-1, 0, 1
	$\epsilon_{3,\rm high}$	0	-1, 0, 1





α_s determination: C-parameter tail fits

$$(1/\sigma)\,\mathrm{d}\sigma/\mathrm{dC} \simeq \mathrm{h}(\mathrm{C} - \Omega_1^{\mathrm{C}}/\mathrm{Q}) = \mathrm{h}(\mathrm{C}) - \mathrm{h}'(\mathrm{C})\,\Omega_1^{\mathrm{C}}/\mathrm{Q} + \ldots$$



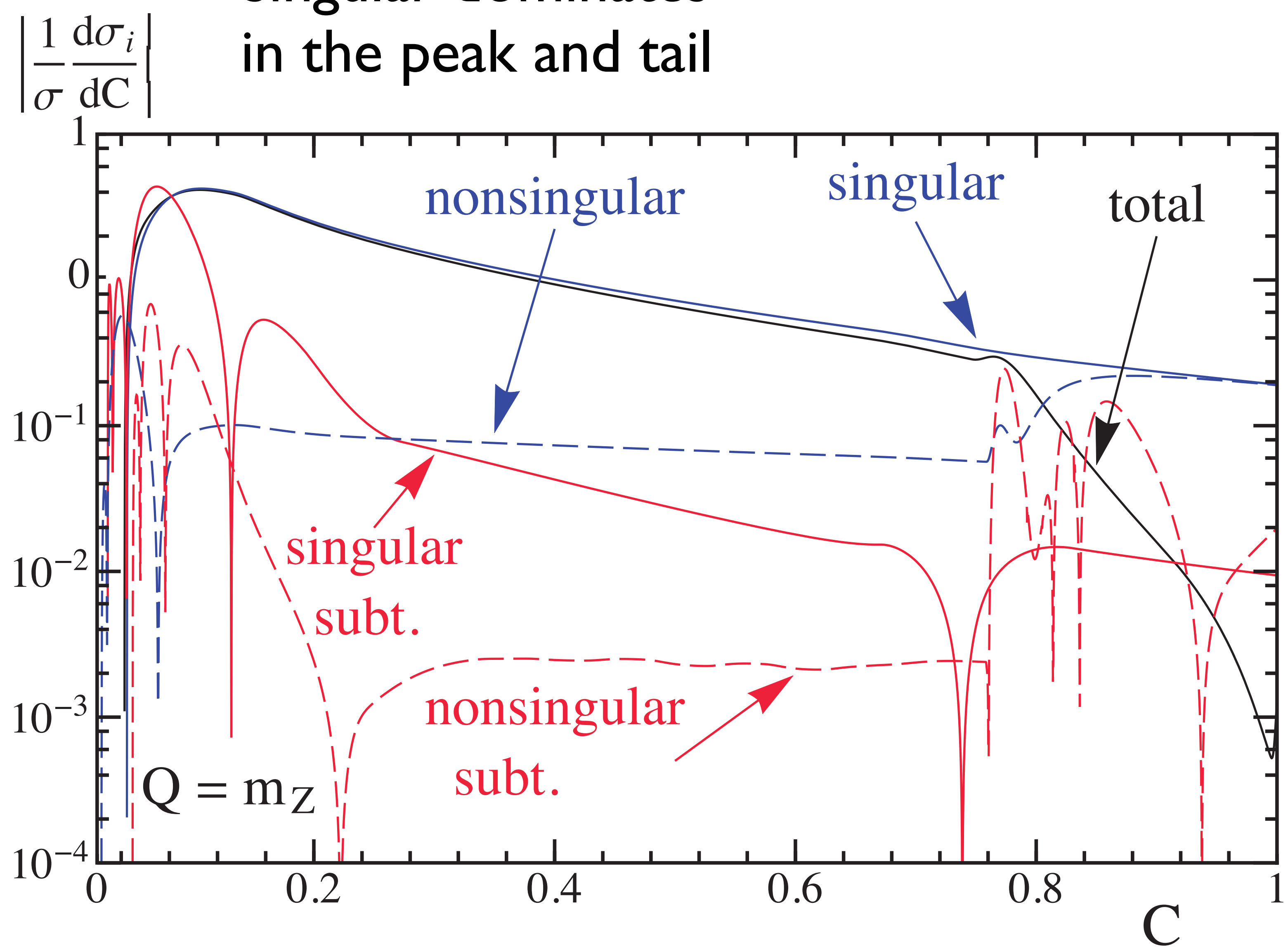
$$\frac{h'(C)}{h(C)} \sim -3.3 \pm 0.8$$

$$2\,\Omega_1^C \sim 3\pi\,\Omega_1^\tau$$

$$\frac{\delta\alpha_s}{\alpha_s} \simeq \frac{\Omega_1^C}{Q}\,\frac{h'(C)}{h(C)} \sim -9\%$$

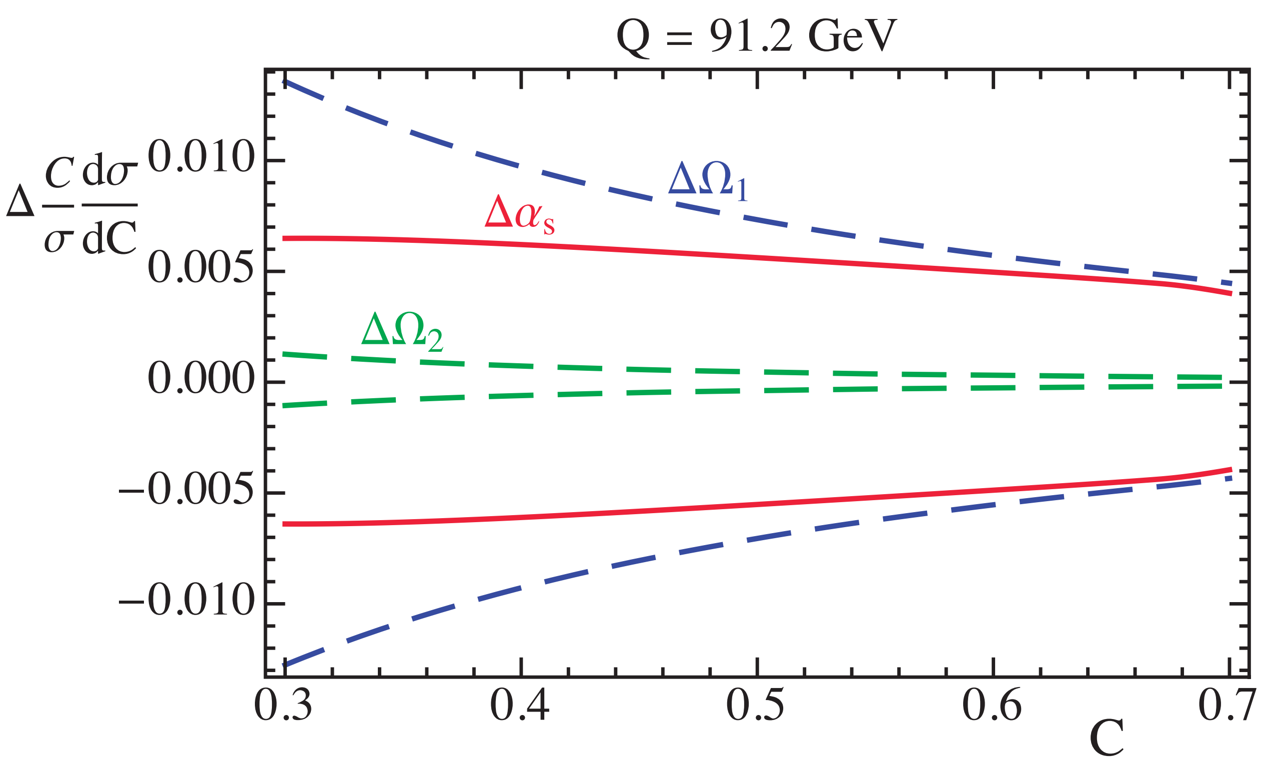
Cross section components

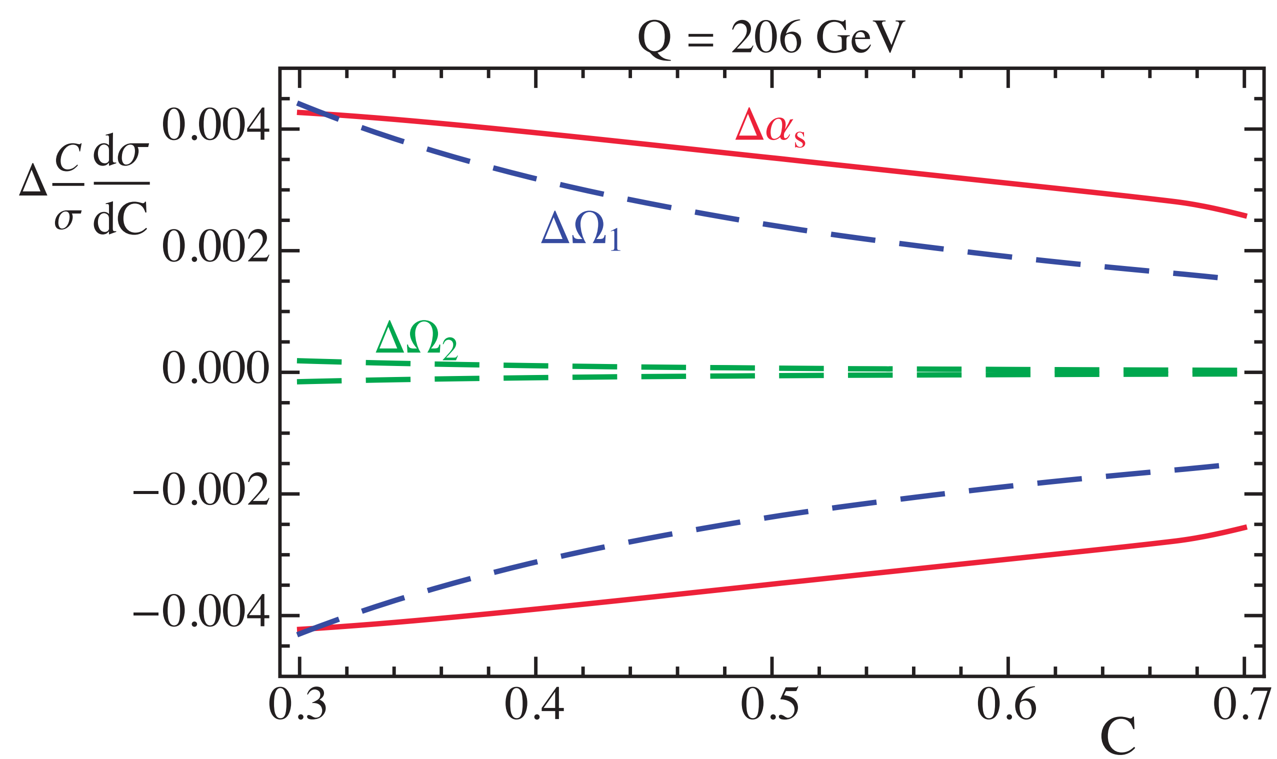
Singular dominates
in the peak and tail

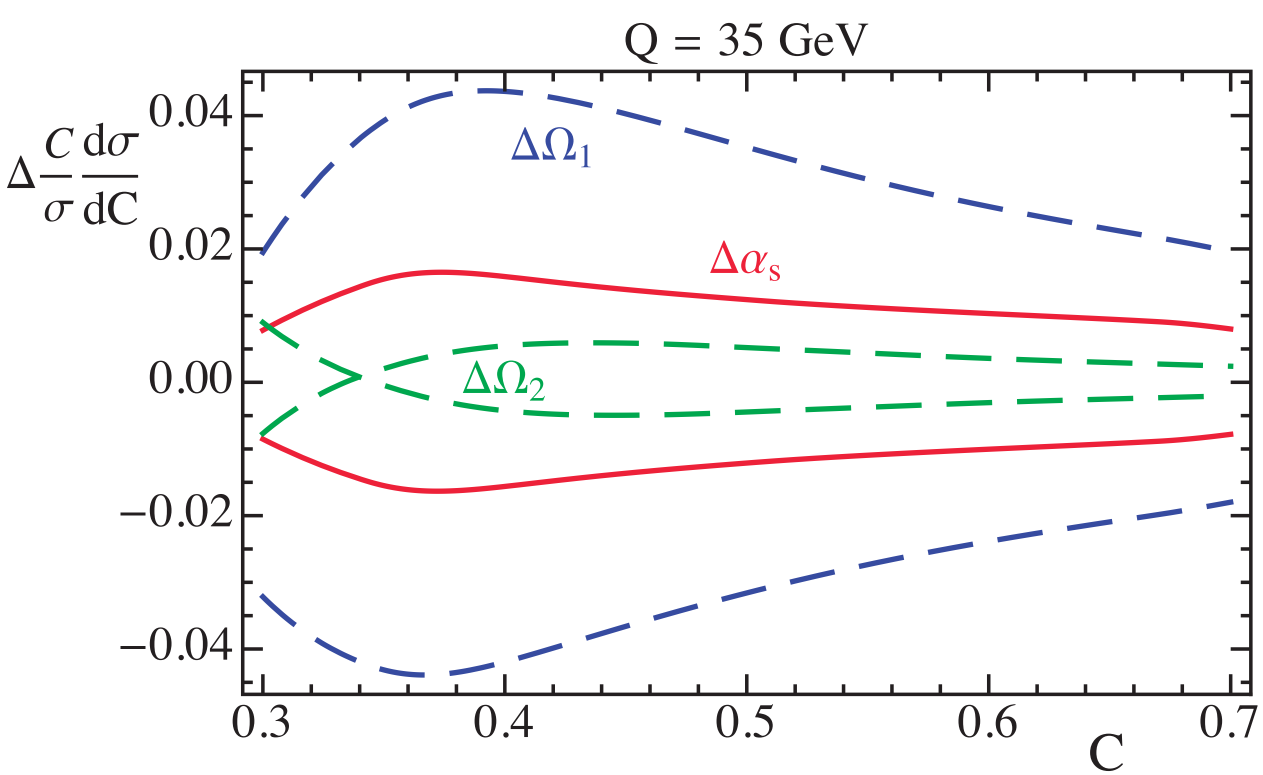


FO results
reproduced
in far tail

α_s determination: C-parameter tail fits







Strong degeneracy between α_s and Ω_1 which is broken if many values of the center of mass energy are included

We perform global fits for energies between 35 and 206 GeV. We restrict ourselves to the tail of the distribution

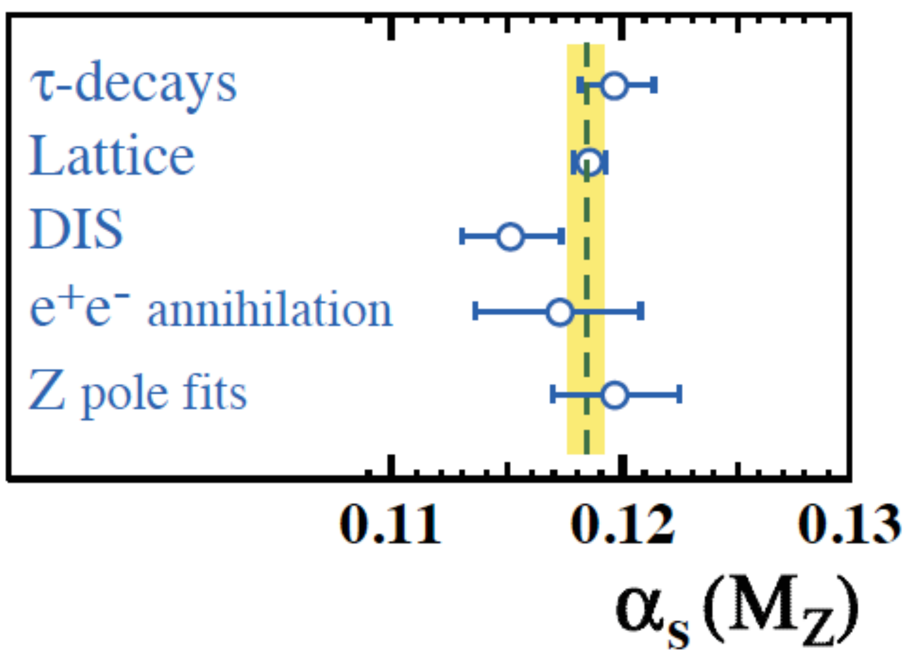
The world average

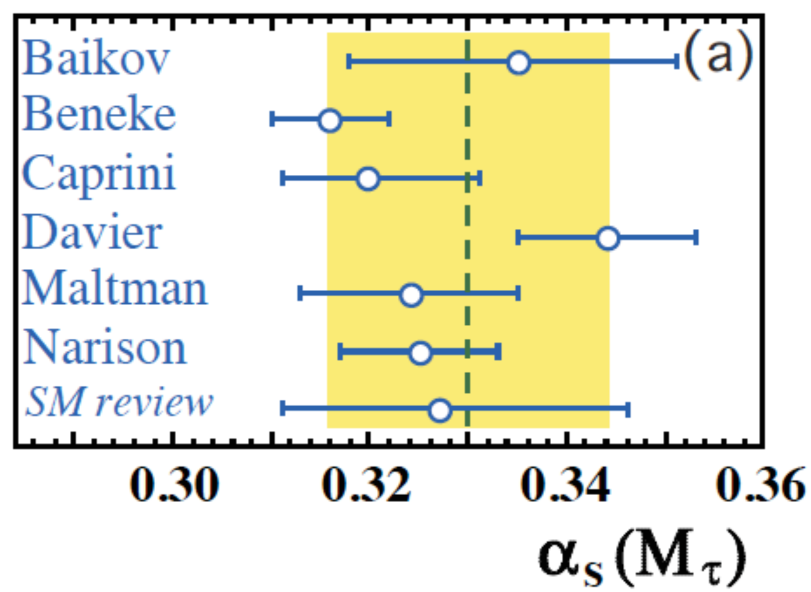
[More details on talk by J. Erler, tuesday, plenary 3]

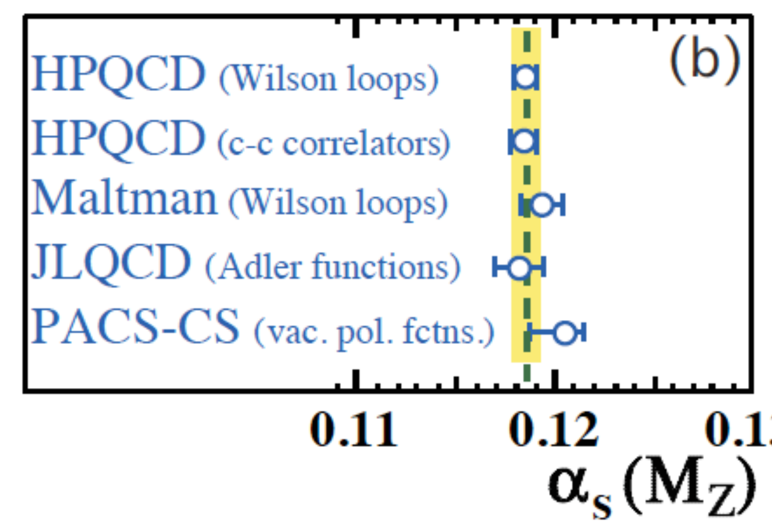
Determinations are first "averaged" within a given process

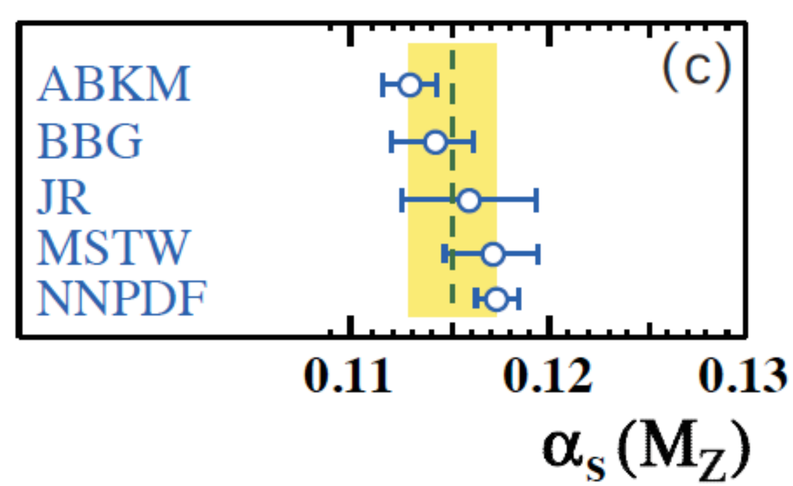
The various averages are later combined together for the final average

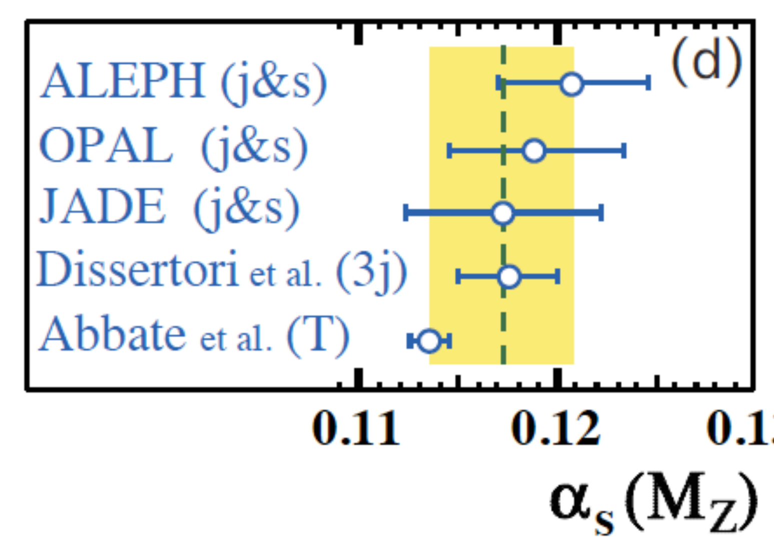
Completely dominated by lattice results !!!









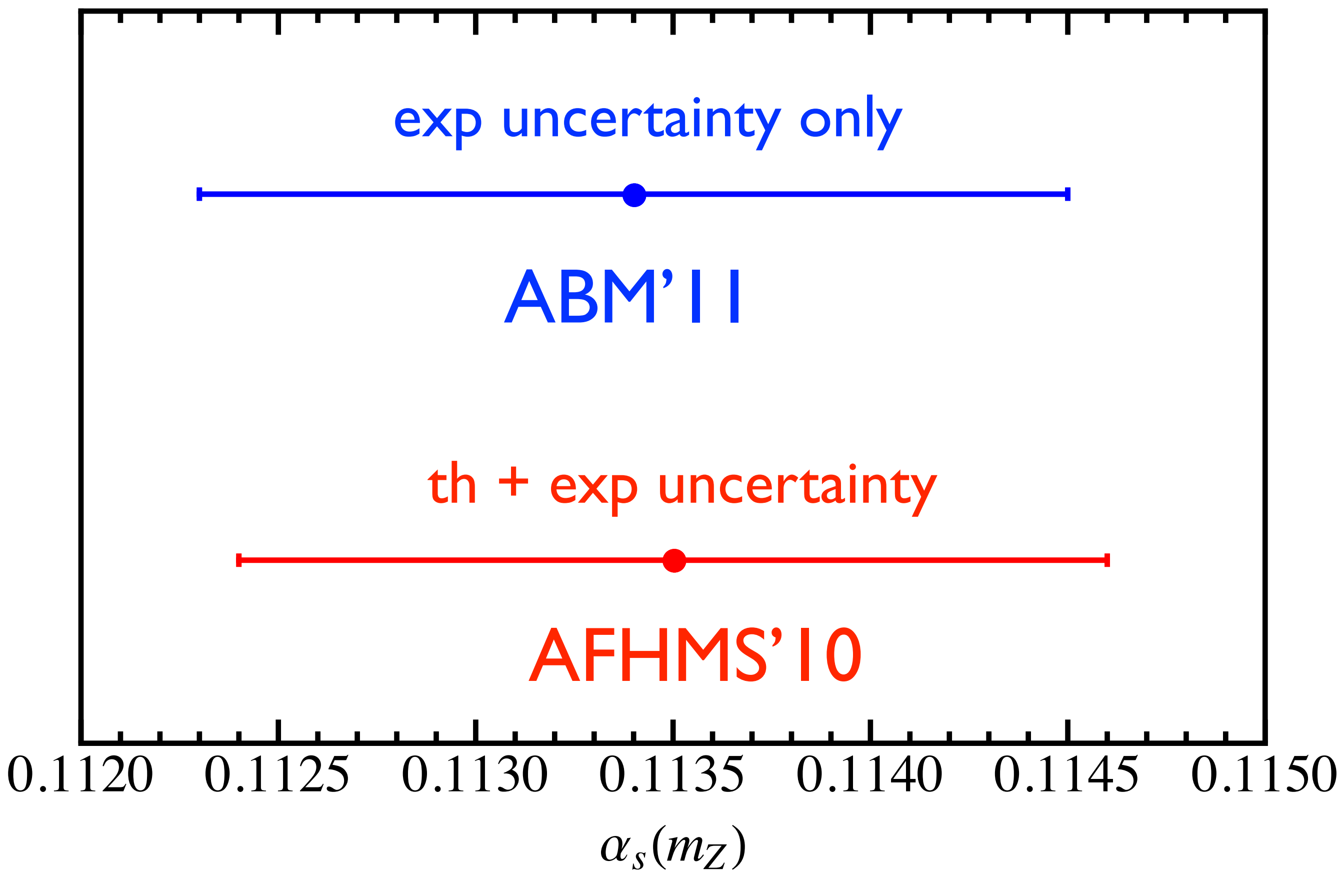


Figures taken from PDG

many details in review *[arXiv: 1110.0016]*

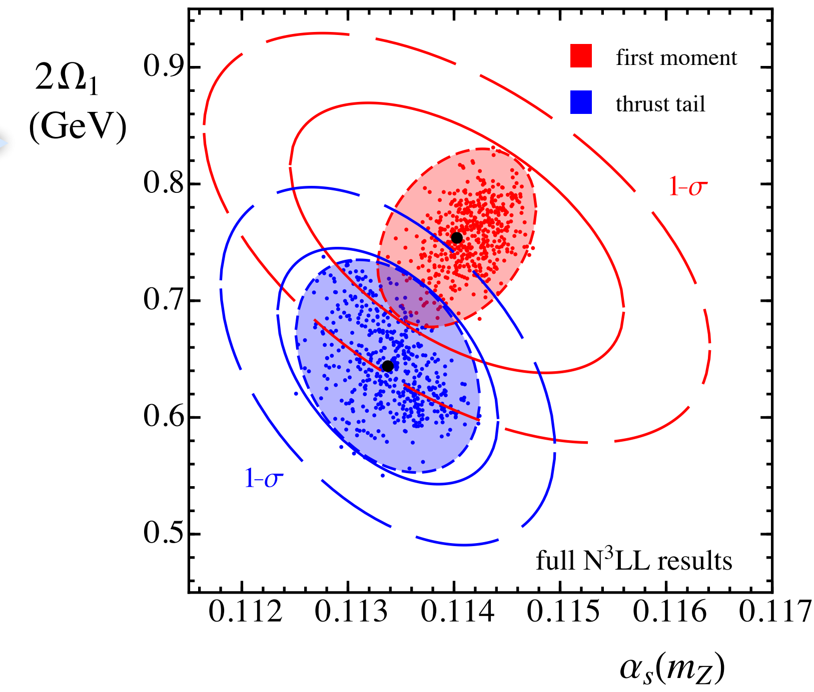
look also *[arXiv:1303.2262]*

DIS analyses of ABM get similarly low and precise determinations (same true for GENEVA MC)



We need to analyze more event-shapes to validate our results.

- ☑ Tail of thrust
- ☑ Moments of thrust distribution
- ☑ Tail of C-parameter (to appear soon)
- ☐ Tail of Heavy Jet Mass (w.i.p.)
- ☐ Moments of C-parameter (w.i.p.) and HJM



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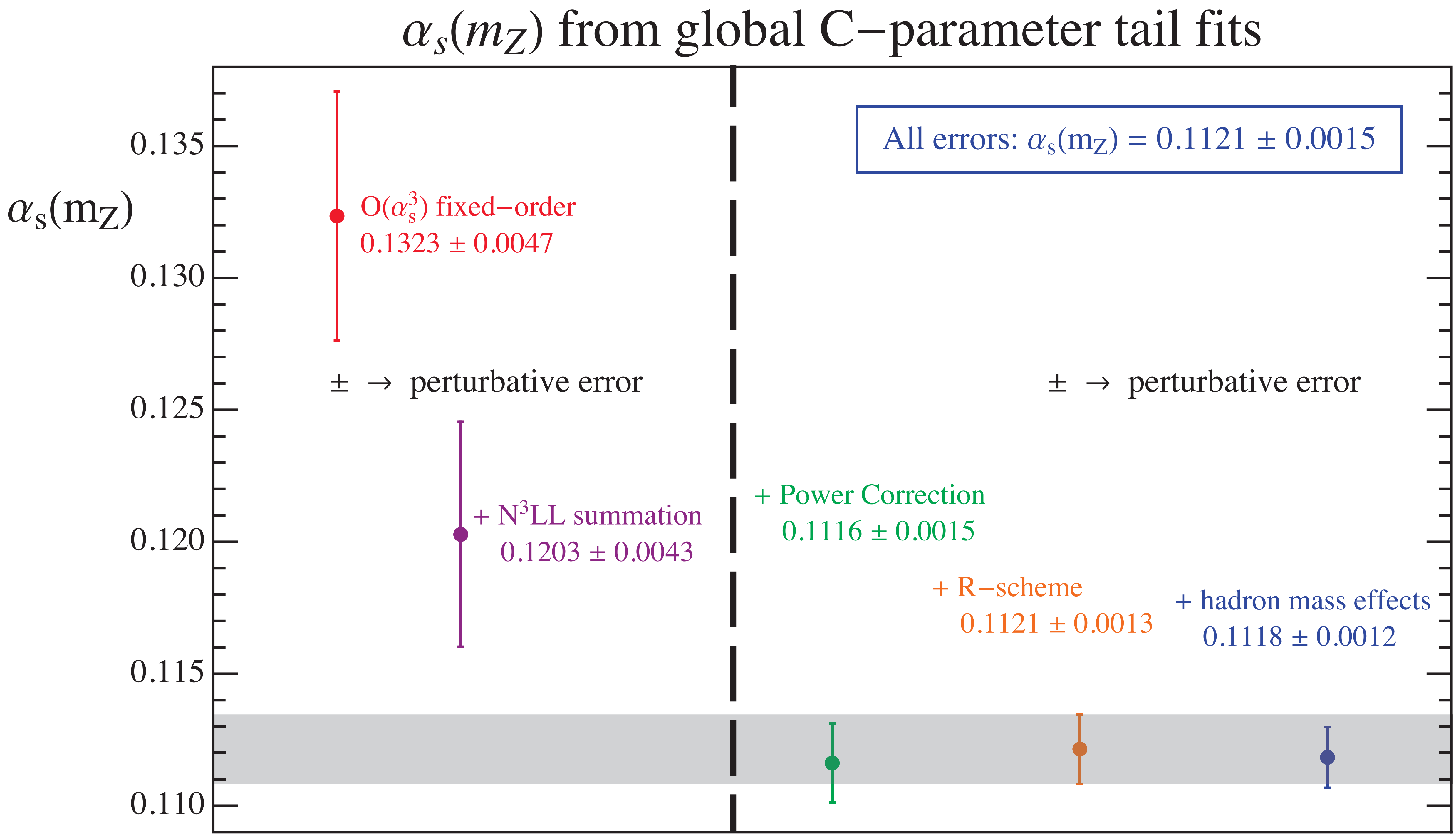

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